Computer Algebra
Independent Integration Tests

Summer 2023 edition

4-Trig-functions/4.1-Sine/68-4.1.11-e-x-^m-a+b-x^n-^p-sin

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [113]. This is test number [68].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (113)	0.00 (0)
Mathematica	100.00 (113)	0.00 (0)
Maple	100.00 (113)	0.00 (0)
Fricas	100.00 (113)	0.00 (0)
Giac	62.83 (71)	37.17 (42)
Maxima	46.90 (53)	53.10 (60)
Sympy	23.01 (26)	76.99 (87)
Mupad	17.70 (20)	82.30 (93)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

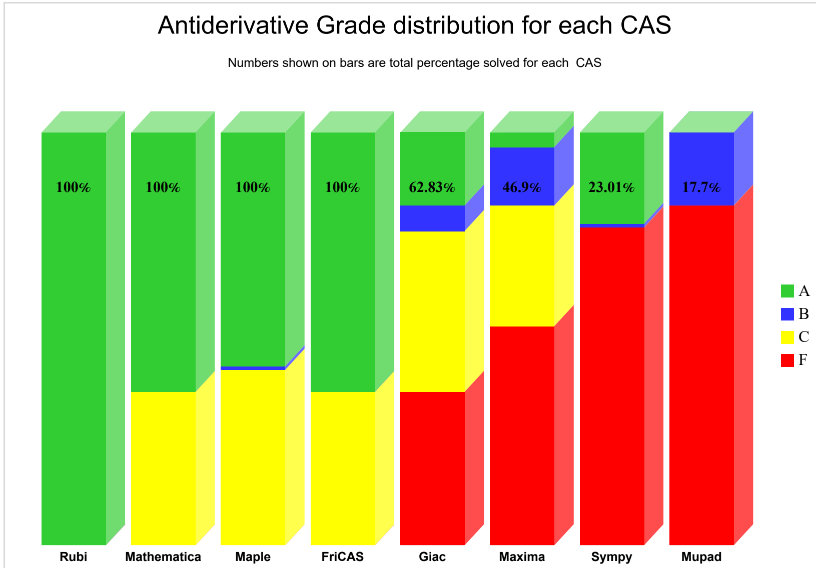
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

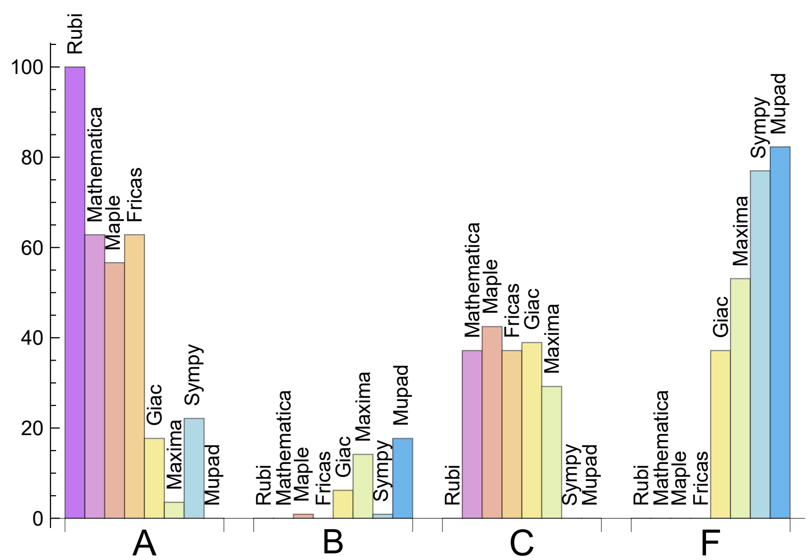
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	62.832	0.000	37.168	0.000
Fricas	62.832	0.000	37.168	0.000
Maple	56.637	0.885	42.478	0.000
Sympy	22.124	0.885	0.000	76.991
Giac	17.699	6.195	38.938	37.168
Maxima	3.540	14.159	29.204	53.097
Mupad	0.000	17.699	0.000	82.301

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	0	0.00	0.00	0.00
Maple	0	0.00	0.00	0.00
Giac	42	100.00	0.00	0.00
Maxima	60	100.00	0.00	0.00
Sympy	87	80.46	19.54	0.00
Mupad	93	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.30
Giac	0.34
Maple	0.42
Rubi	0.43
Mathematica	0.70
Sympy	0.84
Maxima	1.11
Mupad	3.27

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	116.40	0.98	119.50	0.95
Sympy	144.08	1.32	142.50	1.23
Mathematica	214.36	0.82	145.00	0.77
Maxima	249.96	2.85	164.00	1.80
Fricas	274.20	0.97	161.00	0.94
Rubi	281.22	1.00	181.00	1.00
Maple	287.56	1.15	180.00	1.06
Giac	2408.20	13.30	834.00	8.14

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

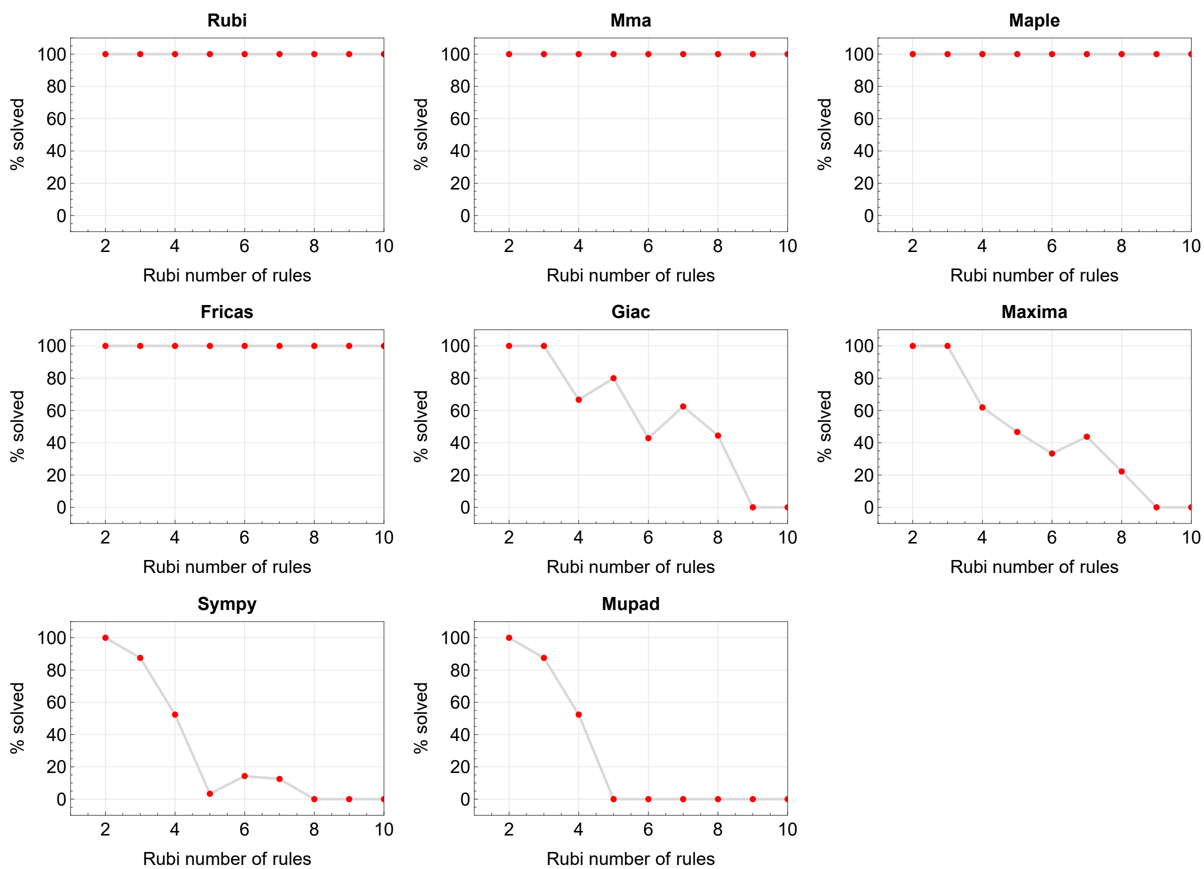


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

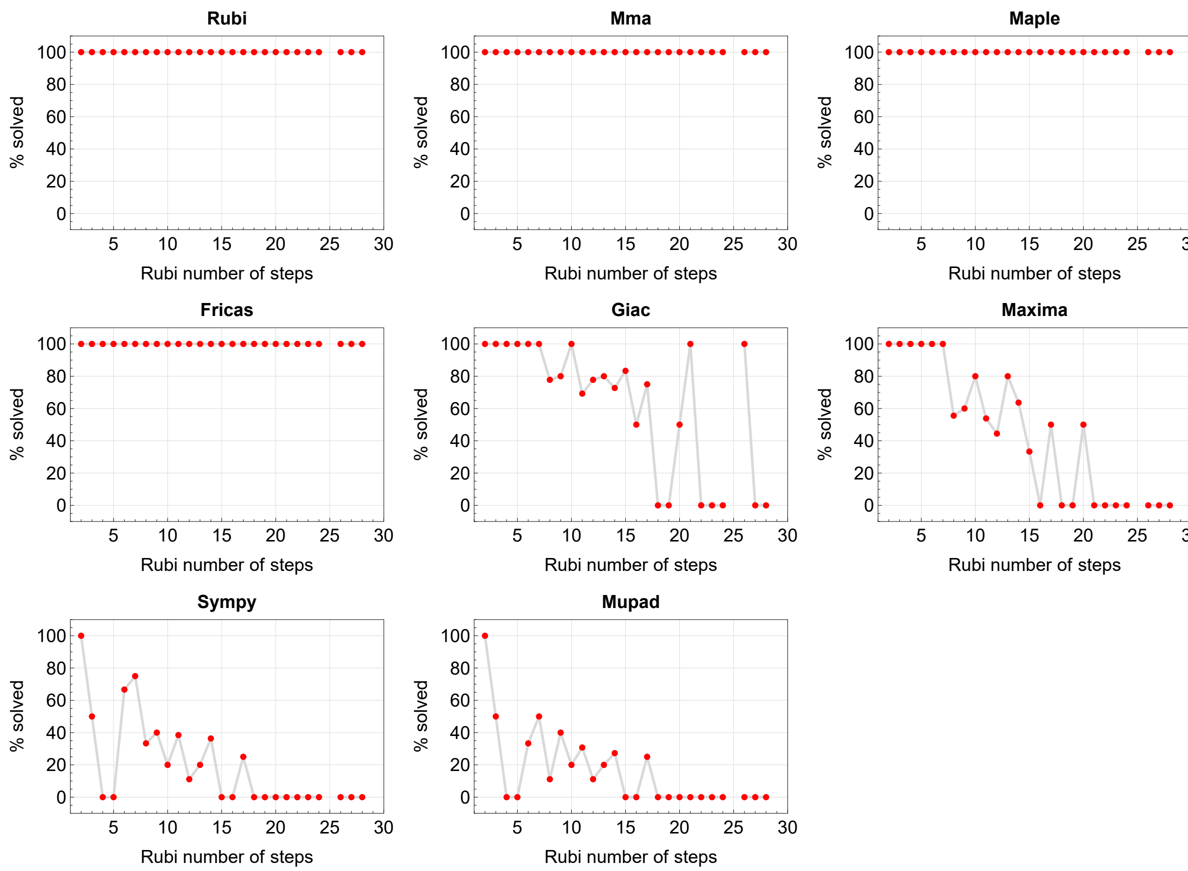


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

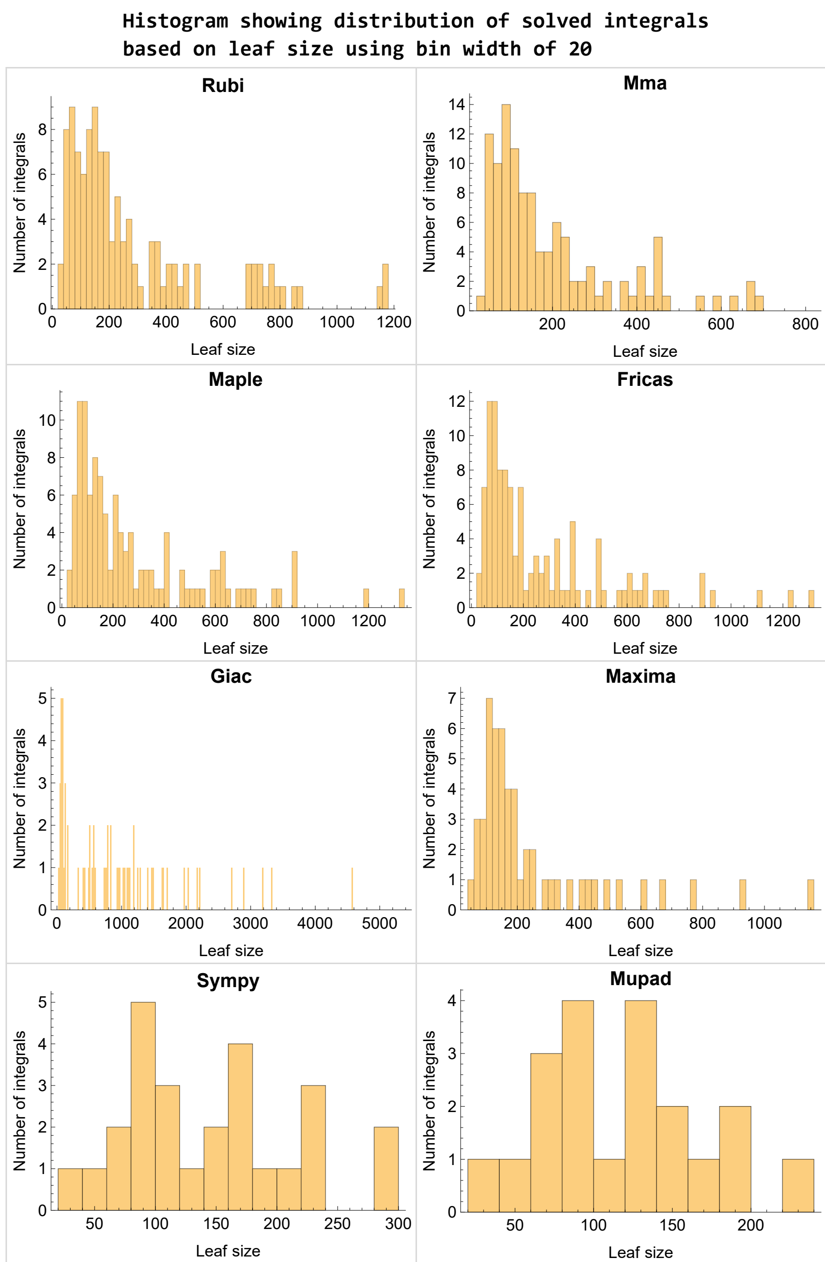


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

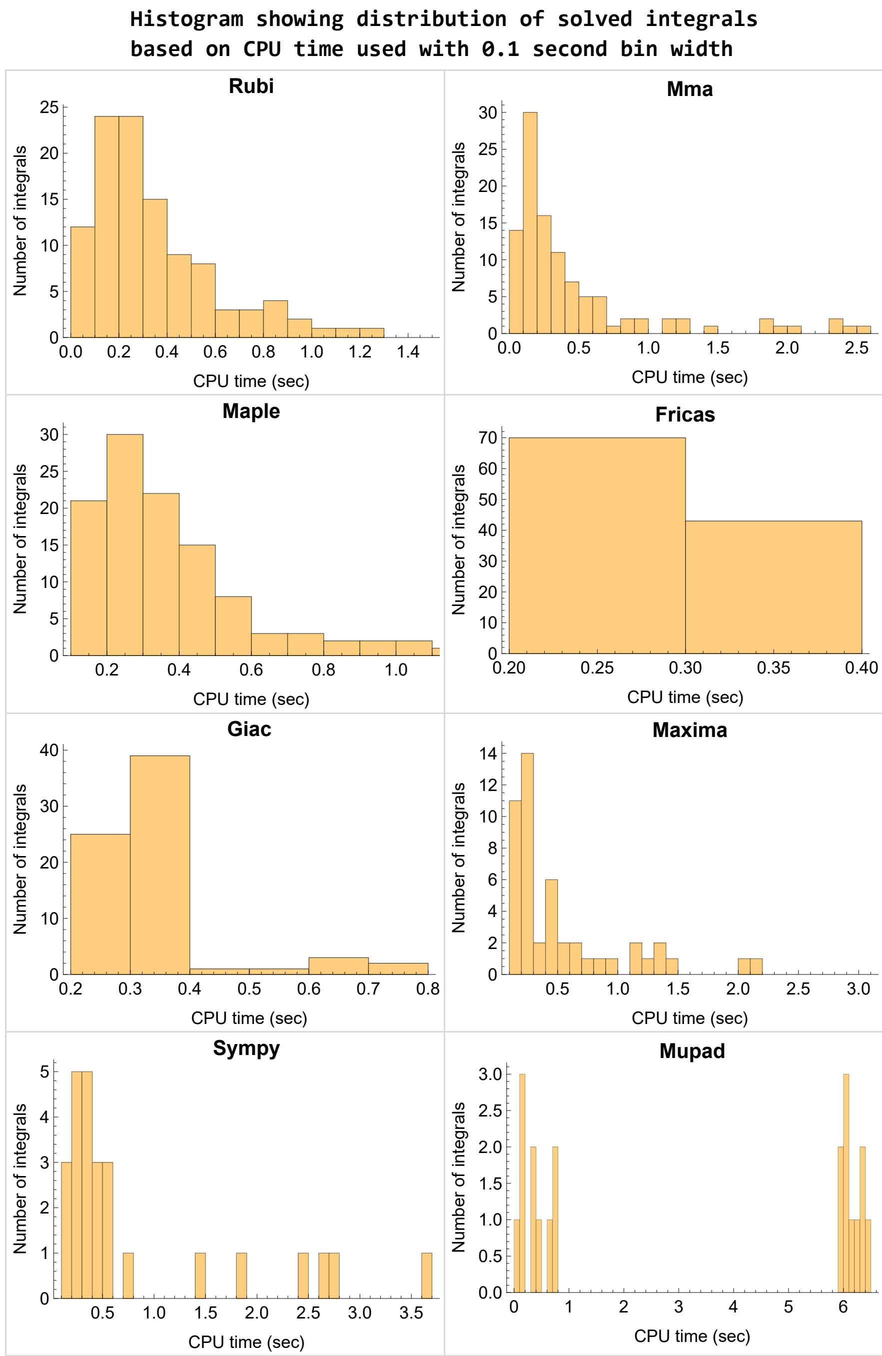


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

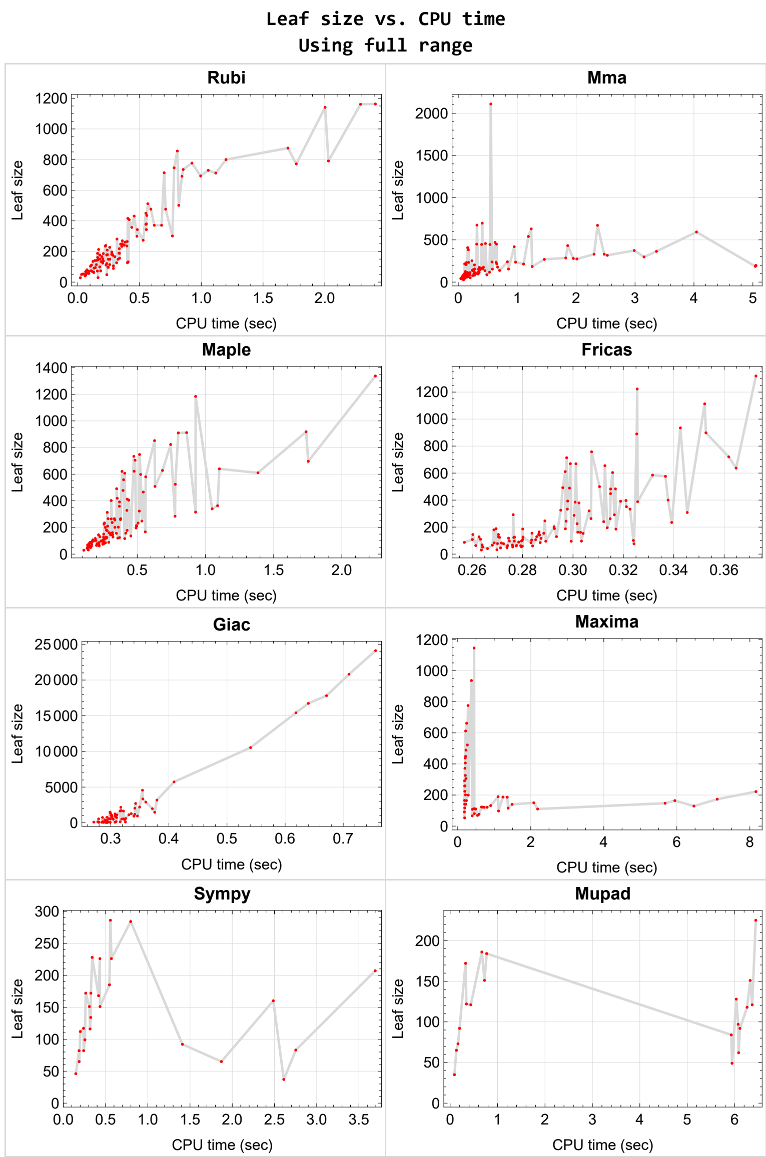


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{111\}$

Maple $\{18, 83, 90, 91\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

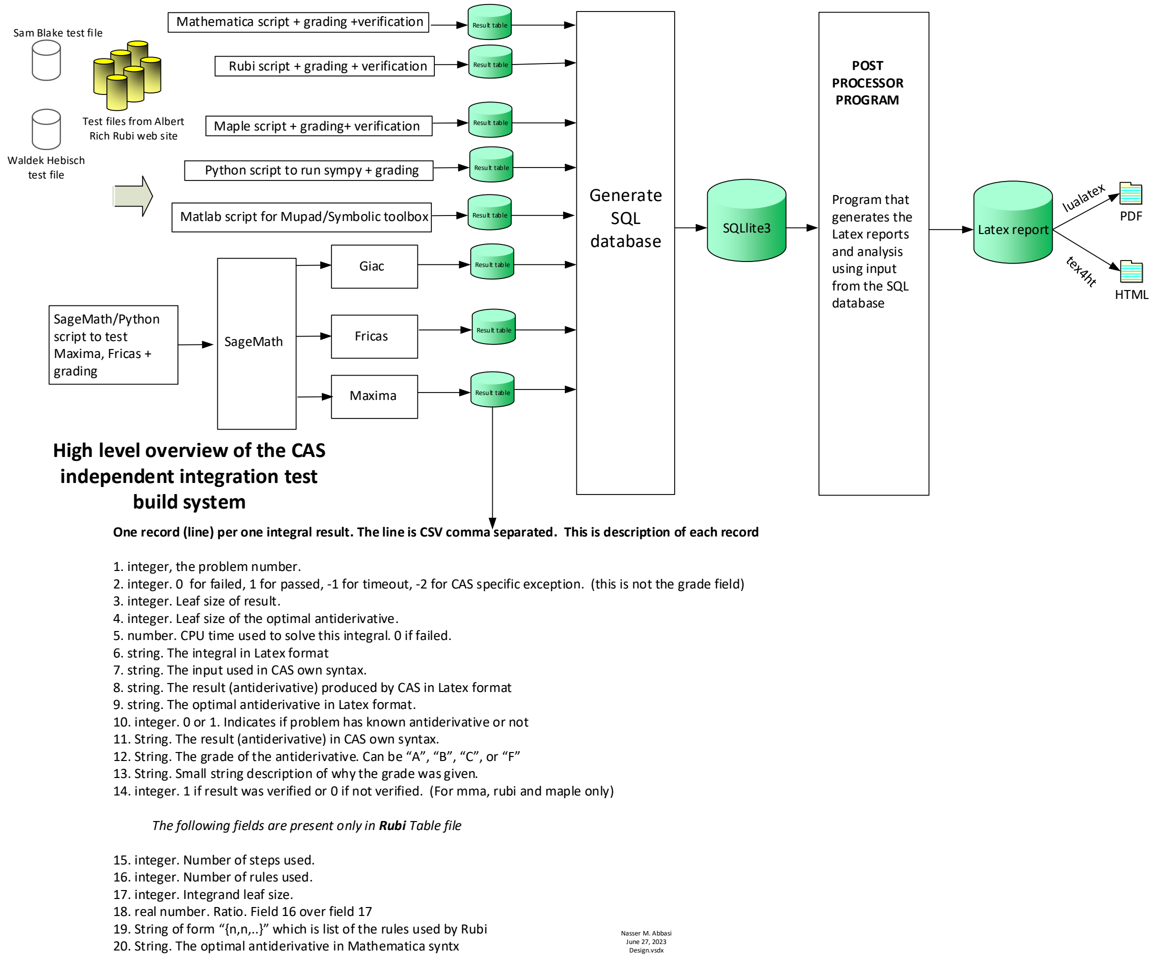
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

B grade { }

C grade { 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 22, 23, 24, 25, 30, 31, 32, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 54, 55, 56, 61, 62, 63, 64, 69, 70, 75, 76, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 92, 93 }

B grade { 35 }

C grade { 18, 19, 20, 21, 26, 27, 28, 29, 33, 34, 52, 57, 58, 59, 60, 65, 66, 67, 68, 71, 72, 73, 74, 77, 83, 89, 90, 91, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

B grade { }

C grade { 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 3, 4, 11, 43 }

B grade { 1, 2, 10, 12, 40, 41, 42, 49, 50, 51, 79, 80, 81, 82, 87, 88 }

C grade { 5, 6, 7, 8, 9, 13, 14, 15, 16, 17, 21, 22, 30, 36, 44, 45, 46, 47, 48, 52, 53, 54, 55, 56, 83, 84, 85, 86, 89, 90, 91, 92, 93 }

F normal fail { 18, 19, 20, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 37, 38, 39, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 10, 11, 12, 40, 41, 42, 43, 49, 50, 51, 79, 80, 81, 82, 87, 88 }

B grade { 26, 27, 28, 29, 30, 31, 32 }

C grade { 5, 6, 7, 8, 9, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 33, 34, 35, 36, 37, 38, 39, 44, 45, 46, 47, 48, 52, 53, 54, 55, 56, 83, 84, 85, 86, 89, 90, 91, 92, 93 }

F normal fail { 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 10, 11, 12, 40, 41, 42, 43, 49, 50, 51, 79, 80, 81, 82, 87, 88 }

C grade { }

F normal fail { }

F(-1) timedout fail { 5, 6, 7, 8, 9, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 44, 45, 46, 47, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 83, 84, 85, 86, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 10, 11, 13, 40, 41, 42, 43, 44, 49, 50, 51, 52, 79, 80, 81, 82, 83, 87, 88, 89 }

B grade { 12 }

C grade { }

F normal fail { 6, 7, 8, 9, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 45, 46, 47, 48, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 76, 84, 85, 86, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 106 }

F(-1) timedout fail { 72, 73, 74, 75, 77, 78, 102, 103, 104, 105, 107, 108, 109, 110, 111, 112, 113 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	82	85	306	85	151	86	122
N.S.	1	1.00	0.65	0.67	2.43	0.67	1.20	0.68	0.97
time (sec)	N/A	0.403	0.151	0.164	0.227	0.287	0.309	0.288	0.343

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	65	67	201	67	117	68	92
N.S.	1	1.00	0.68	0.70	2.09	0.70	1.22	0.71	0.96
time (sec)	N/A	0.248	0.129	0.145	0.212	0.276	0.239	0.285	0.198

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	45	47	117	48	82	49	62
N.S.	1	1.00	0.69	0.72	1.80	0.74	1.26	0.75	0.95
time (sec)	N/A	0.130	0.109	0.136	0.192	0.274	0.187	0.291	6.084

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	27	29	53	30	46	31	35
N.S.	1	1.00	0.96	1.04	1.89	1.07	1.64	1.11	1.25
time (sec)	N/A	0.021	0.089	0.108	0.190	0.264	0.148	0.294	0.090

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	40	31	522	32	37	339	0
N.S.	1	1.00	1.38	1.07	18.00	1.10	1.28	11.69	0.00
time (sec)	N/A	0.164	0.069	0.141	0.263	0.270	2.611	0.294	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	60	56	108	54	0	569	0
N.S.	1	1.00	1.25	1.17	2.25	1.12	0.00	11.85	0.00
time (sec)	N/A	0.235	0.150	0.153	0.404	0.280	0.000	0.297	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	76	88	112	83	0	796	0
N.S.	1	1.00	0.85	0.99	1.26	0.93	0.00	8.94	0.00
time (sec)	N/A	0.288	0.245	0.163	0.439	0.269	0.000	0.302	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	110	117	111	108	0	961	0
N.S.	1	1.00	0.83	0.89	0.84	0.82	0.00	7.28	0.00
time (sec)	N/A	0.408	0.289	0.192	0.460	0.260	0.000	0.308	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	138	145	110	124	0	1108	0
N.S.	1	1.00	0.83	0.87	0.66	0.75	0.00	6.67	0.00
time (sec)	N/A	0.300	0.242	0.216	0.495	0.271	0.000	0.311	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	101	127	406	126	228	128	172
N.S.	1	1.00	0.54	0.68	2.18	0.68	1.23	0.69	0.92
time (sec)	N/A	0.207	0.161	0.253	0.208	0.277	0.342	0.290	0.325

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	87	94	259	95	172	95	128
N.S.	1	1.00	0.64	0.70	1.92	0.70	1.27	0.70	0.95
time (sec)	N/A	0.123	0.127	0.254	0.199	0.289	0.265	0.271	6.034

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	57	61	141	63	112	65	84
N.S.	1	1.00	1.14	1.22	2.82	1.26	2.24	1.30	1.68
time (sec)	N/A	0.031	0.126	0.221	0.194	0.261	0.201	0.279	5.929

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	51	79	80	61	92	551	0
N.S.	1	1.00	0.82	1.27	1.29	0.98	1.48	8.89	0.00
time (sec)	N/A	0.131	0.202	0.256	0.455	0.277	1.410	0.279	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	64	74	122	84	0	743	0
N.S.	1	1.00	0.89	1.03	1.69	1.17	0.00	10.32	0.00
time (sec)	N/A	0.159	0.163	0.247	0.684	0.283	0.000	0.286	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	95	114	189	110	0	1182	0
N.S.	1	1.00	0.79	0.94	1.56	0.91	0.00	9.77	0.00
time (sec)	N/A	0.235	0.273	0.265	1.106	0.263	0.000	0.300	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	154	158	187	145	0	1400	0
N.S.	1	1.00	0.88	0.90	1.07	0.83	0.00	8.00	0.00
time (sec)	N/A	0.285	0.345	0.298	1.238	0.260	0.000	0.299	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	204	201	186	180	0	1712	0
N.S.	1	1.00	0.82	0.81	0.75	0.73	0.00	6.90	0.00
time (sec)	N/A	0.347	0.274	0.336	1.356	0.269	0.000	0.318	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	C	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	218	218	158	328	0	188	0	3337	0
N.S.	1	1.00	0.72	1.50	0.00	0.86	0.00	15.31	0.00
time (sec)	N/A	0.337	0.414	0.424	0.000	0.269	0.000	0.355	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	117	237	0	141	0	2709	0
N.S.	1	1.00	0.77	1.56	0.00	0.93	0.00	17.82	0.00
time (sec)	N/A	0.203	0.354	0.316	0.000	0.287	0.000	0.343	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	87	180	0	108	0	2205	0
N.S.	1	1.00	0.88	1.82	0.00	1.09	0.00	22.27	0.00
time (sec)	N/A	0.178	0.192	0.274	0.000	0.280	0.000	0.350	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	63	150	776	78	0	1647	0
N.S.	1	1.00	0.91	2.17	11.25	1.13	0.00	23.87	0.00
time (sec)	N/A	0.117	0.106	0.254	0.283	0.270	0.000	0.316	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	49	73	141	63	0	597	0
N.S.	1	1.00	0.96	1.43	2.76	1.24	0.00	11.71	0.00
time (sec)	N/A	0.061	0.069	0.219	0.224	0.274	0.000	0.326	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	63	99	0	76	0	838	0
N.S.	1	1.00	0.86	1.36	0.00	1.04	0.00	11.48	0.00
time (sec)	N/A	0.186	0.105	0.236	0.000	0.324	0.000	0.309	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	101	144	0	118	0	2897	0
N.S.	1	1.00	0.89	1.26	0.00	1.04	0.00	25.41	0.00
time (sec)	N/A	0.259	0.265	0.305	0.000	0.274	0.000	0.360	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	176	202	0	186	0	4565	0
N.S.	1	1.00	0.93	1.07	0.00	0.98	0.00	24.15	0.00
time (sec)	N/A	0.334	0.427	0.361	0.000	0.280	0.000	0.355	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	177	734	0	287	0	1973	0
N.S.	1	1.00	0.76	3.15	0.00	1.23	0.00	8.47	0.00
time (sec)	N/A	0.357	0.667	0.475	0.000	0.300	0.000	0.372	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	153	607	0	246	0	1474	0
N.S.	1	1.00	0.85	3.35	0.00	1.36	0.00	8.14	0.00
time (sec)	N/A	0.275	0.578	0.408	0.000	0.289	0.000	0.376	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	117	402	0	202	0	1120	0
N.S.	1	1.00	0.79	2.70	0.00	1.36	0.00	7.52	0.00
time (sec)	N/A	0.247	0.530	0.309	0.000	0.293	0.000	0.331	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	96	313	0	155	0	951	0
N.S.	1	1.00	0.77	2.52	0.00	1.25	0.00	7.67	0.00
time (sec)	N/A	0.203	0.336	0.280	0.000	0.284	0.000	0.346	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	66	107	164	96	0	518	0
N.S.	1	1.00	0.92	1.49	2.28	1.33	0.00	7.19	0.00
time (sec)	N/A	0.068	0.169	0.251	0.238	0.277	0.000	0.299	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	138	210	0	189	0	1281	0
N.S.	1	1.00	0.93	1.41	0.00	1.27	0.00	8.60	0.00
time (sec)	N/A	0.260	0.702	0.283	0.000	0.293	0.000	0.335	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	184	256	0	262	0	3180	0
N.S.	1	1.00	0.98	1.36	0.00	1.39	0.00	16.91	0.00
time (sec)	N/A	0.338	1.254	0.336	0.000	0.315	0.000	0.379	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	235	705	0	388	0	16724	0
N.S.	1	1.00	0.89	2.66	0.00	1.46	0.00	63.11	0.00
time (sec)	N/A	0.401	0.642	0.484	0.000	0.326	0.000	0.640	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	154	621	0	325	0	15410	0
N.S.	1	1.00	0.64	2.58	0.00	1.35	0.00	63.94	0.00
time (sec)	N/A	0.376	0.852	0.388	0.000	0.295	0.000	0.619	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	157	419	0	263	0	10535	0
N.S.	1	1.00	0.88	2.34	0.00	1.47	0.00	58.85	0.00
time (sec)	N/A	0.241	0.388	0.353	0.000	0.307	0.000	0.541	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	87	145	199	164	0	5727	0
N.S.	1	1.00	0.84	1.39	1.91	1.58	0.00	55.07	0.00
time (sec)	N/A	0.104	0.485	0.319	0.289	0.287	0.000	0.409	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	449	359	0	391	0	17806	0
N.S.	1	1.00	1.72	1.38	0.00	1.50	0.00	68.22	0.00
time (sec)	N/A	0.384	0.648	0.369	0.000	0.319	0.000	0.671	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	540	405	0	500	0	20808	0
N.S.	1	1.00	1.81	1.35	0.00	1.67	0.00	69.59	0.00
time (sec)	N/A	0.479	1.190	0.438	0.000	0.311	0.000	0.710	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	630	466	0	585	0	24116	0
N.S.	1	1.00	1.67	1.24	0.00	1.55	0.00	63.97	0.00
time (sec)	N/A	0.552	1.238	0.544	0.000	0.332	0.000	0.756	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	92	96	372	95	168	97	121
N.S.	1	1.00	0.65	0.68	2.64	0.67	1.19	0.69	0.86
time (sec)	N/A	0.140	0.132	0.170	0.198	0.299	0.417	0.295	0.436

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	75	78	258	77	134	79	97
N.S.	1	1.00	0.68	0.70	2.32	0.69	1.21	0.71	0.87
time (sec)	N/A	0.111	0.080	0.146	0.191	0.272	0.324	0.280	6.074

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	57	59	165	60	99	60	73
N.S.	1	1.00	0.71	0.74	2.06	0.75	1.24	0.75	0.91
time (sec)	N/A	0.075	0.065	0.164	0.187	0.284	0.255	0.299	0.166

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	41	43	91	41	65	42	49
N.S.	1	1.00	0.77	0.81	1.72	0.77	1.23	0.79	0.92
time (sec)	N/A	0.043	0.043	0.134	0.182	0.266	0.188	0.294	5.946

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	54	60	66	46	65	432	0
N.S.	1	1.00	1.32	1.46	1.61	1.12	1.59	10.54	0.00
time (sec)	N/A	0.059	0.079	0.151	0.397	0.271	1.872	0.324	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	48	937	53	0	411	0
N.S.	1	1.00	1.00	1.09	21.30	1.20	0.00	9.34	0.00
time (sec)	N/A	0.064	0.057	0.157	0.375	0.276	0.000	0.290	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	82	73	123	62	0	766	0
N.S.	1	1.00	1.11	0.99	1.66	0.84	0.00	10.35	0.00
time (sec)	N/A	0.108	0.109	0.166	0.651	0.279	0.000	0.294	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	95	102	121	82	0	834	0
N.S.	1	1.00	0.90	0.96	1.14	0.77	0.00	7.87	0.00
time (sec)	N/A	0.289	0.097	0.197	0.723	0.272	0.000	0.308	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	125	131	121	101	0	1086	0
N.S.	1	1.00	0.84	0.88	0.81	0.68	0.00	7.29	0.00
time (sec)	N/A	0.327	0.120	0.226	0.804	0.324	0.000	0.300	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	139	160	612	154	286	162	186
N.S.	1	1.00	0.59	0.68	2.59	0.65	1.21	0.69	0.79
time (sec)	N/A	0.398	0.262	0.293	0.217	0.288	0.557	0.286	0.669

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	113	127	438	126	226	129	151
N.S.	1	1.00	0.61	0.69	2.37	0.68	1.22	0.70	0.82
time (sec)	N/A	0.278	0.156	0.273	0.206	0.280	0.433	0.278	6.328

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	86	94	292	97	172	99	118
N.S.	1	1.00	0.62	0.68	2.12	0.70	1.25	0.72	0.86
time (sec)	N/A	0.207	0.116	0.247	0.188	0.275	0.326	0.295	6.263

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	82	128	116	97	160	725	0
N.S.	1	1.00	0.74	1.15	1.05	0.87	1.44	6.53	0.00
time (sec)	N/A	0.229	0.274	0.322	1.380	0.280	2.486	0.310	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	156	97	96	0	1638	0
N.S.	1	1.00	1.00	1.61	1.00	0.99	0.00	16.89	0.00
time (sec)	N/A	0.245	0.178	0.347	1.118	0.303	0.000	0.322	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	99	124	150	107	0	1058	0
N.S.	1	1.00	0.87	1.09	1.32	0.94	0.00	9.28	0.00
time (sec)	N/A	0.151	0.294	0.370	2.085	0.285	0.000	0.301	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	114	120	140	116	0	1032	0
N.S.	1	1.00	0.85	0.90	1.04	0.87	0.00	7.70	0.00
time (sec)	N/A	0.190	0.276	0.357	1.489	0.287	0.000	0.302	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	122	157	222	129	0	1497	0
N.S.	1	1.00	0.69	0.89	1.25	0.73	0.00	8.46	0.00
time (sec)	N/A	0.234	0.341	0.416	8.166	0.293	0.000	0.299	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	269	323	0	240	0	0	0
N.S.	1	1.00	0.99	1.18	0.00	0.88	0.00	0.00	0.00
time (sec)	N/A	0.530	1.461	0.515	0.000	0.312	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	214	266	0	185	0	0	0
N.S.	1	1.00	1.02	1.27	0.00	0.89	0.00	0.00	0.00
time (sec)	N/A	0.239	1.108	0.378	0.000	0.317	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	216	264	0	195	0	0	0
N.S.	1	1.00	0.95	1.16	0.00	0.86	0.00	0.00	0.00
time (sec)	N/A	0.259	0.653	0.309	0.000	0.314	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	155	234	0	146	0	0	0
N.S.	1	1.00	0.88	1.32	0.00	0.82	0.00	0.00	0.00
time (sec)	N/A	0.178	0.277	0.260	0.000	0.286	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	163	225	0	187	0	0	0
N.S.	1	1.00	0.77	1.06	0.00	0.88	0.00	0.00	0.00
time (sec)	N/A	0.169	0.401	0.256	0.000	0.297	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	174	200	0	162	0	0	0
N.S.	1	1.00	0.88	1.02	0.00	0.82	0.00	0.00	0.00
time (sec)	N/A	0.276	0.358	0.266	0.000	0.302	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	238	266	0	234	0	0	0
N.S.	1	1.00	0.95	1.06	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.361	0.573	0.332	0.000	0.339	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	240	259	0	225	0	0	0
N.S.	1	1.00	0.89	0.96	0.00	0.83	0.00	0.00	0.00
time (sec)	N/A	0.361	0.833	0.377	0.000	0.302	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	450	450	298	524	0	351	0	0	0
N.S.	1	1.00	0.66	1.16	0.00	0.78	0.00	0.00	0.00
time (sec)	N/A	0.551	3.153	0.779	0.000	0.321	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	431	431	286	852	0	291	0	0	0
N.S.	1	1.00	0.66	1.98	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.457	1.825	0.627	0.000	0.316	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	274	748	0	333	0	0	0
N.S.	1	1.00	0.66	1.80	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	0.405	2.013	0.517	0.000	0.298	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	236	412	0	244	0	0	0
N.S.	1	1.00	0.99	1.72	0.00	1.02	0.00	0.00	0.00
time (sec)	N/A	0.227	0.972	0.427	0.000	0.297	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	476	476	280	491	0	333	0	0	0
N.S.	1	1.00	0.59	1.03	0.00	0.70	0.00	0.00	0.00
time (sec)	N/A	0.592	1.952	0.349	0.000	0.323	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	435	435	419	478	0	320	0	0	0
N.S.	1	1.00	0.96	1.10	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.560	0.949	0.396	0.000	0.306	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	501	501	330	622	0	400	0	0	0
N.S.	1	1.00	0.66	1.24	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	0.818	2.305	0.477	0.000	0.338	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	476	476	330	640	0	492	0	0	0
N.S.	1	1.00	0.69	1.34	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.712	2.476	1.102	0.000	0.296	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	746	746	364	912	0	604	0	0	0
N.S.	1	1.00	0.49	1.22	0.00	0.81	0.00	0.00	0.00
time (sec)	N/A	0.780	3.364	0.862	0.000	0.316	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	512	512	317	628	0	483	0	0	0
N.S.	1	1.00	0.62	1.23	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.567	2.529	0.685	0.000	0.317	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	856	856	374	598	0	611	0	0	0
N.S.	1	1.00	0.44	0.70	0.00	0.71	0.00	0.00	0.00
time (sec)	N/A	0.806	2.987	0.525	0.000	0.297	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	730	730	672	580	0	637	0	0	0
N.S.	1	1.00	0.92	0.79	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	1.057	2.363	0.562	0.000	0.365	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	875	875	593	910	0	714	0	0	0
N.S.	1	1.00	0.68	1.04	0.00	0.82	0.00	0.00	0.00
time (sec)	N/A	1.701	4.043	0.801	0.000	0.297	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	791	791	432	697	0	758	0	0	0
N.S.	1	1.00	0.55	0.88	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	2.029	1.857	1.754	0.000	0.307	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	101	106	449	104	185	106	151
N.S.	1	1.00	0.65	0.68	2.88	0.67	1.19	0.68	0.97
time (sec)	N/A	0.339	0.145	0.174	0.216	0.284	0.546	0.325	0.724

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	84	88	326	87	151	88	121
N.S.	1	1.00	0.67	0.70	2.59	0.69	1.20	0.70	0.96
time (sec)	N/A	0.315	0.089	0.153	0.207	0.257	0.434	0.286	6.370

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	66	69	224	68	116	69	92
N.S.	1	1.00	0.69	0.73	2.36	0.72	1.22	0.73	0.97
time (sec)	N/A	0.143	0.069	0.136	0.194	0.264	0.316	0.307	6.111

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	50	52	141	52	82	54	65
N.S.	1	1.00	0.74	0.76	2.07	0.76	1.21	0.79	0.96
time (sec)	N/A	0.064	0.055	0.136	0.194	0.264	0.240	0.287	0.132

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	C	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	57	57	50	98	76	58	83	510	0
N.S.	1	1.00	0.88	1.72	1.33	1.02	1.46	8.95	0.00
time (sec)	N/A	0.075	0.118	0.180	0.584	0.280	2.753	0.319	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	79	69	64	0	489	0
N.S.	1	1.00	1.00	1.41	1.23	1.14	0.00	8.73	0.00
time (sec)	N/A	0.091	0.081	0.207	0.543	0.278	0.000	0.304	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	66	65	1146	66	0	564	0
N.S.	1	1.00	0.94	0.93	16.37	0.94	0.00	8.06	0.00
time (sec)	N/A	0.089	0.089	0.201	0.449	0.268	0.000	0.322	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	104	87	132	89	0	796	0
N.S.	1	1.00	1.14	0.96	1.45	0.98	0.00	8.75	0.00
time (sec)	N/A	0.132	0.118	0.220	0.904	0.278	0.000	0.301	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	139	161	662	161	284	161	225
N.S.	1	1.00	0.59	0.69	2.82	0.69	1.21	0.69	0.96
time (sec)	N/A	0.222	0.238	0.318	0.245	0.303	0.798	0.309	6.446

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	112	124	489	129	226	131	184
N.S.	1	1.00	0.60	0.66	2.60	0.69	1.20	0.70	0.98
time (sec)	N/A	0.160	0.199	0.277	0.226	0.286	0.569	0.316	0.771

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	108	178	147	128	207	921	0
N.S.	1	1.00	0.67	1.11	0.91	0.80	1.29	5.72	0.00
time (sec)	N/A	0.161	0.350	0.426	5.679	0.270	3.693	0.340	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	145	145	145	214	129	127	0	2038	0
N.S.	1	1.00	1.00	1.48	0.89	0.88	0.00	14.06	0.00
time (sec)	N/A	0.163	0.250	0.494	6.468	0.263	0.000	0.341	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	142	142	138	210	110	123	0	2171	0
N.S.	1	1.00	0.97	1.48	0.77	0.87	0.00	15.29	0.00
time (sec)	N/A	0.155	0.245	0.496	2.189	0.277	0.000	0.317	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	135	196	173	145	0	1181	0
N.S.	1	1.00	0.89	1.30	1.15	0.96	0.00	7.82	0.00
time (sec)	N/A	0.180	0.452	0.491	7.106	0.270	0.000	0.341	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	148	167	164	153	0	1255	0
N.S.	1	1.00	0.89	1.00	0.98	0.92	0.00	7.51	0.00
time (sec)	N/A	0.195	0.372	0.559	5.946	0.304	0.000	0.307	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	371	231	558	0	397	0	0	0
N.S.	1	1.00	0.62	1.50	0.00	1.07	0.00	0.00	0.00
time (sec)	N/A	0.619	0.139	0.400	0.000	0.321	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	357	216	392	0	393	0	0	0
N.S.	1	1.00	0.61	1.10	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	0.438	0.109	0.370	0.000	0.298	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	186	266	0	292	0	0	0
N.S.	1	1.00	0.66	0.95	0.00	1.04	0.00	0.00	0.00
time (sec)	N/A	0.318	5.041	0.286	0.000	0.276	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	196	176	0	379	0	0	0
N.S.	1	1.00	0.57	0.51	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	0.483	5.052	0.279	0.000	0.302	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	196	85	0	385	0	0	0
N.S.	1	1.00	0.57	0.25	0.00	1.12	0.00	0.00	0.00
time (sec)	N/A	0.555	5.049	0.267	0.000	0.301	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	206	88	0	308	0	0	0
N.S.	1	1.00	0.68	0.29	0.00	1.02	0.00	0.00	0.00
time (sec)	N/A	0.766	0.133	0.291	0.000	0.345	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	380	380	233	116	0	448	0	0	0
N.S.	1	1.00	0.61	0.31	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	0.556	0.175	0.408	0.000	0.315	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	408	408	253	136	0	489	0	0	0
N.S.	1	1.00	0.62	0.33	0.00	1.20	0.00	0.00	0.00
time (sec)	N/A	0.417	0.232	0.453	0.000	0.299	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	714	714	383	1184	0	670	0	0	0
N.S.	1	1.00	0.54	1.66	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.700	0.171	0.928	0.000	0.299	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	371	214	823	0	482	0	0	0
N.S.	1	1.00	0.58	2.22	0.00	1.30	0.00	0.00	0.00
time (sec)	N/A	0.679	0.127	0.745	0.000	0.315	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	691	691	408	508	0	655	0	0	0
N.S.	1	1.00	0.59	0.74	0.00	0.95	0.00	0.00	0.00
time (sec)	N/A	0.844	0.160	0.630	0.000	0.313	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	735	735	406	248	0	669	0	0	0
N.S.	1	1.00	0.55	0.34	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	0.853	0.162	0.534	0.000	0.301	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	693	693	446	233	0	576	0	0	0
N.S.	1	1.00	0.64	0.34	0.00	0.83	0.00	0.00	0.00
time (sec)	N/A	0.995	0.396	0.506	0.000	0.337	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	712	712	445	284	0	720	0	0	0
N.S.	1	1.00	0.62	0.40	0.00	1.01	0.00	0.00	0.00
time (sec)	N/A	1.117	0.538	0.777	0.000	0.362	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	800	800	470	315	0	898	0	0	0
N.S.	1	1.00	0.59	0.39	0.00	1.12	0.00	0.00	0.00
time (sec)	N/A	1.200	0.624	0.928	0.000	0.353	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	772	772	457	1337	0	890	0	0	0
N.S.	1	1.00	0.59	1.73	0.00	1.15	0.00	0.00	0.00
time (sec)	N/A	1.769	0.461	2.246	0.000	0.325	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	777	777	449	918	0	935	0	0	0
N.S.	1	1.00	0.58	1.18	0.00	1.20	0.00	0.00	0.00
time (sec)	N/A	0.924	0.313	1.738	0.000	0.343	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	1141	1141	698	610	0	1319	0	0	0
N.S.	1	1.00	0.61	0.53	0.00	1.16	0.00	0.00	0.00
time (sec)	N/A	2.002	0.407	1.385	0.000	0.373	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1161	1161	675	340	0	1223	0	0	0
N.S.	1	1.00	0.58	0.29	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	2.289	0.317	1.049	0.000	0.325	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1163	1163	2109	363	0	1113	0	0	0
N.S.	1	1.00	1.81	0.31	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	2.409	0.553	1.089	0.000	0.352	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [112] had the largest ratio of [.6250000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	11	4	1.00	15	0.267
2	A	9	4	1.00	15	0.267
3	A	7	4	1.00	13	0.308
4	A	2	2	1.00	12	0.167
5	A	6	5	1.00	15	0.333
6	A	9	5	1.00	15	0.333
7	A	11	5	1.00	15	0.333
8	A	13	5	1.00	15	0.333
9	A	15	5	1.00	15	0.333
10	A	14	4	1.00	17	0.235
11	A	11	4	1.00	15	0.267
12	A	3	2	1.00	14	0.143
13	A	8	7	1.00	17	0.412
14	A	10	6	1.00	17	0.353
15	A	14	5	1.00	17	0.294
16	A	17	5	1.00	17	0.294
17	A	20	5	1.00	17	0.294
18	A	15	7	1.00	17	0.412
19	A	11	7	1.00	17	0.412
20	A	8	7	1.00	17	0.412
21	A	6	5	1.00	15	0.333
22	A	3	3	1.00	14	0.214
23	A	8	4	1.00	17	0.235
24	A	12	5	1.00	17	0.294

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	17	5	1.00	17	0.294
26	A	15	8	1.00	17	0.471
27	A	12	8	1.00	17	0.471
28	A	10	6	1.00	17	0.353
29	A	9	5	1.00	15	0.333
30	A	4	4	1.00	14	0.286
31	A	12	5	1.00	17	0.294
32	A	16	5	1.00	17	0.294
33	A	15	6	1.00	17	0.353
34	A	14	5	1.00	17	0.294
35	A	11	5	1.00	15	0.333
36	A	5	4	1.00	14	0.286
37	A	17	5	1.00	17	0.294
38	A	21	5	1.00	17	0.294
39	A	26	5	1.00	17	0.294
40	A	12	3	1.00	17	0.176
41	A	10	3	1.00	17	0.176
42	A	8	3	1.00	15	0.200
43	A	6	3	1.00	14	0.214
44	A	7	6	1.00	17	0.353
45	A	7	6	1.00	17	0.353
46	A	10	5	1.00	17	0.294
47	A	12	5	1.00	17	0.294
48	A	14	5	1.00	17	0.294
49	A	17	3	1.00	19	0.158
50	A	14	3	1.00	17	0.176
51	A	11	3	1.00	16	0.188
52	A	11	6	1.00	19	0.316
53	A	10	7	1.00	19	0.368
54	A	12	7	1.00	19	0.368
55	A	13	6	1.00	19	0.316
56	A	17	5	1.00	19	0.263
57	A	14	7	1.00	19	0.368
58	A	12	6	1.00	19	0.316
59	A	11	6	1.00	19	0.316

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	8	4	1.00	17	0.235
61	A	8	4	1.00	16	0.250
62	A	13	4	1.00	19	0.210
63	A	14	6	1.00	19	0.316
64	A	18	5	1.00	19	0.263
65	A	24	9	1.00	19	0.474
66	A	20	8	1.00	19	0.421
67	A	17	6	1.00	19	0.316
68	A	9	5	1.00	17	0.294
69	A	18	5	1.00	16	0.312
70	A	22	6	1.00	19	0.316
71	A	32	6	1.00	19	0.316
72	A	27	8	1.00	19	0.421
73	A	28	7	1.00	19	0.368
74	A	19	6	1.00	17	0.353
75	A	28	5	1.00	16	0.312
76	A	41	7	1.00	19	0.368
77	A	60	6	1.00	19	0.316
78	A	46	7	1.00	19	0.368
79	A	13	4	1.00	17	0.235
80	A	11	4	1.00	17	0.235
81	A	9	4	1.00	15	0.267
82	A	7	4	1.00	14	0.286
83	A	8	6	1.00	17	0.353
84	A	8	7	1.00	17	0.412
85	A	8	6	1.00	17	0.353
86	A	11	5	1.00	17	0.294
87	A	17	4	1.00	17	0.235
88	A	14	4	1.00	16	0.250
89	A	14	7	1.00	19	0.368
90	A	13	8	1.00	19	0.421
91	A	12	8	1.00	19	0.421
92	A	14	7	1.00	19	0.368
93	A	15	7	1.00	19	0.368
94	A	15	6	1.00	19	0.316

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	14	6	1.00	19	0.316
96	A	11	4	1.00	19	0.210
97	A	11	4	1.00	17	0.235
98	A	11	4	1.00	16	0.250
99	A	16	4	1.00	19	0.210
100	A	17	5	1.00	19	0.263
101	A	18	6	1.00	19	0.316
102	A	23	6	1.00	19	0.316
103	A	12	5	1.00	19	0.263
104	A	34	7	1.00	17	0.412
105	A	36	8	1.00	16	0.500
106	A	41	8	1.00	19	0.421
107	A	47	7	1.00	19	0.368
108	A	51	8	1.00	19	0.421
109	A	71	10	1.00	19	0.526
110	A	37	9	1.00	19	0.474
111	A	89	9	1.00	17	0.529
112	A	99	10	1.00	16	0.625
113	A	110	9	1.00	19	0.474

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^3(a+bx)\sin(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 126

$$\int x^3(a+bx)\sin(c+dx)\,dx = -\frac{24b\cos(c+dx)}{d^5}+\frac{6ax\cos(c+dx)}{d^3}+\frac{12bx^2\cos(c+dx)}{d^3} -\frac{ax^3\cos(c+dx)}{d}-\frac{bx^4\cos(c+dx)}{d}-\frac{6a\sin(c+dx)}{d^4} -\frac{24bx\sin(c+dx)}{d^4}+\frac{3ax^2\sin(c+dx)}{d^2}+\frac{4bx^3\sin(c+dx)}{d^2}$$

```
[Out] -24*b*cos(d*x+c)/d^5+6*a*x*cos(d*x+c)/d^3+12*b*x^2*cos(d*x+c)/d^3-a*x^3*cos
(d*x+c)/d-b*x^4*cos(d*x+c)/d-6*a*sin(d*x+c)/d^4-24*b*x*sin(d*x+c)/d^4+3*a*x
^2*sin(d*x+c)/d^2+4*b*x^3*sin(d*x+c)/d^2
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {6874, 3377, 2717, 2718}

$$\int x^3(a+bx)\sin(c+dx)\,dx = -\frac{6a\sin(c+dx)}{d^4}+\frac{6ax\cos(c+dx)}{d^3}+\frac{3ax^2\sin(c+dx)}{d^2} -\frac{ax^3\cos(c+dx)}{d}-\frac{24b\cos(c+dx)}{d^5}-\frac{24bx\sin(c+dx)}{d^4} +\frac{12bx^2\cos(c+dx)}{d^3}+\frac{4bx^3\sin(c+dx)}{d^2}-\frac{bx^4\cos(c+dx)}{d}$$

```
[In] Int[x^3*(a + b*x)*Sin[c + d*x],x]
```

```
[Out] (-24*b*Cos[c + d*x])/d^5 + (6*a*x*Cos[c + d*x])/d^3 + (12*b*x^2*Cos[c + d*x
])/d^3 - (a*x^3*Cos[c + d*x])/d - (b*x^4*Cos[c + d*x])/d - (6*a*Sin[c + d*x
```

```
])/d^4 - (24*b*x*Sin[c + d*x])/d^4 + (3*a*x^2*Sin[c + d*x])/d^2 + (4*b*x^3*
Sin[c + d*x])/d^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(ax^3\sin(c+dx)+bx^4\sin(c+dx)\right)\,dx\\
&= a\int x^3\sin(c+dx)\,dx+b\int x^4\sin(c+dx)\,dx\\
&= -\frac{ax^3\cos(c+dx)}{d}-\frac{bx^4\cos(c+dx)}{d}+\frac{(3a)\int x^2\cos(c+dx)\,dx}{d}+\frac{(4b)\int x^3\cos(c+dx)\,dx}{d}\\
&= -\frac{ax^3\cos(c+dx)}{d}-\frac{bx^4\cos(c+dx)}{d}+\frac{3ax^2\sin(c+dx)}{d^2}\\
&\quad+\frac{4bx^3\sin(c+dx)}{d^2}-\frac{(6a)\int x\sin(c+dx)\,dx}{d^2}-\frac{(12b)\int x^2\sin(c+dx)\,dx}{d^2}\\
&= \frac{6ax\cos(c+dx)}{d^3}+\frac{12bx^2\cos(c+dx)}{d^3}-\frac{ax^3\cos(c+dx)}{d}\\
&\quad-\frac{bx^4\cos(c+dx)}{d}+\frac{3ax^2\sin(c+dx)}{d^2}+\frac{4bx^3\sin(c+dx)}{d^2}\\
&\quad-\frac{(6a)\int\cos(c+dx)\,dx}{d^3}-\frac{(24b)\int x\cos(c+dx)\,dx}{d^3}\\
&= \frac{6ax\cos(c+dx)}{d^3}+\frac{12bx^2\cos(c+dx)}{d^3}-\frac{ax^3\cos(c+dx)}{d}-\frac{bx^4\cos(c+dx)}{d}-\frac{6a\sin(c+dx)}{d^4}\\
&\quad-\frac{24bx\sin(c+dx)}{d^4}+\frac{3ax^2\sin(c+dx)}{d^2}+\frac{4bx^3\sin(c+dx)}{d^2}+\frac{(24b)\int\sin(c+dx)\,dx}{d^4}
\end{aligned}
$$

$$= -\frac{24b\cos(c+dx)}{d^5} + \frac{6ax\cos(c+dx)}{d^3} + \frac{12bx^2\cos(c+dx)}{d^3} - \frac{ax^3\cos(c+dx)}{d} - \frac{bx^4\cos(c+dx)}{d} - \frac{6a\sin(c+dx)}{d^4} - \frac{24bx\sin(c+dx)}{d^4} + \frac{3ax^2\sin(c+dx)}{d^2} + \frac{4bx^3\sin(c+dx)}{d^2}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.65

$$\int x^3(a+bx)\sin(c+dx)\,dx = \frac{-((ad^2x(-6+d^2x^2)+b(24-12d^2x^2+d^4x^4))\cos(c+dx))+d(4bx(-6+d^2x^2)+3a(-2+d^2x^2))\sin(c+dx)}{d^5}$$

```
[In] Integrate[x^3*(a + b*x)*Sin[c + d*x],x]
[Out] (-((a*d^2*x*(-6 + d^2*x^2) + b*(24 - 12*d^2*x^2 + d^4*x^4))*Cos[c + d*x]) +
 d*(4*b*x*(-6 + d^2*x^2) + 3*a*(-2 + d^2*x^2))*Sin[c + d*x])/d^5
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.67

method	result
risch	$-\frac{(b\,x^4 d^4+a\,d^4 x^3-12d^2 x^2 b-6a\,d^2 x+24b)\cos(dx+c)}{d^5}+\frac{(4b\,d^2 x^3+3a\,d^2 x^2-24bx-6a)\sin(dx+c)}{d^4}$
parallelrisch	$\frac{(x^2(bx+a)d^2-12bx-6a)x\,d^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6\left(x^2\left(\frac{4bx}{3}+a\right)d^2-8bx-2a\right)d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+(-b\,x^4-a\,x^3)d^4+6x(2bx+c}{d^5\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$
norman	$\frac{\frac{a\,x^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{b\,x^4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{48b}{d^5}-\frac{12a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^4}+\frac{6ax}{d^3}-\frac{a\,x^3}{d}+\frac{12b\,x^2}{d^3}-\frac{b\,x^4}{d}-\frac{6ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d^3}+\frac{6a\,x^2\,\mathrm{ta}}{}}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$
meijerg	$\frac{16b\sqrt{\pi}\,\sin(c)\left(-\frac{x\left(d^2\right)^{\frac{5}{2}}\left(-\frac{5d^2x^2}{2}+15\right)\cos(dx)}{10\sqrt{\pi}\,d^4}+\frac{\left(d^2\right)^{\frac{5}{2}}\left(\frac{5}{8}d^4x^4-\frac{15}{2}d^2x^2+15\right)\sin(dx)}{10\sqrt{\pi}\,d^5}\right)}{d^4\sqrt{d^2}}+\frac{16b\sqrt{\pi}\,\cos(c)\left(\frac{3}{2\sqrt{\pi}}-\frac{\left(\frac{3}{8}d^4x^4\right.}{}\right.}{}$
parts	$-\frac{b\,x^4\cos(dx+c)}{d}-\frac{a\,x^3\cos(dx+c)}{d}+\frac{\frac{3a\,c^2\sin(dx+c)}{d^2}-\frac{6ac(\cos(dx+c)+(dx+c)\sin(dx+c))}{d^2}+\frac{3a\left((dx+c)^2\sin(dx+c)-2\sin(d\right.}{d^2}}{}$
derivativedivides	$\frac{a\,c^3\cos(dx+c)+3a\,c^2(\sin(dx+c)-\cos(dx+c)(dx+c))-3ac\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)+c}{}$
default	$\frac{a\,c^3\cos(dx+c)+3a\,c^2(\sin(dx+c)-\cos(dx+c)(dx+c))-3ac\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)+c}{}$

```
[In] int(x^3*(b*x+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*d^4*x^4+a*d^4*x^3-12*b*d^2*x^2-6*a*d^2*x+24*b)/d^5*cos(d*x+c)+1/d^4*(4*
b*d^2*x^3+3*a*d^2*x^2-24*b*x-6*a)*sin(d*x+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.67

$$\int x^3(a+bx)\sin(c+dx)\,dx = -\frac{(bd^4x^4 + ad^4x^3 - 12\,bd^2x^2 - 6\,ad^2x + 24\,b)\cos(dx+c) - (4\,bd^3x^3 + 3\,ad^3x^2 - 24\,bdx - 6\,ad)\sin(dx+c)}{d^5}$$

```
[In] integrate(x^3*(b*x+a)*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b*d^4*x^4 + a*d^4*x^3 - 12*b*d^2*x^2 - 6*a*d^2*x + 24*b)*cos(d*x + c) -
(4*b*d^3*x^3 + 3*a*d^3*x^2 - 24*b*d*x - 6*a*d)*sin(d*x + c))/d^5
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.20

$$\int x^3(a+bx)\sin(c+dx)\,dx = \begin{cases} -\frac{ax^3\cos(c+dx)}{d} + \frac{3ax^2\sin(c+dx)}{d^2} + \frac{6ax\cos(c+dx)}{d^3} - \frac{6a\sin(c+dx)}{d^4} - \frac{bx^4\cos(c+dx)}{d} + \frac{4bx^3\sin(c+dx)}{d^2} + \frac{12bx^2\cos(c+dx)}{d^3} - \frac{24 \\ \left(\frac{ax^4}{4} + \frac{bx^5}{5}\right)\sin(c) \end{cases}$$

```
[In] integrate(x**3*(b*x+a)*sin(d*x+c),x)
```

```
[Out] Piecewise((-a*x**3*cos(c + d*x)/d + 3*a*x**2*sin(c + d*x)/d**2 + 6*a*x*cos(
c + d*x)/d**3 - 6*a*sin(c + d*x)/d**4 - b*x**4*cos(c + d*x)/d + 4*b*x**3*si
n(c + d*x)/d**2 + 12*b*x**2*cos(c + d*x)/d**3 - 24*b*x*sin(c + d*x)/d**4 -
24*b*cos(c + d*x)/d**5, Ne(d, 0)), ((a*x**4/4 + b*x**5/5)*sin(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. 2(126) = 252.

Time = 0.23 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.43

$$\int x^3(a+bx)\sin(c+dx)\,dx = \frac{ac^3\cos(dx+c) - \frac{bc^4\cos(dx+c)}{d} - 3\left((dx+c)\cos(dx+c) - \sin(dx+c)\right)ac^2 + \frac{4\left((dx+c)\cos(dx+c) - \sin(dx+c)\right)bc^3}{d} +$$

```
[In] integrate(x^3*(b*x+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] (a*c^3*cos(d*x + c) - b*c^4*cos(d*x + c)/d - 3*((d*x + c)*cos(d*x + c) - si
n(d*x + c))*a*c^2 + 4*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c^3/d + 3*(
((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a*c - 6*(((d*x +
 c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b*c^2/d - (((d*x + c)^3
 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*a + 4*(((d
*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*b
*c/d - (((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d*x + c) - 4*((d*x + c)^3 -
 6*d*x - 6*c)*sin(d*x + c))*b/d)/d^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.68

$$\int x^3(a+bx)\sin(c+dx)\,dx = -\frac{(bd^4x^4 + ad^4x^3 - 12\,bd^2x^2 - 6\,ad^2x + 24\,b)\cos(dx+c)}{d^5} + \frac{(4\,bd^3x^3 + 3\,ad^3x^2 - 24\,bdx - 6\,ad)\sin(dx+c)}{d^5}$$

```
[In] integrate(x^3*(b*x+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b*d^4*x^4 + a*d^4*x^3 - 12*b*d^2*x^2 - 6*a*d^2*x + 24*b)*cos(d*x + c)/d^5
 + (4*b*d^3*x^3 + 3*a*d^3*x^2 - 24*b*d*x - 6*a*d)*sin(d*x + c)/d^5
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.97

$$\int x^3(a+bx)\sin(c+dx)\,dx = \frac{6\,a\,x\,\cos(c+d\,x) + 12\,b\,x^2\,\cos(c+d\,x)}{d^3} - \frac{6\,a\,\sin(c+d\,x) + 24\,b\,x\,\sin(c+d\,x)}{d^4} - \frac{a\,x^3\,\cos(c+d\,x) + b\,x^4\,\cos(c+d\,x)}{d} + \frac{3\,a\,x^2\,\sin(c+d\,x) + 4\,b\,x^3\,\sin(c+d\,x)}{d^2} - \frac{24\,b\,\cos(c+d\,x)}{d^5}$$

```
[In] int(x^3*sin(c + d*x)*(a + b*x),x)
```

```
[Out] (6*a*x*cos(c + d*x) + 12*b*x^2*cos(c + d*x))/d^3 - (6*a*sin(c + d*x) + 24*b
*x*sin(c + d*x))/d^4 - (a*x^3*cos(c + d*x) + b*x^4*cos(c + d*x))/d + (3*a*x
^2*sin(c + d*x) + 4*b*x^3*sin(c + d*x))/d^2 - (24*b*cos(c + d*x))/d^5
```

3.2 $\int x^2(a+bx)\sin(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 96

$$\int x^2(a+bx)\sin(c+dx)\,dx=\frac{2a\cos(c+dx)}{d^3}+\frac{6bx\cos(c+dx)}{d^3}-\frac{ax^2\cos(c+dx)}{d}-\frac{bx^3\cos(c+dx)}{d}-\frac{6b\sin(c+dx)}{d^4}+\frac{2ax\sin(c+dx)}{d^2}+\frac{3bx^2\sin(c+dx)}{d^2}$$

```
[Out] 2*a*cos(d*x+c)/d^3+6*b*x*cos(d*x+c)/d^3-a*x^2*cos(d*x+c)/d-b*x^3*cos(d*x+c)
/d-6*b*sin(d*x+c)/d^4+2*a*x*sin(d*x+c)/d^2+3*b*x^2*sin(d*x+c)/d^2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {6874, 3377, 2718, 2717}

$$\int x^2(a+bx)\sin(c+dx)\,dx=\frac{2a\cos(c+dx)}{d^3}+\frac{2ax\sin(c+dx)}{d^2}-\frac{ax^2\cos(c+dx)}{d}-\frac{6b\sin(c+dx)}{d^4}+\frac{6bx\cos(c+dx)}{d^3}+\frac{3bx^2\sin(c+dx)}{d^2}-\frac{bx^3\cos(c+dx)}{d}$$

```
[In] Int[x^2*(a + b*x)*Sin[c + d*x],x]
```

```
[Out] (2*a*Cos[c + d*x])/d^3 + (6*b*x*Cos[c + d*x])/d^3 - (a*x^2*Cos[c + d*x])/d
- (b*x^3*Cos[c + d*x])/d - (6*b*Sin[c + d*x])/d^4 + (2*a*x*Sin[c + d*x])/d^
2 + (3*b*x^2*Sin[c + d*x])/d^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(ax^2\sin(c+dx)+bx^3\sin(c+dx)\right)\,dx\\
&= a\int x^2\sin(c+dx)\,dx+b\int x^3\sin(c+dx)\,dx\\
&= -\frac{ax^2\cos(c+dx)}{d}-\frac{bx^3\cos(c+dx)}{d}+\frac{(2a)\int x\cos(c+dx)\,dx}{d}+\frac{(3b)\int x^2\cos(c+dx)\,dx}{d}\\
&= -\frac{ax^2\cos(c+dx)}{d}-\frac{bx^3\cos(c+dx)}{d}+\frac{2ax\sin(c+dx)}{d^2}\\
&\quad+\frac{3bx^2\sin(c+dx)}{d^2}-\frac{(2a)\int\sin(c+dx)\,dx}{d^2}-\frac{(6b)\int x\sin(c+dx)\,dx}{d^2}\\
&= \frac{2a\cos(c+dx)}{d^3}+\frac{6bx\cos(c+dx)}{d^3}-\frac{ax^2\cos(c+dx)}{d}-\frac{bx^3\cos(c+dx)}{d}\\
&\quad+\frac{2ax\sin(c+dx)}{d^2}+\frac{3bx^2\sin(c+dx)}{d^2}-\frac{(6b)\int\cos(c+dx)\,dx}{d^3}\\
&= \frac{2a\cos(c+dx)}{d^3}+\frac{6bx\cos(c+dx)}{d^3}-\frac{ax^2\cos(c+dx)}{d}-\frac{bx^3\cos(c+dx)}{d}\\
&\quad-\frac{6b\sin(c+dx)}{d^4}+\frac{2ax\sin(c+dx)}{d^2}+\frac{3bx^2\sin(c+dx)}{d^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.68

$$\int x^2(a+bx)\sin(c+dx)\,dx = \frac{-d(bx(-6+d^2x^2)+a(-2+d^2x^2))\cos(c+dx)+(2ad^2x+3b(-2+d^2x^2))\sin(c+dx)}{d^4}$$

```
[In] Integrate[x^2*(a + b*x)*Sin[c + d*x],x]
```

```
[Out] (-(d*(b*x*(-6 + d^2*x^2) + a*(-2 + d^2*x^2))*Cos[c + d*x]) + (2*a*d^2*x + 3
*b*(-2 + d^2*x^2))*Sin[c + d*x])/d^4
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{(b\,d^2x^3+a\,d^2x^2-6bx-2a)\cos(dx+c)}{d^3}+\frac{(3d^2x^2b+2a\,d^2x-6b)\sin(dx+c)}{d^4}$
parallelrisch	$\frac{x(x(bx+a)d^2-6b)d\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+((6b\,x^2+4ax)d^2-12b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-(x^2(bx+a)d^2-6bx-4a)d}{d^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$
norman	$\frac{\frac{4a}{d^3}+\frac{a\,x^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{b\,x^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a\,x^2}{d}-\frac{12b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^4}+\frac{6bx}{d^3}-\frac{b\,x^3}{d}+\frac{4ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^2}-\frac{6bx\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d^3}+ 6}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$
parts	$-\frac{b\,x^3\cos(dx+c)}{d}-\frac{a\,x^2\cos(dx+c)}{d}+\frac{-\frac{2ac\sin(dx+c)}{d}+\frac{2a(\cos(dx+c)+(dx+c)\sin(dx+c))}{d}+\frac{3b\,c^2\sin(dx+c)}{d^2}-\frac{6bc(\cos(dx+c)+}{}}{d^2}$
meijerg	$\frac{8b\sqrt{\pi}\,\sin(c)\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(-\frac{3d^2x^2}{2}+3\right)\cos(dx)}{4\sqrt{\pi}}-\frac{dx\left(-\frac{d^2x^2}{2}+3\right)\sin(dx)}{4\sqrt{\pi}}\right)}{d^4}+\frac{8b\sqrt{\pi}\,\cos(c)\left(\frac{xd\left(-\frac{5d^2x^2}{2}+15\right)\cos(dx)}{20\sqrt{\pi}}-\frac{\left(-\frac{15d^2x}{2}\right.}{}\right.}{d^4}$
derivativedivides	$\frac{-a\,c^2\cos(dx+c)-2ac(\sin(dx+c)-\cos(dx+c)(dx+c))+a\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)+\frac{b\,c^3\,c}{}}{}$
default	$\frac{-a\,c^2\cos(dx+c)-2ac(\sin(dx+c)-\cos(dx+c)(dx+c))+a\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)+\frac{b\,c^3\,c}{}}{}$

```
[In] int(x^2*(b*x+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^3*(b*d^2*x^3+a*d^2*x^2-6*b*x-2*a)*cos(d*x+c)+(3*b*d^2*x^2+2*a*d^2*x-6*
b)/d^4*sin(d*x+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.70

$$\int x^2(a+bx)\sin(c+dx)\,dx = -\frac{(bd^3x^3+ad^3x^2-6\,bdx-2\,ad)\cos{(dx+c)}-(3\,bd^2x^2+2\,ad^2x-6\,b)\sin{(dx+c)}}{d^4}$$

```
[In] integrate(x^2*(b*x+a)*sin(d*x+c),x, algorithm="fricas")
[Out] -((b*d^3*x^3 + a*d^3*x^2 - 6*b*d*x - 2*a*d)*cos(d*x + c) - (3*b*d^2*x^2 + 2
*a*d^2*x - 6*b)*sin(d*x + c))/d^4
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.22

$$\int x^2(a+bx)\sin(c+dx)\,dx = \begin{cases} -\frac{ax^2\cos{(c+dx)}}{d}+\frac{2ax\sin{(c+dx)}}{d^2}+\frac{2a\cos{(c+dx)}}{d^3}-\frac{bx^3\cos{(c+dx)}}{d}+\frac{3bx^2\sin{(c+dx)}}{d^2}+\frac{6bx\cos{(c+dx)}}{d^3}-\frac{6b\sin{(c+dx)}}{d^4} & \text{for } d \\ \left(\frac{ax^3}{3}+\frac{bx^4}{4}\right)\sin{(c)} & \text{other} \end{cases}$$

```
[In] integrate(x**2*(b*x+a)*sin(d*x+c),x)
[Out] Piecewise((-a*x**2*cos(c + d*x)/d + 2*a*x*sin(c + d*x)/d**2 + 2*a*cos(c + d
*x)/d**3 - b*x**3*cos(c + d*x)/d + 3*b*x**2*sin(c + d*x)/d**2 + 6*b*x*cos(c
 + d*x)/d**3 - 6*b*sin(c + d*x)/d**4, Ne(d, 0)), ((a*x**3/3 + b*x**4/4)*sin
(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(96) = 192.

Time = 0.21 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.09

$$\int x^2(a+bx)\sin(c+dx)\,dx = -\frac{ac^2\cos{(dx+c)}-\frac{bc^3\cos(dx+c)}{d}-2\,((dx+c)\cos{(dx+c)}-\sin{(dx+c)})ac+\frac{3\,((dx+c)\cos(dx+c)-\sin(dx+c))bc^2}{d}}{}$$

```
[In] integrate(x^2*(b*x+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a*c^2*cos(d*x + c) - b*c^3*cos(d*x + c)/d - 2*((d*x + c)*cos(d*x + c) - s
in(d*x + c))*a*c + 3*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c^2/d + (((d
*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a - 3*(((d*x + c)^2
 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b*c/d + (((d*x + c)^3 - 6*d*
x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*b/d)/d^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.71

$$\int x^2(a+bx)\sin(c+dx)\,dx = -\frac{(bd^3x^3 + ad^3x^2 - 6\,bdx - 2\,ad)\cos{(dx+c)}}{d^4} + \frac{(3\,bd^2x^2 + 2\,ad^2x - 6\,b)\sin{(dx+c)}}{d^4}$$

```
[In] integrate(x^2*(b*x+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b*d^3*x^3 + a*d^3*x^2 - 6*b*d*x - 2*a*d)*cos(d*x + c)/d^4 + (3*b*d^2*x^2
+ 2*a*d^2*x - 6*b)*sin(d*x + c)/d^4
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.96

$$\int x^2(a+bx)\sin(c+dx)\,dx = \frac{3\,b\,x^2\,\sin{(c+d\,x)} + 2\,a\,x\,\sin{(c+d\,x)}}{d^2} + \frac{2\,a\,\cos{(c+d\,x)} + 6\,b\,x\,\cos{(c+d\,x)}}{d^3} - \frac{a\,x^2\,\cos{(c+d\,x)} + b\,x^3\,\cos{(c+d\,x)}}{d} - \frac{6\,b\,\sin{(c+d\,x)}}{d^4}$$

```
[In] int(x^2*sin(c + d*x)*(a + b*x),x)
```

```
[Out] (3*b*x^2*sin(c + d*x) + 2*a*x*sin(c + d*x))/d^2 + (2*a*cos(c + d*x) + 6*b*x
*cos(c + d*x))/d^3 - (a*x^2*cos(c + d*x) + b*x^3*cos(c + d*x))/d - (6*b*sin
(c + d*x))/d^4
```

3.3 $\int x(a+bx)\sin(c+dx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 65

$$\int x(a+bx)\sin(c+dx)\,dx = \frac{2b\cos(c+dx)}{d^3} - \frac{ax\cos(c+dx)}{d} - \frac{bx^2\cos(c+dx)}{d} + \frac{a\sin(c+dx)}{d^2} + \frac{2bx\sin(c+dx)}{d^2}$$

```
[Out] 2*b*cos(d*x+c)/d^3-a*x*cos(d*x+c)/d-b*x^2*cos(d*x+c)/d+a*sin(d*x+c)/d^2+2*b
*x*sin(d*x+c)/d^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6874, 3377, 2717, 2718}

$$\int x(a+bx)\sin(c+dx)\,dx = \frac{a\sin(c+dx)}{d^2} - \frac{ax\cos(c+dx)}{d} + \frac{2b\cos(c+dx)}{d^3} + \frac{2bx\sin(c+dx)}{d^2} - \frac{bx^2\cos(c+dx)}{d}$$

```
[In] Int[x*(a + b*x)*Sin[c + d*x],x]
```

```
[Out] (2*b*Cos[c + d*x])/d^3 - (a*x*Cos[c + d*x])/d - (b*x^2*Cos[c + d*x])/d + (a
*Sin[c + d*x])/d^2 + (2*b*x*Sin[c + d*x])/d^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(ax\sin(c+dx)+bx^2\sin(c+dx)\right)\,dx\\
&= a\int x\sin(c+dx)\,dx+b\int x^2\sin(c+dx)\,dx\\
&= -\frac{ax\cos(c+dx)}{d}-\frac{bx^2\cos(c+dx)}{d}+\frac{a\int\cos(c+dx)\,dx}{d}+\frac{(2b)\int x\cos(c+dx)\,dx}{d}\\
&= -\frac{ax\cos(c+dx)}{d}-\frac{bx^2\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d^2}+\frac{2bx\sin(c+dx)}{d^2}-\frac{(2b)\int\sin(c+dx)\,dx}{d^2}\\
&= \frac{2b\cos(c+dx)}{d^3}-\frac{ax\cos(c+dx)}{d}-\frac{bx^2\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d^2}+\frac{2bx\sin(c+dx)}{d^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int x(a+bx)\sin(c+dx)\,dx=\frac{-((ad^2x+b(-2+d^2x^2))\cos(c+dx))+d(a+2bx)\sin(c+dx)}{d^3}$$

```
[In] Integrate[x*(a + b*x)*Sin[c + d*x],x]
[Out] (-((a*d^2*x + b*(-2 + d^2*x^2))*Cos[c + d*x]) + d*(a + 2*b*x)*Sin[c + d*x])
/d^3
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{(d^2x^2b+a\,d^2x-2b)\cos(dx+c)}{d^3}+\frac{(2bx+a)\sin(dx+c)}{d^2}$
parallelrisch	$\frac{(x(bx+a)d^2-4b)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2d(2bx+a)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-x(bx+a)d^2}{d^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$
parts	$-\frac{b\,x^2\cos(dx+c)}{d}-\frac{ax\cos(dx+c)}{d}+\frac{a\sin(dx+c)-\frac{2bc\sin(dx+c)}{d}+\frac{2b(\cos(dx+c)+(dx+c)\sin(dx+c))}{d}}{d^2}$
norman	$\frac{\frac{4b}{d^3}+\frac{ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{b\,x^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^2}-\frac{ax}{d}-\frac{b\,x^2}{d}+\frac{4bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^2}}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$
derivativedivides	$\frac{ac\cos(dx+c)+a(\sin(dx+c)-\cos(dx+c)(dx+c))-\frac{b\,c^2\cos(dx+c)}{d}-\frac{2bc(\sin(dx+c)-\cos(dx+c)(dx+c))}{d}+\frac{b\left(-(dx+c)^2\cos(dx+c)+\right.}{}}{d^2}$
default	$\frac{ac\cos(dx+c)+a(\sin(dx+c)-\cos(dx+c)(dx+c))-\frac{b\,c^2\cos(dx+c)}{d}-\frac{2bc(\sin(dx+c)-\cos(dx+c)(dx+c))}{d}+\frac{b\left(-(dx+c)^2\cos(dx+c)+\right.}{}}{d^2}$
meijerg	$\frac{4b\sqrt{\pi}\,\sin(c)\left(\frac{x\left(d^2\right)^{\frac{3}{2}}\cos(dx)}{2\sqrt{\pi}\,d^2}-\frac{\left(d^2\right)^{\frac{3}{2}}\left(-\frac{3d^2x^2}{2}+3\right)\sin(dx)}{6\sqrt{\pi}\,d^3}\right)}{d^2\sqrt{d^2}}+\frac{4b\sqrt{\pi}\,\cos(c)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(-\frac{d^2x^2}{2}+1\right)\cos(dx)}{2\sqrt{\pi}}+\frac{dx\sin(dx)}{2\sqrt{\pi}}\right.}{d^3}$

[In] int(x*(b*x+a)*sin(d*x+c),x,method=_RETURNVERBOSE)

[Out] -(b*d^2*x^2+a*d^2*x-2*b)/d^3*cos(d*x+c)+1/d^2*(2*b*x+a)*sin(d*x+c)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int x(a+bx)\sin(c+dx)\,dx=-\frac{(bd^2x^2+ad^2x-2\,b)\cos(dx+c)-(2\,bdx+ad)\sin(dx+c)}{d^3}$$

[In] integrate(x*(b*x+a)*sin(d*x+c),x, algorithm="fricas")

[Out] -((b*d^2*x^2 + a*d^2*x - 2*b)*cos(d*x + c) - (2*b*d*x + a*d)*sin(d*x + c))/
d^3

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.26

$$\int x(a+bx)\sin(c+dx)\,dx$$
$$=\begin{cases}-\frac{ax\cos{(c+dx)}}{d}+\frac{a\sin{(c+dx)}}{d^2}-\frac{bx^2\cos{(c+dx)}}{d}+\frac{2bx\sin{(c+dx)}}{d^2}+\frac{2b\cos{(c+dx)}}{d^3} & \text{for } d\neq 0\\ \left(\frac{ax^2}{2}+\frac{bx^3}{3}\right)\sin{(c)} & \text{otherwise}\end{cases}$$

```
[In] integrate(x*(b*x+a)*sin(d*x+c),x)
```

```
[Out] Piecewise((-a*x*cos(c + d*x)/d + a*sin(c + d*x)/d**2 - b*x**2*cos(c + d*x)/
d + 2*b*x*sin(c + d*x)/d**2 + 2*b*cos(c + d*x)/d**3, Ne(d, 0)), ((a*x**2/2
+ b*x**3/3)*sin(c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.80

$$\int x(a+bx)\sin(c+dx)\,dx$$
$$=\frac{ac\cos{(dx+c)}-\frac{bc^2\cos(dx+c)}{d}-((dx+c)\cos{(dx+c)}-\sin{(dx+c)})a+\frac{2((dx+c)\cos(dx+c)-\sin(dx+c))bc}{d}-\frac{(((dx\cdots}{}}{d^2}$$

```
[In] integrate(x*(b*x+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] (a*c*cos(d*x + c) - b*c^2*cos(d*x + c)/d - ((d*x + c)*cos(d*x + c) - sin(d*
x + c))*a + 2*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c/d - (((d*x + c)^2
 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b/d)/d^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

$$\int x(a+bx)\sin(c+dx)\,dx=-\frac{(bd^2x^2+ad^2x-2\,b)\cos{(dx+c)}}{d^3}+\frac{(2\,bdx+ad)\sin{(dx+c)}}{d^3}$$

```
[In] integrate(x*(b*x+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b*d^2*x^2 + a*d^2*x - 2*b)*cos(d*x + c)/d^3 + (2*b*d*x + a*d)*sin(d*x + c
)/d^3
```

Mupad [B] (verification not implemented)

Time = 6.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

$$\int x(a+bx)\sin(c+dx)\,dx = \frac{a\,\sin\left(c+d\,x\right)+2\,b\,x\,\sin\left(c+d\,x\right)}{d^2} - \frac{a\,x\,\cos\left(c+d\,x\right)+b\,x^2\,\cos\left(c+d\,x\right)}{d} + \frac{2\,b\,\cos\left(c+d\,x\right)}{d^3}$$

```
[In] int(x*sin(c + d*x)*(a + b*x),x)
```

```
[Out] (a*sin(c + d*x) + 2*b*x*sin(c + d*x))/d^2 - (a*x*cos(c + d*x) + b*x^2*cos(c
 + d*x))/d + (2*b*cos(c + d*x))/d^3
```

3.4 $\int (a+bx)\sin(c+dx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 28

$$\int (a+bx)\sin(c+dx)\,dx = -\frac{(a+bx)\cos(c+dx)}{d} + \frac{b\sin(c+dx)}{d^2}$$

```
[Out] -(b*x+a)*cos(d*x+c)/d+b*sin(d*x+c)/d^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3377, 2717}

$$\int (a+bx)\sin(c+dx)\,dx = \frac{b\sin(c+dx)}{d^2} - \frac{(a+bx)\cos(c+dx)}{d}$$

```
[In] Int[(a + b*x)*Sin[c + d*x],x]
```

```
[Out] -(((a + b*x)*Cos[c + d*x])/d) + (b*Sin[c + d*x])/d^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(a+bx)\cos(c+dx)}{d}+\frac{b\int\cos(c+dx)\,dx}{d} \\
&= -\frac{(a+bx)\cos(c+dx)}{d}+\frac{b\sin(c+dx)}{d^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int(a+bx)\sin(c+dx)\,dx=\frac{-d(a+bx)\cos(c+dx)+b\sin(c+dx)}{d^2}$$

```
[In] Integrate[(a + b*x)*Sin[c + d*x],x]
[Out] (-(d*(a + b*x)*Cos[c + d*x]) + b*Sin[c + d*x])/d^2
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result
risch	$-\frac{(bx+a)\cos(dx+c)}{d}+\frac{b\sin(dx+c)}{d^2}$
parallelrisch	$\frac{-(bx+a)d\cos(dx+c)+da+b\sin(dx+c)}{d^2}$
parts	$-\frac{bx\cos(dx+c)}{d}-\frac{a\cos(dx+c)}{d}+\frac{b\sin(dx+c)}{d^2}$
derivativedivides	$\frac{-a\cos(dx+c)+\frac{bc\cos(dx+c)}{d}+\frac{b(\sin(dx+c)-\cos(dx+c)(dx+c))}{d}}{d}$
default	$\frac{-a\cos(dx+c)+\frac{bc\cos(dx+c)}{d}+\frac{b(\sin(dx+c)-\cos(dx+c)(dx+c))}{d}}{d}$
norman	$\frac{\frac{2a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{bx\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^2}-\frac{bx}{d}}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$
meijerg	$\frac{2b\sqrt{\pi}\,\sin(c)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(dx)}{2\sqrt{\pi}}+\frac{dx\sin(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{2b\sqrt{\pi}\,\cos(c)\left(-\frac{dx\cos(dx)}{2\sqrt{\pi}}+\frac{\sin(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{a\sin(c)\sin(dx)}{d}+\frac{a\sqrt{\pi}\,\cos(c)(}{}$

```
[In] int((b*x+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
[Out] -(b*x+a)*cos(d*x+c)/d+b*sin(d*x+c)/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int (a+bx)\sin(c+dx)\,dx = -\frac{(bdx+ad)\cos(dx+c)-b\sin(dx+c)}{d^2}$$

```
[In] integrate((b*x+a)*sin(d*x+c),x, algorithm="fricas")
[Out] -((b*d*x + a*d)*cos(d*x + c) - b*sin(d*x + c))/d^2
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int (a+bx)\sin(c+dx)\,dx = \begin{cases} -\frac{a\cos(c+dx)}{d} - \frac{bx\cos(c+dx)}{d} + \frac{b\sin(c+dx)}{d^2} & \text{for } d \neq 0 \\ \left(ax + \frac{bx^2}{2}\right)\sin(c) & \text{otherwise} \end{cases}$$

```
[In] integrate((b*x+a)*sin(d*x+c),x)
[Out] Piecewise((-a*cos(c + d*x)/d - b*x*cos(c + d*x)/d + b*sin(c + d*x)/d**2, Ne
(d, 0)), ((a*x + b*x**2/2)*sin(c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.89

$$\int (a+bx)\sin(c+dx)\,dx = -\frac{a\cos(dx+c) - \frac{bc\cos(dx+c)}{d} + \frac{((dx+c)\cos(dx+c)-\sin(dx+c))b}{d}}{d}$$

```
[In] integrate((b*x+a)*sin(d*x+c),x, algorithm="maxima")
[Out] -(a*cos(d*x + c) - b*c*cos(d*x + c)/d + ((d*x + c)*cos(d*x + c) - sin(d*x +
 c))*b/d)/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int (a+bx)\sin(c+dx)\,dx = -\frac{(bdx+ad)\cos(dx+c)}{d^2} + \frac{b\sin(dx+c)}{d^2}$$

```
[In] integrate((b*x+a)*sin(d*x+c),x, algorithm="giac")
[Out] -(b*d*x + a*d)*cos(d*x + c)/d^2 + b*sin(d*x + c)/d^2
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int (a+bx)\sin(c+dx)\,dx = \frac{b\,\sin(c+d\,x)}{d^2} - \frac{a\,\cos(c+d\,x)+b\,x\,\cos(c+d\,x)}{d}$$

```
[In] int(sin(c + d*x)*(a + b*x),x)
[Out] (b*sin(c + d*x))/d^2 - (a*cos(c + d*x) + b*x*cos(c + d*x))/d
```

3.5 $\int \frac{(a+bx)\sin(c+dx)}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 29

$$\int \frac{(a+bx)\sin(c+dx)}{x}\,dx = -\frac{b\cos(c+dx)}{d} + a\,\text{CosIntegral}(dx)\sin(c) + a\cos(c)\text{Si}(dx)$$

```
[Out] -b*cos(d*x+c)/d+a*cos(c)*Si(d*x)+a*Ci(d*x)*sin(c)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6874, 2718, 3384, 3380, 3383}

$$\int \frac{(a+bx)\sin(c+dx)}{x}\,dx = a\sin(c)\,\text{CosIntegral}(dx) + a\cos(c)\text{Si}(dx) - \frac{b\cos(c+dx)}{d}$$

```
[In] Int[((a + b*x)*Sin[c + d*x])/x,x]
```

```
[Out] -((b*Cos[c + d*x])/d) + a*CosIntegral[d*x]*Sin[c] + a*Cos[c]*SinIntegral[d*
x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(b\sin(c+dx) + \frac{a\sin(c+dx)}{x}\right) dx \\
&= a\int \frac{\sin(c+dx)}{x}\,dx + b\int \sin(c+dx)\,dx \\
&= -\frac{b\cos(c+dx)}{d} + (a\cos(c))\int \frac{\sin(dx)}{x}\,dx + (a\sin(c))\int \frac{\cos(dx)}{x}\,dx \\
&= -\frac{b\cos(c+dx)}{d} + a\,\text{CosIntegral}(dx)\sin(c) + a\cos(c)\text{Si}(dx)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.38

$$\int \frac{(a+bx)\sin(c+dx)}{x}\,dx = -\frac{b\cos(c)\cos(dx)}{d} + a\,\text{CosIntegral}(dx)\sin(c) + \frac{b\sin(c)\sin(dx)}{d} + a\cos(c)\text{Si}(dx)$$

```
[In] Integrate[((a + b*x)*Sin[c + d*x])/x,x]
[Out] -((b*Cos[c]*Cos[d*x])/d) + a*CosIntegral[d*x]*Sin[c] + (b*Sin[c]*Sin[d*x])/
d + a*Cos[c]*SinIntegral[d*x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

method	result
derivativedivides	$a(\operatorname{Si}(dx)\cos(c)+\operatorname{Ci}(dx)\sin(c))-\frac{b\cos(dx+c)}{d}$
default	$a(\operatorname{Si}(dx)\cos(c)+\operatorname{Ci}(dx)\sin(c))-\frac{b\cos(dx+c)}{d}$
risch	$\frac{ia\,\mathrm{e}^{ic}\operatorname{Ei}_1(-idx)}{2}-\frac{\pi\ \operatorname{csgn}(dx)\mathrm{e}^{-ic}a}{2}+\operatorname{Si}(dx)\,\mathrm{e}^{-ic}a-\frac{i\mathrm{e}^{-ic}\operatorname{Ei}_1(-idx)a}{2}-\frac{b\cos(dx+c)}{d}$
meijerg	$\frac{b\sin(c)\sin(dx)}{d}+\frac{b\sqrt{\pi}\ \cos(c)\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(dx)}{\sqrt{\pi}}\right)}{d}+\frac{a\sqrt{\pi}\ \sin(c)\left(\frac{2\gamma+2\ln(x)+\ln\left(d^2\right)}{\sqrt{\pi}}-\frac{2\gamma}{\sqrt{\pi}}-\frac{2\ln(2)}{\sqrt{\pi}}-\frac{2\ln\left(\frac{dx}{2}\right)}{\sqrt{\pi}}+\frac{2\ \operatorname{Ci}(dx)}{\sqrt{\pi}}\right)}{2}+$

```
[In] int((b*x+a)*sin(d*x+c)/x,x,method=_RETURNVERBOSE)
```

```
[Out] a*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))-b*cos(d*x+c)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx)\sin(c+dx)}{x}\,dx=\frac{ad\operatorname{Ci}(dx)\sin(c)+ad\cos(c)\operatorname{Si}(dx)-b\cos(dx+c)}{d}$$

```
[In] integrate((b*x+a)*sin(d*x+c)/x,x, algorithm="fricas")
```

```
[Out] (a*d*cos_integral(d*x)*sin(c) + a*d*cos(c)*sin_integral(d*x) - b*cos(d*x +
c))/d
```

Sympy [A] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx)\sin(c+dx)}{x}\,dx=-a(-\sin(c)\operatorname{Ci}(dx)-\cos(c)\operatorname{Si}(dx))-b\left(\begin{cases}-x\sin(c) & \text{for } d=0\\ \frac{\cos(c+dx)}{d} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((b*x+a)*sin(d*x+c)/x,x)
```

```
[Out] -a*(-sin(c)*Ci(d*x) - cos(c)*Si(d*x)) - b*Piecewise((-x*sin(c), Eq(d, 0)),
(cos(c + d*x)/d, True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 522, normalized size of antiderivative = 18.00

$$\int \frac{(a+bx)\sin(c+dx)}{x}\,dx$$
$$= -\frac{1}{2}\left((i\,E_1(i\,dx) - i\,E_1(-i\,dx))\cos(c) + (E_1(i\,dx) + E_1(-i\,dx))\sin(c)\right)a$$
$$+ \frac{((i\,E_1(i\,dx) - i\,E_1(-i\,dx))\cos(c) + (E_1(i\,dx) + E_1(-i\,dx))\sin(c))bc}{2\,d}$$
$$- \frac{\left(2\,(dx+c)\left(\cos(c)^2 + \sin(c)^2\right)\cos(dx+c)^3 + 2\,(dx+c)\left(\cos(c)^2 + \sin(c)^2\right)\cos(dx+c) - (c(E_2(i\,dx)\right.}{}$$

```
[In] integrate((b*x+a)*sin(d*x+c)/x,x, algorithm="maxima")
```

```
[Out] -1/2*((I*exp_integral_e(1, I*d*x) - I*exp_integral_e(1, -I*d*x))*cos(c) + (
exp_integral_e(1, I*d*x) + exp_integral_e(1, -I*d*x))*sin(c))*a + 1/2*((I*e
xp_integral_e(1, I*d*x) - I*exp_integral_e(1, -I*d*x))*cos(c) + (exp_integr
al_e(1, I*d*x) + exp_integral_e(1, -I*d*x))*sin(c))*b*c/d - 1/4*(2*(d*x + c
)*(cos(c)^2 + sin(c)^2)*cos(d*x + c)^3 + 2*(d*x + c)*(cos(c)^2 + sin(c)^2)*
cos(d*x + c) - (c*(exp_integral_e(2, I*d*x) + exp_integral_e(2, -I*d*x))*co
s(c)^3 + c*(exp_integral_e(2, I*d*x) + exp_integral_e(2, -I*d*x))*cos(c)*si
n(c)^2 - c*(I*exp_integral_e(2, I*d*x) - I*exp_integral_e(2, -I*d*x))*sin(c
)^3 + c*(exp_integral_e(2, I*d*x) + exp_integral_e(2, -I*d*x))*cos(c) - (c*
(I*exp_integral_e(2, I*d*x) - I*exp_integral_e(2, -I*d*x))*cos(c)^2 + c*(I*
exp_integral_e(2, I*d*x) - I*exp_integral_e(2, -I*d*x)))*sin(c))*cos(d*x +
c)^2 - (c*(exp_integral_e(2, I*d*x) + exp_integral_e(2, -I*d*x))*cos(c)^3 +
 c*(exp_integral_e(2, I*d*x) + exp_integral_e(2, -I*d*x))*cos(c)*sin(c)^2 -
 c*(I*exp_integral_e(2, I*d*x) - I*exp_integral_e(2, -I*d*x))*sin(c)^3 - 2*
(d*x + c)*(cos(c)^2 + sin(c)^2)*cos(d*x + c) + c*(exp_integral_e(2, I*d*x)
+ exp_integral_e(2, -I*d*x))*cos(c) - (c*(I*exp_integral_e(2, I*d*x) - I*ex
p_integral_e(2, -I*d*x))*cos(c)^2 + c*(I*exp_integral_e(2, I*d*x) - I*exp_i
ntegral_e(2, -I*d*x)))*sin(c))*sin(d*x + c)^2)*b/(((d*x + c)*(cos(c)^2 + si
n(c)^2)*d - (c*cos(c)^2 + c*sin(c)^2)*d)*cos(d*x + c)^2 + ((d*x + c)*(cos(c
)^2 + sin(c)^2)*d - (c*cos(c)^2 + c*sin(c)^2)*d)*sin(d*x + c)^2)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.29 (sec) , antiderivative size = 339, normalized size of antiderivative = 11.69

$$\int \frac{(a+bx)\sin(c+dx)}{x}\,dx =$$

$$-\frac{ad\Im(\mathrm{Ci}\,(dx))\tan\left(\frac{1}{2}\,dx\right)^2\tan\left(\frac{1}{2}\,c\right)^2 - ad\Im(\mathrm{Ci}\,(-dx))\tan\left(\frac{1}{2}\,dx\right)^2\tan\left(\frac{1}{2}\,c\right)^2 + 2\,ad\,\mathrm{Si}\,(dx)\tan\left(\frac{1}{2}\,dx\right)^2\tan$$

```
[In] integrate((b*x+a)*sin(d*x+c)/x,x, algorithm="giac")
```

```
[Out] -1/2*(a*d*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d*im
ag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d*sin_integra
l(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*d*real_part(cos_integral(d*x))*tan
(1/2*d*x)^2*tan(1/2*c) - 2*a*d*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2
*tan(1/2*c) - a*d*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 + a*d*imag_pa
rt(cos_integral(-d*x))*tan(1/2*d*x)^2 - 2*a*d*sin_integral(d*x)*tan(1/2*d*x
)^2 + a*d*imag_part(cos_integral(d*x))*tan(1/2*c)^2 - a*d*imag_part(cos_int
egral(-d*x))*tan(1/2*c)^2 + 2*a*d*sin_integral(d*x)*tan(1/2*c)^2 + 2*b*tan(
1/2*d*x)^2*tan(1/2*c)^2 - 2*a*d*real_part(cos_integral(d*x))*tan(1/2*c) - 2
*a*d*real_part(cos_integral(-d*x))*tan(1/2*c) - a*d*imag_part(cos_integral(
d*x)) + a*d*imag_part(cos_integral(-d*x)) - 2*a*d*sin_integral(d*x) - 2*b*t
an(1/2*d*x)^2 - 8*b*tan(1/2*d*x)*tan(1/2*c) - 2*b*tan(1/2*c)^2 + 2*b)/(d*ta
n(1/2*d*x)^2*tan(1/2*c)^2 + d*tan(1/2*d*x)^2 + d*tan(1/2*c)^2 + d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)\sin(c+dx)}{x}\,dx = a\,\mathrm{cosint}(d\,x)\,\sin{(c)} + a\,\mathrm{sinint}(d\,x)\,\cos{(c)} - \frac{b\,\cos{(c+d\,x)}}{d}$$

```
[In] int((sin(c + d*x)*(a + b*x))/x,x)
```

```
[Out] a*cosint(d*x)*sin(c) + a*sinint(d*x)*cos(c) - (b*cos(c + d*x))/d
```

3.6 $\int \frac{(a+bx)\sin(c+dx)}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 48

$$\int \frac{(a+bx)\sin(c+dx)}{x^2}\,dx = ad\cos(c)\operatorname{CosIntegral}(dx) + b\operatorname{CosIntegral}(dx)\sin(c) - \frac{a\sin(c+dx)}{x} + b\cos(c)\operatorname{Si}(dx) - ad\sin(c)\operatorname{Si}(dx)$$

```
[Out] a*d*Ci(d*x)*cos(c)+b*cos(c)*Si(d*x)+b*Ci(d*x)*sin(c)-a*d*Si(d*x)*sin(c)-a*s
in(d*x+c)/x
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6874, 3378, 3384, 3380, 3383}

$$\int \frac{(a+bx)\sin(c+dx)}{x^2}\,dx = ad\cos(c)\operatorname{CosIntegral}(dx) - ad\sin(c)\operatorname{Si}(dx) - \frac{a\sin(c+dx)}{x} + b\sin(c)\operatorname{CosIntegral}(dx) + b\cos(c)\operatorname{Si}(dx)$$

```
[In] Int[((a + b*x)*Sin[c + d*x])/x^2,x]
```

```
[Out] a*d*Cos[c]*CosIntegral[d*x] + b*CosIntegral[d*x]*Sin[c] - (a*Sin[c + d*x])/
x + b*Cos[c]*SinIntegral[d*x] - a*d*Sin[c]*SinIntegral[d*x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

```
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a\sin(c+dx)}{x^2}+\frac{b\sin(c+dx)}{x}\right)dx\\
&= a\int \frac{\sin(c+dx)}{x^2}\,dx+b\int \frac{\sin(c+dx)}{x}\,dx\\
&= -\frac{a\sin(c+dx)}{x}+(ad)\int \frac{\cos(c+dx)}{x}\,dx+(b\cos(c))\int \frac{\sin(dx)}{x}\,dx+(b\sin(c))\int \frac{\cos(dx)}{x}\,dx\\
&= b\,\text{CosIntegral}(dx)\sin(c)-\frac{a\sin(c+dx)}{x}+b\cos(c)\text{Si}(dx)\\
&\quad+(ad\cos(c))\int \frac{\cos(dx)}{x}\,dx-(ad\sin(c))\int \frac{\sin(dx)}{x}\,dx\\
&= ad\cos(c)\,\text{CosIntegral}(dx)+b\,\text{CosIntegral}(dx)\sin(c)\\
&\quad-\frac{a\sin(c+dx)}{x}+b\cos(c)\text{Si}(dx)-ad\sin(c)\text{Si}(dx)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx)\sin(c+dx)}{x^2}\,dx = -\frac{a\cos(dx)\sin(c)}{x} + b\,\text{CosIntegral}(dx)\sin(c) - \frac{a\cos(c)\sin(dx)}{x} + b\cos(c)\text{Si}(dx) + ad(\cos(c)\,\text{CosIntegral}(dx) - \sin(c)\text{Si}(dx))$$

```
[In] Integrate[((a + b*x)*Sin[c + d*x])/x^2,x]
[Out] -((a*Cos[d*x]*Sin[c])/x) + b*CosIntegral[d*x]*Sin[c] - (a*Cos[c]*Sin[d*x])/
x + b*Cos[c]*SinIntegral[d*x] + a*d*(Cos[c]*CosIntegral[d*x] - Sin[c]*SinIn
tegral[d*x])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.17

method	result
derivativedivides	$d\left(a\left(-\frac{\sin(dx+c)}{dx}-\text{Si}\left(dx\right)\sin\left(c\right)+\text{Ci}\left(dx\right)\cos\left(c\right)\right)+\frac{b(\text{Si}(dx)\cos(c)+\text{Ci}(dx)\sin(c))}{d}\right)$
default	$d\left(a\left(-\frac{\sin(dx+c)}{dx}-\text{Si}\left(dx\right)\sin\left(c\right)+\text{Ci}\left(dx\right)\cos\left(c\right)\right)+\frac{b(\text{Si}(dx)\cos(c)+\text{Ci}(dx)\sin(c))}{d}\right)$
risch	$\frac{i e^{ic}\,\text{Ei}_1(-idx)b}{2}-\frac{a\,e^{ic}\,\text{Ei}_1(-idx)d}{2}-\frac{i e^{-ic}\,\text{Ei}_1(idx)b}{2}-\frac{a\,e^{-ic}\,\text{Ei}_1(idx)d}{2}-\frac{a\sin(dx+c)}{x}$
meijerg	$\frac{b\sqrt{\pi}\,\sin(c)\left(\frac{2\gamma+2\ln(x)+\ln\left(d^2\right)}{\sqrt{\pi}}-\frac{2\gamma}{\sqrt{\pi}}-\frac{2\ln(2)}{\sqrt{\pi}}-\frac{2\ln\left(\frac{dx}{2}\right)}{\sqrt{\pi}}+\frac{2\,\text{Ci}(dx)}{\sqrt{\pi}}\right)}{2}+b\cos\left(c\right)\text{Si}\left(dx\right)+\frac{a\sqrt{\pi}\,\sin(c)d^2\left(-\frac{4d^2 c\cdots}{x(c\cdots}\right.}{4\cdots}$

```
[In] int((b*x+a)*sin(d*x+c)/x^2,x,method=_RETURNVERBOSE)
[Out] d*(a*(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c))+1/d*b*(Si(d*x)*cos(c)+
Ci(d*x)*sin(c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)\sin(c+dx)}{x^2}\,dx = \frac{(adx\,\text{Ci}\left(dx\right)+bx\,\text{Si}\left(dx\right))\cos\left(c\right)-a\sin\left(dx+c\right)-(adx\,\text{Si}\left(dx\right)-bx\,\text{Ci}\left(dx\right))\sin\left(c\right)}{x}$$

```
[In] integrate((b*x+a)*sin(d*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] ((a*d*x*cos_integral(d*x) + b*x*sin_integral(d*x))*cos(c) - a*sin(d*x + c)
- (a*d*x*sin_integral(d*x) - b*x*cos_integral(d*x))*sin(c))/x
```

Sympy [F]

$$\int \frac{(a+bx)\sin(c+dx)}{x^2}\,dx = \int \frac{(a+bx)\sin{(c+dx)}}{x^2}\,dx$$

```
[In] integrate((b*x+a)*sin(d*x+c)/x**2,x)
```

```
[Out] Integral((a + b*x)*sin(c + d*x)/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.25

$$\int \frac{(a+bx)\sin(c+dx)}{x^2}\,dx$$
$$= \frac{((a(\Gamma(-1,i\,dx)+\Gamma(-1,-i\,dx))\cos{(c)}-a(i\,\Gamma(-1,i\,dx)-i\,\Gamma(-1,-i\,dx))\sin{(c)})d^2-(b(-i\,\Gamma(-1,i\,dx)+}{2\,dx}$$

```
[In] integrate((b*x+a)*sin(d*x+c)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(((a*(gamma(-1, I*d*x) + gamma(-1, -I*d*x))*cos(c) - a*(I*gamma(-1, I*d
*x) - I*gamma(-1, -I*d*x))*sin(c))*d^2 - (b*(-I*gamma(-1, I*d*x) + I*gamma(
-1, -I*d*x))*cos(c) - b*(gamma(-1, I*d*x) + gamma(-1, -I*d*x))*sin(c))*d)*x
 - 2*b*cos(d*x + c))/(d*x)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.30 (sec) , antiderivative size = 569, normalized size of antiderivative = 11.85

$$\int \frac{(a+bx)\sin(c+dx)}{x^2}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)*sin(d*x+c)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(a*d*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + a*d*
x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d*x*imag_
part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d*x*imag_part(cos_i
ntegral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a*d*x*sin_integral(d*x)*tan(1/
```

```
2*d*x)^2*tan(1/2*c) + b*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1
/2*c)^2 - b*x*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2
*b*x*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d*x*real_part(cos_in
tegral(d*x))*tan(1/2*d*x)^2 - a*d*x*real_part(cos_integral(-d*x))*tan(1/2*d
*x)^2 - 2*b*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*b*
x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + a*d*x*real_part
(cos_integral(d*x))*tan(1/2*c)^2 + a*d*x*real_part(cos_integral(-d*x))*tan(
1/2*c)^2 - b*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 + b*x*imag_part(
cos_integral(-d*x))*tan(1/2*d*x)^2 - 2*b*x*sin_integral(d*x)*tan(1/2*d*x)^2
 + 2*a*d*x*imag_part(cos_integral(d*x))*tan(1/2*c) - 2*a*d*x*imag_part(cos_
integral(-d*x))*tan(1/2*c) + 4*a*d*x*sin_integral(d*x)*tan(1/2*c) + b*x*ima
g_part(cos_integral(d*x))*tan(1/2*c)^2 - b*x*imag_part(cos_integral(-d*x))*
tan(1/2*c)^2 + 2*b*x*sin_integral(d*x)*tan(1/2*c)^2 - a*d*x*real_part(cos_i
ntegral(d*x)) - a*d*x*real_part(cos_integral(-d*x)) - 2*b*x*real_part(cos_i
ntegral(d*x))*tan(1/2*c) - 2*b*x*real_part(cos_integral(-d*x))*tan(1/2*c) -
 4*a*tan(1/2*d*x)^2*tan(1/2*c) - 4*a*tan(1/2*d*x)*tan(1/2*c)^2 - b*x*imag_p
art(cos_integral(d*x)) + b*x*imag_part(cos_integral(-d*x)) - 2*b*x*sin_inte
gral(d*x) + 4*a*tan(1/2*d*x) + 4*a*tan(1/2*c))/(x*tan(1/2*d*x)^2*tan(1/2*c)
^2 + x*tan(1/2*d*x)^2 + x*tan(1/2*c)^2 + x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)\sin(c+dx)}{x^2}\,dx = \int \frac{\sin\left(c+d\,x\right)\,\left(a+b\,x\right)}{x^2}\,dx$$

```
[In] int((sin(c + d*x)*(a + b*x))/x^2,x)
[Out] int((sin(c + d*x)*(a + b*x))/x^2, x)
```

3.7 $\int \frac{(a+bx)\sin(c+dx)}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 89

$$\int \frac{(a+bx)\sin(c+dx)}{x^3}\,dx = -\frac{ad\cos(c+dx)}{2x} + bd\cos(c)\,\text{CosIntegral}(dx) - \frac{1}{2}ad^2\,\text{CosIntegral}(dx)\sin(c) - \frac{a\sin(c+dx)}{2x^2} - \frac{b\sin(c+dx)}{x} - \frac{1}{2}ad^2\cos(c)\text{Si}(dx) - bd\sin(c)\text{Si}(dx)$$

```
[Out] b*d*Ci(d*x)*cos(c)-1/2*a*d*cos(d*x+c)/x-1/2*a*d^2*cos(c)*Si(d*x)-1/2*a*d^2*
Ci(d*x)*sin(c)-b*d*Si(d*x)*sin(c)-1/2*a*sin(d*x+c)/x^2-b*sin(d*x+c)/x
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6874, 3378, 3384, 3380, 3383}

$$\int \frac{(a+bx)\sin(c+dx)}{x^3}\,dx = -\frac{1}{2}ad^2\sin(c)\,\text{CosIntegral}(dx) - \frac{1}{2}ad^2\cos(c)\text{Si}(dx) - \frac{a\sin(c+dx)}{2x^2} - \frac{ad\cos(c+dx)}{2x} + bd\cos(c)\,\text{CosIntegral}(dx) - bd\sin(c)\text{Si}(dx) - \frac{b\sin(c+dx)}{x}$$

```
[In] Int[((a + b*x)*Sin[c + d*x])/x^3,x]
```

```
[Out] -1/2*(a*d*Cos[c + d*x])/x + b*d*Cos[c]*CosIntegral[d*x] - (a*d^2*CosIntegra
l[d*x]*Sin[c])/2 - (a*Sin[c + d*x])/(2*x^2) - (b*Sin[c + d*x])/x - (a*d^2*C
os[c]*SinIntegral[d*x])/2 - b*d*Sin[c]*SinIntegral[d*x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a\sin(c+dx)}{x^3}+\frac{b\sin(c+dx)}{x^2}\right)dx \\
&= a\int \frac{\sin(c+dx)}{x^3}\,dx+b\int \frac{\sin(c+dx)}{x^2}\,dx \\
&= -\frac{a\sin(c+dx)}{2x^2}-\frac{b\sin(c+dx)}{x}+\frac{1}{2}(ad)\int \frac{\cos(c+dx)}{x^2}\,dx+(bd)\int \frac{\cos(c+dx)}{x}\,dx \\
&= -\frac{ad\cos(c+dx)}{2x}-\frac{a\sin(c+dx)}{2x^2}-\frac{b\sin(c+dx)}{x}-\frac{1}{2}\left(ad^2\right)\int \frac{\sin(c+dx)}{x}\,dx \\
&\quad +(bd\cos(c))\int \frac{\cos(dx)}{x}\,dx-(bd\sin(c))\int \frac{\sin(dx)}{x}\,dx
\end{aligned}
$$

$$= -\frac{ad\cos(c+dx)}{2x} + bd\cos(c)\,\text{CosIntegral}(dx) - \frac{a\sin(c+dx)}{2x^2} - \frac{b\sin(c+dx)}{x} - bd\sin(c)\text{Si}(dx) - \frac{1}{2}\left(ad^2\cos(c)\right)\int\frac{\sin(dx)}{x}\,dx - \frac{1}{2}\left(ad^2\sin(c)\right)\int\frac{\cos(dx)}{x}\,dx$$

$$= -\frac{ad\cos(c+dx)}{2x} + bd\cos(c)\,\text{CosIntegral}(dx) - \frac{1}{2}ad^2\,\text{CosIntegral}(dx)\sin(c) - \frac{a\sin(c+dx)}{2x^2} - \frac{b\sin(c+dx)}{x} - \frac{1}{2}ad^2\cos(c)\text{Si}(dx) - bd\sin(c)\text{Si}(dx)$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.85

$$\int\frac{(a+bx)\sin(c+dx)}{x^3}\,dx = -\frac{adx\cos(c+dx) + dx^2\,\text{CosIntegral}(dx)(-2b\cos(c)+ad\sin(c)) + a\sin(c+dx) + 2bx\sin(c+dx) + dx^2(\cdots}{2x^2}$$

```
[In] Integrate[((a + b*x)*Sin[c + d*x])/x^3,x]
```

```
[Out] -1/2*(a*d*x*Cos[c + d*x] + d*x^2*CosIntegral[d*x]*(-2*b*Cos[c] + a*d*Sin[c]
) + a*Sin[c + d*x] + 2*b*x*Sin[c + d*x] + d*x^2*(a*d*Cos[c] + 2*b*Sin[c])*S
inIntegral[d*x])/x^2
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99

method	result
derivativedivides	$d^2\left(a\left(-\frac{\sin(dx+c)}{2d^2x^2} - \frac{\cos(dx+c)}{2dx} - \frac{\text{Si}(dx)\cos(c)}{2} - \frac{\text{Ci}(dx)\sin(c)}{2}\right) + \frac{b\left(-\frac{\sin(dx+c)}{dx} - \text{Si}(dx)\sin(c) + \text{Ci}(dx)\cos(c\cdots\right.}{d}\right.$
default	$d^2\left(a\left(-\frac{\sin(dx+c)}{2d^2x^2} - \frac{\cos(dx+c)}{2dx} - \frac{\text{Si}(dx)\cos(c)}{2} - \frac{\text{Ci}(dx)\sin(c)}{2}\right) + \frac{b\left(-\frac{\sin(dx+c)}{dx} - \text{Si}(dx)\sin(c) + \text{Ci}(dx)\cos(c\cdots\right.}{d}\right.$
risch	$-\frac{\text{Ei}_1(-idx)\cos(c)bd}{2} - \frac{\cos(c)\,\text{Ei}_1(idx)bd}{2} - \frac{i\,\text{Ei}_1(-idx)\cos(c)a\,d^2}{4} + \frac{i\cos(c)\,\text{Ei}_1(idx)a\,d^2}{4} - \frac{i\,\text{Ei}_1(-idx)\sin(c)bd}{2}\cdots$
meijerg	$\frac{d^2b\sqrt{\pi}\,\sin(c)\left(-\frac{4d^2\cos\left(x\sqrt{d^2}\right)}{x\left(d^2\right)^{\frac{3}{2}}\sqrt{\pi}} - \frac{4\,\text{Si}\left(x\sqrt{d^2}\right)}{\sqrt{\pi}}\right)}{4\sqrt{d^2}} + \frac{db\sqrt{\pi}\,\cos(c)\left(\frac{4\gamma-4+4\ln(x)+4\ln(d)}{\sqrt{\pi}} + \frac{4}{\sqrt{\pi}} - \frac{4\gamma}{\sqrt{\pi}} - \frac{4\ln(2)}{\sqrt{\pi}} - \frac{4\ln\left(\frac{dx}{2}\right)}{\sqrt{\pi}} - \frac{4\,\text{si}\cdots}{\sqrt{\cdots}}\right.}{4}$

```
[In] int((b*x+a)*sin(d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^2*(a*(-1/2*sin(d*x+c)/d^2/x^2-1/2*cos(d*x+c)/d/x-1/2*Si(d*x)*cos(c)-1/2*C
i(d*x)*sin(c))+1/d*b*(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)\sin(c+dx)}{x^3}\,dx = -\frac{adx\cos(dx+c) + (ad^2x^2\operatorname{Si}(dx) - 2\,bdx^2\operatorname{Ci}(dx))\cos(c) + (2\,bx+a)\sin(dx+c) + (ad^2x^2\operatorname{Ci}(dx) + 2}{2\,x^2}$$

```
[In] integrate((b*x+a)*sin(d*x+c)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a*d*x*cos(d*x + c) + (a*d^2*x^2*sin_integral(d*x) - 2*b*d*x^2*cos_int
egral(d*x))*cos(c) + (2*b*x + a)*sin(d*x + c) + (a*d^2*x^2*cos_integral(d*x
) + 2*b*d*x^2*sin_integral(d*x))*sin(c))/x^2
```

Sympy [F]

$$\int \frac{(a+bx)\sin(c+dx)}{x^3}\,dx = \int \frac{(a+bx)\sin(c+dx)}{x^3}\,dx$$

```
[In] integrate((b*x+a)*sin(d*x+c)/x**3,x)
```

```
[Out] Integral((a + b*x)*sin(c + d*x)/x**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx)\sin(c+dx)}{x^3}\,dx = -\frac{((a(-i\,\Gamma(-2,i\,dx) + i\,\Gamma(-2,-i\,dx))\cos(c) - a(\Gamma(-2,i\,dx) + \Gamma(-2,-i\,dx))\sin(c))d^3 + 2\,(b(\Gamma(-2,i\,d}{2\,dx^2}$$

```
[In] integrate((b*x+a)*sin(d*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(((a*(-I*gamma(-2, I*d*x) + I*gamma(-2, -I*d*x))*cos(c) - a*(gamma(-2,
 I*d*x) + gamma(-2, -I*d*x))*sin(c))*d^3 + 2*(b*(gamma(-2, I*d*x) + gamma(-
2, -I*d*x))*cos(c) - b*(I*gamma(-2, I*d*x) - I*gamma(-2, -I*d*x))*sin(c))*d
^2)*x^2 + 2*b*cos(d*x + c))/(d*x^2)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.30 (sec) , antiderivative size = 796, normalized size of antiderivative = 8.94

$$\int \frac{(a+bx)\sin(c+dx)}{x^3} dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)*sin(d*x+c)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(a*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a
*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d^
2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*d^2*x^2*real_part
(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^2*x^2*real_part(cos_i
ntegral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*b*d*x^2*real_part(cos_integral
(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*b*d*x^2*real_part(cos_integral(-d*x)
)*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d^2*x^2*imag_part(cos_integral(d*x))*tan(
1/2*d*x)^2 + a*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 2*a*d
^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2 - 4*b*d*x^2*imag_part(cos_integral(
d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*b*d*x^2*imag_part(cos_integral(-d*x))*t
an(1/2*d*x)^2*tan(1/2*c) - 8*b*d*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1
/2*c) + a*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*c)^2 - a*d^2*x^2*ima
g_part(cos_integral(-d*x))*tan(1/2*c)^2 + 2*a*d^2*x^2*sin_integral(d*x)*tan
(1/2*c)^2 + 2*b*d*x^2*real_part(cos_integral(d*x))*tan(1/2*d*x)^2 + 2*b*d*x
^2*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 2*a*d^2*x^2*real_part(cos
_integral(d*x))*tan(1/2*c) - 2*a*d^2*x^2*real_part(cos_integral(-d*x))*tan(
1/2*c) - 2*b*d*x^2*real_part(cos_integral(d*x))*tan(1/2*c)^2 - 2*b*d*x^2*re
al_part(cos_integral(-d*x))*tan(1/2*c)^2 - 2*a*d*x*tan(1/2*d*x)^2*tan(1/2*c
)^2 - a*d^2*x^2*imag_part(cos_integral(d*x)) + a*d^2*x^2*imag_part(cos_inte
gral(-d*x)) - 2*a*d^2*x^2*sin_integral(d*x) - 4*b*d*x^2*imag_part(cos_integ
ral(d*x))*tan(1/2*c) + 4*b*d*x^2*imag_part(cos_integral(-d*x))*tan(1/2*c) -
 8*b*d*x^2*sin_integral(d*x)*tan(1/2*c) + 2*b*d*x^2*real_part(cos_integral(
d*x)) + 2*b*d*x^2*real_part(cos_integral(-d*x)) + 2*a*d*x*tan(1/2*d*x)^2 +
8*a*d*x*tan(1/2*d*x)*tan(1/2*c) + 8*b*x*tan(1/2*d*x)^2*tan(1/2*c) + 2*a*d*x
*tan(1/2*c)^2 + 8*b*x*tan(1/2*d*x)*tan(1/2*c)^2 + 4*a*tan(1/2*d*x)^2*tan(1/
2*c) + 4*a*tan(1/2*d*x)*tan(1/2*c)^2 - 2*a*d*x - 8*b*x*tan(1/2*d*x) - 8*b*x
*tan(1/2*c) - 4*a*tan(1/2*d*x) - 4*a*tan(1/2*c))/(x^2*tan(1/2*d*x)^2*tan(1/
2*c)^2 + x^2*tan(1/2*d*x)^2 + x^2*tan(1/2*c)^2 + x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)\sin(c+dx)}{x^3}\,dx = \int \frac{\sin\left(c+d\,x\right)\,\left(a+b\,x\right)}{x^3}\,dx$$

```
[In] int((sin(c + d*x)*(a + b*x))/x^3,x)
[Out] int((sin(c + d*x)*(a + b*x))/x^3, x)
```

3.8 $\int \frac{(a+bx)\sin(c+dx)}{x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 132

$$\int \frac{(a+bx)\sin(c+dx)}{x^4}\,dx = -\frac{ad\cos(c+dx)}{6x^2}-\frac{bd\cos(c+dx)}{2x}-\frac{1}{6}ad^3\cos(c)\,\text{CosIntegral}(dx) -\frac{1}{2}bd^2\,\text{CosIntegral}(dx)\sin(c)-\frac{a\sin(c+dx)}{3x^3}-\frac{b\sin(c+dx)}{2x^2} +\frac{ad^2\sin(c+dx)}{6x}-\frac{1}{2}bd^2\cos(c)\text{Si}(dx)+\frac{1}{6}ad^3\sin(c)\text{Si}(dx)$$

```
[Out] -1/6*a*d^3*Ci(d*x)*cos(c)-1/6*a*d*cos(d*x+c)/x^2-1/2*b*d*cos(d*x+c)/x-1/2*b
*d^2*cos(c)*Si(d*x)-1/2*b*d^2*Ci(d*x)*sin(c)+1/6*a*d^3*Si(d*x)*sin(c)-1/3*a
*sin(d*x+c)/x^3-1/2*b*sin(d*x+c)/x^2+1/6*a*d^2*sin(d*x+c)/x
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6874, 3378, 3384, 3380, 3383}

$$\int \frac{(a+bx)\sin(c+dx)}{x^4}\,dx = -\frac{1}{6}ad^3\cos(c)\,\text{CosIntegral}(dx)+\frac{1}{6}ad^3\sin(c)\text{Si}(dx) +\frac{ad^2\sin(c+dx)}{6x}-\frac{a\sin(c+dx)}{3x^3} -\frac{ad\cos(c+dx)}{6x^2}-\frac{1}{2}bd^2\sin(c)\,\text{CosIntegral}(dx) -\frac{1}{2}bd^2\cos(c)\text{Si}(dx)-\frac{b\sin(c+dx)}{2x^2}-\frac{bd\cos(c+dx)}{2x}$$

```
[In] Int[((a + b*x)*Sin[c + d*x])/x^4,x]
```

```
[Out] -1/6*(a*d*Cos[c + d*x])/x^2 - (b*d*Cos[c + d*x])/(2*x) - (a*d^3*Cos[c]*CosI
ntegral[d*x])/6 - (b*d^2*CosIntegral[d*x]*Sin[c])/2 - (a*Sin[c + d*x])/(3*x
^3) - (b*Sin[c + d*x])/(2*x^2) + (a*d^2*Sin[c + d*x])/(6*x) - (b*d^2*Cos[c]
*SinIntegral[d*x])/2 + (a*d^3*Sin[c]*SinIntegral[d*x])/6
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a\sin(c+dx)}{x^4}+\frac{b\sin(c+dx)}{x^3}\right) dx \\
&= a\int \frac{\sin(c+dx)}{x^4}\,dx+b\int \frac{\sin(c+dx)}{x^3}\,dx \\
&= -\frac{a\sin(c+dx)}{3x^3}-\frac{b\sin(c+dx)}{2x^2}+\frac{1}{3}(ad)\int \frac{\cos(c+dx)}{x^3}\,dx+\frac{1}{2}(bd)\int \frac{\cos(c+dx)}{x^2}\,dx \\
&= -\frac{ad\cos(c+dx)}{6x^2}-\frac{bd\cos(c+dx)}{2x}-\frac{a\sin(c+dx)}{3x^3}-\frac{b\sin(c+dx)}{2x^2} \\
&\quad -\frac{1}{6}\left(ad^2\right)\int \frac{\sin(c+dx)}{x^2}\,dx-\frac{1}{2}\left(bd^2\right)\int \frac{\sin(c+dx)}{x}\,dx
\end{aligned}
$$

$$= -\frac{ad\cos(c+dx)}{6x^2} - \frac{bd\cos(c+dx)}{2x} - \frac{a\sin(c+dx)}{3x^3} - \frac{b\sin(c+dx)}{2x^2} + \frac{ad^2\sin(c+dx)}{6x} - \frac{1}{6}(ad^3)\int\frac{\cos(c+dx)}{x}\,dx - \frac{1}{2}(bd^2\cos(c))\int\frac{\sin(dx)}{x}\,dx - \frac{1}{2}(bd^2\sin(c))\int\frac{\cos(dx)}{x}\,dx$$

$$= -\frac{ad\cos(c+dx)}{6x^2} - \frac{bd\cos(c+dx)}{2x} - \frac{1}{2}bd^2\,\mathrm{CosIntegral}(dx)\sin(c) - \frac{a\sin(c+dx)}{3x^3} - \frac{b\sin(c+dx)}{2x^2} + \frac{ad^2\sin(c+dx)}{6x} - \frac{1}{2}bd^2\cos(c)\mathrm{Si}(dx) - \frac{1}{6}(ad^3\cos(c))\int\frac{\cos(dx)}{x}\,dx + \frac{1}{6}(ad^3\sin(c))\int\frac{\sin(dx)}{x}\,dx$$

$$= -\frac{ad\cos(c+dx)}{6x^2} - \frac{bd\cos(c+dx)}{2x} - \frac{1}{6}ad^3\cos(c)\,\mathrm{CosIntegral}(dx) - \frac{1}{2}bd^2\,\mathrm{CosIntegral}(dx)\sin(c) - \frac{a\sin(c+dx)}{3x^3} - \frac{b\sin(c+dx)}{2x^2} + \frac{ad^2\sin(c+dx)}{6x} - \frac{1}{2}bd^2\cos(c)\mathrm{Si}(dx) + \frac{1}{6}ad^3\sin(c)\mathrm{Si}(dx)$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.83

$$\int\frac{(a+bx)\sin(c+dx)}{x^4}\,dx = -\frac{adx\cos(c+dx) + 3bdx^2\cos(c+dx) + d^2x^3\,\mathrm{CosIntegral}(dx)(ad\cos(c) + 3b\sin(c)) + 2a\sin(c+dx) + 3b}{6x^3}$$

```
[In] Integrate[((a + b*x)*Sin[c + d*x])/x^4,x]
```

```
[Out] -1/6*(a*d*x*Cos[c + d*x] + 3*b*d*x^2*Cos[c + d*x] + d^2*x^3*CosIntegral[d*x
]*(a*d*Cos[c] + 3*b*Sin[c]) + 2*a*Sin[c + d*x] + 3*b*x*Sin[c + d*x] - a*d^2
*x^2*Sin[c + d*x] + d^2*x^3*(3*b*Cos[c] - a*d*Sin[c])*SinIntegral[d*x])/x^3
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.89

method	result
derivativedivides	$d^3\left(a\left(-\frac{\sin(dx+c)}{3d^3x^3}-\frac{\cos(dx+c)}{6d^2x^2}+\frac{\sin(dx+c)}{6dx}+\frac{\mathrm{Si}(dx)\sin(c)}{6}-\frac{\mathrm{Ci}(dx)\cos(c)}{6}\right)+\frac{b\left(-\frac{\sin(dx+c)}{2d^2x^2}-\frac{\cos(dx+c)}{2dx}\right.}{}\right.$
default	$d^3\left(a\left(-\frac{\sin(dx+c)}{3d^3x^3}-\frac{\cos(dx+c)}{6d^2x^2}+\frac{\sin(dx+c)}{6dx}+\frac{\mathrm{Si}(dx)\sin(c)}{6}-\frac{\mathrm{Ci}(dx)\cos(c)}{6}\right)+\frac{b\left(-\frac{\sin(dx+c)}{2d^2x^2}-\frac{\cos(dx+c)}{2dx}\right.}{}\right.$
risch	$-\frac{i\cos(c)\,\mathrm{Ei}_1(-idx)b\,d^2}{4}+\frac{i\cos(c)\,\mathrm{Ei}_1(idx)b\,d^2}{4}+\frac{\cos(c)\,\mathrm{Ei}_1(-idx)a\,d^3}{12}+\frac{\cos(c)\,\mathrm{Ei}_1(idx)a\,d^3}{12}+\frac{\sin(c)\,\mathrm{Ei}_1(-idx)}{4}$
meijerg	$\frac{d^2b\sqrt{\pi}\,\sin(c)\left(-\frac{4}{\sqrt{\pi}\,x^2d^2}-\frac{2\left(2\gamma-3+2\ln(x)+\ln\left(d^2\right)\right)}{\sqrt{\pi}}+\frac{-6d^2x^2+4}{\sqrt{\pi}\,x^2d^2}+\frac{4\gamma}{\sqrt{\pi}}+\frac{4\ln(2)}{\sqrt{\pi}}+\frac{4\ln\left(\frac{dx}{2}\right)}{\sqrt{\pi}}-\frac{4\cos(dx)}{\sqrt{\pi}\,d^2x^2}+\frac{4\sin(dx)}{\sqrt{\pi}\,dx}-\frac{4\,\mathrm{Ci}(dx)}{\sqrt{\pi}}\right.}{8}$

```
[In] int((b*x+a)*sin(d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] d^3*(a*(-1/3*sin(d*x+c)/d^3/x^3-1/6*cos(d*x+c)/d^2/x^2+1/6*sin(d*x+c)/d/x+1
/6*Si(d*x)*sin(c)-1/6*Ci(d*x)*cos(c))+1/d*b*(-1/2*sin(d*x+c)/d^2/x^2-1/2*co
s(d*x+c)/d/x-1/2*Si(d*x)*cos(c)-1/2*Ci(d*x)*sin(c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)\sin(c+dx)}{x^4}\,dx = -\frac{(3\,bdx^2+adx)\cos(dx+c)+(ad^3x^3\,\mathrm{Ci}\,(dx)+3\,bd^2x^3\,\mathrm{Si}\,(dx))\cos(c)-(ad^2x^2-3\,bx-2\,a)\sin(dx+c)}{6\,x^3}$$

```
[In] integrate((b*x+a)*sin(d*x+c)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*((3*b*d*x^2 + a*d*x)*cos(d*x + c) + (a*d^3*x^3*cos_integral(d*x) + 3*b
*d^2*x^3*sin_integral(d*x))*cos(c) - (a*d^2*x^2 - 3*b*x - 2*a)*sin(d*x + c)
 - (a*d^3*x^3*sin_integral(d*x) - 3*b*d^2*x^3*cos_integral(d*x))*sin(c))/x^
3
```

Sympy [F]

$$\int \frac{(a+bx)\sin(c+dx)}{x^4}\,dx = \int \frac{(a+bx)\sin(c+dx)}{x^4}\,dx$$

```
[In] integrate((b*x+a)*sin(d*x+c)/x**4,x)
```

```
[Out] Integral((a + b*x)*sin(c + d*x)/x**4, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx)\sin(c+dx)}{x^4}\,dx = -\frac{((a(\Gamma(-3,i\,dx)+\Gamma(-3,-i\,dx))\cos{(c)}+a(-i\,\Gamma(-3,i\,dx)+i\,\Gamma(-3,-i\,dx))\sin{(c)})d^4-3\,(b(-i\,\Gamma(-3,i}{2\,dx^3}$$

```
[In] integrate((b*x+a)*sin(d*x+c)/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*(((a*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*cos(c) + a*(-I*gamma(-3, I
*d*x) + I*gamma(-3, -I*d*x))*sin(c))*d^4 - 3*(b*(-I*gamma(-3, I*d*x) + I*ga
mma(-3, -I*d*x))*cos(c) - b*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*sin(c))*
d^3)*x^3 + 2*b*cos(d*x + c))/(d*x^3)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.31 (sec) , antiderivative size = 961, normalized size of antiderivative = 7.28

$$\int \frac{(a+bx)\sin(c+dx)}{x^4}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)*sin(d*x+c)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*(a*d^3*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
a*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d
^3*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^3*x^3
*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a*d^3*x^3*sin_
integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) + 3*b*d^2*x^3*imag_part(cos_integra
l(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 3*b*d^2*x^3*imag_part(cos_integral(-d
*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 6*b*d^2*x^3*sin_integral(d*x)*tan(1/2*d*
x)^2*tan(1/2*c)^2 - a*d^3*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2 -
 a*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 6*b*d^2*x^3*real_
part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 6*b*d^2*x^3*real_part(c
os_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + a*d^3*x^3*real_part(cos_inte
gral(d*x))*tan(1/2*c)^2 + a*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*c
)^2 - 3*b*d^2*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 + 3*b*d^2*x^3
*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 6*b*d^2*x^3*sin_integral(d*
x)*tan(1/2*d*x)^2 + 2*a*d^3*x^3*imag_part(cos_integral(d*x))*tan(1/2*c) - 2
*a*d^3*x^3*imag_part(cos_integral(-d*x))*tan(1/2*c) + 4*a*d^3*x^3*sin_integ
ral(d*x)*tan(1/2*c) + 3*b*d^2*x^3*imag_part(cos_integral(d*x))*tan(1/2*c)^2
 - 3*b*d^2*x^3*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 + 6*b*d^2*x^3*sin
```

```
_integral(d*x)*tan(1/2*c)^2 - a*d^3*x^3*real_part(cos_integral(d*x)) - a*d^
3*x^3*real_part(cos_integral(-d*x)) - 6*b*d^2*x^3*real_part(cos_integral(d*
x))*tan(1/2*c) - 6*b*d^2*x^3*real_part(cos_integral(-d*x))*tan(1/2*c) - 4*a
*d^2*x^2*tan(1/2*d*x)^2*tan(1/2*c) - 4*a*d^2*x^2*tan(1/2*d*x)*tan(1/2*c)^2
- 6*b*d*x^2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 3*b*d^2*x^3*imag_part(cos_integra
l(d*x)) + 3*b*d^2*x^3*imag_part(cos_integral(-d*x)) - 6*b*d^2*x^3*sin_integ
ral(d*x) - 2*a*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a*d^2*x^2*tan(1/2*d*x) +
 6*b*d*x^2*tan(1/2*d*x)^2 + 4*a*d^2*x^2*tan(1/2*c) + 24*b*d*x^2*tan(1/2*d*x
)*tan(1/2*c) + 6*b*d*x^2*tan(1/2*c)^2 + 2*a*d*x*tan(1/2*d*x)^2 + 8*a*d*x*ta
n(1/2*d*x)*tan(1/2*c) + 12*b*x*tan(1/2*d*x)^2*tan(1/2*c) + 2*a*d*x*tan(1/2*
c)^2 + 12*b*x*tan(1/2*d*x)*tan(1/2*c)^2 - 6*b*d*x^2 + 8*a*tan(1/2*d*x)^2*ta
n(1/2*c) + 8*a*tan(1/2*d*x)*tan(1/2*c)^2 - 2*a*d*x - 12*b*x*tan(1/2*d*x) -
12*b*x*tan(1/2*c) - 8*a*tan(1/2*d*x) - 8*a*tan(1/2*c))/(x^3*tan(1/2*d*x)^2*
tan(1/2*c)^2 + x^3*tan(1/2*d*x)^2 + x^3*tan(1/2*c)^2 + x^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)\sin(c+dx)}{x^4}\,dx = \int \frac{\sin\left(c+d\,x\right)\,\left(a+b\,x\right)}{x^4}\,dx$$

```
[In] int((sin(c + d*x)*(a + b*x))/x^4,x)
[Out] int((sin(c + d*x)*(a + b*x))/x^4, x)
```

3.9 $\int \frac{(a+bx)\sin(c+dx)}{x^5}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 166

$$\begin{aligned}\int \frac{(a+bx)\sin(c+dx)}{x^5}\,dx = & -\frac{ad\cos(c+dx)}{12x^3}-\frac{bd\cos(c+dx)}{6x^2}+\frac{ad^3\cos(c+dx)}{24x} \\ & -\frac{1}{6}bd^3\cos(c)\,\mathrm{CosIntegral}(dx)+\frac{1}{24}ad^4\,\mathrm{CosIntegral}(dx)\sin(c) \\ & -\frac{a\sin(c+dx)}{4x^4}-\frac{b\sin(c+dx)}{3x^3}+\frac{ad^2\sin(c+dx)}{24x^2} \\ & +\frac{bd^2\sin(c+dx)}{6x}+\frac{1}{24}ad^4\cos(c)\mathrm{Si}(dx)+\frac{1}{6}bd^3\sin(c)\mathrm{Si}(dx)\end{aligned}$$

```
[Out] -1/6*b*d^3*Ci(d*x)*cos(c)-1/12*a*d*cos(d*x+c)/x^3-1/6*b*d*cos(d*x+c)/x^2+1/
24*a*d^3*cos(d*x+c)/x+1/24*a*d^4*cos(c)*Si(d*x)+1/24*a*d^4*Ci(d*x)*sin(c)+1
/6*b*d^3*Si(d*x)*sin(c)-1/4*a*sin(d*x+c)/x^4-1/3*b*sin(d*x+c)/x^3+1/24*a*d^
2*sin(d*x+c)/x^2+1/6*b*d^2*sin(d*x+c)/x
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6874, 3378, 3384, 3380, 3383}

$$\int \frac{(a+bx)\sin(c+dx)}{x^5}\,dx = \frac{1}{24}ad^4\sin(c)\,\text{CosIntegral}(dx)+\frac{1}{24}ad^4\cos(c)\text{Si}(dx)+\frac{ad^3\cos(c+dx)}{24x}+\frac{ad^2\sin(c+dx)}{24x^2}-\frac{a\sin(c+dx)}{4x^4}-\frac{ad\cos(c+dx)}{12x^3}-\frac{1}{6}bd^3\cos(c)\,\text{CosIntegral}(dx)+\frac{1}{6}bd^3\sin(c)\text{Si}(dx)+\frac{bd^2\sin(c+dx)}{6x}-\frac{b\sin(c+dx)}{3x^3}-\frac{bd\cos(c+dx)}{6x^2}$$

```
[In] Int[((a + b*x)*Sin[c + d*x])/x^5,x]
```

```
[Out] -1/12*(a*d*Cos[c + d*x])/x^3 - (b*d*Cos[c + d*x])/(6*x^2) + (a*d^3*Cos[c +
d*x])/(24*x) - (b*d^3*Cos[c]*CosIntegral[d*x])/6 + (a*d^4*CosIntegral[d*x]*
Sin[c])/24 - (a*Sin[c + d*x])/(4*x^4) - (b*Sin[c + d*x])/(3*x^3) + (a*d^2*S
in[c + d*x])/(24*x^2) + (b*d^2*Sin[c + d*x])/(6*x) + (a*d^4*Cos[c]*SinInteg
ral[d*x])/24 + (b*d^3*Sin[c]*SinIntegral[d*x])/6
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a\sin(c+dx)}{x^5}+\frac{b\sin(c+dx)}{x^4}\right) dx \\
&= a\int \frac{\sin(c+dx)}{x^5}\,dx+b\int \frac{\sin(c+dx)}{x^4}\,dx \\
&= -\frac{a\sin(c+dx)}{4x^4}-\frac{b\sin(c+dx)}{3x^3}+\frac{1}{4}(ad)\int \frac{\cos(c+dx)}{x^4}\,dx+\frac{1}{3}(bd)\int \frac{\cos(c+dx)}{x^3}\,dx \\
&= -\frac{ad\cos(c+dx)}{12x^3}-\frac{bd\cos(c+dx)}{6x^2}-\frac{a\sin(c+dx)}{4x^4}-\frac{b\sin(c+dx)}{3x^3} \\
&\quad -\frac{1}{12}\left(ad^2\right)\int \frac{\sin(c+dx)}{x^3}\,dx-\frac{1}{6}\left(bd^2\right)\int \frac{\sin(c+dx)}{x^2}\,dx \\
&= -\frac{ad\cos(c+dx)}{12x^3}-\frac{bd\cos(c+dx)}{6x^2}-\frac{a\sin(c+dx)}{4x^4}-\frac{b\sin(c+dx)}{3x^3}+\frac{ad^2\sin(c+dx)}{24x^2} \\
&\quad +\frac{bd^2\sin(c+dx)}{6x}-\frac{1}{24}\left(ad^3\right)\int \frac{\cos(c+dx)}{x^2}\,dx-\frac{1}{6}\left(bd^3\right)\int \frac{\cos(c+dx)}{x}\,dx \\
&= -\frac{ad\cos(c+dx)}{12x^3}-\frac{bd\cos(c+dx)}{6x^2}+\frac{ad^3\cos(c+dx)}{24x}-\frac{a\sin(c+dx)}{4x^4} \\
&\quad -\frac{b\sin(c+dx)}{3x^3}+\frac{ad^2\sin(c+dx)}{24x^2}+\frac{bd^2\sin(c+dx)}{6x}+\frac{1}{24}\left(ad^4\right)\int \frac{\sin(c+dx)}{x}\,dx \\
&\quad -\frac{1}{6}\left(bd^3\cos(c)\right)\int \frac{\cos(dx)}{x}\,dx+\frac{1}{6}\left(bd^3\sin(c)\right)\int \frac{\sin(dx)}{x}\,dx \\
&= -\frac{ad\cos(c+dx)}{12x^3}-\frac{bd\cos(c+dx)}{6x^2}+\frac{ad^3\cos(c+dx)}{24x}-\frac{1}{6}bd^3\cos(c)\,\text{CosIntegral}(dx) \\
&\quad -\frac{a\sin(c+dx)}{4x^4}-\frac{b\sin(c+dx)}{3x^3}+\frac{ad^2\sin(c+dx)}{24x^2}+\frac{bd^2\sin(c+dx)}{6x} \\
&\quad +\frac{1}{6}bd^3\sin(c)\text{Si}(dx)+\frac{1}{24}\left(ad^4\cos(c)\right)\int \frac{\sin(dx)}{x}\,dx+\frac{1}{24}\left(ad^4\sin(c)\right)\int \frac{\cos(dx)}{x}\,dx \\
&= -\frac{ad\cos(c+dx)}{12x^3}-\frac{bd\cos(c+dx)}{6x^2}+\frac{ad^3\cos(c+dx)}{24x}-\frac{1}{6}bd^3\cos(c)\,\text{CosIntegral}(dx) \\
&\quad +\frac{1}{24}ad^4\,\text{CosIntegral}(dx)\sin(c)-\frac{a\sin(c+dx)}{4x^4}-\frac{b\sin(c+dx)}{3x^3} \\
&\quad +\frac{ad^2\sin(c+dx)}{24x^2}+\frac{bd^2\sin(c+dx)}{6x}+\frac{1}{24}ad^4\cos(c)\text{Si}(dx)+\frac{1}{6}bd^3\sin(c)\text{Si}(dx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)\sin(c+dx)}{x^5}\,dx$$
$$= \frac{-2adx\cos(c+dx)-4bdx^2\cos(c+dx)+ad^3x^3\cos(c+dx)+d^3x^4\,\text{CosIntegral}(dx)(-4b\cos(c)+ad\sin(}{}$$

```
[In] Integrate[((a + b*x)*Sin[c + d*x])/x^5,x]
```

```
[Out] (-2*a*d*x*Cos[c + d*x] - 4*b*d*x^2*Cos[c + d*x] + a*d^3*x^3*Cos[c + d*x] +
d^3*x^4*CosIntegral[d*x]*(-4*b*Cos[c] + a*d*Sin[c]) - 6*a*Sin[c + d*x] - 8*
b*x*Sin[c + d*x] + a*d^2*x^2*Sin[c + d*x] + 4*b*d^2*x^3*Sin[c + d*x] + d^3*
x^4*(a*d*Cos[c] + 4*b*Sin[c])*SinIntegral[d*x])/(24*x^4)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.87

method	result
derivativedivides	$d^4\left(a\left(-\frac{\sin(dx+c)}{4d^4x^4}-\frac{\cos(dx+c)}{12d^3x^3}+\frac{\sin(dx+c)}{24d^2x^2}+\frac{\cos(dx+c)}{24dx}+\frac{\text{Si}(dx)\cos(c)}{24}+\frac{\text{Ci}(dx)\sin(c)}{24}\right)+\frac{b\left(-\frac{\sin(dx}{3d^3x}\right.}{}\right.$
default	$d^4\left(a\left(-\frac{\sin(dx+c)}{4d^4x^4}-\frac{\cos(dx+c)}{12d^3x^3}+\frac{\sin(dx+c)}{24d^2x^2}+\frac{\cos(dx+c)}{24dx}+\frac{\text{Si}(dx)\cos(c)}{24}+\frac{\text{Ci}(dx)\sin(c)}{24}\right)+\frac{b\left(-\frac{\sin(dx}{3d^3x}\right.}{}\right.$
risch	$\frac{\cos(c)\,\text{Ei}_1(-idx)b\,d^3}{12}+\frac{\cos(c)\,\text{Ei}_1(idx)b\,d^3}{12}+\frac{i\cos(c)\,\text{Ei}_1(-idx)a\,d^4}{48}-\frac{i\cos(c)\,\text{Ei}_1(idx)a\,d^4}{48}+\frac{i\sin(c)\,\text{Ei}_1(-idx)b}{12}$
meijerg	$\frac{d^4b\sqrt{\pi}\,\sin(c)\left(-\frac{8\left(-d^2x^2+2\right)d^2\cos\left(x\sqrt{d^2}\right)}{3x^3\left(d^2\right)^{\frac{5}{2}}\sqrt{\pi}}+\frac{8\sin\left(x\sqrt{d^2}\right)}{3d^2x^2\sqrt{\pi}}+\frac{8\,\text{Si}\left(x\sqrt{d^2}\right)}{3\sqrt{\pi}}\right)}{16\sqrt{d^2}}+\frac{d^3b\sqrt{\pi}\,\cos(c)\left(-\frac{8}{\sqrt{\pi}\,x^2d^2}-\frac{4\left(2\gamma-\frac{11}{3}+2\ln(\right.}{3\sqrt{\pi}}\right.}{}$

```
[In] int((b*x+a)*sin(d*x+c)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] d^4*(a*(-1/4*sin(d*x+c)/d^4/x^4-1/12*cos(d*x+c)/d^3/x^3+1/24*sin(d*x+c)/d^2
/x^2+1/24*cos(d*x+c)/d/x+1/24*Si(d*x)*cos(c)+1/24*Ci(d*x)*sin(c))+1/d*b*(-1
/3*sin(d*x+c)/d^3/x^3-1/6*cos(d*x+c)/d^2/x^2+1/6*sin(d*x+c)/d/x+1/6*Si(d*x)
*sin(c)-1/6*Ci(d*x)*cos(c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx)\sin(c+dx)}{x^5}\,dx = \frac{(ad^3x^3 - 4\,bdx^2 - 2\,adx)\cos(dx+c) + (ad^4x^4\,\mathrm{Si}(dx) - 4\,bd^3x^4\,\mathrm{Ci}(dx))\cos(c) + (4\,bd^2x^3 + ad^2x^2 - 8\,bx}{24\,x^4}$$

```
[In] integrate((b*x+a)*sin(d*x+c)/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*((a*d^3*x^3 - 4*b*d*x^2 - 2*a*d*x)*cos(d*x + c) + (a*d^4*x^4*sin_integ
ral(d*x) - 4*b*d^3*x^4*cos_integral(d*x))*cos(c) + (4*b*d^2*x^3 + a*d^2*x^2
 - 8*b*x - 6*a)*sin(d*x + c) + (a*d^4*x^4*cos_integral(d*x) + 4*b*d^3*x^4*s
in_integral(d*x))*sin(c))/x^4
```

Sympy [F]

$$\int \frac{(a+bx)\sin(c+dx)}{x^5}\,dx = \int \frac{(a+bx)\sin(c+dx)}{x^5}\,dx$$

```
[In] integrate((b*x+a)*sin(d*x+c)/x**5,x)
```

```
[Out] Integral((a + b*x)*sin(c + d*x)/x**5, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.66

$$\int \frac{(a+bx)\sin(c+dx)}{x^5}\,dx = -\frac{((a(i\,\Gamma(-4,i\,dx) - i\,\Gamma(-4,-i\,dx))\cos(c) + a(\Gamma(-4,i\,dx) + \Gamma(-4,-i\,dx))\sin(c))d^5 - 4\,(b(\Gamma(-4,i\,dx)}{2\,dx^4}$$

```
[In] integrate((b*x+a)*sin(d*x+c)/x^5,x, algorithm="maxima")
```

```
[Out] -1/2*(((a*(I*gamma(-4, I*d*x) - I*gamma(-4, -I*d*x))*cos(c) + a*(gamma(-4,
I*d*x) + gamma(-4, -I*d*x))*sin(c))*d^5 - 4*(b*(gamma(-4, I*d*x) + gamma(-4
, -I*d*x))*cos(c) + b*(-I*gamma(-4, I*d*x) + I*gamma(-4, -I*d*x))*sin(c))*d
^4)*x^4 + 2*b*cos(d*x + c))/(d*x^4)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.31 (sec) , antiderivative size = 1108, normalized size of antiderivative = 6.67

$$\int \frac{(a+bx)\sin(c+dx)}{x^5}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)*sin(d*x+c)/x^5,x, algorithm="giac")
[Out] -1/48*(a*d^4*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 -
 a*d^4*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*
d^4*x^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*d^4*x^4*real_pa
rt(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^4*x^4*real_part(cos
_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 4*b*d^3*x^4*real_part(cos_inte
gral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*b*d^3*x^4*real_part(cos_integral
(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d^4*x^4*imag_part(cos_integral(d*x)
)*tan(1/2*d*x)^2 + a*d^4*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2 -
 2*a*d^4*x^4*sin_integral(d*x)*tan(1/2*d*x)^2 - 8*b*d^3*x^4*imag_part(cos_i
ntegral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 8*b*d^3*x^4*imag_part(cos_integra
l(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 16*b*d^3*x^4*sin_integral(d*x)*tan(1/2
*d*x)^2*tan(1/2*c) + a*d^4*x^4*imag_part(cos_integral(d*x))*tan(1/2*c)^2 -
a*d^4*x^4*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 + 2*a*d^4*x^4*sin_inte
gral(d*x)*tan(1/2*c)^2 + 4*b*d^3*x^4*real_part(cos_integral(d*x))*tan(1/2*d
*x)^2 + 4*b*d^3*x^4*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 2*a*d^4*
x^4*real_part(cos_integral(d*x))*tan(1/2*c) - 2*a*d^4*x^4*real_part(cos_int
egral(-d*x))*tan(1/2*c) - 4*b*d^3*x^4*real_part(cos_integral(d*x))*tan(1/2*
c)^2 - 4*b*d^3*x^4*real_part(cos_integral(-d*x))*tan(1/2*c)^2 - 2*a*d^3*x^3
*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d^4*x^4*imag_part(cos_integral(d*x)) + a*d
^4*x^4*imag_part(cos_integral(-d*x)) - 2*a*d^4*x^4*sin_integral(d*x) - 8*b*
d^3*x^4*imag_part(cos_integral(d*x))*tan(1/2*c) + 8*b*d^3*x^4*imag_part(cos
_integral(-d*x))*tan(1/2*c) - 16*b*d^3*x^4*sin_integral(d*x)*tan(1/2*c) + 4
*b*d^3*x^4*real_part(cos_integral(d*x)) + 4*b*d^3*x^4*real_part(cos_integra
l(-d*x)) + 2*a*d^3*x^3*tan(1/2*d*x)^2 + 8*a*d^3*x^3*tan(1/2*d*x)*tan(1/2*c)
 + 16*b*d^2*x^3*tan(1/2*d*x)^2*tan(1/2*c) + 2*a*d^3*x^3*tan(1/2*c)^2 + 16*b
*d^2*x^3*tan(1/2*d*x)*tan(1/2*c)^2 + 4*a*d^2*x^2*tan(1/2*d*x)^2*tan(1/2*c)
+ 4*a*d^2*x^2*tan(1/2*d*x)*tan(1/2*c)^2 + 8*b*d*x^2*tan(1/2*d*x)^2*tan(1/2*
c)^2 - 2*a*d^3*x^3 - 16*b*d^2*x^3*tan(1/2*d*x) - 16*b*d^2*x^3*tan(1/2*c) +
4*a*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a*d^2*x^2*tan(1/2*d*x) - 8*b*d*x^2*
tan(1/2*d*x)^2 - 4*a*d^2*x^2*tan(1/2*c) - 32*b*d*x^2*tan(1/2*d*x)*tan(1/2*c
) - 8*b*d*x^2*tan(1/2*c)^2 - 4*a*d*x*tan(1/2*d*x)^2 - 16*a*d*x*tan(1/2*d*x)
*tan(1/2*c) - 32*b*x*tan(1/2*d*x)^2*tan(1/2*c) - 4*a*d*x*tan(1/2*c)^2 - 32*
b*x*tan(1/2*d*x)*tan(1/2*c)^2 + 8*b*d*x^2 - 24*a*tan(1/2*d*x)^2*tan(1/2*c)
- 24*a*tan(1/2*d*x)*tan(1/2*c)^2 + 4*a*d*x + 32*b*x*tan(1/2*d*x) + 32*b*x*t
an(1/2*c) + 24*a*tan(1/2*d*x) + 24*a*tan(1/2*c))/(x^4*tan(1/2*d*x)^2*tan(1/
2*c)^2 + x^4*tan(1/2*d*x)^2 + x^4*tan(1/2*c)^2 + x^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)\sin(c+dx)}{x^5}\,dx = \int \frac{\sin\left(c+d\,x\right)\,\left(a+b\,x\right)}{x^5}\,dx$$

```
[In] int((sin(c + d*x)*(a + b*x))/x^5,x)
[Out] int((sin(c + d*x)*(a + b*x))/x^5, x)
```

3.10 $\int x^2(a+bx)^2 \sin(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 186

$$\int x^2(a+bx)^2\sin(c+dx)\,dx = -\frac{24b^2\cos(c+dx)}{d^5}+\frac{2a^2\cos(c+dx)}{d^3}+\frac{12abx\cos(c+dx)}{d^3}+\frac{12b^2x^2\cos(c+dx)}{d^3}-\frac{a^2x^2\cos(c+dx)}{d}-\frac{2abx^3\cos(c+dx)}{d}-\frac{b^2x^4\cos(c+dx)}{d}-\frac{12ab\sin(c+dx)}{d^4}-\frac{24b^2x\sin(c+dx)}{d^4}+\frac{2a^2x\sin(c+dx)}{d^2}+\frac{6abx^2\sin(c+dx)}{d^2}+\frac{4b^2x^3\sin(c+dx)}{d^2}$$

```
[Out] -24*b^2*cos(d*x+c)/d^5+2*a^2*cos(d*x+c)/d^3+12*a*b*x*cos(d*x+c)/d^3+12*b^2*
x^2*cos(d*x+c)/d^3-a^2*x^2*cos(d*x+c)/d-2*a*b*x^3*cos(d*x+c)/d-b^2*x^4*cos(
d*x+c)/d-12*a*b*sin(d*x+c)/d^4-24*b^2*x*sin(d*x+c)/d^4+2*a^2*x*sin(d*x+c)/d
^2+6*a*b*x^2*sin(d*x+c)/d^2+4*b^2*x^3*sin(d*x+c)/d^2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6874, 3377, 2718, 2717}

$$\int x^2(a+bx)^2\sin(c+dx)\,dx = \frac{2a^2\cos(c+dx)}{d^3}+\frac{2a^2x\sin(c+dx)}{d^2}-\frac{a^2x^2\cos(c+dx)}{d}-\frac{12ab\sin(c+dx)}{d^4}+\frac{12abx\cos(c+dx)}{d^3}+\frac{6abx^2\sin(c+dx)}{d^2}-\frac{2abx^3\cos(c+dx)}{d}-\frac{24b^2\cos(c+dx)}{d^5}-\frac{24b^2x\sin(c+dx)}{d^4}+\frac{12b^2x^2\cos(c+dx)}{d^3}+\frac{4b^2x^3\sin(c+dx)}{d^2}-\frac{b^2x^4\cos(c+dx)}{d}$$

```
[In] Int[x^2*(a + b*x)^2*Sin[c + d*x],x]
```

```
[Out] (-24*b^2*Cos[c + d*x])/d^5 + (2*a^2*Cos[c + d*x])/d^3 + (12*a*b*x*Cos[c + d
*x])/d^3 + (12*b^2*x^2*Cos[c + d*x])/d^3 - (a^2*x^2*Cos[c + d*x])/d - (2*a*
b*x^3*Cos[c + d*x])/d - (b^2*x^4*Cos[c + d*x])/d - (12*a*b*Sin[c + d*x])/d^
4 - (24*b^2*x*Sin[c + d*x])/d^4 + (2*a^2*x*Sin[c + d*x])/d^2 + (6*a*b*x^2*S
in[c + d*x])/d^2 + (4*b^2*x^3*Sin[c + d*x])/d^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^2x^2\sin(c+dx)+2abx^3\sin(c+dx)+b^2x^4\sin(c+dx)\right)\,dx\\
&= a^2\int x^2\sin(c+dx)\,dx+(2ab)\int x^3\sin(c+dx)\,dx+b^2\int x^4\sin(c+dx)\,dx\\
&= -\frac{a^2x^2\cos(c+dx)}{d}-\frac{2abx^3\cos(c+dx)}{d}-\frac{b^2x^4\cos(c+dx)}{d}\\
&\quad+\frac{(2a^2)\int x\cos(c+dx)\,dx}{d}+\frac{(6ab)\int x^2\cos(c+dx)\,dx}{d}+\frac{(4b^2)\int x^3\cos(c+dx)\,dx}{d}\\
&= -\frac{a^2x^2\cos(c+dx)}{d}-\frac{2abx^3\cos(c+dx)}{d}-\frac{b^2x^4\cos(c+dx)}{d}+\frac{2a^2x\sin(c+dx)}{d^2}\\
&\quad+\frac{6abx^2\sin(c+dx)}{d^2}+\frac{4b^2x^3\sin(c+dx)}{d^2}-\frac{(2a^2)\int\sin(c+dx)\,dx}{d^2}\\
&\quad-\frac{(12ab)\int x\sin(c+dx)\,dx}{d^2}-\frac{(12b^2)\int x^2\sin(c+dx)\,dx}{d^2}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2a^2\cos(c+dx)}{d^3} + \frac{12abx\cos(c+dx)}{d^3} + \frac{12b^2x^2\cos(c+dx)}{d^3} - \frac{a^2x^2\cos(c+dx)}{d} \\
&\quad - \frac{2abx^3\cos(c+dx)}{d} - \frac{b^2x^4\cos(c+dx)}{d} + \frac{2a^2x\sin(c+dx)}{d^2} + \frac{6abx^2\sin(c+dx)}{d^2} \\
&\quad + \frac{4b^2x^3\sin(c+dx)}{d^2} - \frac{(12ab)\int\cos(c+dx)\,dx}{d^3} - \frac{(24b^2)\int x\cos(c+dx)\,dx}{d^3} \\
&= \frac{2a^2\cos(c+dx)}{d^3} + \frac{12abx\cos(c+dx)}{d^3} + \frac{12b^2x^2\cos(c+dx)}{d^3} - \frac{a^2x^2\cos(c+dx)}{d} \\
&\quad - \frac{2abx^3\cos(c+dx)}{d} - \frac{b^2x^4\cos(c+dx)}{d} - \frac{12ab\sin(c+dx)}{d^4} - \frac{24b^2x\sin(c+dx)}{d^4} \\
&\quad + \frac{2a^2x\sin(c+dx)}{d^2} + \frac{6abx^2\sin(c+dx)}{d^2} + \frac{4b^2x^3\sin(c+dx)}{d^2} + \frac{(24b^2)\int\sin(c+dx)\,dx}{d^4} \\
&= -\frac{24b^2\cos(c+dx)}{d^5} + \frac{2a^2\cos(c+dx)}{d^3} + \frac{12abx\cos(c+dx)}{d^3} + \frac{12b^2x^2\cos(c+dx)}{d^3} \\
&\quad - \frac{a^2x^2\cos(c+dx)}{d} - \frac{2abx^3\cos(c+dx)}{d} - \frac{b^2x^4\cos(c+dx)}{d} - \frac{12ab\sin(c+dx)}{d^4} \\
&\quad - \frac{24b^2x\sin(c+dx)}{d^4} + \frac{2a^2x\sin(c+dx)}{d^2} + \frac{6abx^2\sin(c+dx)}{d^2} + \frac{4b^2x^3\sin(c+dx)}{d^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.54

$$
\int x^2(a+bx)^2\sin(c+dx)\,dx
= \frac{-((2abd^2x(-6+d^2x^2)+a^2d^2(-2+d^2x^2)+b^2(24-12d^2x^2+d^4x^4))\cos(c+dx))+2d(a+2bx)\,(ad^2x+}{d^5}
$$

```
[In] Integrate[x^2*(a + b*x)^2*Sin[c + d*x],x]
```

```
[Out] (-((2*a*b*d^2*x*(-6 + d^2*x^2) + a^2*d^2*(-2 + d^2*x^2) + b^2*(24 - 12*d^2*
x^2 + d^4*x^4))*Cos[c + d*x]) + 2*d*(a + 2*b*x)*(a*d^2*x + b*(-6 + d^2*x^2)
)*Sin[c + d*x])/d^5
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{(b^2x^4d^4+2ab\,d^4x^3+a^2d^4x^2-12d^2x^2b^2-12ab\,d^2x-2d^2a^2+24b^2)\cos(dx+c)}{d^5}+\frac{2(2b^2d^2x^3+3ab\,d^2x^2+a^2d^2x-12b^2x-}{d^4}$
parallelrisch	$\frac{(x(bx+a)d^2-12b)(bx+a)x\,d^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4(x(bx+a)d^2-6b)d(2bx+a)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-x^2(bx+a)^2d^4+4(3x^2b^2+3abx}{d^5\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$
norman	$\frac{\frac{4d^2a^2-48b^2}{d^5}+\frac{b^2x^4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{(d^2a^2-12b^2)x^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d^3}-\frac{b^2x^4}{d}-\frac{(d^2a^2-12b^2)x^2}{d^3}-\frac{24ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^4}+\frac{8b^2x^3\tan(}{d^2}}{1+\tan^2(}$
parts	$-\frac{b^2x^4\cos(dx+c)}{d}-\frac{2ab\,x^3\cos(dx+c)}{d}-\frac{a^2x^2\cos(dx+c)}{d}+\frac{-\frac{2a^2c\sin(dx+c)}{d}+\frac{2a^2(\cos(dx+c)+(dx+c)\sin(dx+c))}{d}+\frac{6ab\,c}{}}{}$
meijerg	$\frac{16b^2\sqrt{\pi}\,\sin(c)\left(-\frac{x\left(d^2\right)^{\frac{5}{2}}\left(-\frac{5d^2x^2}{2}+15\right)\cos(dx)}{10\sqrt{\pi}\,d^4}+\frac{\left(d^2\right)^{\frac{5}{2}}\left(\frac{5}{8}d^4x^4-\frac{15}{2}d^2x^2+15\right)\sin(dx)}{10\sqrt{\pi}\,d^5}\right)}{d^4\sqrt{d^2}}+\frac{16b^2\sqrt{\pi}\,\cos(c)\left(\frac{3}{2\sqrt{\pi}}-\frac{\left(\frac{3}{8}d^4x^4\right.}{}\right.}{}$
derivativedivides	$\frac{-a^2c^2\cos(dx+c)-2a^2c(\sin(dx+c)-\cos(dx+c)(dx+c))+a^2\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)+2}{}$
default	$\frac{-a^2c^2\cos(dx+c)-2a^2c(\sin(dx+c)-\cos(dx+c)(dx+c))+a^2\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)+2}{}$

```
[In] int(x^2*(b*x+a)^2*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -(b^2*d^4*x^4+2*a*b*d^4*x^3+a^2*d^4*x^2-12*b^2*d^2*x^2-12*a*b*d^2*x-2*a^2*d
^2+24*b^2)/d^5*cos(d*x+c)+2/d^4*(2*b^2*d^2*x^3+3*a*b*d^2*x^2+a^2*d^2*x-12*b
^2*x-6*a*b)*sin(d*x+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.68

$$\int x^2(a+bx)^2\sin(c+dx)\,dx = -\frac{(b^2d^4x^4+2\,abd^4x^3-12\,abd^2x-2\,a^2d^2+(a^2d^4-12\,b^2d^2)x^2+24\,b^2)\cos(dx+c)-2\,(2\,b^2d^3x^3+3\,abd^3x}{d^5}$$

```
[In] integrate(x^2*(b*x+a)^2*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b^2*d^4*x^4 + 2*a*b*d^4*x^3 - 12*a*b*d^2*x - 2*a^2*d^2 + (a^2*d^4 - 12*b
^2*d^2)*x^2 + 24*b^2)*cos(d*x + c) - 2*(2*b^2*d^3*x^3 + 3*a*b*d^3*x^2 - 6*a
*b*d + (a^2*d^3 - 12*b^2*d)*x)*sin(d*x + c))/d^5
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.23

$$\int x^2(a+bx)^2\sin(c+dx)\,dx$$
$$=\begin{cases}-\frac{a^2x^2\cos(c+dx)}{d}+\frac{2a^2x\sin(c+dx)}{d^2}+\frac{2a^2\cos(c+dx)}{d^3}-\frac{2abx^3\cos(c+dx)}{d}+\frac{6abx^2\sin(c+dx)}{d^2}+\frac{12abx\cos(c+dx)}{d^3}-\frac{12ab\sin(c+d}{d^4}\\ \left(\frac{a^2x^3}{3}+\frac{abx^4}{2}+\frac{b^2x^5}{5}\right)\sin(c)\end{cases}$$

```
[In] integrate(x**2*(b*x+a)**2*sin(d*x+c),x)
```

```
[Out] Piecewise((-a**2*x**2*cos(c + d*x)/d + 2*a**2*x*sin(c + d*x)/d**2 + 2*a**2*
cos(c + d*x)/d**3 - 2*a*b*x**3*cos(c + d*x)/d + 6*a*b*x**2*sin(c + d*x)/d**
2 + 12*a*b*x*cos(c + d*x)/d**3 - 12*a*b*sin(c + d*x)/d**4 - b**2*x**4*cos(c
 + d*x)/d + 4*b**2*x**3*sin(c + d*x)/d**2 + 12*b**2*x**2*cos(c + d*x)/d**3
- 24*b**2*x*sin(c + d*x)/d**4 - 24*b**2*cos(c + d*x)/d**5, Ne(d, 0)), ((a**
2*x**3/3 + a*b*x**4/2 + b**2*x**5/5)*sin(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. 2(186) = 372.

Time = 0.21 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.18

$$\int x^2(a+bx)^2\sin(c+dx)\,dx =$$
$$-\frac{a^2c^2\cos(dx+c)+\frac{b^2c^4\cos(dx+c)}{d^2}-\frac{2\,abc^3\cos(dx+c)}{d}-2\left((dx+c)\cos(dx+c)-\sin(dx+c)\right)a^2c-\frac{4\left((dx+c)\,c\right.}{}}{}$$

```
[In] integrate(x^2*(b*x+a)^2*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a^2*c^2*cos(d*x + c) + b^2*c^4*cos(d*x + c)/d^2 - 2*a*b*c^3*cos(d*x + c)/
d - 2*((d*x + c)*cos(d*x + c) - sin(d*x + c))*a^2*c - 4*((d*x + c)*cos(d*x
+ c) - sin(d*x + c))*b^2*c^3/d^2 + 6*((d*x + c)*cos(d*x + c) - sin(d*x + c)
)*a*b*c^2/d + (((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a
^2 + 6*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b^2*c^2/
d^2 - 6*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a*b*c/d
 - 4*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*
x + c))*b^2*c/d^2 + 2*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x +
 c)^2 - 2)*sin(d*x + c))*a*b/d + (((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d
*x + c) - 4*((d*x + c)^3 - 6*d*x - 6*c)*sin(d*x + c))*b^2/d^2)/d^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.69

$$\int x^2(a+bx)^2\sin(c+dx)\,dx$$
$$=-\frac{(b^2d^4x^4+2\,abd^4x^3+a^2d^4x^2-12\,b^2d^2x^2-12\,abd^2x-2\,a^2d^2+24\,b^2)\cos{(dx+c)}}{d^5}$$
$$+\frac{2\,(2\,b^2d^3x^3+3\,abd^3x^2+a^2d^3x-12\,b^2dx-6\,abd)\sin{(dx+c)}}{d^5}$$

```
[In] integrate(x^2*(b*x+a)^2*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b^2*d^4*x^4 + 2*a*b*d^4*x^3 + a^2*d^4*x^2 - 12*b^2*d^2*x^2 - 12*a*b*d^2*x
 - 2*a^2*d^2 + 24*b^2)*cos(d*x + c)/d^5 + 2*(2*b^2*d^3*x^3 + 3*a*b*d^3*x^2
+ a^2*d^3*x - 12*b^2*d*x - 6*a*b*d)*sin(d*x + c)/d^5
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.92

$$\int x^2(a+bx)^2\sin(c+dx)\,dx=\frac{4\,b^2\,x^3\,\sin{(c+d\,x)}}{d^2}-\frac{b^2\,x^4\,\cos{(c+d\,x)}}{d}$$
$$-\frac{2\,\cos{(c+d\,x)}\,\left(12\,b^2-a^2\,d^2\right)}{d^5}-\frac{12\,a\,b\,\sin{(c+d\,x)}}{d^4}$$
$$-\frac{2\,x\,\sin{(c+d\,x)}\,\left(12\,b^2-a^2\,d^2\right)}{d^4}$$
$$+\frac{x^2\,\cos{(c+d\,x)}\,\left(12\,b^2-a^2\,d^2\right)}{d^3}-\frac{2\,a\,b\,x^3\,\cos{(c+d\,x)}}{d}$$
$$+\frac{6\,a\,b\,x^2\,\sin{(c+d\,x)}}{d^2}+\frac{12\,a\,b\,x\,\cos{(c+d\,x)}}{d^3}$$

```
[In] int(x^2*sin(c + d*x)*(a + b*x)^2,x)
```

```
[Out] (4*b^2*x^3*sin(c + d*x))/d^2 - (b^2*x^4*cos(c + d*x))/d - (2*cos(c + d*x)*(
12*b^2 - a^2*d^2))/d^5 - (12*a*b*sin(c + d*x))/d^4 - (2*x*sin(c + d*x)*(12*
b^2 - a^2*d^2))/d^4 + (x^2*cos(c + d*x)*(12*b^2 - a^2*d^2))/d^3 - (2*a*b*x^
3*cos(c + d*x))/d + (6*a*b*x^2*sin(c + d*x))/d^2 + (12*a*b*x*cos(c + d*x))/
d^3
```

3.11 $\int x(a+bx)^2 \sin(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 135

$$\int x(a+bx)^2 \sin(c+dx)\,dx = \frac{4ab\cos(c+dx)}{d^3} + \frac{6b^2x\cos(c+dx)}{d^3} - \frac{a^2x\cos(c+dx)}{d} - \frac{2abx^2\cos(c+dx)}{d} - \frac{b^2x^3\cos(c+dx)}{d} - \frac{6b^2\sin(c+dx)}{d^4} + \frac{a^2\sin(c+dx)}{d^2} + \frac{4abx\sin(c+dx)}{d^2} + \frac{3b^2x^2\sin(c+dx)}{d^2}$$

```
[Out] 4*a*b*cos(d*x+c)/d^3+6*b^2*x*cos(d*x+c)/d^3-a^2*x*cos(d*x+c)/d-2*a*b*x^2*co
s(d*x+c)/d-b^2*x^3*cos(d*x+c)/d-6*b^2*sin(d*x+c)/d^4+a^2*sin(d*x+c)/d^2+4*a
*b*x*sin(d*x+c)/d^2+3*b^2*x^2*sin(d*x+c)/d^2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {6874, 3377, 2717, 2718}

$$\int x(a+bx)^2 \sin(c+dx)\,dx = \frac{a^2\sin(c+dx)}{d^2} - \frac{a^2x\cos(c+dx)}{d} + \frac{4ab\cos(c+dx)}{d^3} + \frac{4abx\sin(c+dx)}{d^2} - \frac{2abx^2\cos(c+dx)}{d} - \frac{6b^2\sin(c+dx)}{d^4} + \frac{6b^2x\cos(c+dx)}{d^3} + \frac{3b^2x^2\sin(c+dx)}{d^2} - \frac{b^2x^3\cos(c+dx)}{d}$$

```
[In] Int[x*(a + b*x)^2*Sin[c + d*x],x]
```

```
[Out] (4*a*b*Cos[c + d*x])/d^3 + (6*b^2*x*Cos[c + d*x])/d^3 - (a^2*x*Cos[c + d*x]
)/d - (2*a*b*x^2*Cos[c + d*x])/d - (b^2*x^3*Cos[c + d*x])/d - (6*b^2*Sin[c
```

```
+ d*x])/d^4 + (a^2*Sin[c + d*x])/d^2 + (4*a*b*x*Sin[c + d*x])/d^2 + (3*b^2*
x^2*Sin[c + d*x])/d^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a^2 x \sin(c+dx) + 2abx^2 \sin(c+dx) + b^2 x^3 \sin(c+dx)\right)\, dx \\
&= a^2 \int x \sin(c+dx)\, dx + (2ab) \int x^2 \sin(c+dx)\, dx + b^2 \int x^3 \sin(c+dx)\, dx \\
&= -\frac{a^2 x \cos(c+dx)}{d} - \frac{2abx^2 \cos(c+dx)}{d} - \frac{b^2 x^3 \cos(c+dx)}{d} \\
&\quad + \frac{a^2 \int \cos(c+dx)\, dx}{d} + \frac{(4ab) \int x \cos(c+dx)\, dx}{d} + \frac{(3b^2) \int x^2 \cos(c+dx)\, dx}{d} \\
&= -\frac{a^2 x \cos(c+dx)}{d} - \frac{2abx^2 \cos(c+dx)}{d} - \frac{b^2 x^3 \cos(c+dx)}{d} + \frac{a^2 \sin(c+dx)}{d^2} \\
&\quad + \frac{4abx \sin(c+dx)}{d^2} + \frac{3b^2 x^2 \sin(c+dx)}{d^2} - \frac{(4ab) \int \sin(c+dx)\, dx}{d^2} \\
&\quad - \frac{(6b^2) \int x \sin(c+dx)\, dx}{d^2} \\
&= \frac{4ab \cos(c+dx)}{d^3} + \frac{6b^2 x \cos(c+dx)}{d^3} - \frac{a^2 x \cos(c+dx)}{d} \\
&\quad - \frac{2abx^2 \cos(c+dx)}{d} - \frac{b^2 x^3 \cos(c+dx)}{d} + \frac{a^2 \sin(c+dx)}{d^2} \\
&\quad + \frac{4abx \sin(c+dx)}{d^2} + \frac{3b^2 x^2 \sin(c+dx)}{d^2} - \frac{(6b^2) \int \cos(c+dx)\, dx}{d^3}
\end{aligned}
$$

$$= \frac{4ab\cos(c+dx)}{d^3} + \frac{6b^2x\cos(c+dx)}{d^3} - \frac{a^2x\cos(c+dx)}{d} - \frac{2abx^2\cos(c+dx)}{d} - \frac{b^2x^3\cos(c+dx)}{d} - \frac{6b^2\sin(c+dx)}{d^4} + \frac{a^2\sin(c+dx)}{d^2} + \frac{4abx\sin(c+dx)}{d^2} + \frac{3b^2x^2\sin(c+dx)}{d^2}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.64

$$\int x(a+bx)^2\sin(c+dx)\,dx = \frac{-d(a^2d^2x+b^2x(-6+d^2x^2)+2ab(-2+d^2x^2))\cos(c+dx)+(a^2d^2+4abd^2x+3b^2(-2+d^2x^2))\sin(c+}{d^4}$$

```
[In] Integrate[x*(a + b*x)^2*Sin[c + d*x],x]
```

```
[Out] (-(d*(a^2*d^2*x + b^2*x*(-6 + d^2*x^2) + 2*a*b*(-2 + d^2*x^2))*Cos[c + d*x]
) + (a^2*d^2 + 4*a*b*d^2*x + 3*b^2*(-2 + d^2*x^2))*Sin[c + d*x])/d^4
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{(b^2d^2x^3+2ab\,d^2x^2+a^2d^2x-6b^2x-4ab)\cos(dx+c)}{d^3}+\frac{(3d^2x^2b^2+4ab\,d^2x+d^2a^2-6b^2)\sin(dx+c)}{d^4}$
parallelrisch	$\frac{\left((bx+a)^2d^2-6b^2\right)xd\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2((3x^2b^2+4abx+a^2)d^2-6b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-d\left(x(bx+a)^2d^2-6b^2x-8ab\right)}{d^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$
parts	$-\frac{b^2x^3\cos(dx+c)}{d}-\frac{2ab\,x^2\cos(dx+c)}{d}-\frac{a^2x\cos(dx+c)}{d}+\frac{a^2\sin(dx+c)-\frac{4abc\sin(dx+c)}{d}+\frac{4ab(\cos(dx+c)+(dx+c)\sin(}{d}}{}$
norman	$\frac{\frac{b^2x^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{(d^2a^2-6b^2)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d^3}-\frac{b^2x^3}{d}-\frac{(d^2a^2-6b^2)x}{d^3}+\frac{2(d^2a^2-6b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^4}-\frac{8ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d^3}}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$
derivativedivides	$\frac{a^2c\cos(dx+c)+a^2(\sin(dx+c)-\cos(dx+c)(dx+c))-\frac{2ab\,c^2\cos(dx+c)}{d}-\frac{4abc(\sin(dx+c)-\cos(dx+c)(dx+c))}{d}+\frac{2ab\left(-(dx+c)^2\cos\right.}{}}{}$
default	$\frac{a^2c\cos(dx+c)+a^2(\sin(dx+c)-\cos(dx+c)(dx+c))-\frac{2ab\,c^2\cos(dx+c)}{d}-\frac{4abc(\sin(dx+c)-\cos(dx+c)(dx+c))}{d}+\frac{2ab\left(-(dx+c)^2\cos\right.}{}}{}$
meijerg	$\frac{8b^2\sqrt{\pi}\sin(c)\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(-\frac{3d^2x^2}{2}+3\right)\cos(dx)}{4\sqrt{\pi}}-\frac{dx\left(-\frac{d^2x^2}{2}+3\right)\sin(dx)}{4\sqrt{\pi}}\right)}{d^4}+\frac{8b^2\sqrt{\pi}\cos(c)\left(\frac{xd\left(-\frac{5d^2x^2}{2}+15\right)\cos(dx)}{20\sqrt{\pi}}-\frac{\left(-1\right.}{}\right.}{d^4}$

```
[In] int(x*(b*x+a)^2*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^3*(b^2*d^2*x^3+2*a*b*d^2*x^2+a^2*d^2*x-6*b^2*x-4*a*b)*cos(d*x+c)+(3*b^
2*d^2*x^2+4*a*b*d^2*x+a^2*d^2-6*b^2)/d^4*sin(d*x+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.70

$$\int x(a+bx)^2\sin(c+dx)\,dx = -\frac{(b^2d^3x^3+2\,abd^3x^2-4\,abd+(a^2d^3-6\,b^2d)x)\cos{(dx+c)}-(3\,b^2d^2x^2+4\,abd^2x+a^2d^2-6\,b^2)\sin{(dx+}}{d^4}$$

```
[In] integrate(x*(b*x+a)^2*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b^2*d^3*x^3 + 2*a*b*d^3*x^2 - 4*a*b*d + (a^2*d^3 - 6*b^2*d)*x)*cos(d*x +
 c) - (3*b^2*d^2*x^2 + 4*a*b*d^2*x + a^2*d^2 - 6*b^2)*sin(d*x + c))/d^4
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.27

$$\int x(a+bx)^2\sin(c+dx)\,dx = \begin{cases} -\frac{a^2x\cos{(c+dx)}}{d}+\frac{a^2\sin{(c+dx)}}{d^2}-\frac{2abx^2\cos{(c+dx)}}{d}+\frac{4abx\sin{(c+dx)}}{d^2}+\frac{4ab\cos{(c+dx)}}{d^3}-\frac{b^2x^3\cos{(c+dx)}}{d}+\frac{3b^2x^2\sin{(c+dx)}}{d^2}+ \\ \left(\frac{a^2x^2}{2}+\frac{2abx^3}{3}+\frac{b^2x^4}{4}\right)\sin{(c)} \end{cases}$$

```
[In] integrate(x*(b*x+a)**2*sin(d*x+c),x)
```

```
[Out] Piecewise((-a**2*x*cos(c + d*x)/d + a**2*sin(c + d*x)/d**2 - 2*a*b*x**2*cos
(c + d*x)/d + 4*a*b*x*sin(c + d*x)/d**2 + 4*a*b*cos(c + d*x)/d**3 - b**2*x*
*3*cos(c + d*x)/d + 3*b**2*x**2*sin(c + d*x)/d**2 + 6*b**2*x*cos(c + d*x)/d
**3 - 6*b**2*sin(c + d*x)/d**4, Ne(d, 0)), ((a**2*x**2/2 + 2*a*b*x**3/3 + b
**2*x**4/4)*sin(c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.92

$$\int x(a+bx)^2\sin(c+dx)\,dx$$

$$=\frac{a^2c\cos\left(dx+c\right)+\frac{b^2c^3\cos(dx+c)}{d^2}-\frac{2\,abc^2\cos(dx+c)}{d}-\left(\left(dx+c\right)\cos\left(dx+c\right)-\sin\left(dx+c\right)\right)a^2-\frac{3\,((dx+c)\cos(dx+c}{d^2}}{}$$

```
[In] integrate(x*(b*x+a)^2*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] (a^2*c*cos(d*x + c) + b^2*c^3*cos(d*x + c)/d^2 - 2*a*b*c^2*cos(d*x + c)/d -
 ((d*x + c)*cos(d*x + c) - sin(d*x + c))*a^2 - 3*((d*x + c)*cos(d*x + c) -
sin(d*x + c))*b^2*c^2/d^2 + 4*((d*x + c)*cos(d*x + c) - sin(d*x + c))*a*b*c
/d + 3*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b^2*c/d^
2 - 2*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a*b/d - (
((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c)
)*b^2/d^2)/d^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.70

$$\int x(a+bx)^2\sin(c+dx)\,dx=-\frac{\left(b^2d^3x^3+2\,abd^3x^2+a^2d^3x-6\,b^2dx-4\,abd\right)\cos\left(dx+c\right)}{d^4}+\frac{\left(3\,b^2d^2x^2+4\,abd^2x+a^2d^2-6\,b^2\right)\sin\left(dx+c\right)}{d^4}$$

```
[In] integrate(x*(b*x+a)^2*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b^2*d^3*x^3 + 2*a*b*d^3*x^2 + a^2*d^3*x - 6*b^2*d*x - 4*a*b*d)*cos(d*x +
c)/d^4 + (3*b^2*d^2*x^2 + 4*a*b*d^2*x + a^2*d^2 - 6*b^2)*sin(d*x + c)/d^4
```

Mupad [B] (verification not implemented)

Time = 6.03 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.95

$$\int x(a+bx)^2\sin(c+dx)\,dx=\frac{3\,b^2\,x^2\,\sin\left(c+d\,x\right)}{d^2}-\frac{b^2\,x^3\,\cos\left(c+d\,x\right)}{d}-\frac{\sin\left(c+d\,x\right)\,\left(6\,b^2-a^2\,d^2\right)}{d^4}+\frac{4\,a\,b\,\cos\left(c+d\,x\right)}{d^3}+\frac{x\,\cos\left(c+d\,x\right)\,\left(6\,b^2-a^2\,d^2\right)}{d^3}-\frac{2\,a\,b\,x^2\,\cos\left(c+d\,x\right)}{d}+\frac{4\,a\,b\,x\,\sin\left(c+d\,x\right)}{d^2}$$

```
[In] int(x*sin(c + d*x)*(a + b*x)^2,x)
```

```
[Out] (3*b^2*x^2*sin(c + d*x))/d^2 - (b^2*x^3*cos(c + d*x))/d - (sin(c + d*x)*(6*
b^2 - a^2*d^2))/d^4 + (4*a*b*cos(c + d*x))/d^3 + (x*cos(c + d*x)*(6*b^2 - a
^2*d^2))/d^3 - (2*a*b*x^2*cos(c + d*x))/d + (4*a*b*x*sin(c + d*x))/d^2
```

3.12 $\int (a+bx)^2 \sin(c+dx) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 50

$$\int (a+bx)^2 \sin(c+dx) \, dx = \frac{2b^2 \cos(c+dx)}{d^3} - \frac{(a+bx)^2 \cos(c+dx)}{d} + \frac{2b(a+bx)\sin(c+dx)}{d^2}$$

```
[Out] 2*b^2*cos(d*x+c)/d^3-(b*x+a)^2*cos(d*x+c)/d+2*b*(b*x+a)*sin(d*x+c)/d^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3377, 2718}

$$\int (a+bx)^2 \sin(c+dx) \, dx = \frac{2b(a+bx)\sin(c+dx)}{d^2} - \frac{(a+bx)^2 \cos(c+dx)}{d} + \frac{2b^2 \cos(c+dx)}{d^3}$$

```
[In] Int[(a + b*x)^2*Sin[c + d*x],x]
```

```
[Out] (2*b^2*Cos[c + d*x])/d^3 - ((a + b*x)^2*Cos[c + d*x])/d + (2*b*(a + b*x)*Si
n[c + d*x])/d^2
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(a+bx)^2\cos(c+dx)}{d}+\frac{(2b)\int(a+bx)\cos(c+dx)\,dx}{d}\\&= -\frac{(a+bx)^2\cos(c+dx)}{d}+\frac{2b(a+bx)\sin(c+dx)}{d^2}-\frac{(2b^2)\int\sin(c+dx)\,dx}{d^2}\\&= \frac{2b^2\cos(c+dx)}{d^3}-\frac{(a+bx)^2\cos(c+dx)}{d}+\frac{2b(a+bx)\sin(c+dx)}{d^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.14

$$\int (a+bx)^2\sin(c+dx)\,dx = \frac{-((a^2d^2+2abd^2x+b^2(-2+d^2x^2))\cos(c+dx))+2bd(a+bx)\sin(c+dx)}{d^3}$$

```
[In] Integrate[(a + b*x)^2*Sin[c + d*x],x]
[Out] (-((a^2*d^2 + 2*a*b*d^2*x + b^2*(-2 + d^2*x^2))*Cos[c + d*x]) + 2*b*d*(a +
b*x)*Sin[c + d*x])/d^3
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.22

method	result
risch	$-\frac{\left(d^{2} x^{2} b^{2}+2 a b\, d^{2} x+d^{2} a^{2}-2 b^{2}\right) \cos \left(d x+c\right)}{d^{3}}+\frac{2 b \left(b x+a\right) \sin \left(d x+c\right)}{d^{2}}$
parallelrisch	$\frac{2\left(\frac{b x}{2}+a\right) x\, d^{2} b\left(\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)+4 b d \left(b x+a\right) \tan\left(\frac{d x}{2}+\frac{c}{2}\right)+\left(-x^{2} b^{2}-2 a b x-2 a^{2}\right) d^{2}+4 b^{2}}{d^{3}\left(1+\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}$
parts	$-\frac{b^{2} x^{2} \cos \left(d x+c\right)}{d}-\frac{2 a b x \cos \left(d x+c\right)}{d}-\frac{a^{2} \cos \left(d x+c\right)}{d}+\frac{2 b\left(a \sin \left(d x+c\right)-\frac{b c \sin \left(d x+c\right)}{d}+\frac{b\left(\cos \left(d x+c\right)+\left(d x+c\right) \sin \left(d x+c\right.\right.}{d}\right.}{d^{2}}$
norman	$\frac{\frac{b^{2} x^{2}\left(\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2 d^{2} a^{2}-4 b^{2}}{d^{3}}-\frac{b^{2} x^{2}}{d}+\frac{4 b^{2} x \tan\left(\frac{d x}{2}+\frac{c}{2}\right)}{d^{2}}+\frac{4 a b \tan\left(\frac{d x}{2}+\frac{c}{2}\right)}{d^{2}}-\frac{2 a b x}{d}+\frac{2 a b x\left(\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{d}}{1+\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)}$
derivativedivides	$\frac{-a^{2} \cos \left(d x+c\right)+\frac{2 a b c \cos \left(d x+c\right)}{d}+\frac{2 a b\left(\sin \left(d x+c\right)-\cos \left(d x+c\right)\left(d x+c\right)\right)}{d}-\frac{b^{2} c^{2} \cos \left(d x+c\right)}{d^{2}}-\frac{2 b^{2} c\left(\sin \left(d x+c\right)-\cos \left(d x+c\right)\left(d x+c\right)\right)}{d^{2}}+\frac{b^{2}(}{}}{d}$
default	$\frac{-a^{2} \cos \left(d x+c\right)+\frac{2 a b c \cos \left(d x+c\right)}{d}+\frac{2 a b\left(\sin \left(d x+c\right)-\cos \left(d x+c\right)\left(d x+c\right)\right)}{d}-\frac{b^{2} c^{2} \cos \left(d x+c\right)}{d^{2}}-\frac{2 b^{2} c\left(\sin \left(d x+c\right)-\cos \left(d x+c\right)\left(d x+c\right)\right)}{d^{2}}+\frac{b^{2}(}{}}{d}$
meijerg	$\frac{4 b^{2} \sqrt{\pi}\, \sin \left(c\right)\left(\frac{x\left(d^{2}\right)^{\frac{3}{2}} \cos \left(d x\right)}{2 \sqrt{\pi}\, d^{2}}-\frac{\left(d^{2}\right)^{\frac{3}{2}}\left(-\frac{3 d^{2} x^{2}}{2}+3\right) \sin \left(d x\right)}{6 \sqrt{\pi}\, d^{3}}\right)}{d^{2} \sqrt{d^{2}}}+\frac{4 b^{2} \sqrt{\pi}\, \cos \left(c\right)\left(-\frac{1}{2 \sqrt{\pi}}+\frac{\left(-\frac{d^{2} x^{2}}{2}+1\right) \cos \left(d x\right)}{2 \sqrt{\pi}}+\frac{d x \sin (d}{2 \sqrt{\pi}}\right.}{d^{3}}$

```
[In] int((b*x+a)^2*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2-2*b^2)/d^3*cos(d*x+c)+2*b*(b*x+a)*sin(d*x
+c)/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.26

$$\int (a+bx)^2\sin(c+dx)\,dx = -\frac{\left(b^2d^2x^2+2\,abd^2x+a^2d^2-2\,b^2\right)\cos\left(dx+c\right)-2\left(b^2dx+abd\right)\sin\left(dx+c\right)}{d^3}$$

```
[In] integrate((b*x+a)^2*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2 - 2*b^2)*cos(d*x + c) - 2*(b^2*d*x +
 a*b*d)*sin(d*x + c))/d^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(48) = 96.

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.24

$$\int (a+bx)^2 \sin(c+dx)\,dx$$
$$= \begin{cases} -\frac{a^2\cos(c+dx)}{d} - \frac{2abx\cos(c+dx)}{d} + \frac{2ab\sin(c+dx)}{d^2} - \frac{b^2x^2\cos(c+dx)}{d} + \frac{2b^2x\sin(c+dx)}{d^2} + \frac{2b^2\cos(c+dx)}{d^3} & \text{for } d \neq 0 \\ \left(a^2x + abx^2 + \frac{b^2x^3}{3}\right)\sin(c) & \text{otherwise} \end{cases}$$

```
[In] integrate((b*x+a)**2*sin(d*x+c),x)
[Out] Piecewise((-a**2*cos(c + d*x)/d - 2*a*b*x*cos(c + d*x)/d + 2*a*b*sin(c + d*
x)/d**2 - b**2*x**2*cos(c + d*x)/d + 2*b**2*x*sin(c + d*x)/d**2 + 2*b**2*co
s(c + d*x)/d**3, Ne(d, 0)), ((a**2*x + a*b*x**2 + b**2*x**3/3)*sin(c), True
))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(50) = 100.

Time = 0.19 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.82

$$\int (a+bx)^2 \sin(c+dx)\,dx =$$
$$-\frac{a^2\cos(dx+c) + \frac{b^2c^2\cos(dx+c)}{d^2} - \frac{2\,abc\cos(dx+c)}{d} - \frac{2\,((dx+c)\cos(dx+c)-\sin(dx+c))b^2c}{d^2} + \frac{2\,((dx+c)\cos(dx+c)-\sin(dx+c))ab}{d} -}{d}$$

```
[In] integrate((b*x+a)^2*sin(d*x+c),x, algorithm="maxima")
[Out] -(a^2*cos(d*x + c) + b^2*c^2*cos(d*x + c)/d^2 - 2*a*b*c*cos(d*x + c)/d - 2*
((d*x + c)*cos(d*x + c) - sin(d*x + c))*b^2*c/d^2 + 2*((d*x + c)*cos(d*x +
c) - sin(d*x + c))*a*b/d + (((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*si
n(d*x + c))*b^2/d^2)/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.30

$$\int (a+bx)^2 \sin(c+dx)\,dx = -\frac{(b^2d^2x^2 + 2\,abd^2x + a^2d^2 - 2\,b^2)\cos(dx+c)}{d^3} + \frac{2\,(b^2dx + abd)\sin(dx+c)}{d^3}$$

```
[In] integrate((b*x+a)^2*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2 - 2*b^2)*cos(d*x + c)/d^3 + 2*(b^2*d*
x + a*b*d)*sin(d*x + c)/d^3
```

Mupad [B] (verification not implemented)

Time = 5.93 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.68

$$\int (a+bx)^2 \sin(c+dx)\,dx = \frac{\cos(c+d\,x)\,\left(2\,b^2 - a^2\,d^2\right)}{d^3} - \frac{b^2\,x^2\,\cos(c+d\,x)}{d} + \frac{2\,a\,b\,\sin(c+d\,x)}{d^2} + \frac{2\,b^2\,x\,\sin(c+d\,x)}{d^2} - \frac{2\,a\,b\,x\,\cos(c+d\,x)}{d}$$

```
[In] int(sin(c + d*x)*(a + b*x)^2,x)
```

```
[Out] (cos(c + d*x)*(2*b^2 - a^2*d^2))/d^3 - (b^2*x^2*cos(c + d*x))/d + (2*a*b*si
n(c + d*x))/d^2 + (2*b^2*x*sin(c + d*x))/d^2 - (2*a*b*x*cos(c + d*x))/d
```

3.13 $\int \frac{(a+bx)^2 \sin(c+dx)}{x} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 62

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x} \, dx = -\frac{2ab\cos(c+dx)}{d} - \frac{b^2 x\cos(c+dx)}{d} + a^2 \operatorname{CosIntegral}(dx)\sin(c) + \frac{b^2\sin(c+dx)}{d^2} + a^2\cos(c)\operatorname{Si}(dx)$$

```
[Out] -2*a*b*cos(d*x+c)/d-b^2*x*cos(d*x+c)/d+a^2*cos(c)*Si(d*x)+a^2*Ci(d*x)*sin(c
)+b^2*sin(d*x+c)/d^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {6874, 2718, 3384, 3380, 3383, 3377, 2717}

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x} \, dx = a^2 \sin(c) \operatorname{CosIntegral}(dx) + a^2 \cos(c)\operatorname{Si}(dx) - \frac{2ab\cos(c+dx)}{d} + \frac{b^2\sin(c+dx)}{d^2} - \frac{b^2 x\cos(c+dx)}{d}$$

```
[In] Int[((a + b*x)^2*Sin[c + d*x])/x,x]
```

```
[Out] (-2*a*b*Cos[c + d*x])/d - (b^2*x*Cos[c + d*x])/d + a^2*CosIntegral[d*x]*Sin
[c] + (b^2*Sin[c + d*x])/d^2 + a^2*Cos[c]*SinIntegral[d*x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(2ab\sin(c+dx) + \frac{a^2\sin(c+dx)}{x} + b^2x\sin(c+dx)\right)\,dx \\
&= a^2\int \frac{\sin(c+dx)}{x}\,dx + (2ab)\int \sin(c+dx)\,dx + b^2\int x\sin(c+dx)\,dx \\
&= -\frac{2ab\cos(c+dx)}{d} - \frac{b^2x\cos(c+dx)}{d} + \frac{b^2\int\cos(c+dx)\,dx}{d} \\
&\quad + \left(a^2\cos(c)\right)\int\frac{\sin(dx)}{x}\,dx + \left(a^2\sin(c)\right)\int\frac{\cos(dx)}{x}\,dx
\end{aligned}$$

$$= -\frac{2ab\cos(c+dx)}{d} - \frac{b^2x\cos(c+dx)}{d} + a^2\,\text{CosIntegral}(dx)\sin(c) + \frac{b^2\sin(c+dx)}{d^2} + a^2\cos(c)\text{Si}(dx)$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^2\sin(c+dx)}{x}\,dx = a^2\,\text{CosIntegral}(dx)\sin(c) + \frac{b(-d(2a+bx)\cos(c+dx)+b\sin(c+dx))}{d^2} + a^2\cos(c)\text{Si}(dx)$$

```
[In] Integrate[((a + b*x)^2*Sin[c + d*x])/x,x]
```

```
[Out] a^2*CosIntegral[d*x]*Sin[c] + (b*(-(d*(2*a + b*x)*Cos[c + d*x]) + b*Sin[c +
 d*x]))/d^2 + a^2*Cos[c]*SinIntegral[d*x]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.27

method	result
derivativedivides	$a^2(\text{Si}(dx)\cos(c)+\text{Ci}(dx)\sin(c)) - \frac{2ab\cos(dx+c)}{d} + \frac{2b^2c\cos(dx+c)}{d^2} + \frac{(c+1)b^2(\sin(dx+c)-\cos(dx+c)\cdots}{d^2}$
default	$a^2(\text{Si}(dx)\cos(c)+\text{Ci}(dx)\sin(c)) - \frac{2ab\cos(dx+c)}{d} + \frac{2b^2c\cos(dx+c)}{d^2} + \frac{(c+1)b^2(\sin(dx+c)-\cos(dx+c)\cdots}{d^2}$
risch	$-\frac{e^{-ic}\pi\,\text{csgn}(dx)a^2}{2} - \frac{ie^{-ic}\,\text{Ei}_1(-idx)a^2}{2} + \frac{ia^2e^{ic}\,\text{Ei}_1(-idx)}{2} + e^{-ic}\,\text{Si}(dx)\,a^2 - \frac{b^2x\cos(dx+c)}{d} - \frac{2ab\cos(d\cdots}{d}$
meijerg	$\frac{2b^2\sqrt{\pi}\,\sin(c)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(dx)}{2\sqrt{\pi}}+\frac{dx\sin(dx)}{2\sqrt{\pi}}\right)}{d^2} + \frac{2b^2\sqrt{\pi}\,\cos(c)\left(-\frac{dx\cos(dx)}{2\sqrt{\pi}}+\frac{\sin(dx)}{2\sqrt{\pi}}\right)}{d^2} + \frac{2ab\sin(c)\sin(dx)}{d} + \frac{2ab\sqrt{\pi}\,\text{co}\cdots}{}$

```
[In] int((b*x+a)^2*sin(d*x+c)/x,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))-2*a*b*cos(d*x+c)/d+2/d^2*b^2*c*cos(d*x+
c)+(c+1)/d^2*b^2*(sin(d*x+c)-cos(d*x+c)*(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)^2\sin(c+dx)}{x}\,dx = \frac{a^2d^2\,\mathrm{Ci}\,(dx)\sin{(c)}+a^2d^2\cos{(c)}\,\mathrm{Si}\,(dx)+b^2\sin{(dx+c)}-(b^2dx+2\,abd)\cos{(dx+c)}}{d^2}$$

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x,x, algorithm="fricas")
```

```
[Out] (a^2*d^2*cos_integral(d*x)*sin(c) + a^2*d^2*cos(c)*sin_integral(d*x) + b^2*
sin(d*x + c) - (b^2*d*x + 2*a*b*d)*cos(d*x + c))/d^2
```

Sympy [A] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.48

$$\begin{aligned}\int \frac{(a+bx)^2\sin(c+dx)}{x}\,dx &= a^2\sin{(c)}\,\mathrm{Ci}\,(dx)+a^2\cos{(c)}\,\mathrm{Si}\,(dx)\\ &\quad+2ab\left(\begin{cases}x\sin{(c)} & \text{for } d=0\\ -\frac{\cos{(c+dx)}}{d} & \text{otherwise}\end{cases}\right)\\ &\quad+b^2x\left(\begin{cases}x\sin{(c)} & \text{for } d=0\\ -\frac{\cos{(c+dx)}}{d} & \text{otherwise}\end{cases}\right)\\ &\quad-b^2\left(\begin{cases}\frac{x^2\sin{(c)}}{2} & \text{for } d=0\\ -\frac{\begin{cases}\frac{\sin{(c+dx)}}{d} & \text{for } d\neq 0\\ x\cos{(c)} & \text{otherwise}\end{cases}}{d} & \text{otherwise}\end{cases}\right)\end{aligned}$$

```
[In] integrate((b*x+a)**2*sin(d*x+c)/x,x)
```

```
[Out] a**2*sin(c)*Ci(d*x) + a**2*cos(c)*Si(d*x) + 2*a*b*Piecewise((x*sin(c), Eq(d
, 0)), (-cos(c + d*x)/d, True)) + b**2*x*Piecewise((x*sin(c), Eq(d, 0)), (-
cos(c + d*x)/d, True)) - b**2*Piecewise((x**2*sin(c)/2, Eq(d, 0)), (-Piecew
ise((sin(c + d*x)/d, Ne(d, 0)), (x*cos(c), True))/d, True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.29

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x} dx$$
$$= \frac{(a^2(-i\,\mathrm{Ei}(i\,dx) + i\,\mathrm{Ei}(-i\,dx))\cos(c) + a^2(\mathrm{Ei}(i\,dx) + \mathrm{Ei}(-i\,dx))\sin(c))d^2 + 2\,b^2\sin(dx+c) - 2\,(b^2dx + 2}{2\,d^2}$$

[In] integrate((b*x+a)^2*sin(d*x+c)/x,x, algorithm="maxima")

[Out] 1/2*((a^2*(-I*Ei(I*d*x) + I*Ei(-I*d*x))*cos(c) + a^2*(Ei(I*d*x) + Ei(-I*d*x
))*sin(c))*d^2 + 2*b^2*sin(d*x + c) - 2*(b^2*d*x + 2*a*b*d)*cos(d*x + c))/d
^2

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.28 (sec) , antiderivative size = 551, normalized size of antiderivative = 8.89

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x} dx =$$
$$-\frac{a^2d^2\Im(\mathrm{Ci}\,(dx))\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\tan\left(\frac{1}{2}\,c\right)^2 - a^2d^2\Im(\mathrm{Ci}\,(-dx))\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\tan\left(\frac{1}{2}\,c\right)^2 + 2\,a^2d^2\,\mathrm{Si}\,(d}{}$$

[In] integrate((b*x+a)^2*sin(d*x+c)/x,x, algorithm="giac")

[Out] -1/2*(a^2*d^2*imag_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c
)^2 - a^2*d^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*
c)^2 + 2*a^2*d^2*sin_integral(d*x)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - 2*
a^2*d^2*real_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c) - 2*
a^2*d^2*real_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c) - 2
*b^2*d*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - a^2*d^2*imag_part(cos_integr
al(d*x))*tan(1/2*d*x + 1/2*c)^2 + a^2*d^2*imag_part(cos_integral(-d*x))*tan
(1/2*d*x + 1/2*c)^2 - 2*a^2*d^2*sin_integral(d*x)*tan(1/2*d*x + 1/2*c)^2 +
a^2*d^2*imag_part(cos_integral(d*x))*tan(1/2*c)^2 - a^2*d^2*imag_part(cos_i
ntegral(-d*x))*tan(1/2*c)^2 + 2*a^2*d^2*sin_integral(d*x)*tan(1/2*c)^2 - 4*
a*b*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - 2*b^2*d*x*tan(1/2*d*x + 1/2*c)^
2 - 2*a^2*d^2*real_part(cos_integral(d*x))*tan(1/2*c) - 2*a^2*d^2*real_part
(cos_integral(-d*x))*tan(1/2*c) + 2*b^2*d*x*tan(1/2*c)^2 - a^2*d^2*imag_par
t(cos_integral(d*x)) + a^2*d^2*imag_part(cos_integral(-d*x)) - 2*a^2*d^2*si
n_integral(d*x) - 4*a*b*d*tan(1/2*d*x + 1/2*c)^2 + 4*a*b*d*tan(1/2*c)^2 - 4
*b^2*tan(1/2*d*x + 1/2*c)*tan(1/2*c)^2 + 2*b^2*d*x + 4*a*b*d - 4*b^2*tan(1/
2*d*x + 1/2*c))/(d^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + d^2*tan(1/2*d*x
+ 1/2*c)^2 + d^2*tan(1/2*c)^2 + d^2)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2\sin(c+dx)}{x}\,dx = b^2\cos(c)\left(\frac{\sin(d\,x)}{d^2}-\frac{x\cos(d\,x)}{d}\right) + b^2\sin(c)\left(\frac{\cos(d\,x)}{d^2}+\frac{x\sin(d\,x)}{d}\right) + a^2\,\mathrm{cosint}(d\,x)\sin(c) + a^2\,\mathrm{sinint}(d\,x)\cos(c) - \frac{2\,a\,b\cos(d\,x)\cos(c)}{d} + \frac{2\,a\,b\sin(d\,x)\sin(c)}{d}$$

```
[In] int((sin(c + d*x)*(a + b*x)^2)/x,x)
```

```
[Out] b^2*cos(c)*(sin(d*x)/d^2 - (x*cos(d*x))/d) + b^2*sin(c)*(cos(d*x)/d^2 + (x*
sin(d*x))/d) + a^2*cosint(d*x)*sin(c) + a^2*sinint(d*x)*cos(c) - (2*a*b*cos
(d*x)*cos(c))/d + (2*a*b*sin(d*x)*sin(c))/d
```

3.14 $\int \frac{(a+bx)^2 \sin(c+dx)}{x^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 72

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x^2} dx = -\frac{b^2 \cos(c+dx)}{d} + a^2 d \cos(c) \operatorname{CosIntegral}(dx) + 2ab \operatorname{CosIntegral}(dx) \sin(c) - \frac{a^2 \sin(c+dx)}{x} + 2ab \cos(c) \operatorname{Si}(dx) - a^2 d \sin(c) \operatorname{Si}(dx)$$

```
[Out] a^2*d*Ci(d*x)*cos(c)-b^2*cos(d*x+c)/d+2*a*b*cos(c)*Si(d*x)+2*a*b*Ci(d*x)*si
n(c)-a^2*d*Si(d*x)*sin(c)-a^2*sin(d*x+c)/x
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {6874, 2718, 3378, 3384, 3380, 3383}

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x^2} dx = a^2 d \cos(c) \operatorname{CosIntegral}(dx) - a^2 d \sin(c) \operatorname{Si}(dx) - \frac{a^2 \sin(c+dx)}{x} + 2ab \sin(c) \operatorname{CosIntegral}(dx) + 2ab \cos(c) \operatorname{Si}(dx) - \frac{b^2 \cos(c+dx)}{d}$$

```
[In] Int[((a + b*x)^2*Sin[c + d*x])/x^2,x]
```

```
[Out] -((b^2*Cos[c + d*x])/d) + a^2*d*Cos[c]*CosIntegral[d*x] + 2*a*b*CosIntegral
[d*x]*Sin[c] - (a^2*Sin[c + d*x])/x + 2*a*b*Cos[c]*SinIntegral[d*x] - a^2*d
*Sin[c]*SinIntegral[d*x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(b^2 \sin(c+dx) + \frac{a^2 \sin(c+dx)}{x^2} + \frac{2ab \sin(c+dx)}{x}\right) dx \\
&= a^2 \int \frac{\sin(c+dx)}{x^2} \, dx + (2ab) \int \frac{\sin(c+dx)}{x} \, dx + b^2 \int \sin(c+dx) \, dx \\
&= -\frac{b^2 \cos(c+dx)}{d} - \frac{a^2 \sin(c+dx)}{x} + \left(a^2 d\right) \int \frac{\cos(c+dx)}{x} \, dx \\
&\quad + (2ab \cos(c)) \int \frac{\sin(dx)}{x} \, dx + (2ab \sin(c)) \int \frac{\cos(dx)}{x} \, dx
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b^2\cos(c+dx)}{d} + 2ab\,\text{CosIntegral}(dx)\sin(c) - \frac{a^2\sin(c+dx)}{x} \\
&\quad + 2ab\cos(c)\text{Si}(dx) + \left(a^2d\cos(c)\right)\int\frac{\cos(dx)}{x}\,dx - \left(a^2d\sin(c)\right)\int\frac{\sin(dx)}{x}\,dx \\
&= -\frac{b^2\cos(c+dx)}{d} + a^2d\cos(c)\,\text{CosIntegral}(dx) + 2ab\,\text{CosIntegral}(dx)\sin(c) \\
&\quad - \frac{a^2\sin(c+dx)}{x} + 2ab\cos(c)\text{Si}(dx) - a^2d\sin(c)\text{Si}(dx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89

$$
\begin{aligned}
\int\frac{(a+bx)^2\sin(c+dx)}{x^2}\,dx &= -\frac{b^2\cos(c+dx)}{d} + a\,\text{CosIntegral}(dx)(ad\cos(c)+2b\sin(c)) \\
&\quad - \frac{a^2\sin(c+dx)}{x} - a(-2b\cos(c)+ad\sin(c))\text{Si}(dx)
\end{aligned}
$$

```
[In] Integrate[((a + b*x)^2*Sin[c + d*x])/x^2,x]
```

```
[Out] -((b^2*Cos[c + d*x])/d) + a*CosIntegral[d*x]*(a*d*Cos[c] + 2*b*Sin[c]) - (a
^2*Sin[c + d*x])/x - a*(-2*b*Cos[c] + a*d*Sin[c])*SinIntegral[d*x]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03

method	result
derivativedivides	$d\left(a^2\left(-\frac{\sin(dx+c)}{dx} - \text{Si}\left(dx\right)\sin\left(c\right) + \text{Ci}\left(dx\right)\cos\left(c\right)\right) + \frac{2ab(\text{Si}(dx)\cos(c)+\text{Ci}(dx)\sin(c))}{d} - \frac{b^2\cos(dx\right.$
default	$d\left(a^2\left(-\frac{\sin(dx+c)}{dx} - \text{Si}\left(dx\right)\sin\left(c\right) + \text{Ci}\left(dx\right)\cos\left(c\right)\right) + \frac{2ab(\text{Si}(dx)\cos(c)+\text{Ci}(dx)\sin(c))}{d} - \frac{b^2\cos(dx\right.$
risch	$i\cos\left(c\right)\text{Ei}_1\left(-idx\right)ab - \frac{d\cos(c)a^2\,\text{Ei}_1(-idx)}{2} - i\cos\left(c\right)\text{Ei}_1\left(idx\right)ab - \frac{d\cos(c)a^2\,\text{Ei}_1(idx)}{2} - \sin\left(c\right.$
meijerg	$\frac{b^2\sin(c)\sin(dx)}{d} + \frac{b^2\sqrt{\pi}\,\cos(c)\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(dx)}{\sqrt{\pi}}\right)}{d} + ab\sqrt{\pi}\,\sin\left(c\right)\left(\frac{2\gamma+2\ln(x)+\ln(d^2)}{\sqrt{\pi}} - \frac{2\gamma}{\sqrt{\pi}} - \frac{2\ln(2)}{\sqrt{\pi}} - \frac{2\ln\right.$

```
[In] int((b*x+a)^2*sin(d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] d*(a^2*(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c))+2/d*a*b*(Si(d*x)*cos
(c)+Ci(d*x)*sin(c))-1/d^2*b^2*cos(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx)^2\sin(c+dx)}{x^2}\,dx = -\frac{b^2x\cos(dx+c)+a^2d\sin(dx+c)-(a^2d^2x\,\mathrm{Ci}(dx)+2\,abdx\,\mathrm{Si}(dx))\cos(c)+(a^2d^2x\,\mathrm{Si}(dx)-2\,abdx\,\mathrm{C}}{dx}$$

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] -(b^2*x*cos(d*x + c) + a^2*d*sin(d*x + c) - (a^2*d^2*x*cos_integral(d*x) +
2*a*b*d*x*sin_integral(d*x))*cos(c) + (a^2*d^2*x*sin_integral(d*x) - 2*a*b*
d*x*cos_integral(d*x))*sin(c))/(d*x)
```

Sympy [F]

$$\int \frac{(a+bx)^2\sin(c+dx)}{x^2}\,dx = \int \frac{(a+bx)^2\sin(c+dx)}{x^2}\,dx$$

```
[In] integrate((b*x+a)**2*sin(d*x+c)/x**2,x)
```

```
[Out] Integral((a + b*x)**2*sin(c + d*x)/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.69

$$\int \frac{(a+bx)^2\sin(c+dx)}{x^2}\,dx = \frac{((a^2(\Gamma(-1,i\,dx)+\Gamma(-1,-i\,dx))\cos(c)-a^2(i\,\Gamma(-1,i\,dx)-i\,\Gamma(-1,-i\,dx))\sin(c))d^2+2\,(ab(i\,\Gamma(-1,i\,d}{2\,dx}$$

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(((a^2*(gamma(-1, I*d*x) + gamma(-1, -I*d*x))*cos(c) - a^2*(I*gamma(-1,
 I*d*x) - I*gamma(-1, -I*d*x))*sin(c))*d^2 + 2*(a*b*(I*gamma(-1, I*d*x) - I
*gamma(-1, -I*d*x))*cos(c) + a*b*(gamma(-1, I*d*x) + gamma(-1, -I*d*x))*sin
(c))*d)*x - 2*(b^2*x + 2*a*b)*cos(d*x + c))/(d*x)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.29 (sec) , antiderivative size = 743, normalized size of antiderivative = 10.32

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x^2} \, dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(a^2*d^2*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
a^2*d^2*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^2
*d^2*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^2*x
*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^2*d^2*x*sin_
integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) + 2*a*b*d*x*imag_part(cos_integral(
d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*b*d*x*imag_part(cos_integral(-d*x))
*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a*b*d*x*sin_integral(d*x)*tan(1/2*d*x)^2*t
an(1/2*c)^2 - a^2*d^2*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2 - a^2*d
^2*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 4*a*b*d*x*real_part(cos
_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 4*a*b*d*x*real_part(cos_integra
l(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + a^2*d^2*x*real_part(cos_integral(d*x))
*tan(1/2*c)^2 + a^2*d^2*x*real_part(cos_integral(-d*x))*tan(1/2*c)^2 - 2*a*
b*d*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 + 2*a*b*d*x*imag_part(cos
_integral(-d*x))*tan(1/2*d*x)^2 - 4*a*b*d*x*sin_integral(d*x)*tan(1/2*d*x)^
2 + 2*a^2*d^2*x*imag_part(cos_integral(d*x))*tan(1/2*c) - 2*a^2*d^2*x*imag_
part(cos_integral(-d*x))*tan(1/2*c) + 4*a^2*d^2*x*sin_integral(d*x)*tan(1/2
*c) + 2*a*b*d*x*imag_part(cos_integral(d*x))*tan(1/2*c)^2 - 2*a*b*d*x*imag_
part(cos_integral(-d*x))*tan(1/2*c)^2 + 4*a*b*d*x*sin_integral(d*x)*tan(1/2
*c)^2 + 2*b^2*x*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d^2*x*real_part(cos_integ
ral(d*x)) - a^2*d^2*x*real_part(cos_integral(-d*x)) - 4*a*b*d*x*real_part(c
os_integral(d*x))*tan(1/2*c) - 4*a*b*d*x*real_part(cos_integral(-d*x))*tan(
1/2*c) - 4*a^2*d*tan(1/2*d*x)^2*tan(1/2*c) - 4*a^2*d*tan(1/2*d*x)*tan(1/2*c
)^2 - 2*a*b*d*x*imag_part(cos_integral(d*x)) + 2*a*b*d*x*imag_part(cos_inte
gral(-d*x)) - 4*a*b*d*x*sin_integral(d*x) - 2*b^2*x*tan(1/2*d*x)^2 - 8*b^2*
x*tan(1/2*d*x)*tan(1/2*c) - 2*b^2*x*tan(1/2*c)^2 + 4*a^2*d*tan(1/2*d*x) + 4
*a^2*d*tan(1/2*c) + 2*b^2*x)/(d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + d*x*tan(1/2
*d*x)^2 + d*x*tan(1/2*c)^2 + d*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2\sin(c+dx)}{x^2}\,dx=\int \frac{\sin\left(c+d\,x\right)\,\left(a+b\,x\right)^2}{x^2}\,dx$$

```
[In] int((sin(c + d*x)*(a + b*x)^2)/x^2,x)
[Out] int((sin(c + d*x)*(a + b*x)^2)/x^2, x)
```

3.15 $\int \frac{(a+bx)^2 \sin(c+dx)}{x^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 121

$$\begin{aligned}\int \frac{(a+bx)^2 \sin(c+dx)}{x^3} dx = &-\frac{a^2 d \cos(c+dx)}{2x} + 2abd \cos(c) \operatorname{CosIntegral}(dx) \\ &+ b^2 \operatorname{CosIntegral}(dx) \sin(c) - \frac{1}{2} a^2 d^2 \operatorname{CosIntegral}(dx) \sin(c) \\ &- \frac{a^2 \sin(c+dx)}{2x^2} - \frac{2ab \sin(c+dx)}{x} + b^2 \cos(c) \operatorname{Si}(dx) \\ &- \frac{1}{2} a^2 d^2 \cos(c) \operatorname{Si}(dx) - 2abd \sin(c) \operatorname{Si}(dx)\end{aligned}$$

```
[Out] 2*a*b*d*Ci(d*x)*cos(c)-1/2*a^2*d*cos(d*x+c)/x+b^2*cos(c)*Si(d*x)-1/2*a^2*d^
2*cos(c)*Si(d*x)+b^2*Ci(d*x)*sin(c)-1/2*a^2*d^2*Ci(d*x)*sin(c)-2*a*b*d*Si(d
*x)*sin(c)-1/2*a^2*sin(d*x+c)/x^2-2*a*b*sin(d*x+c)/x
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6874, 3378, 3384, 3380, 3383}

$$\begin{aligned}\int \frac{(a+bx)^2 \sin(c+dx)}{x^3} dx = &-\frac{1}{2} a^2 d^2 \sin(c) \operatorname{CosIntegral}(dx) - \frac{1}{2} a^2 d^2 \cos(c) \operatorname{Si}(dx) \\ &- \frac{a^2 \sin(c+dx)}{2x^2} - \frac{a^2 d \cos(c+dx)}{2x} \\ &+ 2abd \cos(c) \operatorname{CosIntegral}(dx) - 2abd \sin(c) \operatorname{Si}(dx) \\ &- \frac{2ab \sin(c+dx)}{x} + b^2 \sin(c) \operatorname{CosIntegral}(dx) + b^2 \cos(c) \operatorname{Si}(dx)\end{aligned}$$

```
[In] Int[((a + b*x)^2*Sin[c + d*x])/x^3,x]
```

```
[Out] -1/2*(a^2*d*Cos[c + d*x])/x + 2*a*b*d*Cos[c]*CosIntegral[d*x] + b^2*CosInte
gral[d*x]*Sin[c] - (a^2*d^2*CosIntegral[d*x]*Sin[c])/2 - (a^2*Sin[c + d*x])
/(2*x^2) - (2*a*b*Sin[c + d*x])/x + b^2*Cos[c]*SinIntegral[d*x] - (a^2*d^2*
Cos[c]*SinIntegral[d*x])/2 - 2*a*b*d*Sin[c]*SinIntegral[d*x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a^2\sin(c+dx)}{x^3}+\frac{2ab\sin(c+dx)}{x^2}+\frac{b^2\sin(c+dx)}{x}\right)dx\\
&= a^2\int \frac{\sin(c+dx)}{x^3}\,dx+(2ab)\int \frac{\sin(c+dx)}{x^2}\,dx+b^2\int \frac{\sin(c+dx)}{x}\,dx\\
&= -\frac{a^2\sin(c+dx)}{2x^2}-\frac{2ab\sin(c+dx)}{x}+\frac{1}{2}\left(a^2d\right)\int \frac{\cos(c+dx)}{x^2}\,dx\\
&\quad+(2abd)\int \frac{\cos(c+dx)}{x}\,dx+\left(b^2\cos(c)\right)\int \frac{\sin(dx)}{x}\,dx+\left(b^2\sin(c)\right)\int \frac{\cos(dx)}{x}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{a^2 d \cos(c+dx)}{2x} + b^2 \operatorname{CosIntegral}(dx) \sin(c) - \frac{a^2 \sin(c+dx)}{2x^2} \\
&\quad - \frac{2ab \sin(c+dx)}{x} + b^2 \cos(c)\operatorname{Si}(dx) - \frac{1}{2}\left(a^2 d^2\right) \int \frac{\sin(c+dx)}{x}\, dx \\
&\quad + (2abd \cos(c)) \int \frac{\cos(dx)}{x}\, dx - (2abd \sin(c)) \int \frac{\sin(dx)}{x}\, dx \\
&= -\frac{a^2 d \cos(c+dx)}{2x} + 2abd \cos(c) \operatorname{CosIntegral}(dx) + b^2 \operatorname{CosIntegral}(dx) \sin(c) \\
&\quad - \frac{a^2 \sin(c+dx)}{2x^2} - \frac{2ab \sin(c+dx)}{x} + b^2 \cos(c)\operatorname{Si}(dx) - 2abd \sin(c)\operatorname{Si}(dx) \\
&\quad - \frac{1}{2}\left(a^2 d^2 \cos(c)\right) \int \frac{\sin(dx)}{x}\, dx - \frac{1}{2}\left(a^2 d^2 \sin(c)\right) \int \frac{\cos(dx)}{x}\, dx \\
&= -\frac{a^2 d \cos(c+dx)}{2x} + 2abd \cos(c) \operatorname{CosIntegral}(dx) + b^2 \operatorname{CosIntegral}(dx) \sin(c) \\
&\quad - \frac{1}{2} a^2 d^2 \operatorname{CosIntegral}(dx) \sin(c) - \frac{a^2 \sin(c+dx)}{2x^2} - \frac{2ab \sin(c+dx)}{x} \\
&\quad + b^2 \cos(c)\operatorname{Si}(dx) - \frac{1}{2} a^2 d^2 \cos(c)\operatorname{Si}(dx) - 2abd \sin(c)\operatorname{Si}(dx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.79

$$
\begin{aligned}
\int \frac{(a+bx)^2 \sin(c+dx)}{x^3}\, dx = \frac{1}{2}\Bigg(&\operatorname{CosIntegral}(dx) \left(4abd \cos(c) + \left(2b^2 - a^2 d^2\right) \sin(c)\right) \\
&- \frac{a(adx \cos(c+dx) + (a+4bx) \sin(c+dx))}{x^2} \\
&+ \left(\left(2b^2 - a^2 d^2\right) \cos(c) - 4abd \sin(c)\right) \operatorname{Si}(dx)\Bigg)
\end{aligned}
$$

```
[In] Integrate[((a + b*x)^2*Sin[c + d*x])/x^3,x]
```

```
[Out] (CosIntegral[d*x]*(4*a*b*d*Cos[c] + (2*b^2 - a^2*d^2)*Sin[c]) - (a*(a*d*x*C
os[c + d*x] + (a + 4*b*x)*Sin[c + d*x]))/x^2 + ((2*b^2 - a^2*d^2)*Cos[c] -
4*a*b*d*Sin[c])*SinIntegral[d*x])/2
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.94

method	result
derivativedivides	$d^2\left(a^2\left(-\frac{\sin(dx+c)}{2d^2x^2}-\frac{\cos(dx+c)}{2dx}-\frac{\mathrm{Si}(dx)\cos(c)}{2}-\frac{\mathrm{Ci}(dx)\sin(c)}{2}\right)+\frac{2ab\left(-\frac{\sin(dx+c)}{dx}-\mathrm{Si}(dx)\sin(c)+\mathrm{Ci}(dx)\right.}{d}\right.$
default	$d^2\left(a^2\left(-\frac{\sin(dx+c)}{2d^2x^2}-\frac{\cos(dx+c)}{2dx}-\frac{\mathrm{Si}(dx)\cos(c)}{2}-\frac{\mathrm{Ci}(dx)\sin(c)}{2}\right)+\frac{2ab\left(-\frac{\sin(dx+c)}{dx}-\mathrm{Si}(dx)\sin(c)+\mathrm{Ci}(dx)\right.}{d}\right.$
risch	$-\mathrm{Ei}_1(-idx)\cos(c)\,abd-\cos(c)\,\mathrm{Ei}_1(idx)\,abd-\frac{i\,\mathrm{Ei}_1(-idx)\cos(c)a^2d^2}{4}+\frac{i\cos(c)\,\mathrm{Ei}_1(idx)a^2d^2}{4}+$
meijerg	$\frac{b^2\sqrt{\pi}\,\sin(c)\left(\frac{2\gamma+2\ln(x)+\ln(d^2)}{\sqrt{\pi}}-\frac{2\gamma}{\sqrt{\pi}}-\frac{2\ln(2)}{\sqrt{\pi}}-\frac{2\ln\left(\frac{dx}{2}\right)}{\sqrt{\pi}}+\frac{2\,\mathrm{Ci}(dx)}{\sqrt{\pi}}\right)}{2}+b^2\cos(c)\,\mathrm{Si}(dx)+\frac{d^2ab\sqrt{\pi}\,\sin(c)\left(-\frac{4}{\ldots}\right.}{}$

[In] int((b*x+a)^2*sin(d*x+c)/x^3,x,method=_RETURNVERBOSE)

[Out] d^2*(a^2*(-1/2*sin(d*x+c)/d^2/x^2-1/2*cos(d*x+c)/d/x-1/2*Si(d*x)*cos(c)-1/2
*Ci(d*x)*sin(c))+2/d*a*b*(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c))+1/
d^2*b^2*(Si(d*x)*cos(c)+Ci(d*x)*sin(c)))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)^2\sin(c+dx)}{x^3}\,dx =$$
$$-\frac{a^2dx\cos(dx+c)-(4\,abdx^2\,\mathrm{Ci}(dx)-(a^2d^2-2\,b^2)x^2\,\mathrm{Si}(dx))\cos(c)+(4\,abx+a^2)\sin(dx+c)+(4\,a}{2\,x^2}$$

[In] integrate((b*x+a)^2*sin(d*x+c)/x^3,x, algorithm="fricas")

[Out] -1/2*(a^2*d*x*cos(d*x + c) - (4*a*b*d*x^2*cos_integral(d*x) - (a^2*d^2 - 2*
b^2)*x^2*sin_integral(d*x))*cos(c) + (4*a*b*x + a^2)*sin(d*x + c) + (4*a*b*
d*x^2*sin_integral(d*x) + (a^2*d^2 - 2*b^2)*x^2*cos_integral(d*x))*sin(c))/
x^2

Sympy [F]

$$\int \frac{(a+bx)^2\sin(c+dx)}{x^3}\,dx = \int \frac{(a+bx)^2\sin{(c+dx)}}{x^3}\,dx$$

```
[In] integrate((b*x+a)**2*sin(d*x+c)/x**3,x)
```

```
[Out] Integral((a + b*x)**2*sin(c + d*x)/x**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.11 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.56

$$\int \frac{(a+bx)^2\sin(c+dx)}{x^3}\,dx = -\frac{((a^2(-i\,\Gamma(-2,i\,dx)+i\,\Gamma(-2,-i\,dx))\cos{(c)}-a^2(\Gamma(-2,i\,dx)+\Gamma(-2,-i\,dx))\sin{(c)})d^4+4\,(ab(\Gamma(-2,i}{}$$

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(((a^2*(-I*gamma(-2, I*d*x) + I*gamma(-2, -I*d*x))*cos(c) - a^2*(gamma
(-2, I*d*x) + gamma(-2, -I*d*x))*sin(c))*d^4 + 4*(a*b*(gamma(-2, I*d*x) + g
amma(-2, -I*d*x))*cos(c) - a*b*(I*gamma(-2, I*d*x) - I*gamma(-2, -I*d*x))*s
in(c))*d^3 - 2*(b^2*(-I*gamma(-2, I*d*x) + I*gamma(-2, -I*d*x))*cos(c) - b^
2*(gamma(-2, I*d*x) + gamma(-2, -I*d*x))*sin(c))*d^2)*x^2 + 2*b^2*sin(d*x +
 c) + 2*(b^2*d*x + 2*a*b*d)*cos(d*x + c))/(d^2*x^2)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.30 (sec) , antiderivative size = 1182, normalized size of antiderivative = 9.77

$$\int \frac{(a+bx)^2\sin(c+dx)}{x^3}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(a^2*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 -
 a^2*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*
a^2*d^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*d^2*x^2*r
eal_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^2*x^2*real_
part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 4*a*b*d*x^2*real_part(
```

```
cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a*b*d*x^2*real_part(cos_
integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d^2*x^2*imag_part(cos_int
egral(d*x))*tan(1/2*d*x)^2 + a^2*d^2*x^2*imag_part(cos_integral(-d*x))*tan(
1/2*d*x)^2 - 2*a^2*d^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2 - 8*a*b*d*x^2*i
mag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 8*a*b*d*x^2*imag_pa
rt(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 16*a*b*d*x^2*sin_integra
l(d*x)*tan(1/2*d*x)^2*tan(1/2*c) + a^2*d^2*x^2*imag_part(cos_integral(d*x))
*tan(1/2*c)^2 - a^2*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 + 2*
a^2*d^2*x^2*sin_integral(d*x)*tan(1/2*c)^2 - 2*b^2*x^2*imag_part(cos_integr
al(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*b^2*x^2*imag_part(cos_integral(-d*
x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*b^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^
2*tan(1/2*c)^2 + 4*a*b*d*x^2*real_part(cos_integral(d*x))*tan(1/2*d*x)^2 +
4*a*b*d*x^2*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 2*a^2*d^2*x^2*re
al_part(cos_integral(d*x))*tan(1/2*c) - 2*a^2*d^2*x^2*real_part(cos_integra
l(-d*x))*tan(1/2*c) + 4*b^2*x^2*real_part(cos_integral(d*x))*tan(1/2*d*x)^2
*tan(1/2*c) + 4*b^2*x^2*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/
2*c) - 4*a*b*d*x^2*real_part(cos_integral(d*x))*tan(1/2*c)^2 - 4*a*b*d*x^2*
real_part(cos_integral(-d*x))*tan(1/2*c)^2 - 2*a^2*d*x*tan(1/2*d*x)^2*tan(1
/2*c)^2 - a^2*d^2*x^2*imag_part(cos_integral(d*x)) + a^2*d^2*x^2*imag_part(
cos_integral(-d*x)) - 2*a^2*d^2*x^2*sin_integral(d*x) + 2*b^2*x^2*imag_part
(cos_integral(d*x))*tan(1/2*d*x)^2 - 2*b^2*x^2*imag_part(cos_integral(-d*x)
)*tan(1/2*d*x)^2 + 4*b^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2 - 8*a*b*d*x^2
*imag_part(cos_integral(d*x))*tan(1/2*c) + 8*a*b*d*x^2*imag_part(cos_integr
al(-d*x))*tan(1/2*c) - 16*a*b*d*x^2*sin_integral(d*x)*tan(1/2*c) - 2*b^2*x^
2*imag_part(cos_integral(d*x))*tan(1/2*c)^2 + 2*b^2*x^2*imag_part(cos_integ
ral(-d*x))*tan(1/2*c)^2 - 4*b^2*x^2*sin_integral(d*x)*tan(1/2*c)^2 + 4*a*b*
d*x^2*real_part(cos_integral(d*x)) + 4*a*b*d*x^2*real_part(cos_integral(-d*
x)) + 2*a^2*d*x*tan(1/2*d*x)^2 + 4*b^2*x^2*real_part(cos_integral(d*x))*tan
(1/2*c) + 4*b^2*x^2*real_part(cos_integral(-d*x))*tan(1/2*c) + 8*a^2*d*x*ta
n(1/2*d*x)*tan(1/2*c) + 16*a*b*x*tan(1/2*d*x)^2*tan(1/2*c) + 2*a^2*d*x*tan(
1/2*c)^2 + 16*a*b*x*tan(1/2*d*x)*tan(1/2*c)^2 + 2*b^2*x^2*imag_part(cos_int
egral(d*x)) - 2*b^2*x^2*imag_part(cos_integral(-d*x)) + 4*b^2*x^2*sin_integ
ral(d*x) + 4*a^2*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^2*tan(1/2*d*x)*tan(1/2*c)^
2 - 2*a^2*d*x - 16*a*b*x*tan(1/2*d*x) - 16*a*b*x*tan(1/2*c) - 4*a^2*tan(1/2
*d*x) - 4*a^2*tan(1/2*c))/(x^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + x^2*tan(1/2*d*
x)^2 + x^2*tan(1/2*c)^2 + x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x^3}\,dx = \int \frac{\sin(c+d\,x)\,\left(a+b\,x\right)^2}{x^3}\,dx$$

```
[In] int((sin(c + d*x)*(a + b*x)^2)/x^3,x)
[Out] int((sin(c + d*x)*(a + b*x)^2)/x^3, x)
```

3.16 $\int \frac{(a+bx)^2 \sin(c+dx)}{x^4} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 175

$$\begin{aligned}\int \frac{(a+bx)^2 \sin(c+dx)}{x^4} \, dx = & -\frac{a^2 d \cos(c+dx)}{6x^2} - \frac{abd \cos(c+dx)}{x} \\ & + b^2 d \cos(c) \operatorname{CosIntegral}(dx) - \frac{1}{6} a^2 d^3 \cos(c) \operatorname{CosIntegral}(dx) \\ & - abd^2 \operatorname{CosIntegral}(dx) \sin(c) - \frac{a^2 \sin(c+dx)}{3x^3} \\ & - \frac{ab \sin(c+dx)}{x^2} - \frac{b^2 \sin(c+dx)}{x} + \frac{a^2 d^2 \sin(c+dx)}{6x} \\ & - abd^2 \cos(c) \operatorname{Si}(dx) - b^2 d \sin(c) \operatorname{Si}(dx) + \frac{1}{6} a^2 d^3 \sin(c) \operatorname{Si}(dx)\end{aligned}$$

```
[Out] b^2*d*Ci(d*x)*cos(c)-1/6*a^2*d^3*Ci(d*x)*cos(c)-1/6*a^2*d*cos(d*x+c)/x^2-a*
b*d*cos(d*x+c)/x-a*b*d^2*cos(c)*Si(d*x)-a*b*d^2*Ci(d*x)*sin(c)-b^2*d*Si(d*x
)*sin(c)+1/6*a^2*d^3*Si(d*x)*sin(c)-1/3*a^2*sin(d*x+c)/x^3-a*b*sin(d*x+c)/x
^2-b^2*sin(d*x+c)/x+1/6*a^2*d^2*sin(d*x+c)/x
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used

= {6874, 3378, 3384, 3380, 3383}

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x^4} dx = -\frac{1}{6}a^2 d^3 \cos(c) \operatorname{CosIntegral}(dx) + \frac{1}{6}a^2 d^3 \sin(c) \operatorname{Si}(dx) + \frac{a^2 d^2 \sin(c+dx)}{6x} - \frac{a^2 \sin(c+dx)}{3x^3} - \frac{a^2 d \cos(c+dx)}{6x^2} - abd^2 \sin(c) \operatorname{CosIntegral}(dx) - abd^2 \cos(c) \operatorname{Si}(dx) - \frac{ab \sin(c+dx)}{x^2} - \frac{abd \cos(c+dx)}{x} + b^2 d \cos(c) \operatorname{CosIntegral}(dx) - b^2 d \sin(c) \operatorname{Si}(dx) - \frac{b^2 \sin(c+dx)}{x}$$

```
[In] Int[((a + b*x)^2*Sin[c + d*x])/x^4,x]
```

```
[Out] -1/6*(a^2*d*Cos[c + d*x])/x^2 - (a*b*d*Cos[c + d*x])/x + b^2*d*Cos[c]*CosIn
tegral[d*x] - (a^2*d^3*Cos[c]*CosIntegral[d*x])/6 - a*b*d^2*CosIntegral[d*x
]*Sin[c] - (a^2*Sin[c + d*x])/(3*x^3) - (a*b*Sin[c + d*x])/x^2 - (b^2*Sin[c
 + d*x])/x + (a^2*d^2*Sin[c + d*x])/(6*x) - a*b*d^2*Cos[c]*SinIntegral[d*x]
 - b^2*d*Sin[c]*SinIntegral[d*x] + (a^2*d^3*Sin[c]*SinIntegral[d*x])/6
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a^2\sin(c+dx)}{x^4}+\frac{2ab\sin(c+dx)}{x^3}+\frac{b^2\sin(c+dx)}{x^2}\right)dx \\
&= a^2\int \frac{\sin(c+dx)}{x^4}\,dx+(2ab)\int \frac{\sin(c+dx)}{x^3}\,dx+b^2\int \frac{\sin(c+dx)}{x^2}\,dx \\
&= -\frac{a^2\sin(c+dx)}{3x^3}-\frac{ab\sin(c+dx)}{x^2}-\frac{b^2\sin(c+dx)}{x}+\frac{1}{3}\left(a^2d\right)\int \frac{\cos(c+dx)}{x^3}\,dx \\
&\quad +(abd)\int \frac{\cos(c+dx)}{x^2}\,dx+\left(b^2d\right)\int \frac{\cos(c+dx)}{x}\,dx \\
&= -\frac{a^2d\cos(c+dx)}{6x^2}-\frac{abd\cos(c+dx)}{x}-\frac{a^2\sin(c+dx)}{3x^3}-\frac{ab\sin(c+dx)}{x^2} \\
&\quad -\frac{b^2\sin(c+dx)}{x}-\frac{1}{6}\left(a^2d^2\right)\int \frac{\sin(c+dx)}{x^2}\,dx-\left(abd^2\right)\int \frac{\sin(c+dx)}{x}\,dx \\
&\quad +\left(b^2d\cos(c)\right)\int \frac{\cos(dx)}{x}\,dx-\left(b^2d\sin(c)\right)\int \frac{\sin(dx)}{x}\,dx \\
&= -\frac{a^2d\cos(c+dx)}{6x^2}-\frac{abd\cos(c+dx)}{x}+b^2d\cos(c)\,\text{CosIntegral}(dx) \\
&\quad -\frac{a^2\sin(c+dx)}{3x^3}-\frac{ab\sin(c+dx)}{x^2}-\frac{b^2\sin(c+dx)}{x} \\
&\quad +\frac{a^2d^2\sin(c+dx)}{6x}-b^2d\sin(c)\text{Si}(dx)-\frac{1}{6}\left(a^2d^3\right)\int \frac{\cos(c+dx)}{x}\,dx \\
&\quad -\left(abd^2\cos(c)\right)\int \frac{\sin(dx)}{x}\,dx-\left(abd^2\sin(c)\right)\int \frac{\cos(dx)}{x}\,dx \\
&= -\frac{a^2d\cos(c+dx)}{6x^2}-\frac{abd\cos(c+dx)}{x}+b^2d\cos(c)\,\text{CosIntegral}(dx) \\
&\quad -abd^2\,\text{CosIntegral}(dx)\sin(c)-\frac{a^2\sin(c+dx)}{3x^3}-\frac{ab\sin(c+dx)}{x^2} \\
&\quad -\frac{b^2\sin(c+dx)}{x}+\frac{a^2d^2\sin(c+dx)}{6x}-abd^2\cos(c)\text{Si}(dx)-b^2d\sin(c)\text{Si}(dx) \\
&\quad -\frac{1}{6}\left(a^2d^3\cos(c)\right)\int \frac{\cos(dx)}{x}\,dx+\frac{1}{6}\left(a^2d^3\sin(c)\right)\int \frac{\sin(dx)}{x}\,dx \\
&= -\frac{a^2d\cos(c+dx)}{6x^2}-\frac{abd\cos(c+dx)}{x}+b^2d\cos(c)\,\text{CosIntegral}(dx) \\
&\quad -\frac{1}{6}a^2d^3\cos(c)\,\text{CosIntegral}(dx)-abd^2\,\text{CosIntegral}(dx)\sin(c) \\
&\quad -\frac{a^2\sin(c+dx)}{3x^3}-\frac{ab\sin(c+dx)}{x^2}-\frac{b^2\sin(c+dx)}{x}+\frac{a^2d^2\sin(c+dx)}{6x} \\
&\quad -abd^2\cos(c)\text{Si}(dx)-b^2d\sin(c)\text{Si}(dx)+\frac{1}{6}a^2d^3\sin(c)\text{Si}(dx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)^2\sin(c+dx)}{x^4}\,dx =$$
$$-\frac{a^2dx\cos(c+dx)+6abdx^2\cos(c+dx)+dx^3\,\text{CosIntegral}(dx)\left((-6b^2+a^2d^2)\cos(c)+6abd\sin(c)\right)+2a^2\cdots}{\cdots}$$

```
[In] Integrate[((a + b*x)^2*Sin[c + d*x])/x^4,x]
```

```
[Out] -1/6*(a^2*d*x*Cos[c + d*x] + 6*a*b*d*x^2*Cos[c + d*x] + d*x^3*CosIntegral[d
*x]*((-6*b^2 + a^2*d^2)*Cos[c] + 6*a*b*d*Sin[c]) + 2*a^2*Sin[c + d*x] + 6*a
*b*x*Sin[c + d*x] + 6*b^2*x^2*Sin[c + d*x] - a^2*d^2*x^2*Sin[c + d*x] + d*x
^3*(6*a*b*d*Cos[c] + 6*b^2*Sin[c] - a^2*d^2*Sin[c])*SinIntegral[d*x])/x^3
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.90

method	result
derivativedivides	$d^3\left(a^2\left(-\frac{\sin(dx+c)}{3d^3x^3}-\frac{\cos(dx+c)}{6d^2x^2}+\frac{\sin(dx+c)}{6dx}+\frac{\text{Si}(dx)\sin(c)}{6}-\frac{\text{Ci}(dx)\cos(c)}{6}\right)+\frac{2ab\left(-\frac{\sin(dx+c)}{2d^2x^2}-\frac{\cos(dx+c)}{2dx}\cdots\right.}{\cdots}\right.$
default	$d^3\left(a^2\left(-\frac{\sin(dx+c)}{3d^3x^3}-\frac{\cos(dx+c)}{6d^2x^2}+\frac{\sin(dx+c)}{6dx}+\frac{\text{Si}(dx)\sin(c)}{6}-\frac{\text{Ci}(dx)\cos(c)}{6}\right)+\frac{2ab\left(-\frac{\sin(dx+c)}{2d^2x^2}-\frac{\cos(dx+c)}{2dx}\cdots\right.}{\cdots}\right.$
risch	$-\frac{i\cos(c)\,\text{Ei}_1(-idx)ab\,d^2}{2}+\frac{i\cos(c)\,\text{Ei}_1(idx)ab\,d^2}{2}+\frac{\cos(c)\,\text{Ei}_1(-idx)a^2d^3}{12}+\frac{\cos(c)\,\text{Ei}_1(idx)a^2d^3}{12}-\frac{\cos(c)\,\text{Ei}_1(-i\cdots}{2}$
meijerg	$\frac{d^2b^2\sqrt{\pi}\,\sin(c)\left(-\frac{4d^2\cos\left(x\sqrt{d^2}\right)}{x\left(d^2\right)^{\frac{3}{2}}\sqrt{\pi}}-\frac{4\,\text{Si}\left(x\sqrt{d^2}\right)}{\sqrt{\pi}}\right)}{4\sqrt{d^2}}+\frac{d\,b^2\sqrt{\pi}\,\cos(c)\left(\frac{4\gamma-4+4\ln(x)+4\ln(d)}{\sqrt{\pi}}+\frac{4}{\sqrt{\pi}}-\frac{4\gamma}{\sqrt{\pi}}-\frac{4\ln(2)}{\sqrt{\pi}}-\frac{4\ln\left(\frac{dx}{2}\right)}{\sqrt{\pi}}-\cdots\right.}{4}$

```
[In] int((b*x+a)^2*sin(d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] d^3*(a^2*(-1/3*sin(d*x+c)/d^3/x^3-1/6*cos(d*x+c)/d^2/x^2+1/6*sin(d*x+c)/d/x
+1/6*Si(d*x)*sin(c)-1/6*Ci(d*x)*cos(c))+2/d*a*b*(-1/2*sin(d*x+c)/d^2/x^2-1/
2*cos(d*x+c)/d/x-1/2*Si(d*x)*cos(c)-1/2*Ci(d*x)*sin(c))+1/d^2*b^2*(-sin(d*x
+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x^4} \, dx = -\frac{(6\,abdx^2 + a^2dx)\cos(dx+c) + (6\,abd^2x^3\,\mathrm{Si}(dx) + (a^2d^3 - 6\,b^2d)x^3\,\mathrm{Ci}(dx))\cos(c) + (6\,abx - (a^2d^2 -}{6\,x^3}$$

[In] integrate((b*x+a)^2*sin(d*x+c)/x^4,x, algorithm="fricas")

[Out] -1/6*((6*a*b*d*x^2 + a^2*d*x)*cos(d*x + c) + (6*a*b*d^2*x^3*sin_integral(d*x) + (a^2*d^3 - 6*b^2*d)*x^3*cos_integral(d*x))*cos(c) + (6*a*b*x - (a^2*d^2 - 6*b^2)*x^2 + 2*a^2)*sin(d*x + c) + (6*a*b*d^2*x^3*cos_integral(d*x) - (a^2*d^3 - 6*b^2*d)*x^3*sin_integral(d*x))*sin(c))/x^3

Sympy [F]

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x^4} \, dx = \int \frac{(a+bx)^2 \sin(c+dx)}{x^4} \, dx$$

[In] integrate((b*x+a)**2*sin(d*x+c)/x**4,x)

[Out] Integral((a + b*x)**2*sin(c + d*x)/x**4, x)

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.24 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x^4} \, dx = -\frac{((a^2(\Gamma(-3,i\,dx) + \Gamma(-3,-i\,dx))\cos(c) + a^2(-i\,\Gamma(-3,i\,dx) + i\,\Gamma(-3,-i\,dx))\sin(c))d^5 - 6\,(ab(-i\,\Gamma(\cdot}{}$$

[In] integrate((b*x+a)^2*sin(d*x+c)/x^4,x, algorithm="maxima")

[Out] -1/2*(((a^2*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*cos(c) + a^2*(-I*gamma(-3, I*d*x) + I*gamma(-3, -I*d*x))*sin(c))*d^5 - 6*(a*b*(-I*gamma(-3, I*d*x) + I*gamma(-3, -I*d*x))*cos(c) - a*b*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*sin(c))*d^4 - 6*(b^2*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*cos(c) + b^2*(-I*gamma(-3, I*d*x) + I*gamma(-3, -I*d*x))*sin(c))*d^3)*x^3 + 4*b^2*sin(d*x + c) + 2*(b^2*d*x + 2*a*b*d)*cos(d*x + c))/(d^2*x^3)

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.30 (sec) , antiderivative size = 1400, normalized size of antiderivative = 8.00

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x^4} dx = \text{Too large to display}$$

[In] integrate((b*x+a)^2*sin(d*x+c)/x^4,x, algorithm="giac")

[Out] 1/12*(a^2*d^3*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + a^2*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^2*d^3*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^3*x^3*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^2*d^3*x^3*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) + 6*a*b*d^2*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 6*a*b*d^2*x^3*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 12*a*b*d^2*x^3*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d^3*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2 - a^2*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 12*a*b*d^2*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 12*a*b*d^2*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + a^2*d^3*x^3*real_part(cos_integral(d*x))*tan(1/2*c)^2 + a^2*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*c)^2 - 6*b^2*d*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 6*b^2*d*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 6*a*b*d^2*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 + 6*a*b*d^2*x^3*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 12*a*b*d^2*x^3*sin_integral(d*x)*tan(1/2*d*x)^2 + 2*a^2*d^3*x^3*imag_part(cos_integral(d*x))*tan(1/2*c) - 2*a^2*d^3*x^3*imag_part(cos_integral(-d*x))*tan(1/2*c) + 4*a^2*d^3*x^3*sin_integral(d*x)*tan(1/2*c) - 12*b^2*d*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 12*b^2*d*x^3*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 24*b^2*d*x^3*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) + 6*a*b*d^2*x^3*imag_part(cos_integral(d*x))*tan(1/2*c)^2 - 6*a*b*d^2*x^3*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 + 12*a*b*d^2*x^3*sin_integral(d*x)*tan(1/2*c)^2 - a^2*d^3*x^3*real_part(cos_integral(d*x)) - a^2*d^3*x^3*real_part(cos_integral(-d*x)) + 6*b^2*d*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2 + 6*b^2*d*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 12*a*b*d^2*x^3*real_part(cos_integral(d*x))*tan(1/2*c) - 12*a*b*d^2*x^3*real_part(cos_integral(-d*x))*tan(1/2*c) - 4*a^2*d^2*x^2*tan(1/2*d*x)^2*tan(1/2*c) - 6*b^2*d*x^3*real_part(cos_integral(d*x))*tan(1/2*c)^2 - 6*b^2*d*x^3*real_part(cos_integral(-d*x))*tan(1/2*c)^2 - 4*a^2*d^2*x^2*tan(1/2*d*x)*tan(1/2*c)^2 - 12*a*b*d*x^2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 6*a*b*d^2*x^3*imag_part(cos_integral(d*x)) + 6*a*b*d^2*x^3*imag_part(cos_integral(-d*x)) - 12*a*b*d^2*x^3*sin_integral(d*x) - 12*b^2*d*x^3*imag_part(cos_integral(d*x))*tan(1/2*c) + 12*b^2*d*x^3*imag_part(cos_integral(-d*x))*tan(1/2*c) - 24*b^2*d*x^3*sin_integral(d*x)*tan(1/2*c) - 2*a^2*d*x*t

```
an(1/2*d*x)^2*tan(1/2*c)^2 + 6*b^2*d*x^3*real_part(cos_integral(d*x)) + 6*b
^2*d*x^3*real_part(cos_integral(-d*x)) + 4*a^2*d^2*x^2*tan(1/2*d*x) + 12*a*
b*d*x^2*tan(1/2*d*x)^2 + 4*a^2*d^2*x^2*tan(1/2*c) + 48*a*b*d*x^2*tan(1/2*d*
x)*tan(1/2*c) + 24*b^2*x^2*tan(1/2*d*x)^2*tan(1/2*c) + 12*a*b*d*x^2*tan(1/2
*c)^2 + 24*b^2*x^2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*a^2*d*x*tan(1/2*d*x)^2 + 8
*a^2*d*x*tan(1/2*d*x)*tan(1/2*c) + 24*a*b*x*tan(1/2*d*x)^2*tan(1/2*c) + 2*a
^2*d*x*tan(1/2*c)^2 + 24*a*b*x*tan(1/2*d*x)*tan(1/2*c)^2 - 12*a*b*d*x^2 - 2
4*b^2*x^2*tan(1/2*d*x) - 24*b^2*x^2*tan(1/2*c) + 8*a^2*tan(1/2*d*x)^2*tan(1
/2*c) + 8*a^2*tan(1/2*d*x)*tan(1/2*c)^2 - 2*a^2*d*x - 24*a*b*x*tan(1/2*d*x)
 - 24*a*b*x*tan(1/2*c) - 8*a^2*tan(1/2*d*x) - 8*a^2*tan(1/2*c))/(x^3*tan(1/
2*d*x)^2*tan(1/2*c)^2 + x^3*tan(1/2*d*x)^2 + x^3*tan(1/2*c)^2 + x^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2\sin(c+dx)}{x^4}\,dx=\int \frac{\sin\left(c+d\,x\right)\,{\left(a+b\,x\right)}^2}{x^4}\,dx$$

```
[In] int((sin(c + d*x)*(a + b*x)^2)/x^4,x)
[Out] int((sin(c + d*x)*(a + b*x)^2)/x^4, x)
```

3.17 $\int \frac{(a+bx)^2 \sin(c+dx)}{x^5} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 248

$$\begin{aligned}\int \frac{(a+bx)^2 \sin(c+dx)}{x^5} \, dx = & -\frac{a^2 d \cos(c+dx)}{12x^3} - \frac{abd \cos(c+dx)}{3x^2} - \frac{b^2 d \cos(c+dx)}{2x} \\ & + \frac{a^2 d^3 \cos(c+dx)}{24x} - \frac{1}{3} abd^3 \cos(c) \operatorname{CosIntegral}(dx) \\ & - \frac{1}{2} b^2 d^2 \operatorname{CosIntegral}(dx) \sin(c) + \frac{1}{24} a^2 d^4 \operatorname{CosIntegral}(dx) \sin(c) \\ & - \frac{a^2 \sin(c+dx)}{4x^4} - \frac{2ab \sin(c+dx)}{3x^3} - \frac{b^2 \sin(c+dx)}{2x^2} \\ & + \frac{a^2 d^2 \sin(c+dx)}{24x^2} + \frac{abd^2 \sin(c+dx)}{3x} - \frac{1}{2} b^2 d^2 \cos(c) \operatorname{Si}(dx) \\ & + \frac{1}{24} a^2 d^4 \cos(c) \operatorname{Si}(dx) + \frac{1}{3} abd^3 \sin(c) \operatorname{Si}(dx)\end{aligned}$$

```
[Out] -1/3*a*b*d^3*Ci(d*x)*cos(c)-1/12*a^2*d*cos(d*x+c)/x^3-1/3*a*b*d*cos(d*x+c)/
x^2-1/2*b^2*d*cos(d*x+c)/x+1/24*a^2*d^3*cos(d*x+c)/x-1/2*b^2*d^2*cos(c)*Si(
d*x)+1/24*a^2*d^4*cos(c)*Si(d*x)-1/2*b^2*d^2*Ci(d*x)*sin(c)+1/24*a^2*d^4*Ci
(d*x)*sin(c)+1/3*a*b*d^3*Si(d*x)*sin(c)-1/4*a^2*sin(d*x+c)/x^4-2/3*a*b*sin(
d*x+c)/x^3-1/2*b^2*sin(d*x+c)/x^2+1/24*a^2*d^2*sin(d*x+c)/x^2+1/3*a*b*d^2*s
in(d*x+c)/x
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6874, 3378, 3384, 3380, 3383}

$$\int \frac{(a+bx)^2\sin(c+dx)}{x^5}\,dx = \frac{1}{24}a^2d^4\sin(c)\,\text{CosIntegral}(dx)+\frac{1}{24}a^2d^4\cos(c)\text{Si}(dx)+\frac{a^2d^3\cos(c+dx)}{24x}+\frac{a^2d^2\sin(c+dx)}{24x^2}-\frac{a^2\sin(c+dx)}{4x^4}-\frac{a^2d\cos(c+dx)}{12x^3}-\frac{1}{3}abd^3\cos(c)\,\text{CosIntegral}(dx)+\frac{1}{3}abd^3\sin(c)\text{Si}(dx)+\frac{abd^2\sin(c+dx)}{3x}-\frac{2ab\sin(c+dx)}{3x^3}-\frac{abd\cos(c+dx)}{3x^2}-\frac{1}{2}b^2d^2\sin(c)\,\text{CosIntegral}(dx)-\frac{1}{2}b^2d^2\cos(c)\text{Si}(dx)-\frac{b^2\sin(c+dx)}{2x^2}-\frac{b^2d\cos(c+dx)}{2x}$$

```
[In] Int[((a + b*x)^2*Sin[c + d*x])/x^5,x]
```

```
[Out] -1/12*(a^2*d*Cos[c + d*x])/x^3 - (a*b*d*Cos[c + d*x])/(3*x^2) - (b^2*d*Cos[
c + d*x])/(2*x) + (a^2*d^3*Cos[c + d*x])/(24*x) - (a*b*d^3*Cos[c]*CosIntegr
al[d*x])/3 - (b^2*d^2*CosIntegral[d*x]*Sin[c])/2 + (a^2*d^4*CosIntegral[d*x
]*Sin[c])/24 - (a^2*Sin[c + d*x])/(4*x^4) - (2*a*b*Sin[c + d*x])/(3*x^3) -
(b^2*Sin[c + d*x])/(2*x^2) + (a^2*d^2*Sin[c + d*x])/(24*x^2) + (a*b*d^2*Sin
[c + d*x])/(3*x) - (b^2*d^2*Cos[c]*SinIntegral[d*x])/2 + (a^2*d^4*Cos[c]*Si
nIntegral[d*x])/24 + (a*b*d^3*Sin[c]*SinIntegral[d*x])/3
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a^2\sin(c+dx)}{x^5}+\frac{2ab\sin(c+dx)}{x^4}+\frac{b^2\sin(c+dx)}{x^3}\right)dx\\
&= a^2\int \frac{\sin(c+dx)}{x^5}\,dx+(2ab)\int \frac{\sin(c+dx)}{x^4}\,dx+b^2\int \frac{\sin(c+dx)}{x^3}\,dx\\
&= -\frac{a^2\sin(c+dx)}{4x^4}-\frac{2ab\sin(c+dx)}{3x^3}-\frac{b^2\sin(c+dx)}{2x^2}+\frac{1}{4}\left(a^2d\right)\int \frac{\cos(c+dx)}{x^4}\,dx\\
&\quad+\frac{1}{3}(2abd)\int \frac{\cos(c+dx)}{x^3}\,dx+\frac{1}{2}\left(b^2d\right)\int \frac{\cos(c+dx)}{x^2}\,dx\\
&= -\frac{a^2d\cos(c+dx)}{12x^3}-\frac{abd\cos(c+dx)}{3x^2}-\frac{b^2d\cos(c+dx)}{2x}-\frac{a^2\sin(c+dx)}{4x^4}\\
&\quad-\frac{2ab\sin(c+dx)}{3x^3}-\frac{b^2\sin(c+dx)}{2x^2}-\frac{1}{12}\left(a^2d^2\right)\int \frac{\sin(c+dx)}{x^3}\,dx\\
&\quad-\frac{1}{3}\left(abd^2\right)\int \frac{\sin(c+dx)}{x^2}\,dx-\frac{1}{2}\left(b^2d^2\right)\int \frac{\sin(c+dx)}{x}\,dx\\
&= -\frac{a^2d\cos(c+dx)}{12x^3}-\frac{abd\cos(c+dx)}{3x^2}-\frac{b^2d\cos(c+dx)}{2x}-\frac{a^2\sin(c+dx)}{4x^4}-\frac{2ab\sin(c+dx)}{3x^3}\\
&\quad-\frac{b^2\sin(c+dx)}{2x^2}+\frac{a^2d^2\sin(c+dx)}{24x^2}+\frac{abd^2\sin(c+dx)}{3x}-\frac{1}{24}\left(a^2d^3\right)\int \frac{\cos(c+dx)}{x^2}\,dx\\
&\quad-\frac{1}{3}\left(abd^3\right)\int \frac{\cos(c+dx)}{x}\,dx-\frac{1}{2}\left(b^2d^2\cos(c)\right)\int \frac{\sin(dx)}{x}\,dx-\frac{1}{2}\left(b^2d^2\sin(c)\right)\int \frac{\cos(dx)}{x}\,dx\\
&= -\frac{a^2d\cos(c+dx)}{12x^3}-\frac{abd\cos(c+dx)}{3x^2}-\frac{b^2d\cos(c+dx)}{2x}\\
&\quad+\frac{a^2d^3\cos(c+dx)}{24x}-\frac{1}{2}b^2d^2\,\text{CosIntegral}(dx)\sin(c)-\frac{a^2\sin(c+dx)}{4x^4}\\
&\quad-\frac{2ab\sin(c+dx)}{3x^3}-\frac{b^2\sin(c+dx)}{2x^2}+\frac{a^2d^2\sin(c+dx)}{24x^2}\\
&\quad+\frac{abd^2\sin(c+dx)}{3x}-\frac{1}{2}b^2d^2\cos(c)\text{Si}(dx)+\frac{1}{24}\left(a^2d^4\right)\int \frac{\sin(c+dx)}{x}\,dx\\
&\quad-\frac{1}{3}\left(abd^3\cos(c)\right)\int \frac{\cos(dx)}{x}\,dx+\frac{1}{3}\left(abd^3\sin(c)\right)\int \frac{\sin(dx)}{x}\,dx
\end{aligned}$$

$$= -\frac{a^2 d\cos(c+dx)}{12x^3} - \frac{abd\cos(c+dx)}{3x^2} - \frac{b^2 d\cos(c+dx)}{2x} + \frac{a^2 d^3\cos(c+dx)}{24x} - \frac{1}{3}abd^3\cos(c)\,\text{CosIntegral}(dx) - \frac{1}{2}b^2 d^2\,\text{CosIntegral}(dx)\sin(c) - \frac{a^2\sin(c+dx)}{4x^4} - \frac{2ab\sin(c+dx)}{3x^3} - \frac{b^2\sin(c+dx)}{2x^2} + \frac{a^2 d^2\sin(c+dx)}{24x^2} + \frac{abd^2\sin(c+dx)}{3x} - \frac{1}{2}b^2 d^2\cos(c)\text{Si}(dx) + \frac{1}{3}abd^3\sin(c)\text{Si}(dx) + \frac{1}{24}\left(a^2 d^4\cos(c)\right)\int \frac{\sin(dx)}{x}\,dx + \frac{1}{24}\left(a^2 d^4\sin(c)\right)\int \frac{\cos(dx)}{x}\,dx$$

$$= -\frac{a^2 d\cos(c+dx)}{12x^3} - \frac{abd\cos(c+dx)}{3x^2} - \frac{b^2 d\cos(c+dx)}{2x} + \frac{a^2 d^3\cos(c+dx)}{24x} - \frac{1}{3}abd^3\cos(c)\,\text{CosIntegral}(dx) - \frac{1}{2}b^2 d^2\,\text{CosIntegral}(dx)\sin(c) + \frac{1}{24}a^2 d^4\,\text{CosIntegral}(dx)\sin(c) - \frac{a^2\sin(c+dx)}{4x^4} - \frac{2ab\sin(c+dx)}{3x^3} - \frac{b^2\sin(c+dx)}{2x^2} + \frac{a^2 d^2\sin(c+dx)}{24x^2} + \frac{abd^2\sin(c+dx)}{3x} - \frac{1}{2}b^2 d^2\cos(c)\text{Si}(dx) + \frac{1}{24}a^2 d^4\cos(c)\text{Si}(dx) + \frac{1}{3}abd^3\sin(c)\text{Si}(dx)$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^2\sin(c+dx)}{x^5}\,dx = \frac{-2a^2 dx\cos(c+dx) - 8abdx^2\cos(c+dx) - 12b^2 dx^3\cos(c+dx) + a^2 d^3 x^3\cos(c+dx) + d^2 x^4\,\text{CosIntegral}}{}$$

```
[In] Integrate[((a + b*x)^2*Sin[c + d*x])/x^5,x]
```

```
[Out] (-2*a^2*d*x*Cos[c + d*x] - 8*a*b*d*x^2*Cos[c + d*x] - 12*b^2*d*x^3*Cos[c +
d*x] + a^2*d^3*x^3*Cos[c + d*x] + d^2*x^4*CosIntegral[d*x]*(-8*a*b*d*Cos[c]
 + (-12*b^2 + a^2*d^2)*Sin[c]) - 6*a^2*Sin[c + d*x] - 16*a*b*x*Sin[c + d*x]
 - 12*b^2*x^2*Sin[c + d*x] + a^2*d^2*x^2*Sin[c + d*x] + 8*a*b*d^2*x^3*Sin[c
 + d*x] + d^2*x^4*(-12*b^2*Cos[c] + a^2*d^2*Cos[c] + 8*a*b*d*Sin[c])*SinInt
egral[d*x])/(24*x^4)
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.81

method	result
derivativedivides	$d^4\left(a^2\left(-\frac{\sin(dx+c)}{4d^4x^4}-\frac{\cos(dx+c)}{12d^3x^3}+\frac{\sin(dx+c)}{24d^2x^2}+\frac{\cos(dx+c)}{24dx}+\frac{\mathrm{Si}(dx)\cos(c)}{24}+\frac{\mathrm{Ci}(dx)\sin(c)}{24}\right)+\frac{2ab\left(-\frac{\sin(d}{3d^3}\right.}{}\right.$
default	$d^4\left(a^2\left(-\frac{\sin(dx+c)}{4d^4x^4}-\frac{\cos(dx+c)}{12d^3x^3}+\frac{\sin(dx+c)}{24d^2x^2}+\frac{\cos(dx+c)}{24dx}+\frac{\mathrm{Si}(dx)\cos(c)}{24}+\frac{\mathrm{Ci}(dx)\sin(c)}{24}\right)+\frac{2ab\left(-\frac{\sin(d}{3d^3}\right.}{}\right.$
risch	$\frac{\cos(c)\,\mathrm{Ei}_1(idx)ab\,d^3}{6}-\frac{i\cos(c)\,\mathrm{Ei}_1(idx)a^2d^4}{48}+\frac{\cos(c)\,\mathrm{Ei}_1(-idx)ab\,d^3}{6}+\frac{i\cos(c)\,\mathrm{Ei}_1(-idx)a^2d^4}{48}+\frac{i\cos(c)\,\mathrm{Ei}_1(idx)b}{4}$
meijerg	$\frac{d^2b^2\sqrt{\pi}\,\sin(c)\left(-\frac{4}{\sqrt{\pi}\,x^2d^2}-\frac{2\left(2\gamma-3+2\ln(x)+\ln\left(d^2\right)\right)}{\sqrt{\pi}}+\frac{-6d^2x^2+4}{\sqrt{\pi}\,x^2d^2}+\frac{4\gamma}{\sqrt{\pi}}+\frac{4\ln(2)}{\sqrt{\pi}}+\frac{4\ln\left(\frac{dx}{2}\right)}{\sqrt{\pi}}-\frac{4\cos(dx)}{\sqrt{\pi}\,d^2x^2}+\frac{4\sin(dx)}{\sqrt{\pi}\,dx}-\frac{4\,\mathrm{Ci}(dx)}{\sqrt{\pi}}\right)}{8}$

```
[In] int((b*x+a)^2*sin(d*x+c)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] d^4*(a^2*(-1/4*sin(d*x+c)/d^4/x^4-1/12*cos(d*x+c)/d^3/x^3+1/24*sin(d*x+c)/d
^2/x^2+1/24*cos(d*x+c)/d/x+1/24*Si(d*x)*cos(c)+1/24*Ci(d*x)*sin(c))+2*a*b/d
*(-1/3*sin(d*x+c)/d^3/x^3-1/6*cos(d*x+c)/d^2/x^2+1/6*sin(d*x+c)/d/x+1/6*Si(
d*x)*sin(c)-1/6*Ci(d*x)*cos(c))+b^2/d^2*(-1/2*sin(d*x+c)/d^2/x^2-1/2*cos(d*
x+c)/d/x-1/2*Si(d*x)*cos(c)-1/2*Ci(d*x)*sin(c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx)^2\sin(c+dx)}{x^5}\,dx = -\frac{(8\,abdx^2+2\,a^2dx-(a^2d^3-12\,b^2d)x^3)\cos(dx+c)+(8\,abd^3x^4\,\mathrm{Ci}(dx)-(a^2d^4-12\,b^2d^2)x^4\,\mathrm{Si}(dx))\cos}{}$$

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x^5,x, algorithm="fricas")
```

```
[Out] -1/24*((8*a*b*d*x^2 + 2*a^2*d*x - (a^2*d^3 - 12*b^2*d)*x^3)*cos(d*x + c) +
(8*a*b*d^3*x^4*cos_integral(d*x) - (a^2*d^4 - 12*b^2*d^2)*x^4*sin_integral(
d*x))*cos(c) - (8*a*b*d^2*x^3 - 16*a*b*x + (a^2*d^2 - 12*b^2)*x^2 - 6*a^2)*
sin(d*x + c) - (8*a*b*d^3*x^4*sin_integral(d*x) + (a^2*d^4 - 12*b^2*d^2)*x^
4*cos_integral(d*x))*sin(c))/x^4
```

Sympy [F]

$$\int \frac{(a+bx)^2\sin(c+dx)}{x^5}\,dx = \int \frac{(a+bx)^2\sin(c+dx)}{x^5}\,dx$$

```
[In] integrate((b*x+a)**2*sin(d*x+c)/x**5,x)
```

```
[Out] Integral((a + b*x)**2*sin(c + d*x)/x**5, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.36 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx)^2\sin(c+dx)}{x^5}\,dx = -\frac{((a^2(i\Gamma(-4,i\,dx)-i\Gamma(-4,-i\,dx))\cos(c)+a^2(\Gamma(-4,i\,dx)+\Gamma(-4,-i\,dx))\sin(c))d^6-8\,(ab(\Gamma(-4,i}{}$$

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x^5,x, algorithm="maxima")
```

```
[Out] -1/2*(((a^2*(I*gamma(-4, I*d*x) - I*gamma(-4, -I*d*x))*cos(c) + a^2*(gamma(
-4, I*d*x) + gamma(-4, -I*d*x))*sin(c))*d^6 - 8*(a*b*(gamma(-4, I*d*x) + ga
mma(-4, -I*d*x))*cos(c) + a*b*(-I*gamma(-4, I*d*x) + I*gamma(-4, -I*d*x))*s
in(c))*d^5 - 12*(b^2*(I*gamma(-4, I*d*x) - I*gamma(-4, -I*d*x))*cos(c) + b^
2*(gamma(-4, I*d*x) + gamma(-4, -I*d*x))*sin(c))*d^4)*x^4 + 6*b^2*sin(d*x +
 c) + 2*(b^2*d*x + 2*a*b*d)*cos(d*x + c))/(d^2*x^4)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.32 (sec) , antiderivative size = 1712, normalized size of antiderivative = 6.90

$$\int \frac{(a+bx)^2\sin(c+dx)}{x^5}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x^5,x, algorithm="giac")
```

```
[Out] -1/48*(a^2*d^4*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2
 - a^2*d^4*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
2*a^2*d^4*x^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*d^4*x^4
*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^4*x^4*rea
l_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 8*a*b*d^3*x^4*real_p
```

```
art(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 8*a*b*d^3*x^4*real_par
t(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d^4*x^4*imag_part(c
os_integral(d*x))*tan(1/2*d*x)^2 + a^2*d^4*x^4*imag_part(cos_integral(-d*x)
)*tan(1/2*d*x)^2 - 2*a^2*d^4*x^4*sin_integral(d*x)*tan(1/2*d*x)^2 - 16*a*b*
d^3*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 16*a*b*d^3
*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 32*a*b*d^3*x
^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) + a^2*d^4*x^4*imag_part(cos_
integral(d*x))*tan(1/2*c)^2 - a^2*d^4*x^4*imag_part(cos_integral(-d*x))*tan
(1/2*c)^2 + 2*a^2*d^4*x^4*sin_integral(d*x)*tan(1/2*c)^2 - 12*b^2*d^2*x^4*i
mag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 12*b^2*d^2*x^4*im
ag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 24*b^2*d^2*x^4*si
n_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 8*a*b*d^3*x^4*real_part(cos_i
ntegral(d*x))*tan(1/2*d*x)^2 + 8*a*b*d^3*x^4*real_part(cos_integral(-d*x))*
tan(1/2*d*x)^2 - 2*a^2*d^4*x^4*real_part(cos_integral(d*x))*tan(1/2*c) - 2*
a^2*d^4*x^4*real_part(cos_integral(-d*x))*tan(1/2*c) + 24*b^2*d^2*x^4*real_
part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 24*b^2*d^2*x^4*real_par
t(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 8*a*b*d^3*x^4*real_part(c
os_integral(d*x))*tan(1/2*c)^2 - 8*a*b*d^3*x^4*real_part(cos_integral(-d*x)
)*tan(1/2*c)^2 - 2*a^2*d^3*x^3*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d^4*x^4*im
ag_part(cos_integral(d*x)) + a^2*d^4*x^4*imag_part(cos_integral(-d*x)) - 2*
a^2*d^4*x^4*sin_integral(d*x) + 12*b^2*d^2*x^4*imag_part(cos_integral(d*x))
*tan(1/2*d*x)^2 - 12*b^2*d^2*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)
^2 + 24*b^2*d^2*x^4*sin_integral(d*x)*tan(1/2*d*x)^2 - 16*a*b*d^3*x^4*imag_
part(cos_integral(d*x))*tan(1/2*c) + 16*a*b*d^3*x^4*imag_part(cos_integral(
-d*x))*tan(1/2*c) - 32*a*b*d^3*x^4*sin_integral(d*x)*tan(1/2*c) - 12*b^2*d^
2*x^4*imag_part(cos_integral(d*x))*tan(1/2*c)^2 + 12*b^2*d^2*x^4*imag_part(
cos_integral(-d*x))*tan(1/2*c)^2 - 24*b^2*d^2*x^4*sin_integral(d*x)*tan(1/2
*c)^2 + 8*a*b*d^3*x^4*real_part(cos_integral(d*x)) + 8*a*b*d^3*x^4*real_par
t(cos_integral(-d*x)) + 2*a^2*d^3*x^3*tan(1/2*d*x)^2 + 24*b^2*d^2*x^4*real_
part(cos_integral(d*x))*tan(1/2*c) + 24*b^2*d^2*x^4*real_part(cos_integral(
-d*x))*tan(1/2*c) + 8*a^2*d^3*x^3*tan(1/2*d*x)*tan(1/2*c) + 32*a*b*d^2*x^3*
tan(1/2*d*x)^2*tan(1/2*c) + 2*a^2*d^3*x^3*tan(1/2*c)^2 + 32*a*b*d^2*x^3*tan
(1/2*d*x)*tan(1/2*c)^2 + 24*b^2*d*x^3*tan(1/2*d*x)^2*tan(1/2*c)^2 + 12*b^2*
d^2*x^4*imag_part(cos_integral(d*x)) - 12*b^2*d^2*x^4*imag_part(cos_integra
l(-d*x)) + 24*b^2*d^2*x^4*sin_integral(d*x) + 4*a^2*d^2*x^2*tan(1/2*d*x)^2*
tan(1/2*c) + 4*a^2*d^2*x^2*tan(1/2*d*x)*tan(1/2*c)^2 + 16*a*b*d*x^2*tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*a^2*d^3*x^3 - 32*a*b*d^2*x^3*tan(1/2*d*x) - 24*b^2
*d*x^3*tan(1/2*d*x)^2 - 32*a*b*d^2*x^3*tan(1/2*c) - 96*b^2*d*x^3*tan(1/2*d*
x)*tan(1/2*c) - 24*b^2*d*x^3*tan(1/2*c)^2 + 4*a^2*d*x*tan(1/2*d*x)^2*tan(1/
2*c)^2 - 4*a^2*d^2*x^2*tan(1/2*d*x) - 16*a*b*d*x^2*tan(1/2*d*x)^2 - 4*a^2*d
^2*x^2*tan(1/2*c) - 64*a*b*d*x^2*tan(1/2*d*x)*tan(1/2*c) - 48*b^2*x^2*tan(1
/2*d*x)^2*tan(1/2*c) - 16*a*b*d*x^2*tan(1/2*c)^2 - 48*b^2*x^2*tan(1/2*d*x)*
tan(1/2*c)^2 + 24*b^2*d*x^3 - 4*a^2*d*x*tan(1/2*d*x)^2 - 16*a^2*d*x*tan(1/2
*d*x)*tan(1/2*c) - 64*a*b*x*tan(1/2*d*x)^2*tan(1/2*c) - 4*a^2*d*x*tan(1/2*c
)^2 - 64*a*b*x*tan(1/2*d*x)*tan(1/2*c)^2 + 16*a*b*d*x^2 + 48*b^2*x^2*tan(1/
```

```
2*d*x) + 48*b^2*x^2*tan(1/2*c) - 24*a^2*tan(1/2*d*x)^2*tan(1/2*c) - 24*a^2*
tan(1/2*d*x)*tan(1/2*c)^2 + 4*a^2*d*x + 64*a*b*x*tan(1/2*d*x) + 64*a*b*x*ta
n(1/2*c) + 24*a^2*tan(1/2*d*x) + 24*a^2*tan(1/2*c))/(x^4*tan(1/2*d*x)^2*tan
(1/2*c)^2 + x^4*tan(1/2*d*x)^2 + x^4*tan(1/2*c)^2 + x^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2\sin(c+dx)}{x^5}\,dx = \int \frac{\sin\left(c+d\,x\right)\,\left(a+b\,x\right)^2}{x^5}\,dx$$

```
[In] int((sin(c + d*x)*(a + b*x)^2)/x^5,x)
```

```
[Out] int((sin(c + d*x)*(a + b*x)^2)/x^5, x)
```

3.18 $\int \frac{x^4 \sin(c+dx)}{a+bx} dx$

Optimal result

Integrand size = 17, antiderivative size = 218

$$\begin{aligned}\int \frac{x^4 \sin(c+dx)}{a+bx}\,dx = &-\frac{2a\cos(c+dx)}{b^2d^3}+\frac{a^3\cos(c+dx)}{b^4d}+\frac{6x\cos(c+dx)}{bd^3}\\&-\frac{a^2x\cos(c+dx)}{b^3d}+\frac{ax^2\cos(c+dx)}{b^2d}-\frac{x^3\cos(c+dx)}{bd}\\&+\frac{a^4\operatorname{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^5}\\&-\frac{6\sin(c+dx)}{bd^4}+\frac{a^2\sin(c+dx)}{b^3d^2}-\frac{2ax\sin(c+dx)}{b^2d^2}\\&+\frac{3x^2\sin(c+dx)}{bd^2}+\frac{a^4\cos\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(\frac{ad}{b}+dx\right)}{b^5}\end{aligned}$$

```
[Out] -2*a*cos(d*x+c)/b^2/d^3+a^3*cos(d*x+c)/b^4/d+6*x*cos(d*x+c)/b/d^3-a^2*x*cos
(d*x+c)/b^3/d+a*x^2*cos(d*x+c)/b^2/d-x^3*cos(d*x+c)/b/d+a^4*cos(-c+a*d/b)*S
i(a*d/b+d*x)/b^5-a^4*Ci(a*d/b+d*x)*sin(-c+a*d/b)/b^5-6*sin(d*x+c)/b/d^4+a^2
*sin(d*x+c)/b^3/d^2-2*a*x*sin(d*x+c)/b^2/d^2+3*x^2*sin(d*x+c)/b/d^2
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used

= {6874, 2718, 3377, 2717, 3384, 3380, 3383}

$$\int \frac{x^4 \sin(c+dx)}{a+bx} \, dx = \frac{a^4 \sin\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b^5} + \frac{a^4 \cos\left(c-\frac{ad}{b}\right) \text{Si}\left(xd+\frac{ad}{b}\right)}{b^5} + \frac{a^3 \cos(c+dx)}{b^4 d} + \frac{a^2 \sin(c+dx)}{b^3 d^2} - \frac{a^2 x \cos(c+dx)}{b^3 d} - \frac{2a \cos(c+dx)}{b^2 d^3} - \frac{2ax \sin(c+dx)}{b^2 d^2} + \frac{ax^2 \cos(c+dx)}{b^2 d} - \frac{6 \sin(c+dx)}{bd^4} + \frac{6x \cos(c+dx)}{bd^3} + \frac{3x^2 \sin(c+dx)}{bd^2} - \frac{x^3 \cos(c+dx)}{bd}$$

```
[In] Int[(x^4*Sin[c + d*x])/(a + b*x),x]
```

```
[Out] (-2*a*Cos[c + d*x])/(b^2*d^3) + (a^3*Cos[c + d*x])/(b^4*d) + (6*x*Cos[c + d
*x])/(b*d^3) - (a^2*x*Cos[c + d*x])/(b^3*d) + (a*x^2*Cos[c + d*x])/(b^2*d)
- (x^3*Cos[c + d*x])/(b*d) + (a^4*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/
b])/b^5 - (6*Sin[c + d*x])/(b*d^4) + (a^2*Sin[c + d*x])/(b^3*d^2) - (2*a*x*
Sin[c + d*x])/(b^2*d^2) + (3*x^2*Sin[c + d*x])/(b*d^2) + (a^4*Cos[c - (a*d)
/b]*SinIntegral[(a*d)/b + d*x])/b^5
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{a^3\sin(c+dx)}{b^4}+\frac{a^2x\sin(c+dx)}{b^3}-\frac{ax^2\sin(c+dx)}{b^2}+\frac{x^3\sin(c+dx)}{b}+\frac{a^4\sin(c+dx)}{b^4(a+bx)}\right)dx$$

$$= -\frac{a^3\int\sin(c+dx)\,dx}{b^4}+\frac{a^4\int\frac{\sin(c+dx)}{a+bx}\,dx}{b^4}+\frac{a^2\int x\sin(c+dx)\,dx}{b^3}-\frac{a\int x^2\sin(c+dx)\,dx}{b^2}+\frac{\int x^3\sin(c+dx)\,dx}{b}$$

$$= \frac{a^3\cos(c+dx)}{b^4d}-\frac{a^2x\cos(c+dx)}{b^3d}+\frac{ax^2\cos(c+dx)}{b^2d}-\frac{x^3\cos(c+dx)}{bd}+\frac{a^2\int\cos(c+dx)\,dx}{b^3d}-\frac{(2a)\int x\cos(c+dx)\,dx}{b^2d}+\frac{3\int x^2\cos(c+dx)\,dx}{bd}+\frac{\left(a^4\cos\left(c-\frac{ad}{b}\right)\right)\int\frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^4}+\frac{\left(a^4\sin\left(c-\frac{ad}{b}\right)\right)\int\frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^4}$$

$$= \frac{a^3\cos(c+dx)}{b^4d}-\frac{a^2x\cos(c+dx)}{b^3d}+\frac{ax^2\cos(c+dx)}{b^2d}-\frac{x^3\cos(c+dx)}{bd}+\frac{a^4\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^5}+\frac{a^2\sin(c+dx)}{b^3d^2}-\frac{2ax\sin(c+dx)}{b^2d^2}+\frac{3x^2\sin(c+dx)}{bd^2}+\frac{a^4\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^5}+\frac{(2a)\int\sin(c+dx)\,dx}{b^2d^2}-\frac{6\int x\sin(c+dx)\,dx}{bd^2}$$

$$
\begin{aligned}
&= -\frac{2a\cos(c+dx)}{b^2d^3} + \frac{a^3\cos(c+dx)}{b^4d} + \frac{6x\cos(c+dx)}{bd^3} \\
&\quad - \frac{a^2x\cos(c+dx)}{b^3d} + \frac{ax^2\cos(c+dx)}{b^2d} - \frac{x^3\cos(c+dx)}{bd} \\
&\quad + \frac{a^4 \operatorname{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^5} + \frac{a^2\sin(c+dx)}{b^3d^2} - \frac{2ax\sin(c+dx)}{b^2d^2} \\
&\quad + \frac{3x^2\sin(c+dx)}{bd^2} + \frac{a^4\cos\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(\frac{ad}{b}+dx\right)}{b^5} - \frac{6\int\cos(c+dx)\,dx}{bd^3} \\
&= -\frac{2a\cos(c+dx)}{b^2d^3} + \frac{a^3\cos(c+dx)}{b^4d} + \frac{6x\cos(c+dx)}{bd^3} \\
&\quad - \frac{a^2x\cos(c+dx)}{b^3d} + \frac{ax^2\cos(c+dx)}{b^2d} - \frac{x^3\cos(c+dx)}{bd} \\
&\quad + \frac{a^4 \operatorname{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^5} - \frac{6\sin(c+dx)}{bd^4} + \frac{a^2\sin(c+dx)}{b^3d^2} \\
&\quad - \frac{2ax\sin(c+dx)}{b^2d^2} + \frac{3x^2\sin(c+dx)}{bd^2} + \frac{a^4\cos\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(\frac{ad}{b}+dx\right)}{b^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.72

$$
\int \frac{x^4\sin(c+dx)}{a+bx}\,dx
= \frac{a^4d^4 \operatorname{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)\sin\left(c-\frac{ad}{b}\right) + b(d(a^3d^2-a^2bd^2x+b^3x(6-d^2x^2)+ab^2(-2+d^2x^2))\cos(c+}{b^5d^4}
$$

```
[In] Integrate[(x^4*Sin[c + d*x])/(a + b*x),x]
```

```
[Out] (a^4*d^4*CosIntegral[d*(a/b + x)]*Sin[c - (a*d)/b] + b*(d*(a^3*d^2 - a^2*b*
d^2*x + b^3*x*(6 - d^2*x^2) + a*b^2*(-2 + d^2*x^2))*Cos[c + d*x] + b*(a^2*d
^2 - 2*a*b*d^2*x + 3*b^2*(-2 + d^2*x^2))*Sin[c + d*x]) + a^4*d^4*Cos[c - (a
*d)/b]*SinIntegral[d*(a/b + x)])/(b^5*d^4)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.42 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.50

method	result
risch	$-\frac{i\pi\ \mathrm{csgn}\left(\frac{d(bx+a)}{b}\right)\sin\left(\frac{da-cb}{b}\right)a^4}{2b^5}-\frac{x^3\cos(dx+c)}{bd}-\frac{\pi\ \mathrm{csgn}\left(\frac{d(bx+a)}{b}\right)\cos\left(\frac{da-cb}{b}\right)a^4}{2b^5}+\frac{i\,\mathrm{Si}\left(\frac{d(bx+a)}{b}\right)\sin\left(\frac{da-cb}{b}\right.}{b^5}$
derivativedivides	$d\,c^4\left(\frac{\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}-\frac{\mathrm{Ci}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}\right)+\frac{4(da-cb)d\,c^3\left(\frac{\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}-\mathrm{Ci}\left(dx+\right.\right.}{b}$
default	$d\,c^4\left(\frac{\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}-\frac{\mathrm{Ci}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}\right)+\frac{4(da-cb)d\,c^3\left(\frac{\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}-\mathrm{Ci}\left(dx+\right.\right.}{b}$

```
[In] int(x^4*sin(d*x+c)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/b^5*Pi*csgn(d*(b*x+a)/b)*sin((a*d-b*c)/b)*a^4-x^3*cos(d*x+c)/b/d-1/2
/b^5*Pi*csgn(d*(b*x+a)/b)*cos((a*d-b*c)/b)*a^4+I/b^5*Si(d*(b*x+a)/b)*sin((a
*d-b*c)/b)*a^4+a*x^2*cos(d*x+c)/b^2/d+1/b^5*Si(d*(b*x+a)/b)*cos((a*d-b*c)/b
)*a^4+1/b^5*Ei(1,-I*d*(b*x+a)/b)*sin((a*d-b*c)/b)*a^4+3*x^2*sin(d*x+c)/b/d^
2-a^2*x*cos(d*x+c)/b^3/d-2*a*x*sin(d*x+c)/b^2/d^2+a^3*cos(d*x+c)/b^4/d+a^2*
sin(d*x+c)/b^3/d^2+6*x*cos(d*x+c)/b/d^3-2*a*cos(d*x+c)/b^2/d^3-6*sin(d*x+c)
/b/d^4
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.86

$$\int \frac{x^4\sin(c+dx)}{a+bx}\,dx = -\frac{a^4d^4\,\mathrm{Ci}\left(\frac{bdx+ad}{b}\right)\sin\left(-\frac{bc-ad}{b}\right)-a^4d^4\cos\left(-\frac{bc-ad}{b}\right)\mathrm{Si}\left(\frac{bdx+ad}{b}\right)+\left(b^4d^3x^3-ab^3d^3x^2-a^3bd^3+2\,ab^3d+(a^2\cdots\right.}{b^5d^4}$$

```
[In] integrate(x^4*sin(d*x+c)/(b*x+a),x, algorithm="fricas")
```

```
[Out] -(a^4*d^4*cos_integral((b*d*x + a*d)/b)*sin(-(b*c - a*d)/b) - a^4*d^4*cos(-
(b*c - a*d)/b)*sin_integral((b*d*x + a*d)/b) + (b^4*d^3*x^3 - a*b^3*d^3*x^2
 - a^3*b*d^3 + 2*a*b^3*d + (a^2*b^2*d^3 - 6*b^4*d)*x)*cos(d*x + c) - (3*b^4
*d^2*x^2 - 2*a*b^3*d^2*x + a^2*b^2*d^2 - 6*b^4)*sin(d*x + c))/(b^5*d^4)
```

Sympy [F]

$$\int \frac{x^4 \sin(c+dx)}{a+bx}\,dx = \int \frac{x^4 \sin(c+dx)}{a+bx}\,dx$$

```
[In] integrate(x**4*sin(d*x+c)/(b*x+a),x)
```

```
[Out] Integral(x**4*sin(c + d*x)/(a + b*x), x)
```

Maxima [F]

$$\int \frac{x^4 \sin(c+dx)}{a+bx}\,dx = \int \frac{x^4 \sin(dx+c)}{bx+a}\,dx$$

```
[In] integrate(x^4*sin(d*x+c)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(((6*a*b^2*(I*exp_integral_e(2, (I*b*d*x + I*a*d)/b) - I*exp_integral_
e(2, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + 6*a*b^2*(I*exp_integral_e(2, (I*b*d*
x + I*a*d)/b) - I*exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^2 - (a^3*
(-I*exp_integral_e(2, (I*b*d*x + I*a*d)/b) + I*exp_integral_e(2, -(I*b*d*x
+ I*a*d)/b))*cos(c)^2 + a^3*(-I*exp_integral_e(2, (I*b*d*x + I*a*d)/b) + I*
exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*d^2 - 4*(a^2*b*(exp_inte
gral_e(2, (I*b*d*x + I*a*d)/b) + exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*c
os(c)^2 + a^2*b*(exp_integral_e(2, (I*b*d*x + I*a*d)/b) + exp_integral_e(2,
 -(I*b*d*x + I*a*d)/b))*sin(c)^2)*d)*cos(-(b*c - a*d)/b) - (6*a*b^2*(exp_in
tegral_e(2, (I*b*d*x + I*a*d)/b) + exp_integral_e(2, -(I*b*d*x + I*a*d)/b))
*cos(c)^2 + 6*a*b^2*(exp_integral_e(2, (I*b*d*x + I*a*d)/b) + exp_integral_
e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^2 + (a^3*(exp_integral_e(2, (I*b*d*x + I
*a*d)/b) + exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a^3*(exp_int
egral_e(2, (I*b*d*x + I*a*d)/b) + exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*
sin(c)^2)*d^2 - 4*(a^2*b*(-I*exp_integral_e(2, (I*b*d*x + I*a*d)/b) + I*exp
_integral_e(2, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a^2*b*(-I*exp_integral_e(2
, (I*b*d*x + I*a*d)/b) + I*exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^
2)*d)*sin(-(b*c - a*d)/b))*cos(d*x + c)^2 + ((6*a*b^2*(I*exp_integral_e(2,
(I*b*d*x + I*a*d)/b) - I*exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*cos(c)^2
+ 6*a*b^2*(I*exp_integral_e(2, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(2, -
(I*b*d*x + I*a*d)/b))*sin(c)^2 - (a^3*(-I*exp_integral_e(2, (I*b*d*x + I*a*
d)/b) + I*exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a^3*(-I*exp_i
ntegral_e(2, (I*b*d*x + I*a*d)/b) + I*exp_integral_e(2, -(I*b*d*x + I*a*d)/
b))*sin(c)^2)*d^2 - 4*(a^2*b*(exp_integral_e(2, (I*b*d*x + I*a*d)/b) + exp_
integral_e(2, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a^2*b*(exp_integral_e(2, (I
*b*d*x + I*a*d)/b) + exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*d)*
cos(-(b*c - a*d)/b) - (6*a*b^2*(exp_integral_e(2, (I*b*d*x + I*a*d)/b) + ex
```

```
p_integral_e(2, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + 6*a*b^2*(exp_integral_e(2
, (I*b*d*x + I*a*d)/b) + exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^2
+ (a^3*(exp_integral_e(2, (I*b*d*x + I*a*d)/b) + exp_integral_e(2, -(I*b*d*
x + I*a*d)/b))*cos(c)^2 + a^3*(exp_integral_e(2, (I*b*d*x + I*a*d)/b) + exp
_integral_e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*d^2 - 4*(a^2*b*(-I*exp_inte
gral_e(2, (I*b*d*x + I*a*d)/b) + I*exp_integral_e(2, -(I*b*d*x + I*a*d)/b))
*cos(c)^2 + a^2*b*(-I*exp_integral_e(2, (I*b*d*x + I*a*d)/b) + I*exp_integr
al_e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*d)*sin(-(b*c - a*d)/b))*sin(d*x +
c)^2 + ((b^3*d^3*x^4*cos(c) + 3*b^3*d^2*x^3*sin(c) + (a*b^2*d^2*sin(c) - 6*
b^3*d*cos(c))*x^2 - (a^2*b*d^2*sin(c) + 4*a*b^2*d*cos(c) + 6*b^3*sin(c))*x)
*cos(d*x + c)^2 + (b^3*d^3*x^4*cos(c) + 3*b^3*d^2*x^3*sin(c) + (a*b^2*d^2*s
in(c) - 6*b^3*d*cos(c))*x^2 - (a^2*b*d^2*sin(c) + 4*a*b^2*d*cos(c) + 6*b^3*
sin(c))*x)*sin(d*x + c)^2)*cos(d*x + 2*c) + ((b^3*cos(c)^2 + b^3*sin(c)^2)*
d^3*x^4 - 6*(b^3*cos(c)^2 + b^3*sin(c)^2)*d*x^2 - 4*(a*b^2*cos(c)^2 + a*b^2
*sin(c)^2)*d*x)*cos(d*x + c) - 2*(((a^4*b^3*cos(c)^2 + a^4*b^3*sin(c)^2)*d^
7 - 6*(a^2*b^5*cos(c)^2 + a^2*b^5*sin(c)^2)*d^5 + ((a^3*b^4*cos(c)^2 + a^3*
b^4*sin(c)^2)*d^7 - 6*(a*b^6*cos(c)^2 + a*b^6*sin(c)^2)*d^5)*x)*cos(d*x + c
)^2 + ((a^4*b^3*cos(c)^2 + a^4*b^3*sin(c)^2)*d^7 - 6*(a^2*b^5*cos(c)^2 + a^
2*b^5*sin(c)^2)*d^5 + ((a^3*b^4*cos(c)^2 + a^3*b^4*sin(c)^2)*d^7 - 6*(a*b^6
*cos(c)^2 + a*b^6*sin(c)^2)*d^5)*x)*sin(d*x + c)^2)*integrate(1/2*x*cos(d*x
 + c)/(b^4*d^4*x^2 + 2*a*b^3*d^4*x + a^2*b^2*d^4), x) - 2*(((a^4*b^3*cos(c)
^2 + a^4*b^3*sin(c)^2)*d^7 - 6*(a^2*b^5*cos(c)^2 + a^2*b^5*sin(c)^2)*d^5 +
((a^3*b^4*cos(c)^2 + a^3*b^4*sin(c)^2)*d^7 - 6*(a*b^6*cos(c)^2 + a*b^6*sin(
c)^2)*d^5)*x)*cos(d*x + c)^2 + ((a^4*b^3*cos(c)^2 + a^4*b^3*sin(c)^2)*d^7 -
 6*(a^2*b^5*cos(c)^2 + a^2*b^5*sin(c)^2)*d^5 + ((a^3*b^4*cos(c)^2 + a^3*b^4
*sin(c)^2)*d^7 - 6*(a*b^6*cos(c)^2 + a*b^6*sin(c)^2)*d^5)*x)*sin(d*x + c)^2
)*integrate(1/2*x*cos(d*x + c)/((b^4*d^4*x^2 + 2*a*b^3*d^4*x + a^2*b^2*d^4)
*cos(d*x + c)^2 + (b^4*d^4*x^2 + 2*a*b^3*d^4*x + a^2*b^2*d^4)*sin(d*x + c)^
2), x) - 4*(((a^2*b^5*cos(c)^2 + a^2*b^5*sin(c)^2)*d^6*x + (a^3*b^4*cos(c)^
2 + a^3*b^4*sin(c)^2)*d^6)*cos(d*x + c)^2 + ((a^2*b^5*cos(c)^2 + a^2*b^5*si
n(c)^2)*d^6*x + (a^3*b^4*cos(c)^2 + a^3*b^4*sin(c)^2)*d^6)*sin(d*x + c)^2)*
integrate(1/2*x*sin(d*x + c)/(b^4*d^4*x^2 + 2*a*b^3*d^4*x + a^2*b^2*d^4), x
) - 4*(((a^2*b^5*cos(c)^2 + a^2*b^5*sin(c)^2)*d^6*x + (a^3*b^4*cos(c)^2 + a
^3*b^4*sin(c)^2)*d^6)*cos(d*x + c)^2 + ((a^2*b^5*cos(c)^2 + a^2*b^5*sin(c)^
2)*d^6*x + (a^3*b^4*cos(c)^2 + a^3*b^4*sin(c)^2)*d^6)*sin(d*x + c)^2)*integ
rate(1/2*x*sin(d*x + c)/((b^4*d^4*x^2 + 2*a*b^3*d^4*x + a^2*b^2*d^4)*cos(d*
x + c)^2 + (b^4*d^4*x^2 + 2*a*b^3*d^4*x + a^2*b^2*d^4)*sin(d*x + c)^2), x)
+ ((b^3*d^3*x^4*sin(c) - 3*b^3*d^2*x^3*cos(c) - (a*b^2*d^2*cos(c) + 6*b^3*d
*sin(c))*x^2 + (a^2*b*d^2*cos(c) - 4*a*b^2*d*sin(c) + 6*b^3*cos(c))*x)*cos(
d*x + c)^2 + (b^3*d^3*x^4*sin(c) - 3*b^3*d^2*x^3*cos(c) - (a*b^2*d^2*cos(c)
 + 6*b^3*d*sin(c))*x^2 + (a^2*b*d^2*cos(c) - 4*a*b^2*d*sin(c) + 6*b^3*cos(c
))*x)*sin(d*x + c)^2)*sin(d*x + 2*c) - (3*(b^3*cos(c)^2 + b^3*sin(c)^2)*d^2
*x^3 + (a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^2*x^2 - (6*b^3*cos(c)^2 + 6*b^3*
sin(c)^2 + (a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^2)*x)*sin(d*x + c))/(((b^4*c
os(c)^2 + b^4*sin(c)^2)*d^4*x + (a*b^3*cos(c)^2 + a*b^3*sin(c)^2)*d^4)*cos(
```

```
d*x + c)^2 + ((b^4*cos(c)^2 + b^4*sin(c)^2)*d^4*x + (a*b^3*cos(c)^2 + a*b^3
*sin(c)^2)*d^4)*sin(d*x + c)^2)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.36 (sec) , antiderivative size = 3337, normalized size of antiderivative = 15.31

$$\int \frac{x^4 \sin(c+dx)}{a+bx} \, dx = \text{Too large to display}$$

```
[In] integrate(x^4*sin(d*x+c)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^4*d^3*x^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2
*a*b^3*d^3*x^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^4*d
^4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2
*tan(1/2*a*d/b)^2 - a^4*d^4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d
*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^4*d^4*sin_integral((b*d*x
 + a*d)/b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^4*d^3
*x^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + 2*a^4*d^4*real_part(cos_integral
(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^4*d
^4*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^
2*tan(1/2*a*d/b) + 2*b^4*d^3*x^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)^2 -
2*a^4*d^4*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1
/2*c)*tan(1/2*a*d/b)^2 - 2*a^4*d^4*real_part(cos_integral(-d*x - a*d/b))*ta
n(1/2*d*x + 1/2*c)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*b^4*d^3*x^3*tan(1/2*c)
^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d^3*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*t
an(1/2*a*d/b)^2 - 2*a*b^3*d^3*x^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - a^4
*d^4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)
^2 + a^4*d^4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/2*c)^2*t
an(1/2*c)^2 - 2*a^4*d^4*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x + 1/2*c)^
2*tan(1/2*c)^2 + 4*a^4*d^4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x
 + 1/2*c)^2*tan(1/2*c)*tan(1/2*a*d/b) - 4*a^4*d^4*imag_part(cos_integral(-d
*x - a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^4*d^4*s
in_integral((b*d*x + a*d)/b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)*tan(1/2*a*d/
b) - 2*a*b^3*d^3*x^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)^2 - a^4*d^4*imag
_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)^2 +
a^4*d^4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/
2*a*d/b)^2 - 2*a^4*d^4*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x + 1/2*c)^2
*tan(1/2*a*d/b)^2 + 2*a*b^3*d^3*x^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^4*d^4
*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^4*d
^4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*
a^4*d^4*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 12*b^
4*d^2*x^2*tan(1/2*d*x + 1/2*c)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^3*b*d^3*
tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^4*d^3*x^3*tan(1/
```

2*d*x + 1/2*c)^2 + 2*a^4*d^4*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d
*x + 1/2*c)^2*tan(1/2*c) + 2*a^4*d^4*real_part(cos_integral(-d*x - a*d/b))*
tan(1/2*d*x + 1/2*c)^2*tan(1/2*c) - 2*b^4*d^3*x^3*tan(1/2*c)^2 + 2*a^2*b^2*
d^3*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - 2*a^4*d^4*real_part(cos_integra
l(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b) - 2*a^4*d^4*real_part
(cos_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b) + 2*a^4*
d^4*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^
4*d^4*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 2
*b^4*d^3*x^3*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d^3*x*tan(1/2*d*x + 1/2*c)^2*tan(
1/2*a*d/b)^2 - 2*a^4*d^4*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*ta
n(1/2*a*d/b)^2 - 2*a^4*d^4*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)
*tan(1/2*a*d/b)^2 - 2*a^2*b^2*d^3*x*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 8*a*b^3
*d^2*x*tan(1/2*d*x + 1/2*c)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 12*b^4*d*x*tan(
1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a*b^3*d^3*x^2*tan(1/2*
d*x + 1/2*c)^2 + a^4*d^4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x +
 1/2*c)^2 - a^4*d^4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/2
*c)^2 + 2*a^4*d^4*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x + 1/2*c)^2 + 2*
a*b^3*d^3*x^2*tan(1/2*c)^2 - a^4*d^4*imag_part(cos_integral(d*x + a*d/b))*t
an(1/2*c)^2 + a^4*d^4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 -
2*a^4*d^4*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2 + 12*b^4*d^2*x^2*tan(1
/2*d*x + 1/2*c)*tan(1/2*c)^2 - 2*a^3*b*d^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c
)^2 + 4*a^4*d^4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d
/b) - 4*a^4*d^4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*
d/b) + 8*a^4*d^4*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) +
2*a*b^3*d^3*x^2*tan(1/2*a*d/b)^2 - a^4*d^4*imag_part(cos_integral(d*x + a*d
/b))*tan(1/2*a*d/b)^2 + a^4*d^4*imag_part(cos_integral(-d*x - a*d/b))*tan(1
/2*a*d/b)^2 - 2*a^4*d^4*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 + 12
*b^4*d^2*x^2*tan(1/2*d*x + 1/2*c)*tan(1/2*a*d/b)^2 - 2*a^3*b*d^3*tan(1/2*d*
x + 1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^3*b*d^3*tan(1/2*c)^2*tan(1/2*a*d/b)^2 +
 4*a^2*b^2*d^2*tan(1/2*d*x + 1/2*c)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a*b^3
*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*b^4*d^3*x^3 + 2
*a^2*b^2*d^3*x*tan(1/2*d*x + 1/2*c)^2 + 2*a^4*d^4*real_part(cos_integral(d*
x + a*d/b))*tan(1/2*c) + 2*a^4*d^4*real_part(cos_integral(-d*x - a*d/b))*ta
n(1/2*c) - 2*a^2*b^2*d^3*x*tan(1/2*c)^2 - 8*a*b^3*d^2*x*tan(1/2*d*x + 1/2*c
)*tan(1/2*c)^2 - 12*b^4*d*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - 2*a^4*d^4
*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 2*a^4*d^4*real_part(
cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) - 2*a^2*b^2*d^3*x*tan(1/2*a*d/b)
^2 - 8*a*b^3*d^2*x*tan(1/2*d*x + 1/2*c)*tan(1/2*a*d/b)^2 - 12*b^4*d*x*tan(1
/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)^2 + 12*b^4*d*x*tan(1/2*c)^2*tan(1/2*a*d/b)
^2 + 2*a*b^3*d^3*x^2 + a^4*d^4*imag_part(cos_integral(d*x + a*d/b)) - a^4*d
^4*imag_part(cos_integral(-d*x - a*d/b)) + 2*a^4*d^4*sin_integral((b*d*x +
a*d)/b) + 12*b^4*d^2*x^2*tan(1/2*d*x + 1/2*c) - 2*a^3*b*d^3*tan(1/2*d*x + 1
/2*c)^2 + 2*a^3*b*d^3*tan(1/2*c)^2 + 4*a^2*b^2*d^2*tan(1/2*d*x + 1/2*c)*tan
(1/2*c)^2 + 4*a*b^3*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + 2*a^3*b*d^3*tan
(1/2*a*d/b)^2 + 4*a^2*b^2*d^2*tan(1/2*d*x + 1/2*c)*tan(1/2*a*d/b)^2 + 4*a*b

```
^3*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)^2 - 4*a*b^3*d*tan(1/2*c)^2*tan(1
/2*a*d/b)^2 - 24*b^4*tan(1/2*d*x + 1/2*c)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2
*a^2*b^2*d^3*x - 8*a*b^3*d^2*x*tan(1/2*d*x + 1/2*c) - 12*b^4*d*x*tan(1/2*d*
x + 1/2*c)^2 + 12*b^4*d*x*tan(1/2*c)^2 + 12*b^4*d*x*tan(1/2*a*d/b)^2 + 2*a^
3*b*d^3 + 4*a^2*b^2*d^2*tan(1/2*d*x + 1/2*c) + 4*a*b^3*d*tan(1/2*d*x + 1/2*
c)^2 - 4*a*b^3*d*tan(1/2*c)^2 - 24*b^4*tan(1/2*d*x + 1/2*c)*tan(1/2*c)^2 -
4*a*b^3*d*tan(1/2*a*d/b)^2 - 24*b^4*tan(1/2*d*x + 1/2*c)*tan(1/2*a*d/b)^2 +
 12*b^4*d*x - 4*a*b^3*d - 24*b^4*tan(1/2*d*x + 1/2*c))/(b^5*d^4*tan(1/2*d*x
 + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + b^5*d^4*tan(1/2*d*x + 1/2*c)^2*
tan(1/2*c)^2 + b^5*d^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)^2 + b^5*d^4*ta
n(1/2*c)^2*tan(1/2*a*d/b)^2 + b^5*d^4*tan(1/2*d*x + 1/2*c)^2 + b^5*d^4*tan(
1/2*c)^2 + b^5*d^4*tan(1/2*a*d/b)^2 + b^5*d^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \sin(c+dx)}{a+bx} \, dx = \int \frac{x^4 \sin(c+d\,x)}{a+b\,x} \,dx$$

```
[In] int((x^4*sin(c + d*x))/(a + b*x),x)
[Out] int((x^4*sin(c + d*x))/(a + b*x), x)
```

3.19 $\int \frac{x^3 \sin(c+dx)}{a+bx} dx$

Optimal result

Integrand size = 17, antiderivative size = 152

$$\int \frac{x^3 \sin(c+dx)}{a+bx} dx = \frac{2\cos(c+dx)}{bd^3} - \frac{a^2\cos(c+dx)}{b^3d} + \frac{ax\cos(c+dx)}{b^2d} - \frac{x^2\cos(c+dx)}{bd} - \frac{a^3 \operatorname{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^4} - \frac{a\sin(c+dx)}{b^2d^2} + \frac{2x\sin(c+dx)}{bd^2} - \frac{a^3\cos\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(\frac{ad}{b}+dx\right)}{b^4}$$

```
[Out] 2*cos(d*x+c)/b/d^3-a^2*cos(d*x+c)/b^3/d+a*x*cos(d*x+c)/b^2/d-x^2*cos(d*x+c)
/b/d-a^3*cos(-c+a*d/b)*Si(a*d/b+d*x)/b^4+a^3*Ci(a*d/b+d*x)*sin(-c+a*d/b)/b^
4-a*sin(d*x+c)/b^2/d^2+2*x*sin(d*x+c)/b/d^2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {6874, 2718, 3377, 2717, 3384, 3380, 3383}

$$\int \frac{x^3 \sin(c+dx)}{a+bx} dx = -\frac{a^3\sin\left(c-\frac{ad}{b}\right)\operatorname{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b^4} - \frac{a^3\cos\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(xd+\frac{ad}{b}\right)}{b^4} - \frac{a^2\cos(c+dx)}{b^3d} - \frac{a\sin(c+dx)}{b^2d^2} + \frac{ax\cos(c+dx)}{b^2d} + \frac{2\cos(c+dx)}{bd^3} + \frac{2x\sin(c+dx)}{bd^2} - \frac{x^2\cos(c+dx)}{bd}$$

```
[In] Int[(x^3*Sin[c + d*x])/(a + b*x),x]
```

```
[Out] (2*Cos[c + d*x])/(b*d^3) - (a^2*Cos[c + d*x])/(b^3*d) + (a*x*Cos[c + d*x])/
(b^2*d) - (x^2*Cos[c + d*x])/(b*d) - (a^3*CosIntegral[(a*d)/b + d*x]*Sin[c
- (a*d)/b])/b^4 - (a*Sin[c + d*x])/(b^2*d^2) + (2*x*Sin[c + d*x])/(b*d^2) -
 (a^3*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^4
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a^2\sin(c+dx)}{b^3} - \frac{ax\sin(c+dx)}{b^2} + \frac{x^2\sin(c+dx)}{b} - \frac{a^3\sin(c+dx)}{b^3(a+bx)}\right) dx$$

$$= \frac{a^2\int \sin(c+dx)\,dx}{b^3} - \frac{a^3\int \frac{\sin(c+dx)}{a+bx}\,dx}{b^3} - \frac{a\int x\sin(c+dx)\,dx}{b^2} + \frac{\int x^2\sin(c+dx)\,dx}{b}$$

$$= -\frac{a^2\cos(c+dx)}{b^3d} + \frac{ax\cos(c+dx)}{b^2d} - \frac{x^2\cos(c+dx)}{bd} - \frac{a\int \cos(c+dx)\,dx}{b^2d} + \frac{2\int x\cos(c+dx)\,dx}{bd} - \frac{\left(a^3\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^3} - \frac{\left(a^3\sin\left(c-\frac{ad}{b}\right)\right)\int \frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^3}$$

$$= -\frac{a^2\cos(c+dx)}{b^3d} + \frac{ax\cos(c+dx)}{b^2d} - \frac{x^2\cos(c+dx)}{bd} - \frac{a^3\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^4} - \frac{a\sin(c+dx)}{b^2d^2} + \frac{2x\sin(c+dx)}{bd^2} - \frac{a^3\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^4} - \frac{2\int \sin(c+dx)\,dx}{bd^2}$$

$$= \frac{2\cos(c+dx)}{bd^3} - \frac{a^2\cos(c+dx)}{b^3d} + \frac{ax\cos(c+dx)}{b^2d} - \frac{x^2\cos(c+dx)}{bd} - \frac{a^3\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^4} - \frac{a\sin(c+dx)}{b^2d^2} + \frac{2x\sin(c+dx)}{bd^2} - \frac{a^3\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^4}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.77

$$\int \frac{x^3\sin(c+dx)}{a+bx}\,dx = -\frac{a^3d^3\,\text{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)\sin\left(c-\frac{ad}{b}\right) + b((a^2d^2-abd^2x+b^2(-2+d^2x^2))\cos(c+dx)+bd(a-2bx)}{b^4d^3}$$

```
[In] Integrate[(x^3*Sin[c + d*x])/(a + b*x),x]
```

```
[Out] -((a^3*d^3*CosIntegral[d*(a/b + x)]*Sin[c - (a*d)/b] + b*((a^2*d^2 - a*b*d^
2*x + b^2*(-2 + d^2*x^2))*Cos[c + d*x] + b*d*(a - 2*b*x)*Sin[c + d*x]) + a^
3*d^3*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)])/(b^4*d^3))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.56

method	result
risch	$\frac{i\,\mathrm{Ei}_1\left(\frac{id(bx+a)}{b}\right)\cos\left(\frac{da-cb}{b}\right)a^3}{2b^4}-\frac{i\,\mathrm{Ei}_1\left(-\frac{id(bx+a)}{b}\right)\cos\left(\frac{da-cb}{b}\right)a^3}{2b^4}-\frac{x^2\cos(dx+c)}{bd}-\frac{\mathrm{Ei}_1\left(\frac{id(bx+a)}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{2b^4}$
derivativedivides	$-d\,c^3\left(\frac{\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}-\frac{\mathrm{Ci}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}\right)-\frac{3(da-cb)d\,c^2\left(\frac{\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}-\mathrm{Ci}(\right.}{b}$
default	$-d\,c^3\left(\frac{\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}-\frac{\mathrm{Ci}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}\right)-\frac{3(da-cb)d\,c^2\left(\frac{\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}-\mathrm{Ci}(\right.}{b}$

```
[In] int(x^3*sin(d*x+c)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I/b^4*Ei(1,I*d*(b*x+a)/b)*cos((a*d-b*c)/b)*a^3-1/2*I/b^4*Ei(1,-I*d*(b*x
+a)/b)*cos((a*d-b*c)/b)*a^3-x^2*cos(d*x+c)/b/d-1/2/b^4*Ei(1,I*d*(b*x+a)/b)*
sin((a*d-b*c)/b)*a^3-1/2/b^4*Ei(1,-I*d*(b*x+a)/b)*sin((a*d-b*c)/b)*a^3+a*x*
cos(d*x+c)/b^2/d+2*x*sin(d*x+c)/b/d^2-a^2*cos(d*x+c)/b^3/d-a*sin(d*x+c)/b^2
/d^2+2*cos(d*x+c)/b/d^3
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.93

$$\int \frac{x^3\sin(c+dx)}{a+bx}\,dx$$
$$=\frac{a^3d^3\,\mathrm{Ci}\left(\frac{bdx+ad}{b}\right)\sin\left(-\frac{bc-ad}{b}\right)-a^3d^3\cos\left(-\frac{bc-ad}{b}\right)\mathrm{Si}\left(\frac{bdx+ad}{b}\right)-(b^3d^2x^2-ab^2d^2x+a^2bd^2-2\,b^3)\cos\left(dx+\right.}{b^4d^3}$$

```
[In] integrate(x^3*sin(d*x+c)/(b*x+a),x, algorithm="fricas")
```

```
[Out] (a^3*d^3*cos_integral((b*d*x + a*d)/b)*sin(-(b*c - a*d)/b) - a^3*d^3*cos(-(
b*c - a*d)/b)*sin_integral((b*d*x + a*d)/b) - (b^3*d^2*x^2 - a*b^2*d^2*x +
a^2*b*d^2 - 2*b^3)*cos(d*x + c) + (2*b^3*d*x - a*b^2*d)*sin(d*x + c))/(b^4*
d^3)
```

Sympy [F]

$$\int \frac{x^3 \sin(c+dx)}{a+bx} \, dx = \int \frac{x^3 \sin(c+dx)}{a+bx} \, dx$$

```
[In] integrate(x**3*sin(d*x+c)/(b*x+a),x)
```

```
[Out] Integral(x**3*sin(c + d*x)/(a + b*x), x)
```

Maxima [F]

$$\int \frac{x^3 \sin(c+dx)}{a+bx} \, dx = \int \frac{x^3 \sin(dx+c)}{bx+a} \, dx$$

```
[In] integrate(x^3*sin(d*x+c)/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(((2*a*b*(exp_integral_e(2, (I*b*d*x + I*a*d)/b) + exp_integral_e(2, -(
I*b*d*x + I*a*d)/b))*cos(c)^2 + 2*a*b*(exp_integral_e(2, (I*b*d*x + I*a*d)/
b) + exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^2 - (a^2*(-I*exp_integ
ral_e(2, (I*b*d*x + I*a*d)/b) + I*exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*
cos(c)^2 + a^2*(-I*exp_integral_e(2, (I*b*d*x + I*a*d)/b) + I*exp_integral_
e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*d)*cos(-(b*c - a*d)/b) + (2*a*b*(I*ex
p_integral_e(2, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(2, -(I*b*d*x + I*a*
d)/b))*cos(c)^2 + 2*a*b*(I*exp_integral_e(2, (I*b*d*x + I*a*d)/b) - I*exp_i
ntegral_e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^2 - (a^2*(exp_integral_e(2, (I*b
*d*x + I*a*d)/b) + exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a^2*
(exp_integral_e(2, (I*b*d*x + I*a*d)/b) + exp_integral_e(2, -(I*b*d*x + I*a
*d)/b))*sin(c)^2)*d)*sin(-(b*c - a*d)/b))*cos(d*x + c)^2 + ((2*a*b*(exp_int
egral_e(2, (I*b*d*x + I*a*d)/b) + exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*
cos(c)^2 + 2*a*b*(exp_integral_e(2, (I*b*d*x + I*a*d)/b) + exp_integral_e(2
, -(I*b*d*x + I*a*d)/b))*sin(c)^2 - (a^2*(-I*exp_integral_e(2, (I*b*d*x + I
*a*d)/b) + I*exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a^2*(-I*ex
p_integral_e(2, (I*b*d*x + I*a*d)/b) + I*exp_integral_e(2, -(I*b*d*x + I*a*
d)/b))*sin(c)^2)*d)*cos(-(b*c - a*d)/b) + (2*a*b*(I*exp_integral_e(2, (I*b*
d*x + I*a*d)/b) - I*exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + 2*a
*b*(I*exp_integral_e(2, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(2, -(I*b*d*
x + I*a*d)/b))*sin(c)^2 - (a^2*(exp_integral_e(2, (I*b*d*x + I*a*d)/b) + ex
p_integral_e(2, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a^2*(exp_integral_e(2, (I
*b*d*x + I*a*d)/b) + exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*d)*
sin(-(b*c - a*d)/b))*sin(d*x + c)^2 - ((b^2*d^2*x^3*cos(c) + 2*b^2*d*x^2*si
n(c) + (a*b*d*sin(c) - 2*b^2*cos(c))*x)*cos(d*x + c)^2 + (b^2*d^2*x^3*cos(c
) + 2*b^2*d*x^2*sin(c) + (a*b*d*sin(c) - 2*b^2*cos(c))*x)*sin(d*x + c)^2)*c
os(d*x + 2*c) - ((b^2*cos(c)^2 + b^2*sin(c)^2)*d^2*x^3 - 2*(b^2*cos(c)^2 +
```

```
b^2*sin(c)^2)*x)*cos(d*x + c) - 2*(((a^2*b^3*cos(c)^2 + a^2*b^3*sin(c)^2)*d
^5*x + (a^3*b^2*cos(c)^2 + a^3*b^2*sin(c)^2)*d^5)*cos(d*x + c)^2 + ((a^2*b^
3*cos(c)^2 + a^2*b^3*sin(c)^2)*d^5*x + (a^3*b^2*cos(c)^2 + a^3*b^2*sin(c)^2
)*d^5)*sin(d*x + c)^2)*integrate(1/2*x*cos(d*x + c)/(b^3*d^3*x^2 + 2*a*b^2*
d^3*x + a^2*b*d^3), x) - 2*(((a^2*b^3*cos(c)^2 + a^2*b^3*sin(c)^2)*d^5*x +
(a^3*b^2*cos(c)^2 + a^3*b^2*sin(c)^2)*d^5)*cos(d*x + c)^2 + ((a^2*b^3*cos(c
)^2 + a^2*b^3*sin(c)^2)*d^5*x + (a^3*b^2*cos(c)^2 + a^3*b^2*sin(c)^2)*d^5)*
sin(d*x + c)^2)*integrate(1/2*x*cos(d*x + c)/((b^3*d^3*x^2 + 2*a*b^2*d^3*x
+ a^2*b*d^3)*cos(d*x + c)^2 + (b^3*d^3*x^2 + 2*a*b^2*d^3*x + a^2*b*d^3)*sin
(d*x + c)^2), x) - 4*(((a*b^4*cos(c)^2 + a*b^4*sin(c)^2)*d^4*x + (a^2*b^3*c
os(c)^2 + a^2*b^3*sin(c)^2)*d^4)*cos(d*x + c)^2 + ((a*b^4*cos(c)^2 + a*b^4*
sin(c)^2)*d^4*x + (a^2*b^3*cos(c)^2 + a^2*b^3*sin(c)^2)*d^4)*sin(d*x + c)^2
)*integrate(1/2*x*sin(d*x + c)/(b^3*d^3*x^2 + 2*a*b^2*d^3*x + a^2*b*d^3), x
) - 4*(((a*b^4*cos(c)^2 + a*b^4*sin(c)^2)*d^4*x + (a^2*b^3*cos(c)^2 + a^2*b
^3*sin(c)^2)*d^4)*cos(d*x + c)^2 + ((a*b^4*cos(c)^2 + a*b^4*sin(c)^2)*d^4*x
 + (a^2*b^3*cos(c)^2 + a^2*b^3*sin(c)^2)*d^4)*sin(d*x + c)^2)*integrate(1/2
*x*sin(d*x + c)/((b^3*d^3*x^2 + 2*a*b^2*d^3*x + a^2*b*d^3)*cos(d*x + c)^2 +
 (b^3*d^3*x^2 + 2*a*b^2*d^3*x + a^2*b*d^3)*sin(d*x + c)^2), x) - ((b^2*d^2*
x^3*sin(c) - 2*b^2*d*x^2*cos(c) - (a*b*d*cos(c) + 2*b^2*sin(c))*x)*cos(d*x
+ c)^2 + (b^2*d^2*x^3*sin(c) - 2*b^2*d*x^2*cos(c) - (a*b*d*cos(c) + 2*b^2*s
in(c))*x)*sin(d*x + c)^2)*sin(d*x + 2*c) + (2*(b^2*cos(c)^2 + b^2*sin(c)^2)
*d*x^2 + (a*b*cos(c)^2 + a*b*sin(c)^2)*d*x)*sin(d*x + c))/(((b^3*cos(c)^2 +
 b^3*sin(c)^2)*d^3*x + (a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^3)*cos(d*x + c)^
2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d^3*x + (a*b^2*cos(c)^2 + a*b^2*sin(c)^2
)*d^3)*sin(d*x + c)^2)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.34 (sec) , antiderivative size = 2709, normalized size of antiderivative = 17.82

$$\int \frac{x^3 \sin(c+dx)}{a+bx}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*sin(d*x+c)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^3*d^2*x^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a
^3*d^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*
c)^2*tan(1/2*a*d/b)^2 + a^3*d^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1
/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^3*d^3*sin_integral((b
*d*x + a*d)/b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^3
*d^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)
^2*tan(1/2*a*d/b) - 2*a^3*d^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2
*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^3*d^3*real_part(cos_integ
ral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*a^
```

3*d^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*
c)*tan(1/2*a*d/b)^2 - 2*a*b^2*d^2*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan
(1/2*a*d/b)^2 + 2*b^3*d^2*x^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + a^3*d^3
*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 -
 a^3*d^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1
/2*c)^2 + 2*a^3*d^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x + 1/2*c)^2*ta
n(1/2*c)^2 - 4*a^3*d^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x + 1
/2*c)^2*tan(1/2*c)*tan(1/2*a*d/b) + 4*a^3*d^3*imag_part(cos_integral(-d*x -
 a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)*tan(1/2*a*d/b) - 8*a^3*d^3*sin_i
ntegral((b*d*x + a*d)/b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)*tan(1/2*a*d/b) +
 2*b^3*d^2*x^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)^2 + a^3*d^3*imag_part(
cos_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)^2 - a^3*d^
3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/
b)^2 + 2*a^3*d^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x + 1/2*c)^2*tan(1
/2*a*d/b)^2 - 2*b^3*d^2*x^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^3*d^3*imag_pa
rt(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^3*d^3*imag_
part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^3*d^3*
sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b*d^2*t
an(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^3*d^3*real_part(c
os_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c) - 2*a^3*d^3*rea
l_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c) - 2*a*
b^2*d^2*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + 2*a^3*d^3*real_part(cos_int
egral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b) + 2*a^3*d^3*real_
part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b) - 2*
a^3*d^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) -
2*a^3*d^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)
 - 2*a*b^2*d^2*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^3*d^3*real_p
art(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*a^3*d^3*real
_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*a*b^2*d^2
*x*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 8*b^3*d*x*tan(1/2*d*x + 1/2*c)*tan(1/2*c
)^2*tan(1/2*a*d/b)^2 + 2*b^3*d^2*x^2*tan(1/2*d*x + 1/2*c)^2 - a^3*d^3*imag_
part(cos_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2 + a^3*d^3*imag_part(
cos_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/2*c)^2 - 2*a^3*d^3*sin_integral
((b*d*x + a*d)/b)*tan(1/2*d*x + 1/2*c)^2 - 2*b^3*d^2*x^2*tan(1/2*c)^2 + a^3
*d^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 - a^3*d^3*imag_part(
cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 + 2*a^3*d^3*sin_integral((b*d*x +
a*d)/b)*tan(1/2*c)^2 + 2*a^2*b*d^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - 4*
a^3*d^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 4*
a^3*d^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 8
*a^3*d^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) - 2*b^3*d^
2*x^2*tan(1/2*a*d/b)^2 + a^3*d^3*imag_part(cos_integral(d*x + a*d/b))*tan(1
/2*a*d/b)^2 - a^3*d^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^
2 + 2*a^3*d^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 + 2*a^2*b*d^2*
tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^2*b*d^2*tan(1/2*c)^2*tan(1/2*
a*d/b)^2 - 4*a*b^2*d*tan(1/2*d*x + 1/2*c)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4

```
*b^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a*b^2*d^2*x*t
an(1/2*d*x + 1/2*c)^2 - 2*a^3*d^3*real_part(cos_integral(d*x + a*d/b))*tan(
1/2*c) - 2*a^3*d^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) + 2*a*b
^2*d^2*x*tan(1/2*c)^2 + 8*b^3*d*x*tan(1/2*d*x + 1/2*c)*tan(1/2*c)^2 + 2*a^3
*d^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) + 2*a^3*d^3*real_p
art(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) + 2*a*b^2*d^2*x*tan(1/2*a*d/
b)^2 + 8*b^3*d*x*tan(1/2*d*x + 1/2*c)*tan(1/2*a*d/b)^2 - 2*b^3*d^2*x^2 - a^
3*d^3*imag_part(cos_integral(d*x + a*d/b)) + a^3*d^3*imag_part(cos_integral
(-d*x - a*d/b)) - 2*a^3*d^3*sin_integral((b*d*x + a*d)/b) + 2*a^2*b*d^2*tan
(1/2*d*x + 1/2*c)^2 - 2*a^2*b*d^2*tan(1/2*c)^2 - 4*a*b^2*d*tan(1/2*d*x + 1/
2*c)*tan(1/2*c)^2 - 4*b^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - 2*a^2*b*d^2
*tan(1/2*a*d/b)^2 - 4*a*b^2*d*tan(1/2*d*x + 1/2*c)*tan(1/2*a*d/b)^2 - 4*b^3
*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)^2 + 4*b^3*tan(1/2*c)^2*tan(1/2*a*d/b
)^2 + 2*a*b^2*d^2*x + 8*b^3*d*x*tan(1/2*d*x + 1/2*c) - 2*a^2*b*d^2 - 4*a*b^
2*d*tan(1/2*d*x + 1/2*c) - 4*b^3*tan(1/2*d*x + 1/2*c)^2 + 4*b^3*tan(1/2*c)^
2 + 4*b^3*tan(1/2*a*d/b)^2 + 4*b^3)/(b^4*d^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2
*c)^2*tan(1/2*a*d/b)^2 + b^4*d^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + b^4*
d^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)^2 + b^4*d^3*tan(1/2*c)^2*tan(1/2*
a*d/b)^2 + b^4*d^3*tan(1/2*d*x + 1/2*c)^2 + b^4*d^3*tan(1/2*c)^2 + b^4*d^3*
tan(1/2*a*d/b)^2 + b^4*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \sin(c+dx)}{a+bx} \, dx = \int \frac{x^3 \sin(c+d\,x)}{a+b\,x} \, dx$$

```
[In] int((x^3*sin(c + d*x))/(a + b*x),x)
[Out] int((x^3*sin(c + d*x))/(a + b*x), x)
```

3.20 $\int \frac{x^2 \sin(c+dx)}{a+bx} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 99

$$\int \frac{x^2 \sin(c+dx)}{a+bx} \, dx = \frac{a\cos(c+dx)}{b^2 d} - \frac{x\cos(c+dx)}{bd} + \frac{a^2 \operatorname{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^3} + \frac{\sin(c+dx)}{bd^2} + \frac{a^2\cos\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(\frac{ad}{b}+dx\right)}{b^3}$$

```
[Out] a*cos(d*x+c)/b^2/d-x*cos(d*x+c)/b/d+a^2*cos(-c+a*d/b)*Si(a*d/b+d*x)/b^3-a^2
*Ci(a*d/b+d*x)*sin(-c+a*d/b)/b^3+sin(d*x+c)/b/d^2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {6874, 2718, 3377, 2717, 3384, 3380, 3383}

$$\int \frac{x^2 \sin(c+dx)}{a+bx} \, dx = \frac{a^2 \sin\left(c-\frac{ad}{b}\right)\operatorname{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b^3} + \frac{a^2\cos\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(xd+\frac{ad}{b}\right)}{b^3} + \frac{a\cos(c+dx)}{b^2 d} + \frac{\sin(c+dx)}{bd^2} - \frac{x\cos(c+dx)}{bd}$$

```
[In] Int[(x^2*Sin[c + d*x])/(a + b*x),x]
```

```
[Out] (a*Cos[c + d*x])/(b^2*d) - (x*Cos[c + d*x])/(b*d) + (a^2*CosIntegral[(a*d)/
b + d*x]*Sin[c - (a*d)/b])/b^3 + Sin[c + d*x]/(b*d^2) + (a^2*Cos[c - (a*d)/
b]*SinIntegral[(a*d)/b + d*x])/b^3
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{a\sin(c+dx)}{b^2}+\frac{x\sin(c+dx)}{b}+\frac{a^2\sin(c+dx)}{b^2(a+bx)}\right)\,dx\\
&= -\frac{a\int \sin(c+dx)\,dx}{b^2}+\frac{a^2\int \frac{\sin(c+dx)}{a+bx}\,dx}{b^2}+\frac{\int x\sin(c+dx)\,dx}{b}
\end{aligned}$$

$$= \frac{a\cos(c+dx)}{b^2d} - \frac{x\cos(c+dx)}{bd} + \frac{\int \cos(c+dx)\,dx}{bd} + \frac{\left(a^2\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^2} + \frac{\left(a^2\sin\left(c-\frac{ad}{b}\right)\right)\int \frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^2}$$

$$= \frac{a\cos(c+dx)}{b^2d} - \frac{x\cos(c+dx)}{bd} + \frac{a^2\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^3} + \frac{\sin(c+dx)}{bd^2} + \frac{a^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^3}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.88

$$\int \frac{x^2\sin(c+dx)}{a+bx}\,dx = \frac{a^2d^2\,\text{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)\sin\left(c-\frac{ad}{b}\right)+b(d(a-bx)\cos(c+dx)+b\sin(c+dx))+a^2d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}(}{b^3d^2}$$

```
[In] Integrate[(x^2*Sin[c + d*x])/(a + b*x),x]
```

```
[Out] (a^2*d^2*CosIntegral[d*(a/b + x)]*Sin[c - (a*d)/b] + b*(d*(a - b*x)*Cos[c +
 d*x] + b*Sin[c + d*x]) + a^2*d^2*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)]
)/(b^3*d^2)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.82

method	result
risch	$-\frac{(dxb-da)\cos(dx+c)}{d^2b^2}+\frac{\sin(dx+c)}{b\,d^2}-\frac{ia^2\cos\left(\frac{da-cb}{b}\right)\text{Ei}_1\left(\frac{id(bx+a)}{b}\right)}{2b^3}+\frac{ia^2\cos\left(\frac{da-cb}{b}\right)\text{Ei}_1\left(-\frac{id(bx+a)}{b}\right)}{2b^3}+\frac{a^2\,\text{si}}{}$
derivativedivides	$\frac{c^2d\left(\frac{\text{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}-\frac{\text{Ci}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}\right)+\frac{2(da-cb)cd\left(\frac{\text{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}-\frac{\text{Ci}\left(dx+c+\right.}{}\right.}{b}}{}$
default	$\frac{c^2d\left(\frac{\text{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}-\frac{\text{Ci}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}\right)+\frac{2(da-cb)cd\left(\frac{\text{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}-\frac{\text{Ci}\left(dx+c+\right.}{}\right.}{b}}{}$

```
[In] int(x^2*sin(d*x+c)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^2*(b*d*x-a*d)/b^2*cos(d*x+c)+sin(d*x+c)/b/d^2-1/2*I*a^2/b^3*cos((a*d-b
*c)/b)*Ei(1,I*d*(b*x+a)/b)+1/2*I*a^2/b^3*cos((a*d-b*c)/b)*Ei(1,-I*d*(b*x+a)
```

```
/b)+1/2*a^2/b^3*sin((a*d-b*c)/b)*Ei(1,I*d*(b*x+a)/b)+1/2*a^2/b^3*sin((a*d-b
*c)/b)*Ei(1,-I*d*(b*x+a)/b)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.09

$$\int \frac{x^2 \sin(c+dx)}{a+bx}\,dx = -\frac{a^2d^2\operatorname{Ci}\left(\frac{bdx+ad}{b}\right)\sin\left(-\frac{bc-ad}{b}\right) - a^2d^2\cos\left(-\frac{bc-ad}{b}\right)\operatorname{Si}\left(\frac{bdx+ad}{b}\right) - b^2\sin(dx+c) + (b^2dx-abd)\cos(dx+}{b^3d^2}$$

```
[In] integrate(x^2*sin(d*x+c)/(b*x+a),x, algorithm="fricas")
```

```
[Out] -(a^2*d^2*cos_integral((b*d*x + a*d)/b)*sin(-(b*c - a*d)/b) - a^2*d^2*cos(-
(b*c - a*d)/b)*sin_integral((b*d*x + a*d)/b) - b^2*sin(d*x + c) + (b^2*d*x
- a*b*d)*cos(d*x + c))/(b^3*d^2)
```

Sympy [F]

$$\int \frac{x^2 \sin(c+dx)}{a+bx}\,dx = \int \frac{x^2 \sin(c+dx)}{a+bx}\,dx$$

```
[In] integrate(x**2*sin(d*x+c)/(b*x+a),x)
```

```
[Out] Integral(x**2*sin(c + d*x)/(a + b*x), x)
```

Maxima [F]

$$\int \frac{x^2 \sin(c+dx)}{a+bx}\,dx = \int \frac{x^2 \sin(dx+c)}{bx+a}\,dx$$

```
[In] integrate(x^2*sin(d*x+c)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*((b*cos(c)^2 + b*sin(c)^2)*d*x^2*cos(d*x + c) - ((a*(I*exp_integral_e(
2, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*cos(c)
^2 + a*(I*exp_integral_e(2, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(2, -(I*
b*d*x + I*a*d)/b))*sin(c)^2)*cos(-(b*c - a*d)/b) - (a*(exp_integral_e(2, (I
*b*d*x + I*a*d)/b) + exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a*
(exp_integral_e(2, (I*b*d*x + I*a*d)/b) + exp_integral_e(2, -(I*b*d*x + I*a
*d)/b))*sin(c)^2)*sin(-(b*c - a*d)/b))*cos(d*x + c)^2 - (b*cos(c)^2 + b*sin
```

```
(c)^2)*x*sin(d*x + c) - ((a*(I*exp_integral_e(2, (I*b*d*x + I*a*d)/b) - I*e
xp_integral_e(2, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a*(I*exp_integral_e(2, (
I*b*d*x + I*a*d)/b) - I*exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*
cos(-(b*c - a*d)/b) - (a*(exp_integral_e(2, (I*b*d*x + I*a*d)/b) + exp_inte
gral_e(2, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a*(exp_integral_e(2, (I*b*d*x +
 I*a*d)/b) + exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*sin(-(b*c -
 a*d)/b))*sin(d*x + c)^2 + ((b*d*x^2*cos(c) + b*x*sin(c))*cos(d*x + c)^2 +
(b*d*x^2*cos(c) + b*x*sin(c))*sin(d*x + c)^2)*cos(d*x + 2*c) - 2*(((a*b^2*c
os(c)^2 + a*b^2*sin(c)^2)*d^3*x + (a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^3)*co
s(d*x + c)^2 + ((a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^3*x + (a^2*b*cos(c)^2 +
 a^2*b*sin(c)^2)*d^3)*sin(d*x + c)^2)*integrate(1/2*x*cos(d*x + c)/(b^2*d^2
*x^2 + 2*a*b*d^2*x + a^2*d^2), x) - 2*(((a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d
^3*x + (a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^3)*cos(d*x + c)^2 + ((a*b^2*cos(
c)^2 + a*b^2*sin(c)^2)*d^3*x + (a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^3)*sin(d
*x + c)^2)*integrate(1/2*x*cos(d*x + c)/((b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d
^2)*cos(d*x + c)^2 + (b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*sin(d*x + c)^2),
 x) + ((b*d*x^2*sin(c) - b*x*cos(c))*cos(d*x + c)^2 + (b*d*x^2*sin(c) - b*x
*cos(c))*sin(d*x + c)^2)*sin(d*x + 2*c))/(((b^2*cos(c)^2 + b^2*sin(c)^2)*d^
2*x + (a*b*cos(c)^2 + a*b*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b^2*cos(c)^2 +
b^2*sin(c)^2)*d^2*x + (a*b*cos(c)^2 + a*b*sin(c)^2)*d^2)*sin(d*x + c)^2)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.35 (sec) , antiderivative size = 2205, normalized size of antiderivative = 22.27

$$\int \frac{x^2 \sin(c + dx)}{a + bx} \, dx = \text{Too large to display}$$

```
[In] integrate(x^2*sin(d*x+c)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(a^2*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*ta
n(1/2*c)^2*tan(1/2*a*d/b)^2 - a^2*d^2*imag_part(cos_integral(-d*x - a*d/b))
*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*d^2*sin_integ
ral((b*d*x + a*d)/b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 +
 2*a^2*d^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(
1/2*c)^2*tan(1/2*a*d/b) + 2*a^2*d^2*real_part(cos_integral(-d*x - a*d/b))*t
an(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a^2*d^2*real_part(cos
_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)*tan(1/2*a*d/b)^2
- 2*a^2*d^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/2*c)^2*ta
n(1/2*c)*tan(1/2*a*d/b)^2 + 2*b^2*d*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*t
an(1/2*a*d/b)^2 - a^2*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x
+ 1/2*c)^2*tan(1/2*c)^2 + a^2*d^2*imag_part(cos_integral(-d*x - a*d/b))*tan
(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - 2*a^2*d^2*sin_integral((b*d*x + a*d)/b)*
tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + 4*a^2*d^2*imag_part(cos_integral(d*x
```

+ a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)*tan(1/2*a*d/b) - 4*a^2*d^2*imag
_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)*tan(1/2
*a*d/b) + 8*a^2*d^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x + 1/2*c)^2*ta
n(1/2*c)*tan(1/2*a*d/b) - a^2*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(
1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)^2 + a^2*d^2*imag_part(cos_integral(-d*x -
 a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^2*d^2*sin_integral((
b*d*x + a*d)/b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)^2 + a^2*d^2*imag_part
(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^2*d^2*imag_pa
rt(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*d^2*si
n_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a*b*d*tan(1/2
*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*d^2*real_part(cos_int
egral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c) + 2*a^2*d^2*real_part
(cos_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c) + 2*b^2*d*x*
tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - 2*a^2*d^2*real_part(cos_integral(d*x
+ a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b) - 2*a^2*d^2*real_part(cos_i
ntegral(-d*x - a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b) + 2*a^2*d^2*re
al_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^2*d^2*
real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*b^2*d
*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^2*d^2*real_part(cos_integr
al(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^2*d^2*real_part(cos_inte
gral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*b^2*d*x*tan(1/2*c)^2*ta
n(1/2*a*d/b)^2 + a^2*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x +
 1/2*c)^2 - a^2*d^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/2
*c)^2 + 2*a^2*d^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x + 1/2*c)^2 - a^
2*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 + a^2*d^2*imag_part
(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 - 2*a^2*d^2*sin_integral((b*d*x +
 a*d)/b)*tan(1/2*c)^2 - 2*a*b*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + 4*a^2
*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 4*a^2
*d^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^
2*d^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) - a^2*d^2*ima
g_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 + a^2*d^2*imag_part(cos_
integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - 2*a^2*d^2*sin_integral((b*d*x +
a*d)/b)*tan(1/2*a*d/b)^2 - 2*a*b*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)^2
+ 2*a*b*d*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*b^2*tan(1/2*d*x + 1/2*c)*tan(1/
2*c)^2*tan(1/2*a*d/b)^2 + 2*b^2*d*x*tan(1/2*d*x + 1/2*c)^2 + 2*a^2*d^2*real
_part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 2*a^2*d^2*real_part(cos_integ
ral(-d*x - a*d/b))*tan(1/2*c) - 2*b^2*d*x*tan(1/2*c)^2 - 2*a^2*d^2*real_par
t(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 2*a^2*d^2*real_part(cos_integ
ral(-d*x - a*d/b))*tan(1/2*a*d/b) - 2*b^2*d*x*tan(1/2*a*d/b)^2 + a^2*d^2*im
ag_part(cos_integral(d*x + a*d/b)) - a^2*d^2*imag_part(cos_integral(-d*x -
a*d/b)) + 2*a^2*d^2*sin_integral((b*d*x + a*d)/b) - 2*a*b*d*tan(1/2*d*x + 1
/2*c)^2 + 2*a*b*d*tan(1/2*c)^2 + 4*b^2*tan(1/2*d*x + 1/2*c)*tan(1/2*c)^2 +
2*a*b*d*tan(1/2*a*d/b)^2 + 4*b^2*tan(1/2*d*x + 1/2*c)*tan(1/2*a*d/b)^2 - 2*
b^2*d*x + 2*a*b*d + 4*b^2*tan(1/2*d*x + 1/2*c))/(b^3*d^2*tan(1/2*d*x + 1/2*
c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + b^3*d^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2

```
*c)^2 + b^3*d^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)^2 + b^3*d^2*tan(1/2*c
)^2*tan(1/2*a*d/b)^2 + b^3*d^2*tan(1/2*d*x + 1/2*c)^2 + b^3*d^2*tan(1/2*c)^
2 + b^3*d^2*tan(1/2*a*d/b)^2 + b^3*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \sin(c+dx)}{a+bx} \, dx = \int \frac{x^2 \, \sin\left(c+d\,x\right)}{a+b\,x} \,dx$$

```
[In] int((x^2*sin(c + d*x))/(a + b*x),x)
[Out] int((x^2*sin(c + d*x))/(a + b*x), x)
```

3.21 $\int \frac{x\sin(c+dx)}{a+bx}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 69

$$\int \frac{x\sin(c+dx)}{a+bx}\,dx = -\frac{\cos(c+dx)}{bd} - \frac{a\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^2} - \frac{a\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^2}$$

```
[Out] -cos(d*x+c)/b/d-a*cos(-c+a*d/b)*Si(a*d/b+d*x)/b^2+a*Ci(a*d/b+d*x)*sin(-c+a*
d/b)/b^2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6874, 2718, 3384, 3380, 3383}

$$\int \frac{x\sin(c+dx)}{a+bx}\,dx = -\frac{a\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b^2} - \frac{a\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b^2} - \frac{\cos(c+dx)}{bd}$$

```
[In] Int[(x*Sin[c + d*x])/(a + b*x),x]
```

```
[Out] -(Cos[c + d*x]/(b*d)) - (a*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/b^2
 - (a*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^2
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\sin(c+dx)}{b} - \frac{a\sin(c+dx)}{b(a+bx)}\right) dx \\
&= \frac{\int \sin(c+dx)\,dx}{b} - \frac{a\int \frac{\sin(c+dx)}{a+bx}\,dx}{b} \\
&= -\frac{\cos(c+dx)}{bd} - \frac{\left(a\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b} - \frac{\left(a\sin\left(c-\frac{ad}{b}\right)\right)\int \frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b} \\
&= -\frac{\cos(c+dx)}{bd} - \frac{a\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^2} - \frac{a\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \frac{x\sin(c+dx)}{a+bx}\,dx = -\frac{b\cos(c+dx) + ad\,\text{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)\sin\left(c-\frac{ad}{b}\right) + ad\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(d\left(\frac{a}{b}+x\right)\right)}{b^2 d}$$

```
[In] Integrate[(x*Sin[c + d*x])/(a + b*x),x]
[Out] -((b*Cos[c + d*x] + a*d*CosIntegral[d*(a/b + x)]*Sin[c - (a*d)/b] + a*d*Cos
[c - (a*d)/b]*SinIntegral[d*(a/b + x)])/(b^2*d))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.17

method	result
risch	$-\frac{\cos(dx+c)}{bd}+\frac{ia\cos\left(\frac{da-cb}{b}\right)\mathrm{Ei}_1\left(\frac{id(bx+a)}{b}\right)}{2b^2}-\frac{ia\cos\left(\frac{da-cb}{b}\right)\mathrm{Ei}_1\left(-\frac{id(bx+a)}{b}\right)}{2b^2}-\frac{a\sin\left(\frac{da-cb}{b}\right)\mathrm{Ei}_1\left(\frac{id(bx+a)}{b}\right)}{2b^2}$
derivativedivides	$\frac{-\frac{(da-cb)d\left(\frac{\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}-\frac{\mathrm{Ci}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}\right)}{b}-\frac{d\cos(dx+c)}{b}-dc\left(\frac{\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-c}{b}\right.}{b}\right.}{d^2}$
default	$\frac{-\frac{(da-cb)d\left(\frac{\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}-\frac{\mathrm{Ci}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}\right)}{b}-\frac{d\cos(dx+c)}{b}-dc\left(\frac{\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-c}{b}\right.}{b}\right.}{d^2}$

```
[In] int(x*sin(d*x+c)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -cos(d*x+c)/b/d+1/2*I*a/b^2*cos((a*d-b*c)/b)*Ei(1,I*d*(b*x+a)/b)-1/2*I*a/b^
2*cos((a*d-b*c)/b)*Ei(1,-I*d*(b*x+a)/b)-1/2*a/b^2*sin((a*d-b*c)/b)*Ei(1,I*d
*(b*x+a)/b)-1/2*a/b^2*sin((a*d-b*c)/b)*Ei(1,-I*d*(b*x+a)/b)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int \frac{x\sin(c+dx)}{a+bx}\,dx = \frac{ad\,\mathrm{Ci}\left(\frac{bdx+ad}{b}\right)\sin\left(-\frac{bc-ad}{b}\right)-ad\cos\left(-\frac{bc-ad}{b}\right)\mathrm{Si}\left(\frac{bdx+ad}{b}\right)-b\cos(dx+c)}{b^2d}$$

```
[In] integrate(x*sin(d*x+c)/(b*x+a),x, algorithm="fricas")
```

```
[Out] (a*d*cos_integral((b*d*x + a*d)/b)*sin(-(b*c - a*d)/b) - a*d*cos(-(b*c - a*
d)/b)*sin_integral((b*d*x + a*d)/b) - b*cos(d*x + c))/(b^2*d)
```

Sympy [F]

$$\int \frac{x\sin(c+dx)}{a+bx}\,dx = \int \frac{x\sin{(c+dx)}}{a+bx}\,dx$$

[In] integrate(x*sin(d*x+c)/(b*x+a),x)

[Out] Integral(x*sin(c + d*x)/(a + b*x), x)

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 776, normalized size of antiderivative = 11.25

$$\int \frac{x\sin(c+dx)}{a+bx}\,dx = -\frac{\frac{\left(d\left(-iE_1\left(\frac{i(dx+c)b-ibc+iad}{b}\right)+iE_1\left(-\frac{i(dx+c)b-ibc+iad}{b}\right)\right)\cos\left(-\frac{bc-ad}{b}\right)+d\left(E_1\left(\frac{i(dx+c)b-ibc+iad}{b}\right)+E_1\left(-\frac{i(dx+c)b-ibc+iad}{b}\right)\right)\sin\left(-\frac{bc-}{b}\right.}{b}}{}$$

[In] integrate(x*sin(d*x+c)/(b*x+a),x, algorithm="maxima")

[Out] -1/2*((d*(-I*exp_integral_e(1, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + I*exp_integral_e(1, -(I*(d*x + c)*b - I*b*c + I*a*d)/b))*cos(-(b*c - a*d)/b) + d*(exp_integral_e(1, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + exp_integral_e(1, -(I*(d*x + c)*b - I*b*c + I*a*d)/b))*sin(-(b*c - a*d)/b))*c/b + ((d*x + c)*b*d*cos(d*x + c)^3 + (d*x + c)*b*d*cos(d*x + c) - ((b*c*d*(exp_integral_e(2, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + exp_integral_e(2, -(I*(d*x + c)*b - I*b*c + I*a*d)/b)) - a*d^2*(exp_integral_e(2, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + exp_integral_e(2, -(I*(d*x + c)*b - I*b*c + I*a*d)/b)))*cos(-(b*c - a*d)/b) - (a*d^2*(I*exp_integral_e(2, (I*(d*x + c)*b - I*b*c + I*a*d)/b) - I*exp_integral_e(2, -(I*(d*x + c)*b - I*b*c + I*a*d)/b)) + b*c*d*(-I*exp_integral_e(2, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + I*exp_integral_e(2, -(I*(d*x + c)*b - I*b*c + I*a*d)/b)))*sin(-(b*c - a*d)/b))*cos(d*x + c)^2 + ((d*x + c)*b*d*cos(d*x + c) - (b*c*d*(exp_integral_e(2, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + exp_integral_e(2, -(I*(d*x + c)*b - I*b*c + I*a*d)/b)) - a*d^2*(exp_integral_e(2, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + exp_integral_e(2, -(I*(d*x + c)*b - I*b*c + I*a*d)/b)))*cos(-(b*c - a*d)/b) + (a*d^2*(I*exp_integral_e(2, (I*(d*x + c)*b - I*b*c + I*a*d)/b) - I*exp_integral_e(2, -(I*(d*x + c)*b - I*b*c + I*a*d)/b)) + b*c*d*(-I*exp_integral_e(2, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + I*exp_integral_e(2, -(I*(d*x + c)*b - I*b*c + I*a*d)/b)))*sin(-(b*c - a*d)/b))*sin(d*x + c)^2)/(((d*x + c)*b^2 - b^2*c + a*b*d)*cos(d*x + c)^2 + ((d*x + c)*b^2 - b^2*c + a*b*d)*sin(d*x + c)^2))/d^2

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.32 (sec) , antiderivative size = 1647, normalized size of antiderivative = 23.87

$$\int \frac{x\sin(c+dx)}{a+bx}\,dx = \text{Too large to display}$$

```
[In] integrate(x*sin(d*x+c)/(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(a*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*
tan(1/2*a*d/b)^2 - a*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2
*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*d*sin_integral((b*d*x + a*d)/b)*tan(1/
2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*d*real_part(cos_integral(d*x +
 a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a*d*real_part(cos_i
ntegral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a*d*r
eal_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b
)^2 - 2*a*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)
*tan(1/2*a*d/b)^2 - a*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2
*tan(1/2*c)^2 + a*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*ta
n(1/2*c)^2 - 2*a*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^
2 + 4*a*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*ta
n(1/2*a*d/b) - 4*a*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*t
an(1/2*c)*tan(1/2*a*d/b) + 8*a*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)
^2*tan(1/2*c)*tan(1/2*a*d/b) - a*d*imag_part(cos_integral(d*x + a*d/b))*tan
(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a*d*imag_part(cos_integral(-d*x - a*d/b))*ta
n(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*a*d*sin_integral((b*d*x + a*d)/b)*tan(1/2
*d*x)^2*tan(1/2*a*d/b)^2 + a*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2
*c)^2*tan(1/2*a*d/b)^2 - a*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*
c)^2*tan(1/2*a*d/b)^2 + 2*a*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*ta
n(1/2*a*d/b)^2 + 2*b*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*d*r
eal_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 2*a*d*real_
part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d*real_par
t(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 2*a*d*real_par
t(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 2*a*d*real_pa
rt(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a*d*real_part
(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a*d*real_part(
cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a*d*real_part(co
s_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + a*d*imag_part(cos_i
ntegral(d*x + a*d/b))*tan(1/2*d*x)^2 - a*d*imag_part(cos_integral(-d*x - a*
d/b))*tan(1/2*d*x)^2 + 2*a*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2 -
 a*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 + a*d*imag_part(cos_
integral(-d*x - a*d/b))*tan(1/2*c)^2 - 2*a*d*sin_integral((b*d*x + a*d)/b)*
tan(1/2*c)^2 + 2*b*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a*d*imag_part(cos_integr
al(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 4*a*d*imag_part(cos_integral(-
```

```
d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 8*a*d*sin_integral((b*d*x + a*d)/
b)*tan(1/2*c)*tan(1/2*a*d/b) - a*d*imag_part(cos_integral(d*x + a*d/b))*tan
(1/2*a*d/b)^2 + a*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2
- 2*a*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 - 2*b*tan(1/2*d*x)^2
*tan(1/2*a*d/b)^2 - 8*b*tan(1/2*d*x)*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*b*tan(
1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*d*real_part(cos_integral(d*x + a*d/b))*tan(
1/2*c) + 2*a*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) - 2*a*d*rea
l_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 2*a*d*real_part(cos_inte
gral(-d*x - a*d/b))*tan(1/2*a*d/b) + a*d*imag_part(cos_integral(d*x + a*d/b
)) - a*d*imag_part(cos_integral(-d*x - a*d/b)) + 2*a*d*sin_integral((b*d*x
+ a*d)/b) - 2*b*tan(1/2*d*x)^2 - 8*b*tan(1/2*d*x)*tan(1/2*c) - 2*b*tan(1/2*
c)^2 + 2*b*tan(1/2*a*d/b)^2 + 2*b)/(b^2*d*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1
/2*a*d/b)^2 + b^2*d*tan(1/2*d*x)^2*tan(1/2*c)^2 + b^2*d*tan(1/2*d*x)^2*tan(
1/2*a*d/b)^2 + b^2*d*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + b^2*d*tan(1/2*d*x)^2 +
 b^2*d*tan(1/2*c)^2 + b^2*d*tan(1/2*a*d/b)^2 + b^2*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sin(c+dx)}{a+bx}\,dx = \int \frac{x\,\sin\left(c+d\,x\right)}{a+b\,x}\,dx$$

```
[In] int((x*sin(c + d*x))/(a + b*x),x)
[Out] int((x*sin(c + d*x))/(a + b*x), x)
```

3.22 $\int \frac{\sin(c+dx)}{a+bx} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 51

$$\int \frac{\sin(c+dx)}{a+bx} \, dx = \frac{\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b} + \frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b}$$

```
[Out] cos(-c+a*d/b)*Si(a*d/b+d*x)/b-Ci(a*d/b+d*x)*sin(-c+a*d/b)/b
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3384, 3380, 3383}

$$\int \frac{\sin(c+dx)}{a+bx} \, dx = \frac{\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b} + \frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b}$$

```
[In] Int[Sin[c + d*x]/(a + b*x),x]
```

```
[Out] (CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/b + (Cos[c - (a*d)/b]*SinInte
gral[(a*d)/b + d*x])/b
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\text{integral} = \cos\left(c-\frac{ad}{b}\right)\int\frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx+\sin\left(c-\frac{ad}{b}\right)\int\frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx$$
$$=\frac{\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b}+\frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int\frac{\sin(c+dx)}{a+bx}\,dx=\frac{\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)+\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b}$$

```
[In] Integrate[Sin[c + d*x]/(a + b*x),x]
[Out] (CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b] + Cos[c - (a*d)/b]*SinIntegral
[(a*d)/b + d*x])/b
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.43

method	result	size
derivativedivides	$\frac{\text{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}-\frac{\text{Ci}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}$	73
default	$\frac{\text{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}-\frac{\text{Ci}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}$	73
risch	$\frac{i\,e^{-\frac{i(da-cb)}{b}}\,\text{Ei}_1\left(-idx-ic-\frac{iad-icb}{b}\right)}{2b}-\frac{i\,e^{\frac{i(da-cb)}{b}}\,\text{Ei}_1\left(idx+ic+\frac{i(da-cb)}{b}\right)}{2b}$	98

```
[In] int(sin(d*x+c)/(b*x+a),x,method=_RETURNVERBOSE)
[Out] Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c
)/b)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24

$$\int \frac{\sin(c+dx)}{a+bx}\,dx = -\frac{\operatorname{Ci}\left(\frac{bdx+ad}{b}\right)\sin\left(-\frac{bc-ad}{b}\right) - \cos\left(-\frac{bc-ad}{b}\right)\operatorname{Si}\left(\frac{bdx+ad}{b}\right)}{b}$$

```
[In] integrate(sin(d*x+c)/(b*x+a),x, algorithm="fricas")
```

```
[Out] -(cos_integral((b*d*x + a*d)/b)*sin(-(b*c - a*d)/b) - cos(-(b*c - a*d)/b)*s
in_integral((b*d*x + a*d)/b))/b
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{a+bx}\,dx = \int \frac{\sin(c+dx)}{a+bx}\,dx$$

```
[In] integrate(sin(d*x+c)/(b*x+a),x)
```

```
[Out] Integral(sin(c + d*x)/(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.76

$$\int \frac{\sin(c+dx)}{a+bx}\,dx = \frac{d\left(-i\,E_1\left(\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)+i\,E_1\left(-\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)\right)\cos\left(-\frac{bc-ad}{b}\right)+d\left(E_1\left(\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)+E_1\left(-\frac{i\,(d}{}\right.\right.}{2\,bd}$$

```
[In] integrate(sin(d*x+c)/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(d*(-I*exp_integral_e(1, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + I*exp_int
egral_e(1, -(I*(d*x + c)*b - I*b*c + I*a*d)/b))*cos(-(b*c - a*d)/b) + d*(ex
p_integral_e(1, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + exp_integral_e(1, -(I*
(d*x + c)*b - I*b*c + I*a*d)/b))*sin(-(b*c - a*d)/b))/(b*d)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.33 (sec) , antiderivative size = 597, normalized size of antiderivative = 11.71

$$\int \frac{\sin(c+dx)}{a+bx}\,dx$$
$$= \frac{\Im\left(\mathrm{Ci}\left(dx+\frac{ad}{b}\right)\right)\tan\left(\frac{1}{2}c\right)^2\tan\left(\frac{ad}{2b}\right)^2 - \Im\left(\mathrm{Ci}\left(-dx-\frac{ad}{b}\right)\right)\tan\left(\frac{1}{2}c\right)^2\tan\left(\frac{ad}{2b}\right)^2 + 2\,\mathrm{Si}\left(\frac{bdx+ad}{b}\right)\tan\left(\frac{1}{2}c\right)^2\tan}{}$$

```
[In] integrate(sin(d*x+c)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - i
mag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*sin_
integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*real_part(cos_i
ntegral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*real_part(cos_integra
l(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*real_part(cos_integral(d*x
 + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*real_part(cos_integral(-d*x - a*
d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - imag_part(cos_integral(d*x + a*d/b))*ta
n(1/2*c)^2 + imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 - 2*sin_int
egral((b*d*x + a*d)/b)*tan(1/2*c)^2 + 4*imag_part(cos_integral(d*x + a*d/b)
)*tan(1/2*c)*tan(1/2*a*d/b) - 4*imag_part(cos_integral(-d*x - a*d/b))*tan(1
/2*c)*tan(1/2*a*d/b) + 8*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a
*d/b) - imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 + imag_part(c
os_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - 2*sin_integral((b*d*x + a*d)/
b)*tan(1/2*a*d/b)^2 + 2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 2
*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) - 2*real_part(cos_integra
l(d*x + a*d/b))*tan(1/2*a*d/b) - 2*real_part(cos_integral(-d*x - a*d/b))*ta
n(1/2*a*d/b) + imag_part(cos_integral(d*x + a*d/b)) - imag_part(cos_integra
l(-d*x - a*d/b)) + 2*sin_integral((b*d*x + a*d)/b))/(b*tan(1/2*c)^2*tan(1/2
*a*d/b)^2 + b*tan(1/2*c)^2 + b*tan(1/2*a*d/b)^2 + b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{a+bx}\,dx = \int \frac{\sin(c+d\,x)}{a+b\,x}\,dx$$

```
[In] int(sin(c + d*x)/(a + b*x),x)
```

```
[Out] int(sin(c + d*x)/(a + b*x), x)
```

3.23 $\int \frac{\sin(c+dx)}{x(a+bx)} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 73

$$\int \frac{\sin(c+dx)}{x(a+bx)} \, dx = \frac{\text{CosIntegral}(dx)\sin(c)}{a} - \frac{\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a} + \frac{\cos(c)\text{Si}(dx)}{a} - \frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a}$$

```
[Out] cos(c)*Si(d*x)/a-cos(-c+a*d/b)*Si(a*d/b+d*x)/a+Ci(d*x)*sin(c)/a+Ci(a*d/b+d*
x)*sin(-c+a*d/b)/a
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6874, 3384, 3380, 3383}

$$\int \frac{\sin(c+dx)}{x(a+bx)} \, dx = -\frac{\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{a} - \frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a} + \frac{\sin(c)\,\text{CosIntegral}(dx)}{a} + \frac{\cos(c)\text{Si}(dx)}{a}$$

```
[In] Int[Sin[c + d*x]/(x*(a + b*x)),x]
```

```
[Out] (CosIntegral[d*x]*Sin[c])/a - (CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])
/a + (Cos[c]*SinIntegral[d*x])/a - (Cos[c - (a*d)/b]*SinIntegral[(a*d)/b +
d*x])/a
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{\sin(c+dx)}{ax} - \frac{b\sin(c+dx)}{a(a+bx)}\right) dx \\
&= \frac{\int \frac{\sin(c+dx)}{x}\,dx}{a} - \frac{b\int \frac{\sin(c+dx)}{a+bx}\,dx}{a} \\
&= \frac{\cos(c)\int \frac{\sin(dx)}{x}\,dx}{a} - \frac{\left(b\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a} \\
&\quad + \frac{\sin(c)\int \frac{\cos(dx)}{x}\,dx}{a} - \frac{\left(b\sin\left(c-\frac{ad}{b}\right)\right)\int \frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a} \\
&= \frac{\text{CosIntegral}(dx)\sin(c)}{a} - \frac{\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a} \\
&\quad + \frac{\cos(c)\text{Si}(dx)}{a} - \frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \frac{\sin(c+dx)}{x(a+bx)}\,dx = \frac{\text{CosIntegral}(dx)\sin(c) - \text{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)\sin\left(c-\frac{ad}{b}\right) + \cos(c)\text{Si}(dx) - \cos\left(c-\frac{ad}{b}\right)\text{Si}\left(d\left(\frac{a}{b}+x\right)\right)}{a}$$

```
[In] Integrate[Sin[c + d*x]/(x*(a + b*x)),x]
[Out] (CosIntegral[d*x]*Sin[c] - CosIntegral[d*(a/b + x)]*Sin[c - (a*d)/b] + Cos[
c]*SinIntegral[d*x] - Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)])/a
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.36

method	result
derivativedivides	$\frac{\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin(c)}{a} - \frac{b\left(\frac{\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b} - \frac{\mathrm{Ci}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}\right)}{a}$
default	$\frac{\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin(c)}{a} - \frac{b\left(\frac{\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b} - \frac{\mathrm{Ci}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}\right)}{a}$
risch	$\frac{i e^{ic}\,\mathrm{Ei}_1(-idx)}{2a} - \frac{i e^{-\frac{i(da-cb)}{b}}\,\mathrm{Ei}_1\left(-idx-ic-\frac{iad-icb}{b}\right)}{2a} - \frac{e^{-ic}\pi\,\mathrm{csgn}(dx)}{2a} + \frac{e^{-ic}\,\mathrm{Si}(dx)}{a} - \frac{i e^{-ic}\,\mathrm{Ei}_1(-idx)}{2a} + \ldots$

```
[In] int(sin(d*x+c)/x/(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/a*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))-b/a*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c
)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04

$$\int \frac{\sin(c+dx)}{x(a+bx)}\,dx = \frac{\mathrm{Ci}\,(dx)\sin\,(c) + \mathrm{Ci}\left(\frac{bdx+ad}{b}\right)\sin\left(-\frac{bc-ad}{b}\right) + \cos\,(c)\,\mathrm{Si}\,(dx) - \cos\left(-\frac{bc-ad}{b}\right)\mathrm{Si}\left(\frac{bdx+ad}{b}\right)}{a}$$

```
[In] integrate(sin(d*x+c)/x/(b*x+a),x, algorithm="fricas")
```

```
[Out] (cos_integral(d*x)*sin(c) + cos_integral((b*d*x + a*d)/b)*sin(-(b*c - a*d)/
b) + cos(c)*sin_integral(d*x) - cos(-(b*c - a*d)/b)*sin_integral((b*d*x + a
*d)/b))/a
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{x(a+bx)}\,dx = \int \frac{\sin(c+dx)}{x(a+bx)}\,dx$$

```
[In] integrate(sin(d*x+c)/x/(b*x+a),x)
[Out] Integral(sin(c + d*x)/(x*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x(a+bx)}\,dx = \int \frac{\sin(dx+c)}{(bx+a)x}\,dx$$

```
[In] integrate(sin(d*x+c)/x/(b*x+a),x, algorithm="maxima")
[Out] integrate(sin(d*x + c)/((b*x + a)*x), x)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.31 (sec) , antiderivative size = 838, normalized size of antiderivative = 11.48

$$\int \frac{\sin(c+dx)}{x(a+bx)}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)/x/(b*x+a),x, algorithm="giac")
[Out] -1/2*(imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 +
imag_part(cos_integral(d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - imag_part(cos_
integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - imag_part(cos_integ
ral(-d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*sin_integral(d*x)*tan(1/2*c)^2
*tan(1/2*a*d/b)^2 + 2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*
d/b)^2 + 2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)
 + 2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*
real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*real_p
art(cos_integral(d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*real_part(cos_integr
al(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*real_part(cos_integral(-d
```

```
*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 - imag_part(cos_integral(d*x + a*d/b))*tan
(1/2*c)^2 + imag_part(cos_integral(d*x))*tan(1/2*c)^2 + imag_part(cos_integ
ral(-d*x - a*d/b))*tan(1/2*c)^2 - imag_part(cos_integral(-d*x))*tan(1/2*c)^
2 + 2*sin_integral(d*x)*tan(1/2*c)^2 - 2*sin_integral((b*d*x + a*d)/b)*tan(
1/2*c)^2 + 4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)
 - 4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 8*si
n_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) - imag_part(cos_integ
ral(d*x + a*d/b))*tan(1/2*a*d/b)^2 - imag_part(cos_integral(d*x))*tan(1/2*a
*d/b)^2 + imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 + imag_par
t(cos_integral(-d*x))*tan(1/2*a*d/b)^2 - 2*sin_integral(d*x)*tan(1/2*a*d/b)
^2 - 2*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 + 2*real_part(cos_int
egral(d*x + a*d/b))*tan(1/2*c) - 2*real_part(cos_integral(d*x))*tan(1/2*c)
+ 2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) - 2*real_part(cos_inte
gral(-d*x))*tan(1/2*c) - 2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d
/b) - 2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) + imag_part(co
s_integral(d*x + a*d/b)) - imag_part(cos_integral(d*x)) - imag_part(cos_int
egral(-d*x - a*d/b)) + imag_part(cos_integral(-d*x)) - 2*sin_integral(d*x)
+ 2*sin_integral((b*d*x + a*d)/b))/(a*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a*tan
(1/2*c)^2 + a*tan(1/2*a*d/b)^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x(a+bx)}\,dx=\int \frac{\sin\left(c+d\,x\right)}{x\,\left(a+b\,x\right)} \,d x$$

```
[In] int(sin(c + d*x)/(x*(a + b*x)),x)
[Out] int(sin(c + d*x)/(x*(a + b*x)), x)
```

3.24 $\int \frac{\sin(c+dx)}{x^2(a+bx)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 114

$$\int \frac{\sin(c+dx)}{x^2(a+bx)}\,dx = \frac{d\cos(c)\,\text{CosIntegral}(dx)}{a} - \frac{b\,\text{CosIntegral}(dx)\sin(c)}{a^2} + \frac{b\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^2} - \frac{\sin(c+dx)}{ax} - \frac{b\cos(c)\text{Si}(dx)}{a^2} - \frac{d\sin(c)\text{Si}(dx)}{a} + \frac{b\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^2}$$

```
[Out] d*Ci(d*x)*cos(c)/a-b*cos(c)*Si(d*x)/a^2+b*cos(-c+a*d/b)*Si(a*d/b+d*x)/a^2-b
*Ci(d*x)*sin(c)/a^2-d*Si(d*x)*sin(c)/a-b*Ci(a*d/b+d*x)*sin(-c+a*d/b)/a^2-si
n(d*x+c)/a/x
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6874, 3378, 3384, 3380, 3383}

$$\int \frac{\sin(c+dx)}{x^2(a+bx)}\,dx = -\frac{b\sin(c)\,\text{CosIntegral}(dx)}{a^2} + \frac{b\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{a^2} - \frac{b\cos(c)\text{Si}(dx)}{a^2} + \frac{b\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^2} + \frac{d\cos(c)\,\text{CosIntegral}(dx)}{a} - \frac{d\sin(c)\text{Si}(dx)}{a} - \frac{\sin(c+dx)}{ax}$$

```
[In] Int[Sin[c + d*x]/(x^2*(a + b*x)),x]
```

```
[Out] (d*Cos[c]*CosIntegral[d*x])/a - (b*CosIntegral[d*x]*Sin[c])/a^2 + (b*CosInt
egral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/a^2 - Sin[c + d*x]/(a*x) - (b*Cos[c]
*SinIntegral[d*x])/a^2 - (d*Sin[c]*SinIntegral[d*x])/a + (b*Cos[c - (a*d)/b
]*SinIntegral[(a*d)/b + d*x])/a^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\sin(c+dx)}{ax^2} - \frac{b\sin(c+dx)}{a^2x} + \frac{b^2\sin(c+dx)}{a^2(a+bx)}\right) dx \\
&= \frac{\int \frac{\sin(c+dx)}{x^2}\,dx}{a} - \frac{b\int \frac{\sin(c+dx)}{x}\,dx}{a^2} + \frac{b^2\int \frac{\sin(c+dx)}{a+bx}\,dx}{a^2}
\end{aligned}$$

$$= -\frac{\sin(c+dx)}{ax} + \frac{d\int \frac{\cos(c+dx)}{x}\,dx}{a} - \frac{(b\cos(c))\int \frac{\sin(dx)}{x}\,dx}{a^2}$$
$$+ \frac{\left(b^2\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^2}$$
$$- \frac{(b\sin(c))\int \frac{\cos(dx)}{x}\,dx}{a^2} + \frac{\left(b^2\sin\left(c-\frac{ad}{b}\right)\right)\int \frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^2}$$
$$= -\frac{b\,\mathrm{CosIntegral}(dx)\sin(c)}{a^2} + \frac{b\,\mathrm{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^2}$$
$$- \frac{\sin(c+dx)}{ax} - \frac{b\cos(c)\mathrm{Si}(dx)}{a^2} + \frac{b\cos\left(c-\frac{ad}{b}\right)\mathrm{Si}\left(\frac{ad}{b}+dx\right)}{a^2}$$
$$+ \frac{(d\cos(c))\int \frac{\cos(dx)}{x}\,dx}{a} - \frac{(d\sin(c))\int \frac{\sin(dx)}{x}\,dx}{a}$$
$$= \frac{d\cos(c)\,\mathrm{CosIntegral}(dx)}{a} - \frac{b\,\mathrm{CosIntegral}(dx)\sin(c)}{a^2} + \frac{b\,\mathrm{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^2}$$
$$- \frac{\sin(c+dx)}{ax} - \frac{b\cos(c)\mathrm{Si}(dx)}{a^2} - \frac{d\sin(c)\mathrm{Si}(dx)}{a} + \frac{b\cos\left(c-\frac{ad}{b}\right)\mathrm{Si}\left(\frac{ad}{b}+dx\right)}{a^2}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.89

$$\int \frac{\sin(c+dx)}{x^2(a+bx)}\,dx$$
$$= \frac{x\,\mathrm{CosIntegral}(dx)(ad\cos(c)-b\sin(c)) + bx\,\mathrm{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)\sin\left(c-\frac{ad}{b}\right) - a\sin(c+dx) - bx\cos(c}{a^2x}$$

```
[In] Integrate[Sin[c + d*x]/(x^2*(a + b*x)),x]
```

```
[Out] (x*CosIntegral[d*x]*(a*d*Cos[c] - b*Sin[c]) + b*x*CosIntegral[d*(a/b + x)]*
Sin[c - (a*d)/b] - a*Sin[c + d*x] - b*x*Cos[c]*SinIntegral[d*x] - a*d*x*Sin
[c]*SinIntegral[d*x] + b*x*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)])/(a^2*
x)
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.26

method	result
derivativedivides	$d\left(-\frac{b(\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin(c))}{a^2 d}+\frac{-\frac{\sin(dx+c)}{dx}-\mathrm{Si}(dx)\sin(c)+\mathrm{Ci}(dx)\cos(c)}{a}+\frac{b^2\left(\frac{\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}\right.}{}\right.$
default	$d\left(-\frac{b(\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin(c))}{a^2 d}+\frac{-\frac{\sin(dx+c)}{dx}-\mathrm{Si}(dx)\sin(c)+\mathrm{Ci}(dx)\cos(c)}{a}+\frac{b^2\left(\frac{\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}\right.}{}\right.$
risch	$\frac{ib\,\mathrm{e}^{-\frac{i(da-cb)}{b}}\,\mathrm{Ei}_1\left(-idx-ic-\frac{iad-icb}{b}\right)}{2a^2}-\frac{ib\,\mathrm{e}^{ic}\,\mathrm{Ei}_1(-idx)}{2a^2}-\frac{\mathrm{e}^{ic}\,\mathrm{Ei}_1(-idx)d}{2a}-\frac{d\,\mathrm{e}^{-ic}\,\mathrm{Ei}_1(idx)}{2a}+\frac{i\mathrm{e}^{-ic}\,\mathrm{Ei}_1(idx)b}{2a^2}$

[In] int(sin(d*x+c)/x^2/(b*x+a),x,method=_RETURNVERBOSE)

[Out] d*(-b/a^2/d*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))+1/a*(-sin(d*x+c)/d/x-Si(d*x)*si
n(c)+Ci(d*x)*cos(c))+b^2/a^2/d*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci
(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04

$$\int \frac{\sin(c+dx)}{x^2(a+bx)}\,dx =$$
$$-\frac{bx\,\mathrm{Ci}\left(\frac{bdx+ad}{b}\right)\sin\left(-\frac{bc-ad}{b}\right)-bx\cos\left(-\frac{bc-ad}{b}\right)\mathrm{Si}\left(\frac{bdx+ad}{b}\right)-(adx\,\mathrm{Ci}\,(dx)-bx\,\mathrm{Si}\,(dx))\cos(c)+a\sin(dx}{a^2x}$$

[In] integrate(sin(d*x+c)/x^2/(b*x+a),x, algorithm="fricas")

[Out] -(b*x*cos_integral((b*d*x + a*d)/b)*sin(-(b*c - a*d)/b) - b*x*cos(-(b*c - a
*d)/b)*sin_integral((b*d*x + a*d)/b) - (a*d*x*cos_integral(d*x) - b*x*sin_i
ntegral(d*x))*cos(c) + a*sin(d*x + c) + (a*d*x*sin_integral(d*x) + b*x*cos_
integral(d*x))*sin(c))/(a^2*x)

Sympy [F]

$$\int \frac{\sin(c+dx)}{x^2(a+bx)}\,dx = \int \frac{\sin(c+dx)}{x^2(a+bx)}\,dx$$

```
[In] integrate(sin(d*x+c)/x**2/(b*x+a),x)
[Out] Integral(sin(c + d*x)/(x**2*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x^2(a+bx)}\,dx = \int \frac{\sin(dx+c)}{(bx+a)x^2}\,dx$$

```
[In] integrate(sin(d*x+c)/x^2/(b*x+a),x, algorithm="maxima")
[Out] integrate(sin(d*x + c)/((b*x + a)*x^2), x)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.36 (sec) , antiderivative size = 2897, normalized size of antiderivative = 25.41

$$\int \frac{\sin(c+dx)}{x^2(a+bx)}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)/x^2/(b*x+a),x, algorithm="giac")
[Out] -1/2*(a*d*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/
2*a*d/b)^2 + a*d*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^
2*tan(1/2*a*d/b)^2 + 2*a*d*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*ta
n(1/2*c)*tan(1/2*a*d/b)^2 - 2*a*d*x*imag_part(cos_integral(-d*x))*tan(1/2*d
*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a*d*x*sin_integral(d*x)*tan(1/2*d*x)^
2*tan(1/2*c)*tan(1/2*a*d/b)^2 - b*x*imag_part(cos_integral(d*x + a*d/b))*ta
n(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - b*x*imag_part(cos_integral(d*x
))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + b*x*imag_part(cos_integra
l(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + b*x*imag_pa
rt(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*b*x
*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*b*x*sin
_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a
*d*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + a*d*x*real_
part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*b*x*real_part(cos_
integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*b*x*r
```

```
eal_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*
d/b) - a*d*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 -
 a*d*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*b*
x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*
d/b)^2 + 2*b*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1
/2*a*d/b)^2 + 2*b*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*ta
n(1/2*c)*tan(1/2*a*d/b)^2 + 2*b*x*real_part(cos_integral(-d*x))*tan(1/2*d*x
)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + a*d*x*real_part(cos_integral(d*x))*tan(1/
2*c)^2*tan(1/2*a*d/b)^2 + a*d*x*real_part(cos_integral(-d*x))*tan(1/2*c)^2*
tan(1/2*a*d/b)^2 + 2*a*d*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(
1/2*c) - 2*a*d*x*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) +
4*a*d*x*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) + b*x*imag_part(cos_int
egral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - b*x*imag_part(cos_integra
l(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - b*x*imag_part(cos_integral(-d*x - a*d
/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + b*x*imag_part(cos_integral(-d*x))*tan(1/
2*d*x)^2*tan(1/2*c)^2 - 2*b*x*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2
 + 2*b*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*b*x*
imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/
b) + 4*b*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*
tan(1/2*a*d/b) - 8*b*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2
*c)*tan(1/2*a*d/b) + b*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^
2*tan(1/2*a*d/b)^2 + b*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/
2*a*d/b)^2 - b*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1
/2*a*d/b)^2 - b*x*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/
b)^2 + 2*b*x*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*b*x*sin_
integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*a*d*x*imag_pa
rt(cos_integral(d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a*d*x*imag_part(cos_i
ntegral(-d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a*d*x*sin_integral(d*x)*tan(
1/2*c)*tan(1/2*a*d/b)^2 - b*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*
c)^2*tan(1/2*a*d/b)^2 - b*x*imag_part(cos_integral(d*x))*tan(1/2*c)^2*tan(1
/2*a*d/b)^2 + b*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/
2*a*d/b)^2 + b*x*imag_part(cos_integral(-d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^
2 - 2*b*x*sin_integral(d*x)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*b*x*sin_integ
ral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a*d*x*real_part(cos_in
tegral(d*x))*tan(1/2*d*x)^2 - a*d*x*real_part(cos_integral(-d*x))*tan(1/2*d
*x)^2 - 2*b*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c
) + 2*b*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*b*x*re
al_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 2*b*x*real_
part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + a*d*x*real_part(cos_in
tegral(d*x))*tan(1/2*c)^2 + a*d*x*real_part(cos_integral(-d*x))*tan(1/2*c)^
2 + 2*b*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b
) + 2*b*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/
b) - 2*b*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)
 - 2*b*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)
- a*d*x*real_part(cos_integral(d*x))*tan(1/2*a*d/b)^2 - a*d*x*real_part(cos
```

```
_integral(-d*x))*tan(1/2*a*d/b)^2 + 2*b*x*real_part(cos_integral(d*x + a*d/
b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*b*x*real_part(cos_integral(d*x))*tan(1/
2*c)*tan(1/2*a*d/b)^2 + 2*b*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2
*c)*tan(1/2*a*d/b)^2 + 2*b*x*real_part(cos_integral(-d*x))*tan(1/2*c)*tan(1
/2*a*d/b)^2 - 4*a*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a*tan(1/2*
d*x)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - b*x*imag_part(cos_integral(d*x + a*d/b
))*tan(1/2*d*x)^2 + b*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 + b*x*i
mag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 - b*x*imag_part(cos_int
egral(-d*x))*tan(1/2*d*x)^2 + 2*b*x*sin_integral(d*x)*tan(1/2*d*x)^2 - 2*b*
x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2 + 2*a*d*x*imag_part(cos_inte
gral(d*x))*tan(1/2*c) - 2*a*d*x*imag_part(cos_integral(-d*x))*tan(1/2*c) +
4*a*d*x*sin_integral(d*x)*tan(1/2*c) + b*x*imag_part(cos_integral(d*x + a*d
/b))*tan(1/2*c)^2 - b*x*imag_part(cos_integral(d*x))*tan(1/2*c)^2 - b*x*ima
g_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 + b*x*imag_part(cos_integra
l(-d*x))*tan(1/2*c)^2 - 2*b*x*sin_integral(d*x)*tan(1/2*c)^2 + 2*b*x*sin_in
tegral((b*d*x + a*d)/b)*tan(1/2*c)^2 - 4*b*x*imag_part(cos_integral(d*x + a
*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 4*b*x*imag_part(cos_integral(-d*x - a*d/
b))*tan(1/2*c)*tan(1/2*a*d/b) - 8*b*x*sin_integral((b*d*x + a*d)/b)*tan(1/2
*c)*tan(1/2*a*d/b) + b*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b
)^2 + b*x*imag_part(cos_integral(d*x))*tan(1/2*a*d/b)^2 - b*x*imag_part(cos
_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - b*x*imag_part(cos_integral(-d*x
))*tan(1/2*a*d/b)^2 + 2*b*x*sin_integral(d*x)*tan(1/2*a*d/b)^2 + 2*b*x*sin_
integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 - a*d*x*real_part(cos_integral(d
*x)) - a*d*x*real_part(cos_integral(-d*x)) - 2*b*x*real_part(cos_integral(d
*x + a*d/b))*tan(1/2*c) + 2*b*x*real_part(cos_integral(d*x))*tan(1/2*c) - 2
*b*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) + 2*b*x*real_part(cos
_integral(-d*x))*tan(1/2*c) - 4*a*tan(1/2*d*x)^2*tan(1/2*c) - 4*a*tan(1/2*d
*x)*tan(1/2*c)^2 + 2*b*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b
) + 2*b*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) + 4*a*tan(1/
2*d*x)*tan(1/2*a*d/b)^2 + 4*a*tan(1/2*c)*tan(1/2*a*d/b)^2 - b*x*imag_part(c
os_integral(d*x + a*d/b)) + b*x*imag_part(cos_integral(d*x)) + b*x*imag_par
t(cos_integral(-d*x - a*d/b)) - b*x*imag_part(cos_integral(-d*x)) + 2*b*x*s
in_integral(d*x) - 2*b*x*sin_integral((b*d*x + a*d)/b) + 4*a*tan(1/2*d*x) +
 4*a*tan(1/2*c))/(a^2*x*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^2*
x*tan(1/2*d*x)^2*tan(1/2*c)^2 + a^2*x*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a^2
*x*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^2*x*tan(1/2*d*x)^2 + a^2*x*tan(1/2*c)^
2 + a^2*x*tan(1/2*a*d/b)^2 + a^2*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x^2(a+bx)}\,dx = \int \frac{\sin\left(c+d\,x\right)}{x^2\,\left(a+b\,x\right)}\,dx$$

```
[In] int(sin(c + d*x)/(x^2*(a + b*x)),x)
[Out] int(sin(c + d*x)/(x^2*(a + b*x)), x)
```

3.25 $\int \frac{\sin(c+dx)}{x^3(a+bx)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 189

$$\begin{aligned}\int \frac{\sin(c+dx)}{x^3(a+bx)}\,dx = &-\frac{d\cos(c+dx)}{2ax}-\frac{bd\cos(c)\operatorname{CosIntegral}(dx)}{a^2}+\frac{b^2\operatorname{CosIntegral}(dx)\sin(c)}{a^3}\\&-\frac{d^2\operatorname{CosIntegral}(dx)\sin(c)}{2a}-\frac{b^2\operatorname{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^3}\\&-\frac{\sin(c+dx)}{2ax^2}+\frac{b\sin(c+dx)}{a^2x}+\frac{b^2\cos(c)\operatorname{Si}(dx)}{a^3}\\&-\frac{d^2\cos(c)\operatorname{Si}(dx)}{2a}+\frac{bd\sin(c)\operatorname{Si}(dx)}{a^2}-\frac{b^2\cos\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(\frac{ad}{b}+dx\right)}{a^3}\end{aligned}$$

```
[Out] -b*d*Ci(d*x)*cos(c)/a^2-1/2*d*cos(d*x+c)/a/x+b^2*cos(c)*Si(d*x)/a^3-1/2*d^2
*cos(c)*Si(d*x)/a-b^2*cos(-c+a*d/b)*Si(a*d/b+d*x)/a^3+b^2*Ci(d*x)*sin(c)/a^
3-1/2*d^2*Ci(d*x)*sin(c)/a+b*d*Si(d*x)*sin(c)/a^2+b^2*Ci(a*d/b+d*x)*sin(-c+
a*d/b)/a^3-1/2*sin(d*x+c)/a/x^2+b*sin(d*x+c)/a^2/x
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used

$= \{6874, 3378, 3384, 3380, 3383\}$

$$\int \frac{\sin(c+dx)}{x^3(a+bx)}\,dx = \frac{b^2\sin(c)\,\text{CosIntegral}(dx)}{a^3} - \frac{b^2\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{a^3} + \frac{b^2\cos(c)\text{Si}(dx)}{a^3} - \frac{b^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^3} - \frac{bd\cos(c)\,\text{CosIntegral}(dx)}{a^2} + \frac{bd\sin(c)\text{Si}(dx)}{a^2} + \frac{b\sin(c+dx)}{a^2x} - \frac{d^2\sin(c)\,\text{CosIntegral}(dx)}{2a} - \frac{d^2\cos(c)\text{Si}(dx)}{2a} - \frac{\sin(c+dx)}{2ax^2} - \frac{d\cos(c+dx)}{2ax}$$

```
[In] Int[Sin[c + d*x]/(x^3*(a + b*x)),x]
```

```
[Out] -1/2*(d*Cos[c + d*x])/(a*x) - (b*d*Cos[c]*CosIntegral[d*x])/a^2 + (b^2*CosI
ntegral[d*x]*Sin[c])/a^3 - (d^2*CosIntegral[d*x]*Sin[c])/(2*a) - (b^2*CosIn
tegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/a^3 - Sin[c + d*x]/(2*a*x^2) + (b*S
in[c + d*x])/(a^2*x) + (b^2*Cos[c]*SinIntegral[d*x])/a^3 - (d^2*Cos[c]*SinI
ntegral[d*x])/(2*a) + (b*d*Sin[c]*SinIntegral[d*x])/a^2 - (b^2*Cos[c - (a*d
)/b]*SinIntegral[(a*d)/b + d*x])/a^3
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\sin(c+dx)}{ax^3} - \frac{b\sin(c+dx)}{a^2x^2} + \frac{b^2\sin(c+dx)}{a^3x} - \frac{b^3\sin(c+dx)}{a^3(a+bx)}\right)dx \\
&= \frac{\int \frac{\sin(c+dx)}{x^3}\,dx}{a} - \frac{b\int \frac{\sin(c+dx)}{x^2}\,dx}{a^2} + \frac{b^2\int \frac{\sin(c+dx)}{x}\,dx}{a^3} - \frac{b^3\int \frac{\sin(c+dx)}{a+bx}\,dx}{a^3} \\
&= -\frac{\sin(c+dx)}{2ax^2} + \frac{b\sin(c+dx)}{a^2x} + \frac{d\int \frac{\cos(c+dx)}{x^2}\,dx}{2a} - \frac{(bd)\int \frac{\cos(c+dx)}{x}\,dx}{a^2} \\
&\quad + \frac{\left(b^2\cos(c)\right)\int \frac{\sin(dx)}{x}\,dx}{a^3} - \frac{\left(b^3\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^3} \\
&\quad + \frac{\left(b^2\sin(c)\right)\int \frac{\cos(dx)}{x}\,dx}{a^3} - \frac{\left(b^3\sin\left(c-\frac{ad}{b}\right)\right)\int \frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^3} \\
&= -\frac{d\cos(c+dx)}{2ax} + \frac{b^2\,\text{CosIntegral}(dx)\sin(c)}{a^3} - \frac{b^2\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^3} \\
&\quad - \frac{\sin(c+dx)}{2ax^2} + \frac{b\sin(c+dx)}{a^2x} + \frac{b^2\cos(c)\text{Si}(dx)}{a^3} - \frac{b^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^3} \\
&\quad - \frac{d^2\int \frac{\sin(c+dx)}{x}\,dx}{2a} - \frac{(bd\cos(c))\int \frac{\cos(dx)}{x}\,dx}{a^2} + \frac{(bd\sin(c))\int \frac{\sin(dx)}{x}\,dx}{a^2} \\
&= -\frac{d\cos(c+dx)}{2ax} - \frac{bd\cos(c)\,\text{CosIntegral}(dx)}{a^2} + \frac{b^2\,\text{CosIntegral}(dx)\sin(c)}{a^3} \\
&\quad - \frac{b^2\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^3} - \frac{\sin(c+dx)}{2ax^2} \\
&\quad + \frac{b\sin(c+dx)}{a^2x} + \frac{b^2\cos(c)\text{Si}(dx)}{a^3} + \frac{bd\sin(c)\text{Si}(dx)}{a^2} \\
&\quad - \frac{b^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^3} - \frac{\left(d^2\cos(c)\right)\int \frac{\sin(dx)}{x}\,dx}{2a} - \frac{\left(d^2\sin(c)\right)\int \frac{\cos(dx)}{x}\,dx}{2a} \\
&= -\frac{d\cos(c+dx)}{2ax} - \frac{bd\cos(c)\,\text{CosIntegral}(dx)}{a^2} + \frac{b^2\,\text{CosIntegral}(dx)\sin(c)}{a^3} \\
&\quad - \frac{d^2\,\text{CosIntegral}(dx)\sin(c)}{2a} - \frac{b^2\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^3} \\
&\quad - \frac{\sin(c+dx)}{2ax^2} + \frac{b\sin(c+dx)}{a^2x} + \frac{b^2\cos(c)\text{Si}(dx)}{a^3} \\
&\quad - \frac{d^2\cos(c)\text{Si}(dx)}{2a} + \frac{bd\sin(c)\text{Si}(dx)}{a^2} - \frac{b^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.93

$$\int \frac{\sin(c+dx)}{x^3(a+bx)}\,dx =$$
$$-\frac{a^2dx\cos(c+dx)+x^2\,\text{CosIntegral}(dx)\left(2abd\cos(c)+(-2b^2+a^2d^2)\sin(c)\right)+2b^2x^2\,\text{CosIntegral}\left(d\left(\frac{a}{b}+\right.\right.}{}$$

```
[In] Integrate[Sin[c + d*x]/(x^3*(a + b*x)),x]
```

```
[Out] -1/2*(a^2*d*x*Cos[c + d*x] + x^2*CosIntegral[d*x]*(2*a*b*d*Cos[c] + (-2*b^2
 + a^2*d^2)*Sin[c]) + 2*b^2*x^2*CosIntegral[d*(a/b + x)]*Sin[c - (a*d)/b] +
 a^2*Sin[c + d*x] - 2*a*b*x*Sin[c + d*x] - 2*b^2*x^2*Cos[c]*SinIntegral[d*x
] + a^2*d^2*x^2*Cos[c]*SinIntegral[d*x] - 2*a*b*d*x^2*Sin[c]*SinIntegral[d*
x] + 2*b^2*x^2*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)])/(a^3*x^2)
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.07

method	result
derivativedivides	$d^2\left(-\frac{b^3\left(\frac{\text{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}-\frac{\text{Ci}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}\right)}{d^2a^3}-\frac{b\left(-\frac{\sin(dx+c)}{dx}-\text{Si}(dx)\sin(c)+\text{Ci}(dx)\cos\right.}{a^2d}\right.$
default	$d^2\left(-\frac{b^3\left(\frac{\text{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}-\frac{\text{Ci}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}\right)}{d^2a^3}-\frac{b\left(-\frac{\sin(dx+c)}{dx}-\text{Si}(dx)\sin(c)+\text{Ci}(dx)\cos\right.}{a^2d}\right.$
risch	$-\frac{id^2e^{ic}\,\text{Ei}_1(-idx)}{4a}+\frac{ib^2e^{ic}\,\text{Ei}_1(-idx)}{2a^3}-\frac{ib^2e^{-\frac{i(da-cb)}{b}}\,\text{Ei}_1\left(-idx-ic-\frac{iad-icb}{b}\right)}{2a^3}+\frac{db\,e^{ic}\,\text{Ei}_1(-idx)}{2a^2}+\frac{d\,e^{-ic}\,\text{Ei}}{2a}$

```
[In] int(sin(d*x+c)/x^3/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] d^2*(-1/d^2*b^3/a^3*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d
-b*c)/b)*sin((a*d-b*c)/b)/b)-b/a^2/d*(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x
)*cos(c))+b^2/a^3/d^2*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))+1/a*(-1/2*sin(d*x+c)/
d^2/x^2-1/2*cos(d*x+c)/d/x-1/2*Si(d*x)*cos(c)-1/2*Ci(d*x)*sin(c))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.98

$$\int \frac{\sin(c+dx)}{x^3(a+bx)}\,dx$$
$$= \frac{2\,b^2x^2\,\mathrm{Ci}\left(\frac{bdx+ad}{b}\right)\sin\left(-\frac{bc-ad}{b}\right) - 2\,b^2x^2\cos\left(-\frac{bc-ad}{b}\right)\mathrm{Si}\left(\frac{bdx+ad}{b}\right) - a^2dx\cos\left(dx+c\right) - \left(2\,abdx^2\,\mathrm{Ci}\left(dx\right) + (a}{2\,a}$$

```
[In] integrate(sin(d*x+c)/x^3/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*x^2*cos_integral((b*d*x + a*d)/b)*sin(-(b*c - a*d)/b) - 2*b^2*x^
2*cos(-(b*c - a*d)/b)*sin_integral((b*d*x + a*d)/b) - a^2*d*x*cos(d*x + c)
- (2*a*b*d*x^2*cos_integral(d*x) + (a^2*d^2 - 2*b^2)*x^2*sin_integral(d*x))
*cos(c) + (2*a*b*x - a^2)*sin(d*x + c) + (2*a*b*d*x^2*sin_integral(d*x) - (
a^2*d^2 - 2*b^2)*x^2*cos_integral(d*x))*sin(c))/(a^3*x^2)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{x^3(a+bx)}\,dx = \int \frac{\sin\left(c+dx\right)}{x^3\left(a+bx\right)}\,dx$$

```
[In] integrate(sin(d*x+c)/x**3/(b*x+a),x)
```

```
[Out] Integral(sin(c + d*x)/(x**3*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x^3(a+bx)}\,dx = \int \frac{\sin\left(dx+c\right)}{(bx+a)x^3}\,dx$$

```
[In] integrate(sin(d*x+c)/x^3/(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x + a)*x^3), x)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.35 (sec) , antiderivative size = 4565, normalized size of antiderivative = 24.15

$$\int \frac{\sin(c+dx)}{x^3(a+bx)}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)/x^3/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*(a^2*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*t
an(1/2*a*d/b)^2 - a^2*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*
tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*d^2*x^2*sin_integral(d*x)*tan(1/2*d*x
)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^2*d^2*x^2*real_part(cos_integral(d*
x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^2*d^2*x^2*real_part(co
s_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*a*b*d*x^2*
real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 +
 2*a*b*d*x^2*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(
1/2*a*d/b)^2 + a^2*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(
1/2*c)^2 - a^2*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2
*c)^2 + 2*a^2*d^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d
^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a^2*d
^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*a^
2*d^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 4*a*b*d*x^2*i
mag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*
a*b*d*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a
*d/b)^2 + 8*a*b*d*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a
*d/b)^2 + a^2*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*c)^2*tan(1/2*a*d
/b)^2 - a^2*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*c)^2*tan(1/2*a*d/
b)^2 + 2*a^2*d^2*x^2*sin_integral(d*x)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*b^
2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(
1/2*a*d/b)^2 - 2*b^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/
2*c)^2*tan(1/2*a*d/b)^2 + 2*b^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*t
an(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^2*x^2*imag_part(cos_integ
ral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*b^2*x^2*sin_int
egral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*b^2*x^2*sin_int
egral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^2
*d^2*x^2*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^2
*x^2*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 2*a*b*d*x^2*
real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*b*d*x^2*real
_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*b^2*x^2*real_part
(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 4*
b^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*t
an(1/2*a*d/b) - 2*a*b*d*x^2*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan
(1/2*a*d/b)^2 - 2*a*b*d*x^2*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*ta
```

```
n(1/2*a*d/b)^2 - 2*a^2*d^2*x^2*real_part(cos_integral(d*x))*tan(1/2*c)*tan(
1/2*a*d/b)^2 - 2*a^2*d^2*x^2*real_part(cos_integral(-d*x))*tan(1/2*c)*tan(1
/2*a*d/b)^2 + 4*b^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2
*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*b^2*x^2*real_part(cos_integral(d*x))*tan(1
/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*b^2*x^2*real_part(cos_integral(-d
*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*b^2*x^2*real_pa
rt(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*a*b*d
*x^2*real_part(cos_integral(d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*b*d*x
^2*real_part(cos_integral(-d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^2*d*x*
tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^2*d^2*x^2*imag_part(cos_in
tegral(d*x))*tan(1/2*d*x)^2 + a^2*d^2*x^2*imag_part(cos_integral(-d*x))*tan
(1/2*d*x)^2 - 2*a^2*d^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2 + 4*a*b*d*x^2*
imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 4*a*b*d*x^2*imag_p
art(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 8*a*b*d*x^2*sin_integra
l(d*x)*tan(1/2*d*x)^2*tan(1/2*c) + a^2*d^2*x^2*imag_part(cos_integral(d*x))
*tan(1/2*c)^2 - a^2*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 + 2*
a^2*d^2*x^2*sin_integral(d*x)*tan(1/2*c)^2 + 2*b^2*x^2*imag_part(cos_integr
al(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*b^2*x^2*imag_part(cos_inte
gral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*b^2*x^2*imag_part(cos_integral(-
d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*b^2*x^2*imag_part(cos_integra
l(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*b^2*x^2*sin_integral(d*x)*tan(1/2*
d*x)^2*tan(1/2*c)^2 + 4*b^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^
2*tan(1/2*c)^2 - 8*b^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x
)^2*tan(1/2*c)*tan(1/2*a*d/b) + 8*b^2*x^2*imag_part(cos_integral(-d*x - a*d
/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 16*b^2*x^2*sin_integral((b*
d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - a^2*d^2*x^2*imag_p
art(cos_integral(d*x))*tan(1/2*a*d/b)^2 + a^2*d^2*x^2*imag_part(cos_integra
l(-d*x))*tan(1/2*a*d/b)^2 - 2*a^2*d^2*x^2*sin_integral(d*x)*tan(1/2*a*d/b)^
2 + 2*b^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a
*d/b)^2 + 2*b^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*a*d
/b)^2 - 2*b^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(
1/2*a*d/b)^2 - 2*b^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1
/2*a*d/b)^2 + 4*b^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 +
 4*b^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 +
4*a*b*d*x^2*imag_part(cos_integral(d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a*
b*d*x^2*imag_part(cos_integral(-d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 8*a*b*d
*x^2*sin_integral(d*x)*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*b^2*x^2*imag_part(co
s_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*b^2*x^2*imag_par
t(cos_integral(d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^2*x^2*imag_part(co
s_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^2*x^2*imag_pa
rt(cos_integral(-d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*b^2*x^2*sin_integr
al(d*x)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*b^2*x^2*sin_integral((b*d*x + a*d
)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a*b*d*x^2*real_part(cos_integral(d*x
))*tan(1/2*d*x)^2 - 2*a*b*d*x^2*real_part(cos_integral(-d*x))*tan(1/2*d*x)^
2 - 2*a^2*d^2*x^2*real_part(cos_integral(d*x))*tan(1/2*c) - 2*a^2*d^2*x^2*r
```

```
eal_part(cos_integral(-d*x))*tan(1/2*c) - 4*b^2*x^2*real_part(cos_integral(
d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 4*b^2*x^2*real_part(cos_integral(
d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 4*b^2*x^2*real_part(cos_integral(-d*x - a
*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 4*b^2*x^2*real_part(cos_integral(-d*x))*
tan(1/2*d*x)^2*tan(1/2*c) + 2*a*b*d*x^2*real_part(cos_integral(d*x))*tan(1/
2*c)^2 + 2*a*b*d*x^2*real_part(cos_integral(-d*x))*tan(1/2*c)^2 - 2*a^2*d*x
*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*b^2*x^2*real_part(cos_integral(d*x + a*d/b
))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4*b^2*x^2*real_part(cos_integral(-d*x -
a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 4*b^2*x^2*real_part(cos_integral(d*
x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 4*b^2*x^2*real_part(cos_integral(
-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a*b*d*x^2*real_part(cos_inte
gral(d*x))*tan(1/2*a*d/b)^2 - 2*a*b*d*x^2*real_part(cos_integral(-d*x))*tan
(1/2*a*d/b)^2 + 2*a^2*d*x*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 4*b^2*x^2*real_
part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*b^2*x^2*rea
l_part(cos_integral(d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*b^2*x^2*real_part
(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*b^2*x^2*real_p
art(cos_integral(-d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 8*a^2*d*x*tan(1/2*d*x
)*tan(1/2*c)*tan(1/2*a*d/b)^2 - 8*a*b*x*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a
*d/b)^2 + 2*a^2*d*x*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 8*a*b*x*tan(1/2*d*x)*ta
n(1/2*c)^2*tan(1/2*a*d/b)^2 - a^2*d^2*x^2*imag_part(cos_integral(d*x)) + a^
2*d^2*x^2*imag_part(cos_integral(-d*x)) - 2*a^2*d^2*x^2*sin_integral(d*x) -
 2*b^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2 + 2*b^2*x^2*
imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 + 2*b^2*x^2*imag_part(cos_integ
ral(-d*x - a*d/b))*tan(1/2*d*x)^2 - 2*b^2*x^2*imag_part(cos_integral(-d*x))
*tan(1/2*d*x)^2 + 4*b^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2 - 4*b^2*x^2*si
n_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2 + 4*a*b*d*x^2*imag_part(cos_inte
gral(d*x))*tan(1/2*c) - 4*a*b*d*x^2*imag_part(cos_integral(-d*x))*tan(1/2*c
) + 8*a*b*d*x^2*sin_integral(d*x)*tan(1/2*c) + 2*b^2*x^2*imag_part(cos_inte
gral(d*x + a*d/b))*tan(1/2*c)^2 - 2*b^2*x^2*imag_part(cos_integral(d*x))*ta
n(1/2*c)^2 - 2*b^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 +
 2*b^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 - 4*b^2*x^2*sin_integ
ral(d*x)*tan(1/2*c)^2 + 4*b^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^
2 - 8*b^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b
) + 8*b^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/
b) - 16*b^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) + 2
*b^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 + 2*b^2*x^2*
imag_part(cos_integral(d*x))*tan(1/2*a*d/b)^2 - 2*b^2*x^2*imag_part(cos_int
egral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - 2*b^2*x^2*imag_part(cos_integral(-d
*x))*tan(1/2*a*d/b)^2 + 4*b^2*x^2*sin_integral(d*x)*tan(1/2*a*d/b)^2 + 4*b^
2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 + 4*a^2*tan(1/2*d*x)^2
*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a^2*tan(1/2*d*x)*tan(1/2*c)^2*tan(1/2*a*d/
b)^2 - 2*a*b*d*x^2*real_part(cos_integral(d*x)) - 2*a*b*d*x^2*real_part(cos
_integral(-d*x)) + 2*a^2*d*x*tan(1/2*d*x)^2 - 4*b^2*x^2*real_part(cos_integ
ral(d*x + a*d/b))*tan(1/2*c) + 4*b^2*x^2*real_part(cos_integral(d*x))*tan(1
/2*c) - 4*b^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) + 4*b^2*
```

```
x^2*real_part(cos_integral(-d*x))*tan(1/2*c) + 8*a^2*d*x*tan(1/2*d*x)*tan(1
/2*c) - 8*a*b*x*tan(1/2*d*x)^2*tan(1/2*c) + 2*a^2*d*x*tan(1/2*c)^2 - 8*a*b*
x*tan(1/2*d*x)*tan(1/2*c)^2 + 4*b^2*x^2*real_part(cos_integral(d*x + a*d/b)
)*tan(1/2*a*d/b) + 4*b^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*
a*d/b) - 2*a^2*d*x*tan(1/2*a*d/b)^2 + 8*a*b*x*tan(1/2*d*x)*tan(1/2*a*d/b)^2
 + 8*a*b*x*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*b^2*x^2*imag_part(cos_integral(d
*x + a*d/b)) + 2*b^2*x^2*imag_part(cos_integral(d*x)) + 2*b^2*x^2*imag_part
(cos_integral(-d*x - a*d/b)) - 2*b^2*x^2*imag_part(cos_integral(-d*x)) + 4*
b^2*x^2*sin_integral(d*x) - 4*b^2*x^2*sin_integral((b*d*x + a*d)/b) + 4*a^2
*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^2*tan(1/2*d*x)*tan(1/2*c)^2 - 4*a^2*tan(1/
2*d*x)*tan(1/2*a*d/b)^2 - 4*a^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^2*d*x + 8
*a*b*x*tan(1/2*d*x) + 8*a*b*x*tan(1/2*c) - 4*a^2*tan(1/2*d*x) - 4*a^2*tan(1
/2*c))/(a^3*x^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^3*x^2*tan(
1/2*d*x)^2*tan(1/2*c)^2 + a^3*x^2*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a^3*x^2
*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^3*x^2*tan(1/2*d*x)^2 + a^3*x^2*tan(1/2*c
)^2 + a^3*x^2*tan(1/2*a*d/b)^2 + a^3*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x^3(a+bx)}\,dx=\int \frac{\sin\left(c+d\,x\right)}{x^3\,\left(a+b\,x\right)}\,dx$$

```
[In] int(sin(c + d*x)/(x^3*(a + b*x)),x)
[Out] int(sin(c + d*x)/(x^3*(a + b*x)), x)
```

3.26 $\int \frac{x^4 \sin(c+dx)}{(a+bx)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 233

$$\begin{aligned}\int \frac{x^4 \sin(c+dx)}{(a+bx)^2} dx = &\frac{2\cos(c+dx)}{b^2d^3} - \frac{3a^2\cos(c+dx)}{b^4d} + \frac{2ax\cos(c+dx)}{b^3d} \\ &- \frac{x^2\cos(c+dx)}{b^2d} + \frac{a^4d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^6} \\ &- \frac{4a^3\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^5} \\ &- \frac{2a\sin(c+dx)}{b^3d^2} + \frac{2x\sin(c+dx)}{b^2d^2} - \frac{a^4\sin(c+dx)}{b^5(a+bx)} \\ &- \frac{4a^3\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^5} - \frac{a^4d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^6}\end{aligned}$$

```
[Out] a^4*d*Ci(a*d/b+d*x)*cos(-c+a*d/b)/b^6+2*cos(d*x+c)/b^2/d^3-3*a^2*cos(d*x+c)
/b^4/d+2*a*x*cos(d*x+c)/b^3/d-x^2*cos(d*x+c)/b^2/d-4*a^3*cos(-c+a*d/b)*Si(a
*d/b+d*x)/b^5+4*a^3*Ci(a*d/b+d*x)*sin(-c+a*d/b)/b^5+a^4*d*Si(a*d/b+d*x)*sin
(-c+a*d/b)/b^6-2*a*sin(d*x+c)/b^3/d^2+2*x*sin(d*x+c)/b^2/d^2-a^4*sin(d*x+c)
/b^5/(b*x+a)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {6874, 2718, 3377, 2717, 3378, 3384, 3380, 3383}

$$\int \frac{x^4\sin(c+dx)}{(a+bx)^2}\,dx = \frac{a^4d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b^6} - \frac{a^4d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b^6} - \frac{a^4\sin(c+dx)}{b^5(a+bx)} - \frac{4a^3\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b^5} - \frac{4a^3\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b^5} - \frac{3a^2\cos(c+dx)}{b^4d} - \frac{2a\sin(c+dx)}{b^3d^2} + \frac{2ax\cos(c+dx)}{b^3d} + \frac{2\cos(c+dx)}{b^2d^3} + \frac{2x\sin(c+dx)}{b^2d^2} - \frac{x^2\cos(c+dx)}{b^2d}$$

```
[In] Int[(x^4*Sin[c + d*x])/(a + b*x)^2,x]
```

```
[Out] (2*Cos[c + d*x])/(b^2*d^3) - (3*a^2*Cos[c + d*x])/(b^4*d) + (2*a*x*Cos[c +
d*x])/(b^3*d) - (x^2*Cos[c + d*x])/(b^2*d) + (a^4*d*Cos[c - (a*d)/b]*CosInt
egral[(a*d)/b + d*x])/b^6 - (4*a^3*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)
/b])/b^5 - (2*a*Sin[c + d*x])/(b^3*d^2) + (2*x*Sin[c + d*x])/(b^2*d^2) - (a
^4*Sin[c + d*x])/(b^5*(a + b*x)) - (4*a^3*Cos[c - (a*d)/b]*SinIntegral[(a*d
)/b + d*x])/b^5 - (a^4*d*Sin[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^6
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int \left(\frac{3a^2\sin(c+dx)}{b^4} - \frac{2ax\sin(c+dx)}{b^3} + \frac{x^2\sin(c+dx)}{b^2} + \frac{a^4\sin(c+dx)}{b^4(a+bx)^2} - \frac{4a^3\sin(c+dx)}{b^4(a+bx)}\right) dx$$

$$= \frac{(3a^2)\int \sin(c+dx)\,dx}{b^4} - \frac{(4a^3)\int \frac{\sin(c+dx)}{a+bx}\,dx}{b^4} + \frac{a^4\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx}{b^4} - \frac{(2a)\int x\sin(c+dx)\,dx}{b^3} + \frac{\int x^2\sin(c+dx)\,dx}{b^2}$$

$$= -\frac{3a^2\cos(c+dx)}{b^4d} + \frac{2ax\cos(c+dx)}{b^3d} - \frac{x^2\cos(c+dx)}{b^2d} - \frac{a^4\sin(c+dx)}{b^5(a+bx)} - \frac{(2a)\int \cos(c+dx)\,dx}{b^3d} + \frac{2\int x\cos(c+dx)\,dx}{b^2d} + \frac{(a^4d)\int \frac{\cos(c+dx)}{a+bx}\,dx}{b^5} - \frac{\left(4a^3\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^4} - \frac{\left(4a^3\sin\left(c-\frac{ad}{b}\right)\right)\int \frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^4}$$

$$= -\frac{3a^2\cos(c+dx)}{b^4d} + \frac{2ax\cos(c+dx)}{b^3d} - \frac{x^2\cos(c+dx)}{b^2d} - \frac{4a^3\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^5} - \frac{2a\sin(c+dx)}{b^3d^2} + \frac{2x\sin(c+dx)}{b^2d^2} - \frac{a^4\sin(c+dx)}{b^5(a+bx)} - \frac{4a^3\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^5} - \frac{2\int\sin(c+dx)\,dx}{b^2d^2} + \frac{\left(a^4d\cos\left(c-\frac{ad}{b}\right)\right)\int\frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^5} - \frac{\left(a^4d\sin\left(c-\frac{ad}{b}\right)\right)\int\frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^5}$$

$$= \frac{2\cos(c+dx)}{b^2d^3} - \frac{3a^2\cos(c+dx)}{b^4d} + \frac{2ax\cos(c+dx)}{b^3d} - \frac{x^2\cos(c+dx)}{b^2d} + \frac{a^4d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^6} - \frac{4a^3\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^5} - \frac{2a\sin(c+dx)}{b^3d^2} + \frac{2x\sin(c+dx)}{b^2d^2} - \frac{a^4\sin(c+dx)}{b^5(a+bx)} - \frac{4a^3\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^5} - \frac{a^4d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^6}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.76

$$\int\frac{x^4\sin(c+dx)}{(a+bx)^2}\,dx = \frac{a^3\,\text{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)\left(ad\cos\left(c-\frac{ad}{b}\right)-4b\sin\left(c-\frac{ad}{b}\right)\right) - \frac{b\left(b(a+bx)\left(3a^2d^2-2abd^2x+b^2\left(-2+d^2x^2\right)\right)\cos(c+dx)+d(\cdots\right.}{d^3(a+bx)}}{b^6}$$

```
[In] Integrate[(x^4*Sin[c + d*x])/(a + b*x)^2,x]
```

```
[Out] (a^3*CosIntegral[d*(a/b + x)]*(a*d*Cos[c - (a*d)/b] - 4*b*Sin[c - (a*d)/b])
 - (b*(b*(a + b*x)*(3*a^2*d^2 - 2*a*b*d^2*x + b^2*(-2 + d^2*x^2))*Cos[c + d
*x] + d*(2*a^2*b^2 + a^4*d^2 - 2*b^4*x^2)*Sin[c + d*x]))/(d^3*(a + b*x)) -
a^3*(4*b*Cos[c - (a*d)/b] + a*d*Sin[c - (a*d)/b])*SinIntegral[d*(a/b + x)])
/b^6
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 734, normalized size of antiderivative = 3.15

method	result
risch	$-\frac{i\left(-2ib^6d^5x^5+4ia\,b^5d^5x^4-6ib^6c\,d^4x^4-8ia^3b^3d^5x^2+10ia^4b^2d^5x-24ia^3b^3c\,d^4x+4ib^6d^3x^3+12ia^5b\,d^5-18ia^4b^2c\,d^4+1\right.}{2b^5d^4(bx+a)(-dxb-da)(-dxb+2da-3cb)}$
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int(x^4*sin(d*x+c)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/b^5/d^4*(-8*I*a^3*b^3*d^5*x^2+4*I*a*b^5*d^5*x^4-6*I*b^6*c*d^4*x^4+10
*I*a^4*b^2*d^5*x-18*I*a^4*b^2*c*d^4+12*I*b^6*c*d^2*x^2+24*I*a*b^5*c*d^2*x-2
4*I*a^3*b^3*c*d^4*x+4*I*b^6*d^3*x^3+12*I*a^5*b*d^5-8*I*a^3*b^3*d^3-2*I*b^6*
d^5*x^5-12*I*a^2*b^4*d^3*x+12*I*a^2*b^4*c*d^2)/(b*x+a)/(-b*d*x-a*d)/(-b*d*x
+2*a*d-3*b*c)*cos(d*x+c)+1/2/b^5/d^4*(-2*a^4*b^2*d^6*x^2+4*b^6*d^4*x^4+2*a^
5*b*d^6*x-6*a^4*b^2*c*d^5*x-4*a*b^5*d^4*x^3+12*b^6*c*d^3*x^3+4*a^6*d^6-6*a^
5*b*c*d^5-12*a^2*b^4*d^4*x^2+12*a*b^5*c*d^3*x^2+4*a^3*b^3*d^4*x-12*a^2*b^4*
c*d^3*x+8*a^4*b^2*d^4-12*a^3*b^3*c*d^3)/(b*x+a)/(-b*d*x-a*d)/(-b*d*x+2*a*d-
3*b*c)*sin(d*x+c)-2*I/b^5*cos((a*d-b*c)/b)*Ei(1,-I*d*(b*x+a)/b)*a^3-1/2*d/b
^6*cos((a*d-b*c)/b)*Ei(1,-I*d*(b*x+a)/b)*a^4+2*I/b^5*cos((a*d-b*c)/b)*Ei(1,
I*d*(b*x+a)/b)*a^3-1/2*d/b^6*cos((a*d-b*c)/b)*Ei(1,I*d*(b*x+a)/b)*a^4-2/b^5
*sin((a*d-b*c)/b)*Ei(1,-I*d*(b*x+a)/b)*a^3+1/2*I*d/b^6*sin((a*d-b*c)/b)*Ei(
1,-I*d*(b*x+a)/b)*a^4-2/b^5*sin((a*d-b*c)/b)*Ei(1,I*d*(b*x+a)/b)*a^3-1/2*I*
d/b^6*sin((a*d-b*c)/b)*Ei(1,I*d*(b*x+a)/b)*a^4
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.23

$$\int \frac{x^4\sin(c+dx)}{(a+bx)^2}\,dx =$$
$$-\frac{(b^5d^2x^3-ab^4d^2x^2+3\,a^3b^2d^2-2\,ab^4+(a^2b^3d^2-2\,b^5)x)\cos\left(dx+c\right)-\left((a^4bd^4x+a^5d^4)\operatorname{Ci}\left(\frac{bdx+ad}{b}\right)-\right.}{}$$

```
[In] integrate(x^4*sin(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -((b^5*d^2*x^3 - a*b^4*d^2*x^2 + 3*a^3*b^2*d^2 - 2*a*b^4 + (a^2*b^3*d^2 - 2
*b^5)*x)*cos(d*x + c) - ((a^4*b*d^4*x + a^5*d^4)*cos_integral((b*d*x + a*d)
/b) - 4*(a^3*b^2*d^3*x + a^4*b*d^3)*sin_integral((b*d*x + a*d)/b))*cos(-(b*
c - a*d)/b) + (a^4*b*d^3 - 2*b^5*d*x^2 + 2*a^2*b^3*d)*sin(d*x + c) - (4*(a^
```

```
3*b^2*d^3*x + a^4*b*d^3)*cos_integral((b*d*x + a*d)/b) + (a^4*b*d^4*x + a^5
*d^4)*sin_integral((b*d*x + a*d)/b))*sin(-(b*c - a*d)/b))/(b^7*d^3*x + a*b^
6*d^3)
```

Sympy [F]

$$\int \frac{x^4 \sin(c+dx)}{(a+bx)^2}\,dx = \int \frac{x^4 \sin{(c+dx)}}{(a+bx)^2}\,dx$$

```
[In] integrate(x**4*sin(d*x+c)/(b*x+a)**2,x)
[Out] Integral(x**4*sin(c + d*x)/(a + b*x)**2, x)
```

Maxima [F]

$$\int \frac{x^4 \sin(c+dx)}{(a+bx)^2}\,dx = \int \frac{x^4 \sin{(dx+c)}}{(bx+a)^2}\,dx$$

```
[In] integrate(x^4*sin(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
[Out] 1/2*(2*((2*a^2*b*(exp_integral_e(3, (I*b*d*x + I*a*d)/b) + exp_integral_e(3
, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + 2*a^2*b*(exp_integral_e(3, (I*b*d*x + I
*a*d)/b) + exp_integral_e(3, -(I*b*d*x + I*a*d)/b))*sin(c)^2 + (a^3*(-I*exp
_integral_e(3, (I*b*d*x + I*a*d)/b) + I*exp_integral_e(3, -(I*b*d*x + I*a*d
)/b))*cos(c)^2 + a^3*(-I*exp_integral_e(3, (I*b*d*x + I*a*d)/b) + I*exp_int
egral_e(3, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*d)*cos(-(b*c - a*d)/b) + (2*a^2
*b*(I*exp_integral_e(3, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(3, -(I*b*d*
x + I*a*d)/b))*cos(c)^2 + 2*a^2*b*(I*exp_integral_e(3, (I*b*d*x + I*a*d)/b)
 - I*exp_integral_e(3, -(I*b*d*x + I*a*d)/b))*sin(c)^2 + (a^3*(exp_integral
_e(3, (I*b*d*x + I*a*d)/b) + exp_integral_e(3, -(I*b*d*x + I*a*d)/b))*cos(c
)^2 + a^3*(exp_integral_e(3, (I*b*d*x + I*a*d)/b) + exp_integral_e(3, -(I*b
*d*x + I*a*d)/b))*sin(c)^2)*d)*sin(-(b*c - a*d)/b))*cos(d*x + c)^2 + 2*((2*
a^2*b*(exp_integral_e(3, (I*b*d*x + I*a*d)/b) + exp_integral_e(3, -(I*b*d*x
 + I*a*d)/b))*cos(c)^2 + 2*a^2*b*(exp_integral_e(3, (I*b*d*x + I*a*d)/b) +
exp_integral_e(3, -(I*b*d*x + I*a*d)/b))*sin(c)^2 + (a^3*(-I*exp_integral_e
(3, (I*b*d*x + I*a*d)/b) + I*exp_integral_e(3, -(I*b*d*x + I*a*d)/b))*cos(c
)^2 + a^3*(-I*exp_integral_e(3, (I*b*d*x + I*a*d)/b) + I*exp_integral_e(3,
-(I*b*d*x + I*a*d)/b))*sin(c)^2)*d)*cos(-(b*c - a*d)/b) + (2*a^2*b*(I*exp_i
ntegral_e(3, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(3, -(I*b*d*x + I*a*d)/
b))*cos(c)^2 + 2*a^2*b*(I*exp_integral_e(3, (I*b*d*x + I*a*d)/b) - I*exp_in
tegral_e(3, -(I*b*d*x + I*a*d)/b))*sin(c)^2 + (a^3*(exp_integral_e(3, (I*b*
d*x + I*a*d)/b) + exp_integral_e(3, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a^3*(
```

```
exp_integral_e(3, (I*b*d*x + I*a*d)/b) + exp_integral_e(3, -(I*b*d*x + I*a*
d)/b))*sin(c)^2)*d)*sin(-(b*c - a*d)/b))*sin(d*x + c)^2 - ((b^3*d^2*x^4*cos
(c) + 2*b^3*d*x^3*sin(c) + 2*(a*b^2*d*sin(c) - b^3*cos(c))*x^2 - 2*(a^2*b*d
*sin(c) + 2*a*b^2*cos(c))*x)*cos(d*x + c)^2 + (b^3*d^2*x^4*cos(c) + 2*b^3*d
*x^3*sin(c) + 2*(a*b^2*d*sin(c) - b^3*cos(c))*x^2 - 2*(a^2*b*d*sin(c) + 2*a
*b^2*cos(c))*x)*sin(d*x + c)^2)*cos(d*x + 2*c) - ((b^3*cos(c)^2 + b^3*sin(c
)^2)*d^2*x^4 - 2*(b^3*cos(c)^2 + b^3*sin(c)^2)*x^2 - 4*(a*b^2*cos(c)^2 + a*
b^2*sin(c)^2)*x)*cos(d*x + c) + 2*(((a^3*b^5*cos(c)^2 + a^3*b^5*sin(c)^2)*d
^5*x^2 + 2*(a^4*b^4*cos(c)^2 + a^4*b^4*sin(c)^2)*d^5*x + (a^5*b^3*cos(c)^2
+ a^5*b^3*sin(c)^2)*d^5)*cos(d*x + c)^2 + ((a^3*b^5*cos(c)^2 + a^3*b^5*sin(
c)^2)*d^5*x^2 + 2*(a^4*b^4*cos(c)^2 + a^4*b^4*sin(c)^2)*d^5*x + (a^5*b^3*co
s(c)^2 + a^5*b^3*sin(c)^2)*d^5)*sin(d*x + c)^2)*integrate(x*cos(d*x + c)/(b
^5*d^3*x^3 + 3*a*b^4*d^3*x^2 + 3*a^2*b^3*d^3*x + a^3*b^2*d^3), x) + 2*(((a^
3*b^5*cos(c)^2 + a^3*b^5*sin(c)^2)*d^5*x^2 + 2*(a^4*b^4*cos(c)^2 + a^4*b^4*
sin(c)^2)*d^5*x + (a^5*b^3*cos(c)^2 + a^5*b^3*sin(c)^2)*d^5)*cos(d*x + c)^2
 + ((a^3*b^5*cos(c)^2 + a^3*b^5*sin(c)^2)*d^5*x^2 + 2*(a^4*b^4*cos(c)^2 + a
^4*b^4*sin(c)^2)*d^5*x + (a^5*b^3*cos(c)^2 + a^5*b^3*sin(c)^2)*d^5)*sin(d*x
 + c)^2)*integrate(x*cos(d*x + c)/((b^5*d^3*x^3 + 3*a*b^4*d^3*x^2 + 3*a^2*b
^3*d^3*x + a^3*b^2*d^3)*cos(d*x + c)^2 + (b^5*d^3*x^3 + 3*a*b^4*d^3*x^2 + 3
*a^2*b^3*d^3*x + a^3*b^2*d^3)*sin(d*x + c)^2), x) - 2*(((a^2*b^6*cos(c)^2 +
 a^2*b^6*sin(c)^2)*d^4*x^2 + 2*(a^3*b^5*cos(c)^2 + a^3*b^5*sin(c)^2)*d^4*x
+ (a^4*b^4*cos(c)^2 + a^4*b^4*sin(c)^2)*d^4)*cos(d*x + c)^2 + ((a^2*b^6*cos
(c)^2 + a^2*b^6*sin(c)^2)*d^4*x^2 + 2*(a^3*b^5*cos(c)^2 + a^3*b^5*sin(c)^2)
*d^4*x + (a^4*b^4*cos(c)^2 + a^4*b^4*sin(c)^2)*d^4)*sin(d*x + c)^2)*integra
te(x*sin(d*x + c)/(b^5*d^3*x^3 + 3*a*b^4*d^3*x^2 + 3*a^2*b^3*d^3*x + a^3*b^
2*d^3), x) - 2*(((a^2*b^6*cos(c)^2 + a^2*b^6*sin(c)^2)*d^4*x^2 + 2*(a^3*b^5
*cos(c)^2 + a^3*b^5*sin(c)^2)*d^4*x + (a^4*b^4*cos(c)^2 + a^4*b^4*sin(c)^2)
*d^4)*cos(d*x + c)^2 + ((a^2*b^6*cos(c)^2 + a^2*b^6*sin(c)^2)*d^4*x^2 + 2*(
a^3*b^5*cos(c)^2 + a^3*b^5*sin(c)^2)*d^4*x + (a^4*b^4*cos(c)^2 + a^4*b^4*si
n(c)^2)*d^4)*sin(d*x + c)^2)*integrate(x*sin(d*x + c)/((b^5*d^3*x^3 + 3*a*b
^4*d^3*x^2 + 3*a^2*b^3*d^3*x + a^3*b^2*d^3)*cos(d*x + c)^2 + (b^5*d^3*x^3 +
 3*a*b^4*d^3*x^2 + 3*a^2*b^3*d^3*x + a^3*b^2*d^3)*sin(d*x + c)^2), x) - ((b
^3*d^2*x^4*sin(c) - 2*b^3*d*x^3*cos(c) - 2*(a*b^2*d*cos(c) + b^3*sin(c))*x^
2 + 2*(a^2*b*d*cos(c) - 2*a*b^2*sin(c))*x)*cos(d*x + c)^2 + (b^3*d^2*x^4*si
n(c) - 2*b^3*d*x^3*cos(c) - 2*(a*b^2*d*cos(c) + b^3*sin(c))*x^2 + 2*(a^2*b*
d*cos(c) - 2*a*b^2*sin(c))*x)*sin(d*x + c)^2)*sin(d*x + 2*c) + 2*((b^3*cos(
c)^2 + b^3*sin(c)^2)*d*x^3 + (a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x^2 - (a^2
*b*cos(c)^2 + a^2*b*sin(c)^2)*d*x)*sin(d*x + c))/(((b^5*cos(c)^2 + b^5*sin(
c)^2)*d^3*x^2 + 2*(a*b^4*cos(c)^2 + a*b^4*sin(c)^2)*d^3*x + (a^2*b^3*cos(c)
^2 + a^2*b^3*sin(c)^2)*d^3)*cos(d*x + c)^2 + ((b^5*cos(c)^2 + b^5*sin(c)^2)
*d^3*x^2 + 2*(a*b^4*cos(c)^2 + a*b^4*sin(c)^2)*d^3*x + (a^2*b^3*cos(c)^2 +
a^2*b^3*sin(c)^2)*d^3)*sin(d*x + c)^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1973 vs. $2(236) = 472$.

Time = 0.37 (sec) , antiderivative size = 1973, normalized size of antiderivative = 8.47

$$\int \frac{x^4 \sin(c+dx)}{(a+bx)^2} \, dx = \text{Too large to display}$$

```
[In] integrate(x^4*sin(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)*a^4*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^4*cos(-(b*c - a*d)/b)*
cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)
 - a^4*b*c*d^4*cos(-(b*c - a*d)/b)*cos_integral(((b*x + a)*(b*c/(b*x + a) -
 a*d/(b*x + a) + d) - b*c + a*d)/b) + a^5*d^5*cos(-(b*c - a*d)/b)*cos_integ
ral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + (b*x +
 a)*a^4*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^4*sin(-(b*c - a*d)/b)*sin_int
egral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) - a^4*
b*c*d^4*sin(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b
*x + a) + d) - b*c + a*d)/b) + a^5*d^5*sin(-(b*c - a*d)/b)*sin_integral(((b
*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + 4*(b*x + a)*a
^3*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^3*cos_integral(((b*x + a)*(b*c/(
b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*sin(-(b*c - a*d)/b) - 4*a^3*b
^2*c*d^3*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c
+ a*d)/b)*sin(-(b*c - a*d)/b) + 4*a^4*b*d^4*cos_integral(((b*x + a)*(b*c/(b
*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*sin(-(b*c - a*d)/b) - 4*(b*x +
 a)*a^3*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^3*cos(-(b*c - a*d)/b)*sin_i
ntegral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + 4*
a^3*b^2*c*d^3*cos(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) -
a*d/(b*x + a) + d) - b*c + a*d)/b) - 4*a^4*b*d^4*cos(-(b*c - a*d)/b)*sin_in
tegral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + a^4
*b*d^4*sin(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - (b*x + a)^3*
b^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)^3*cos(-(b*x + a)*(b*c/(b*x + a) - a
*d/(b*x + a) + d)/b) + 3*(b*x + a)^2*b^3*(b*c/(b*x + a) - a*d/(b*x + a) + d
)^2*c*cos(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 3*(b*x + a)*b
^4*(b*c/(b*x + a) - a*d/(b*x + a) + d)*c^2*cos(-(b*x + a)*(b*c/(b*x + a) -
a*d/(b*x + a) + d)/b) + b^5*c^3*cos(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x +
a) + d)/b) + (b*x + a)^2*a*b^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*d*cos(
-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 2*(b*x + a)*a*b^3*(b*c/
(b*x + a) - a*d/(b*x + a) + d)*c*d*cos(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x
 + a) + d)/b) + a*b^4*c^2*d*cos(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) +
 d)/b) - (b*x + a)*a^2*b^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*cos(-(b*
x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + a^2*b^3*c*d^2*cos(-(b*x + a
)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 3*a^3*b^2*d^3*cos(-(b*x + a)*(b*
c/(b*x + a) - a*d/(b*x + a) + d)/b) - 2*(b*x + a)^2*b^3*(b*c/(b*x + a) - a*
d/(b*x + a) + d)^2*sin(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) +
```

```
4*(b*x + a)*b^4*(b*c/(b*x + a) - a*d/(b*x + a) + d)*c*sin(-(b*x + a)*(b*c/(
b*x + a) - a*d/(b*x + a) + d)/b) - 2*b^5*c^2*sin(-(b*x + a)*(b*c/(b*x + a)
- a*d/(b*x + a) + d)/b) + 2*a^2*b^3*d^2*sin(-(b*x + a)*(b*c/(b*x + a) - a*d
/(b*x + a) + d)/b) + 2*(b*x + a)*b^4*(b*c/(b*x + a) - a*d/(b*x + a) + d)*co
s(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 2*b^5*c*cos(-(b*x + a
)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + 2*a*b^4*d*cos(-(b*x + a)*(b*c/(b
*x + a) - a*d/(b*x + a) + d)/b))*b^2/(((b*x + a)*b^8*(b*c/(b*x + a) - a*d/(
b*x + a) + d)*d^2 - b^9*c*d^2 + a*b^8*d^3)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \sin(c+dx)}{(a+bx)^2}\,dx = \int \frac{x^4\,\sin\left(c+d\,x\right)}{{\left(a+b\,x\right)}^2}\,dx$$

```
[In] int((x^4*sin(c + d*x))/(a + b*x)^2,x)
[Out] int((x^4*sin(c + d*x))/(a + b*x)^2, x)
```

3.27 $\int \frac{x^3 \sin(c+dx)}{(a+bx)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 181

$$\int \frac{x^3 \sin(c+dx)}{(a+bx)^2} dx = \frac{2a\cos(c+dx)}{b^3d} - \frac{x\cos(c+dx)}{b^2d} - \frac{a^3d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^5} + \frac{3a^2\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^4} + \frac{\sin(c+dx)}{b^2d^2} + \frac{a^3\sin(c+dx)}{b^4(a+bx)} + \frac{3a^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^4} + \frac{a^3d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^5}$$

```
[Out] -a^3*d*Ci(a*d/b+d*x)*cos(-c+a*d/b)/b^5+2*a*cos(d*x+c)/b^3/d-x*cos(d*x+c)/b^
2/d+3*a^2*cos(-c+a*d/b)*Si(a*d/b+d*x)/b^4-3*a^2*Ci(a*d/b+d*x)*sin(-c+a*d/b)
/b^4-a^3*d*Si(a*d/b+d*x)*sin(-c+a*d/b)/b^5+sin(d*x+c)/b^2/d^2+a^3*sin(d*x+c
)/b^4/(b*x+a)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used

= {6874, 2718, 3377, 2717, 3378, 3384, 3380, 3383}

$$\int \frac{x^3 \sin(c+dx)}{(a+bx)^2} dx = -\frac{a^3 d \cos\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b^5} + \frac{a^3 d \sin\left(c-\frac{ad}{b}\right) \text{Si}\left(xd+\frac{ad}{b}\right)}{b^5} + \frac{a^3 \sin(c+dx)}{b^4(a+bx)} + \frac{3a^2 \sin\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b^4} + \frac{3a^2 \cos\left(c-\frac{ad}{b}\right) \text{Si}\left(xd+\frac{ad}{b}\right)}{b^4} + \frac{2a\cos(c+dx)}{b^3 d} + \frac{\sin(c+dx)}{b^2 d^2} - \frac{x\cos(c+dx)}{b^2 d}$$

```
[In] Int[(x^3*Sin[c + d*x])/(a + b*x)^2,x]
```

```
[Out] (2*a*Cos[c + d*x])/(b^3*d) - (x*Cos[c + d*x])/(b^2*d) - (a^3*d*Cos[c - (a*d
)/b]*CosIntegral[(a*d)/b + d*x])/b^5 + (3*a^2*CosIntegral[(a*d)/b + d*x]*Si
n[c - (a*d)/b])/b^4 + Sin[c + d*x]/(b^2*d^2) + (a^3*Sin[c + d*x])/(b^4*(a +
 b*x)) + (3*a^2*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^4 + (a^3*d*S
in[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^5
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{2a\sin(c+dx)}{b^3}+\frac{x\sin(c+dx)}{b^2}-\frac{a^3\sin(c+dx)}{b^3(a+bx)^2}+\frac{3a^2\sin(c+dx)}{b^3(a+bx)}\right)dx\\
&=-\frac{(2a)\int\sin(c+dx)\,dx}{b^3}+\frac{(3a^2)\int\frac{\sin(c+dx)}{a+bx}\,dx}{b^3}-\frac{a^3\int\frac{\sin(c+dx)}{(a+bx)^2}\,dx}{b^3}+\frac{\int x\sin(c+dx)\,dx}{b^2}\\
&=\frac{2a\cos(c+dx)}{b^3d}-\frac{x\cos(c+dx)}{b^2d}+\frac{a^3\sin(c+dx)}{b^4(a+bx)}+\frac{\int\cos(c+dx)\,dx}{b^2d}\\
&\quad-\frac{(a^3d)\int\frac{\cos(c+dx)}{a+bx}\,dx}{b^4}+\frac{\left(3a^2\cos\left(c-\frac{ad}{b}\right)\right)\int\frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^3}\\
&\quad+\frac{\left(3a^2\sin\left(c-\frac{ad}{b}\right)\right)\int\frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^3}\\
&=\frac{2a\cos(c+dx)}{b^3d}-\frac{x\cos(c+dx)}{b^2d}+\frac{3a^2\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^4}\\
&\quad+\frac{\sin(c+dx)}{b^2d^2}+\frac{a^3\sin(c+dx)}{b^4(a+bx)}+\frac{3a^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^4}\\
&\quad-\frac{\left(a^3d\cos\left(c-\frac{ad}{b}\right)\right)\int\frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^4}+\frac{\left(a^3d\sin\left(c-\frac{ad}{b}\right)\right)\int\frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^4}
\end{aligned}
$$

$$= \frac{2a\cos(c+dx)}{b^3d} - \frac{x\cos(c+dx)}{b^2d} - \frac{a^3d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^5} + \frac{3a^2\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^4} + \frac{\sin(c+dx)}{b^2d^2} + \frac{a^3\sin(c+dx)}{b^4(a+bx)} + \frac{3a^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^4} + \frac{a^3d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^5}$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.85

$$\int \frac{x^3\sin(c+dx)}{(a+bx)^2}\,dx$$

$$= \frac{-a^2\,\text{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)\left(ad\cos\left(c-\frac{ad}{b}\right)-3b\sin\left(c-\frac{ad}{b}\right)\right)+\frac{b(bd(2a^2+abx-b^2x^2)\cos(c+dx)+(ab^2+a^3d^2+b^3x)\,\text{si}}{d^2(a+bx)}}{b^5}$$

```
[In] Integrate[(x^3*Sin[c + d*x])/(a + b*x)^2,x]
```

```
[Out] (-(a^2*CosIntegral[d*(a/b + x)]*(a*d*Cos[c - (a*d)/b] - 3*b*Sin[c - (a*d)/b
])) + (b*(b*d*(2*a^2 + a*b*x - b^2*x^2)*Cos[c + d*x] + (a*b^2 + a^3*d^2 + b
^3*x)*Sin[c + d*x]))/(d^2*(a + b*x)) + a^2*(3*b*Cos[c - (a*d)/b] + a*d*Sin[
c - (a*d)/b])*SinIntegral[d*(a/b + x)])/b^5
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 607, normalized size of antiderivative = 3.35

method	result
risch	$-\frac{i\left(-2ib^4d^3x^4+4ia\,b^3d^3x^3-6ib^4c\,d^2x^3+6ia^2b^2d^3x^2-8ia^3b\,d^3x+18ia^2b^2c\,d^2x-8ia^4d^3+12ia^3bc\,d^2\right)\cos(dx+c)}{2d^2b^3(bx+a)(-dxb+2da-3cb)(-dxb-da)} + \frac{(2a}{}$
derivativedivides	$\frac{-d^2c^3\left(-\frac{\sin(dx+c)}{(da-cb+b(dx+c))b}+\frac{\frac{\text{Si}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}+\frac{\text{Ci}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}}{b}\right)+\frac{3d^2c^2\left(\frac{\text{Si}\left(dx+c+\frac{da-cb}{b}\right)c}{b}\right.}{}}{}$
default	$\frac{-d^2c^3\left(-\frac{\sin(dx+c)}{(da-cb+b(dx+c))b}+\frac{\frac{\text{Si}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}+\frac{\text{Ci}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}}{b}\right)+\frac{3d^2c^2\left(\frac{\text{Si}\left(dx+c+\frac{da-cb}{b}\right)c}{b}\right.}{}}{}$

```
[In] int(x^3*sin(d*x+c)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/d^2/b^3*(18*I*a^2*b^2*c*d^2*x-2*I*b^4*d^3*x^4-8*I*a^4*d^3+4*I*a*b^3*
d^3*x^3-6*I*b^4*c*d^2*x^3+6*I*a^2*b^2*d^3*x^2-8*I*a^3*b*d^3*x+12*I*a^3*b*c*
d^2)/(b*x+a)/(-b*d*x+2*a*d-3*b*c)/(-b*d*x-a*d)*cos(d*x+c)+1/2/d^2/b^4*(2*a^
3*b^2*d^4*x^2-2*a^4*b*d^4*x+6*a^3*b^2*c*d^3*x+2*b^5*d^2*x^3-4*a^5*d^4+6*a^4
*b*c*d^3+6*b^5*c*d*x^2-6*a^2*b^3*d^2*x+12*a*b^4*c*d*x-4*a^3*b^2*d^2+6*a^2*b
^3*c*d)/(b*x+a)/(-b*d*x+2*a*d-3*b*c)/(-b*d*x-a*d)*sin(d*x+c)+1/2*d/b^5*cos(
(a*d-b*c)/b)*Ei(1,I*d*(b*x+a)/b)*a^3+1/2*d/b^5*cos((a*d-b*c)/b)*Ei(1,-I*d*(
b*x+a)/b)*a^3-3/2*I/b^4*cos((a*d-b*c)/b)*Ei(1,I*d*(b*x+a)/b)*a^2+3/2*I/b^4*
cos((a*d-b*c)/b)*Ei(1,-I*d*(b*x+a)/b)*a^2+1/2*I*d/b^5*sin((a*d-b*c)/b)*Ei(1
,I*d*(b*x+a)/b)*a^3-1/2*I*d/b^5*sin((a*d-b*c)/b)*Ei(1,-I*d*(b*x+a)/b)*a^3+3
/2/b^4*sin((a*d-b*c)/b)*Ei(1,I*d*(b*x+a)/b)*a^2+3/2/b^4*sin((a*d-b*c)/b)*Ei
(1,-I*d*(b*x+a)/b)*a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.36

$$\int \frac{x^3\sin(c+dx)}{(a+bx)^2}\,dx = -\frac{\left(b^4dx^2-ab^3dx-2\,a^2b^2d\right)\cos\left(dx+c\right)+\left(\left(a^3bd^3x+a^4d^3\right)\operatorname{Ci}\left(\frac{bdx+ad}{b}\right)-3\left(a^2b^2d^2x+a^3bd^2\right)\operatorname{Si}\left(\frac{bdx+ad}{b}\right)\right)}{}$$

```
[In] integrate(x^3*sin(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -((b^4*d*x^2 - a*b^3*d*x - 2*a^2*b^2*d)*cos(d*x + c) + ((a^3*b*d^3*x + a^4*
d^3)*cos_integral((b*d*x + a*d)/b) - 3*(a^2*b^2*d^2*x + a^3*b*d^2)*sin_inte
gral((b*d*x + a*d)/b))*cos(-(b*c - a*d)/b) - (a^3*b*d^2 + b^4*x + a*b^3)*si
n(d*x + c) + (3*(a^2*b^2*d^2*x + a^3*b*d^2)*cos_integral((b*d*x + a*d)/b) +
 (a^3*b*d^3*x + a^4*d^3)*sin_integral((b*d*x + a*d)/b))*sin(-(b*c - a*d)/b)
)/(b^6*d^2*x + a*b^5*d^2)
```

Sympy [F]

$$\int \frac{x^3\sin(c+dx)}{(a+bx)^2}\,dx = \int \frac{x^3\sin\left(c+dx\right)}{\left(a+bx\right)^2}\,dx$$

```
[In] integrate(x**3*sin(d*x+c)/(b*x+a)**2,x)
```

```
[Out] Integral(x**3*sin(c + d*x)/(a + b*x)**2, x)
```

Maxima [F]

$$\int \frac{x^3 \sin(c+dx)}{(a+bx)^2} \, dx = \int \frac{x^3 \sin(dx+c)}{(bx+a)^2} \, dx$$

```
[In] integrate(x^3*sin(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((b^2*cos(c)^2 + b^2*sin(c)^2)*d*x^3*cos(d*x + c) - 2*((a^2*(I*exp_int
egral_e(3, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(3, -(I*b*d*x + I*a*d)/b)
)*cos(c)^2 + a^2*(I*exp_integral_e(3, (I*b*d*x + I*a*d)/b) - I*exp_integral
_e(3, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*cos(-(b*c - a*d)/b) - (a^2*(exp_inte
gral_e(3, (I*b*d*x + I*a*d)/b) + exp_integral_e(3, -(I*b*d*x + I*a*d)/b))*c
os(c)^2 + a^2*(exp_integral_e(3, (I*b*d*x + I*a*d)/b) + exp_integral_e(3, -
(I*b*d*x + I*a*d)/b))*sin(c)^2)*sin(-(b*c - a*d)/b))*cos(d*x + c)^2 - 2*((a
^2*(I*exp_integral_e(3, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(3, -(I*b*d*
x + I*a*d)/b))*cos(c)^2 + a^2*(I*exp_integral_e(3, (I*b*d*x + I*a*d)/b) - I
*exp_integral_e(3, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*cos(-(b*c - a*d)/b) - (
a^2*(exp_integral_e(3, (I*b*d*x + I*a*d)/b) + exp_integral_e(3, -(I*b*d*x +
 I*a*d)/b))*cos(c)^2 + a^2*(exp_integral_e(3, (I*b*d*x + I*a*d)/b) + exp_in
tegral_e(3, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*sin(-(b*c - a*d)/b))*sin(d*x +
 c)^2 + ((b^2*d*x^3*cos(c) + b^2*x^2*sin(c) + 2*a*b*x*sin(c))*cos(d*x + c)^
2 + (b^2*d*x^3*cos(c) + b^2*x^2*sin(c) + 2*a*b*x*sin(c))*sin(d*x + c)^2)*co
s(d*x + 2*c) + 2*(((a^2*b^4*cos(c)^2 + a^2*b^4*sin(c)^2)*d^3*x^2 + 2*(a^3*b
^3*cos(c)^2 + a^3*b^3*sin(c)^2)*d^3*x + (a^4*b^2*cos(c)^2 + a^4*b^2*sin(c)^
2)*d^3)*cos(d*x + c)^2 + ((a^2*b^4*cos(c)^2 + a^2*b^4*sin(c)^2)*d^3*x^2 + 2
*(a^3*b^3*cos(c)^2 + a^3*b^3*sin(c)^2)*d^3*x + (a^4*b^2*cos(c)^2 + a^4*b^2*
sin(c)^2)*d^3)*sin(d*x + c)^2)*integrate(x*cos(d*x + c)/(b^4*d^2*x^3 + 3*a*
b^3*d^2*x^2 + 3*a^2*b^2*d^2*x + a^3*b*d^2), x) + 2*(((a^2*b^4*cos(c)^2 + a^
2*b^4*sin(c)^2)*d^3*x^2 + 2*(a^3*b^3*cos(c)^2 + a^3*b^3*sin(c)^2)*d^3*x + (
a^4*b^2*cos(c)^2 + a^4*b^2*sin(c)^2)*d^3)*cos(d*x + c)^2 + ((a^2*b^4*cos(c)
^2 + a^2*b^4*sin(c)^2)*d^3*x^2 + 2*(a^3*b^3*cos(c)^2 + a^3*b^3*sin(c)^2)*d^
3*x + (a^4*b^2*cos(c)^2 + a^4*b^2*sin(c)^2)*d^3)*sin(d*x + c)^2)*integrate(
x*cos(d*x + c)/((b^4*d^2*x^3 + 3*a*b^3*d^2*x^2 + 3*a^2*b^2*d^2*x + a^3*b*d^
2)*cos(d*x + c)^2 + (b^4*d^2*x^3 + 3*a*b^3*d^2*x^2 + 3*a^2*b^2*d^2*x + a^3*
b*d^2)*sin(d*x + c)^2), x) + ((b^2*d*x^3*sin(c) - b^2*x^2*cos(c) - 2*a*b*x*
cos(c))*cos(d*x + c)^2 + (b^2*d*x^3*sin(c) - b^2*x^2*cos(c) - 2*a*b*x*cos(c
))*sin(d*x + c)^2)*sin(d*x + 2*c) - ((b^2*cos(c)^2 + b^2*sin(c)^2)*x^2 + 2*
(a*b*cos(c)^2 + a*b*sin(c)^2)*x)*sin(d*x + c))/(((b^4*cos(c)^2 + b^4*sin(c)
^2)*d^2*x^2 + 2*(a*b^3*cos(c)^2 + a*b^3*sin(c)^2)*d^2*x + (a^2*b^2*cos(c)^2
 + a^2*b^2*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b^4*cos(c)^2 + b^4*sin(c)^2)*d
^2*x^2 + 2*(a*b^3*cos(c)^2 + a*b^3*sin(c)^2)*d^2*x + (a^2*b^2*cos(c)^2 + a^
2*b^2*sin(c)^2)*d^2)*sin(d*x + c)^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1474 vs. $2(186) = 372$.

Time = 0.38 (sec) , antiderivative size = 1474, normalized size of antiderivative = 8.14

$$\int \frac{x^3 \sin(c+dx)}{(a+bx)^2} \, dx = \text{Too large to display}$$

```
[In] integrate(x^3*sin(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -((b*x + a)*a^3*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^3*cos(-(b*c - a*d)/b)
*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b
) - a^3*b*c*d^3*cos(-(b*c - a*d)/b)*cos_integral(((b*x + a)*(b*c/(b*x + a)
- a*d/(b*x + a) + d) - b*c + a*d)/b) + a^4*d^4*cos(-(b*c - a*d)/b)*cos_inte
gral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + (b*x
+ a)*a^3*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^3*sin(-(b*c - a*d)/b)*sin_in
tegral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) - a^3
*b*c*d^3*sin(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(
b*x + a) + d) - b*c + a*d)/b) + a^4*d^4*sin(-(b*c - a*d)/b)*sin_integral(((
b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + 3*(b*x + a)*
a^2*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*cos_integral(((b*x + a)*(b*c/
(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*sin(-(b*c - a*d)/b) - 3*a^2*
b^2*c*d^2*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c
 + a*d)/b)*sin(-(b*c - a*d)/b) + 3*a^3*b*d^3*cos_integral(((b*x + a)*(b*c/(
b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*sin(-(b*c - a*d)/b) - 3*(b*x
+ a)*a^2*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*cos(-(b*c - a*d)/b)*sin_
integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + 3
*a^2*b^2*c*d^2*cos(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) -
 a*d/(b*x + a) + d) - b*c + a*d)/b) - 3*a^3*b*d^3*cos(-(b*c - a*d)/b)*sin_i
ntegral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + a^
3*b*d^3*sin(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + (b*x + a)^2
*b^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*cos(-(b*x + a)*(b*c/(b*x + a) -
a*d/(b*x + a) + d)/b) - 2*(b*x + a)*b^3*(b*c/(b*x + a) - a*d/(b*x + a) + d)
*c*cos(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + b^4*c^2*cos(-(b*
x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - (b*x + a)*a*b^2*(b*c/(b*x +
 a) - a*d/(b*x + a) + d)*d*cos(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) +
d)/b) + a*b^3*c*d*cos(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 2
*a^2*b^2*d^2*cos(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + (b*x +
 a)*b^3*(b*c/(b*x + a) - a*d/(b*x + a) + d)*sin(-(b*x + a)*(b*c/(b*x + a) -
 a*d/(b*x + a) + d)/b) - b^4*c*sin(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a
) + d)/b) + a*b^3*d*sin(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b))*
b^2/(((b*x + a)*b^7*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d - b^8*c*d + a*b^7
*d^2)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \sin(c+dx)}{(a+bx)^2}\,dx = \int \frac{x^3\, \sin\left(c+d\,x\right)}{{\left(a+b\,x\right)}^2}\,dx$$

```
[In] int((x^3*sin(c + d*x))/(a + b*x)^2,x)
[Out] int((x^3*sin(c + d*x))/(a + b*x)^2, x)
```

3.28 $\int \frac{x^2\sin(c+dx)}{(a+bx)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 149

$$\int \frac{x^2\sin(c+dx)}{(a+bx)^2}\,dx = -\frac{\cos(c+dx)}{b^2d}+\frac{a^2d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^4} - \frac{2a\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^3}-\frac{a^2\sin(c+dx)}{b^3(a+bx)} - \frac{2a\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^3}-\frac{a^2d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^4}$$

```
[Out] a^2*d*Ci(a*d/b+d*x)*cos(-c+a*d/b)/b^4-cos(d*x+c)/b^2/d-2*a*cos(-c+a*d/b)*Si
(a*d/b+d*x)/b^3+2*a*Ci(a*d/b+d*x)*sin(-c+a*d/b)/b^3+a^2*d*Si(a*d/b+d*x)*sin
(-c+a*d/b)/b^4-a^2*sin(d*x+c)/b^3/(b*x+a)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {6874, 2718, 3378, 3384, 3380, 3383}

$$\int \frac{x^2\sin(c+dx)}{(a+bx)^2}\,dx = \frac{a^2d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b^4}-\frac{a^2d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b^4} - \frac{a^2\sin(c+dx)}{b^3(a+bx)}-\frac{2a\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b^3} - \frac{2a\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b^3}-\frac{\cos(c+dx)}{b^2d}$$

```
[In] Int[(x^2*Sin[c + d*x])/(a + b*x)^2,x]
```

```
[Out] -(Cos[c + d*x]/(b^2*d)) + (a^2*d*Cos[c - (a*d)/b]*CosIntegral[(a*d)/b + d*x
])/b^4 - (2*a*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/b^3 - (a^2*Sin[c
 + d*x])/(b^3*(a + b*x)) - (2*a*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x]
)/b^3 - (a^2*d*Sin[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^4
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\sin(c+dx)}{b^2} + \frac{a^2\sin(c+dx)}{b^2(a+bx)^2} - \frac{2a\sin(c+dx)}{b^2(a+bx)}\right) dx \\
&= \frac{\int \sin(c+dx)\,dx}{b^2} - \frac{(2a)\int \frac{\sin(c+dx)}{a+bx}\,dx}{b^2} + \frac{a^2\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx}{b^2}
\end{aligned}$$

$$= -\frac{\cos(c+dx)}{b^2d} - \frac{a^2\sin(c+dx)}{b^3(a+bx)} + \frac{(a^2d)\int\frac{\cos(c+dx)}{a+bx}\,dx}{b^3} - \frac{\left(2a\cos\left(c-\frac{ad}{b}\right)\right)\int\frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^2} - \frac{\left(2a\sin\left(c-\frac{ad}{b}\right)\right)\int\frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^2}$$

$$= -\frac{\cos(c+dx)}{b^2d} - \frac{2a\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^3} - \frac{a^2\sin(c+dx)}{b^3(a+bx)} - \frac{2a\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^3} + \frac{\left(a^2d\cos\left(c-\frac{ad}{b}\right)\right)\int\frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^3} - \frac{\left(a^2d\sin\left(c-\frac{ad}{b}\right)\right)\int\frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^3}$$

$$= -\frac{\cos(c+dx)}{b^2d} + \frac{a^2d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^4} - \frac{2a\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^3} - \frac{a^2\sin(c+dx)}{b^3(a+bx)} - \frac{2a\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^3} - \frac{a^2d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^4}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.79

$$\int\frac{x^2\sin(c+dx)}{(a+bx)^2}\,dx$$

$$= \frac{a\,\text{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)\left(ad\cos\left(c-\frac{ad}{b}\right)-2b\sin\left(c-\frac{ad}{b}\right)\right)+b\left(-\frac{b\cos(c+dx)}{d}-\frac{a^2\sin(c+dx)}{a+bx}\right)-a\left(2b\cos\left(c-\right.\right.}{b^4}$$

```
[In] Integrate[(x^2*Sin[c + d*x])/(a + b*x)^2,x]
```

```
[Out] (a*CosIntegral[d*(a/b + x)]*(a*d*Cos[c - (a*d)/b] - 2*b*Sin[c - (a*d)/b]) +
 b*(-((b*Cos[c + d*x])/d) - (a^2*Sin[c + d*x])/(a + b*x)) - a*(2*b*Cos[c -
(a*d)/b] + a*d*Sin[c - (a*d)/b])*SinIntegral[d*(a/b + x)])/b^4
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 402, normalized size of antiderivative = 2.70

method	result
risch	$-\frac{i(2ib^3d^2x^2+4iab^2d^2x+2ia^2bd^2)\cos(dx+c)}{2b^3d^2(bx+a)(-dxb-da)}+\frac{(2a^2bd^3x+2d^3a^3)\sin(dx+c)}{2b^3d^2(bx+a)(-dxb-da)}-\frac{i\cos\left(\frac{da-cb}{b}\right)\mathrm{Ei}_1\left(-\frac{id(bx+a)}{b}\right)a}{b^3}$
derivativedivides	$c^2d^2\left(-\frac{\sin(dx+c)}{(da-cb+b(dx+c))b}+\frac{\frac{\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}+\frac{\mathrm{Ci}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}}{b}\right)-\frac{2cd^2\left(\frac{\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos(}{b}\right.}{}$
default	$c^2d^2\left(-\frac{\sin(dx+c)}{(da-cb+b(dx+c))b}+\frac{\frac{\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}+\frac{\mathrm{Ci}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}}{b}\right)-\frac{2cd^2\left(\frac{\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos(}{b}\right.}{}$

```
[In] int(x^2*sin(d*x+c)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/b^3/d^2*(2*I*a^2*b*d^2+4*I*a*b^2*d^2*x+2*I*b^3*d^2*x^2)/(b*x+a)/(-b*
d*x-a*d)*cos(d*x+c)+1/2/b^3/d^2*(2*a^2*b*d^3*x+2*a^3*d^3)/(b*x+a)/(-b*d*x-a
*d)*sin(d*x+c)-I/b^3*cos((a*d-b*c)/b)*Ei(1,-I*d*(b*x+a)/b)*a+I/b^3*cos((a*d
-b*c)/b)*Ei(1,I*d*(b*x+a)/b)*a-1/2*d/b^4*cos((a*d-b*c)/b)*Ei(1,-I*d*(b*x+a)
/b)*a^2-1/2*d/b^4*cos((a*d-b*c)/b)*Ei(1,I*d*(b*x+a)/b)*a^2-1/b^3*sin((a*d-b
*c)/b)*Ei(1,-I*d*(b*x+a)/b)*a-1/b^3*sin((a*d-b*c)/b)*Ei(1,I*d*(b*x+a)/b)*a+
1/2*I*d/b^4*sin((a*d-b*c)/b)*Ei(1,-I*d*(b*x+a)/b)*a^2-1/2*I*d/b^4*sin((a*d-
b*c)/b)*Ei(1,I*d*(b*x+a)/b)*a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.36

$$\int \frac{x^2\sin(c+dx)}{(a+bx)^2}\,dx = -\frac{a^2bd\sin(dx+c)+(b^3x+ab^2)\cos(dx+c)-\left((a^2bd^2x+a^3d^2)\,\mathrm{Ci}\left(\frac{bdx+ad}{b}\right)-2\,(ab^2dx+a^2bd)\,\mathrm{Si}\left(\frac{bdx+a}{b}\right.\right.}{b^5dx+ab^4}$$

```
[In] integrate(x^2*sin(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(a^2*b*d*sin(d*x + c) + (b^3*x + a*b^2)*cos(d*x + c) - ((a^2*b*d^2*x + a^3
*d^2)*cos_integral((b*d*x + a*d)/b) - 2*(a*b^2*d*x + a^2*b*d)*sin_integral(
(b*d*x + a*d)/b))*cos(-(b*c - a*d)/b) - (2*(a*b^2*d*x + a^2*b*d)*cos_integr
al((b*d*x + a*d)/b) + (a^2*b*d^2*x + a^3*d^2)*sin_integral((b*d*x + a*d)/b)
)*sin(-(b*c - a*d)/b))/(b^5*d*x + a*b^4*d)
```

Sympy [F]

$$\int \frac{x^2 \sin(c+dx)}{(a+bx)^2} \, dx = \int \frac{x^2 \sin{(c+dx)}}{(a+bx)^2} \, dx$$

```
[In] integrate(x**2*sin(d*x+c)/(b*x+a)**2,x)
```

```
[Out] Integral(x**2*sin(c + d*x)/(a + b*x)**2, x)
```

Maxima [F]

$$\int \frac{x^2 \sin(c+dx)}{(a+bx)^2} \, dx = \int \frac{x^2 \sin{(dx+c)}}{(bx+a)^2} \, dx$$

```
[In] integrate(x^2*sin(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((cos(c)^2 + sin(c)^2)*x^2*cos(d*x + c) + (x^2*cos(d*x + c)^2*cos(c) +
 x^2*cos(c)*sin(d*x + c)^2)*cos(d*x + 2*c) - 2*(((a*b^2*cos(c)^2 + a*b^2*si
n(c)^2)*d*x^2 + 2*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d*x + (a^3*cos(c)^2 + a
^3*sin(c)^2)*d)*cos(d*x + c)^2 + ((a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x^2 +
 2*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d*x + (a^3*cos(c)^2 + a^3*sin(c)^2)*d)
*sin(d*x + c)^2)*integrate(x*cos(d*x + c)/(b^3*d*x^3 + 3*a*b^2*d*x^2 + 3*a^
2*b*d*x + a^3*d), x) - 2*(((a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x^2 + 2*(a^2
*b*cos(c)^2 + a^2*b*sin(c)^2)*d*x + (a^3*cos(c)^2 + a^3*sin(c)^2)*d)*cos(d*
x + c)^2 + ((a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x^2 + 2*(a^2*b*cos(c)^2 + a
^2*b*sin(c)^2)*d*x + (a^3*cos(c)^2 + a^3*sin(c)^2)*d)*sin(d*x + c)^2)*integ
rate(x*cos(d*x + c)/((b^3*d*x^3 + 3*a*b^2*d*x^2 + 3*a^2*b*d*x + a^3*d)*cos(
d*x + c)^2 + (b^3*d*x^3 + 3*a*b^2*d*x^2 + 3*a^2*b*d*x + a^3*d)*sin(d*x + c)
^2), x) + (x^2*cos(d*x + c)^2*sin(c) + x^2*sin(d*x + c)^2*sin(c))*sin(d*x +
 2*c))/(((b^2*cos(c)^2 + b^2*sin(c)^2)*d*x^2 + 2*(a*b*cos(c)^2 + a*b*sin(c)
^2)*d*x + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*cos(d*x + c)^2 + ((b^2*cos(c)^2
+ b^2*sin(c)^2)*d*x^2 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d*x + (a^2*cos(c)^2
 + a^2*sin(c)^2)*d)*sin(d*x + c)^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1120 vs. $2(152) = 304$.

Time = 0.33 (sec) , antiderivative size = 1120, normalized size of antiderivative = 7.52

$$\int \frac{x^2 \sin(c+dx)}{(a+bx)^2} \, dx = \text{Too large to display}$$

```
[In] integrate(x^2*sin(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)*a^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*cos(-(b*c - a*d)/b)*
cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)
 - a^2*b*c*d^2*cos(-(b*c - a*d)/b)*cos_integral(((b*x + a)*(b*c/(b*x + a) -
 a*d/(b*x + a) + d) - b*c + a*d)/b) + a^3*d^3*cos(-(b*c - a*d)/b)*cos_integ
ral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + (b*x +
 a)*a^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*sin(-(b*c - a*d)/b)*sin_int
egral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) - a^2*
b*c*d^2*sin(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b
*x + a) + d) - b*c + a*d)/b) + a^3*d^3*sin(-(b*c - a*d)/b)*sin_integral(((b
*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + 2*(b*x + a)*a
*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d*cos_integral(((b*x + a)*(b*c/(b*x
+ a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*sin(-(b*c - a*d)/b) - 2*a*b^2*c*d
*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b
)*sin(-(b*c - a*d)/b) + 2*a^2*b*d^2*cos_integral(((b*x + a)*(b*c/(b*x + a)
- a*d/(b*x + a) + d) - b*c + a*d)/b)*sin(-(b*c - a*d)/b) - 2*(b*x + a)*a*b*
(b*c/(b*x + a) - a*d/(b*x + a) + d)*d*cos(-(b*c - a*d)/b)*sin_integral(((b*
x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + 2*a*b^2*c*d*co
s(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) +
d) - b*c + a*d)/b) - 2*a^2*b*d^2*cos(-(b*c - a*d)/b)*sin_integral(((b*x + a
)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + a^2*b*d^2*sin(-(b*x
 + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - (b*x + a)*b^2*(b*c/(b*x + a)
 - a*d/(b*x + a) + d)*cos(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b)
 + b^3*c*cos(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - a*b^2*d*co
s(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b))*b^2/(((b*x + a)*b^6*(b
*c/(b*x + a) - a*d/(b*x + a) + d) - b^7*c + a*b^6*d)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \sin(c + dx)}{(a + bx)^2} \, dx = \int \frac{x^2 \, \sin(c + d\,x)}{(a + b\,x)^2} \, dx$$

```
[In] int((x^2*sin(c + d*x))/(a + b*x)^2,x)
```

```
[Out] int((x^2*sin(c + d*x))/(a + b*x)^2, x)
```

3.29 $\int \frac{x\sin(c+dx)}{(a+bx)^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 124

$$\int \frac{x\sin(c+dx)}{(a+bx)^2}\,dx = -\frac{ad\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^3} + \frac{\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^2} + \frac{a\sin(c+dx)}{b^2(a+bx)} + \frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^2} + \frac{ad\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^3}$$

```
[Out] -a*d*Ci(a*d/b+d*x)*cos(-c+a*d/b)/b^3+cos(-c+a*d/b)*Si(a*d/b+d*x)/b^2-Ci(a*d
/b+d*x)*sin(-c+a*d/b)/b^2-a*d*Si(a*d/b+d*x)*sin(-c+a*d/b)/b^3+a*sin(d*x+c)/
b^2/(b*x+a)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6874, 3378, 3384, 3380, 3383}

$$\int \frac{x\sin(c+dx)}{(a+bx)^2}\,dx = -\frac{ad\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b^3} + \frac{ad\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b^3} + \frac{\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b^2} + \frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b^2} + \frac{a\sin(c+dx)}{b^2(a+bx)}$$

```
[In] Int[(x*Sin[c + d*x])/(a + b*x)^2,x]
```

```
[Out] -((a*d*Cos[c - (a*d)/b]*CosIntegral[(a*d)/b + d*x])/b^3) + (CosIntegral[(a*
d)/b + d*x]*Sin[c - (a*d)/b])/b^2 + (a*Sin[c + d*x])/(b^2*(a + b*x)) + (Cos
[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^2 + (a*d*Sin[c - (a*d)/b]*SinIn
tegral[(a*d)/b + d*x])/b^3
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{a\sin(c+dx)}{b(a+bx)^2}+\frac{\sin(c+dx)}{b(a+bx)}\right)dx \\
&= \frac{\int \frac{\sin(c+dx)}{a+bx}\,dx}{b}-\frac{a\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx}{b} \\
&= \frac{a\sin(c+dx)}{b^2(a+bx)}-\frac{(ad)\int \frac{\cos(c+dx)}{a+bx}\,dx}{b^2} \\
&\quad+\frac{\cos\left(c-\frac{ad}{b}\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b}+\frac{\sin\left(c-\frac{ad}{b}\right)\int \frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b}
\end{aligned}
$$

$$= \frac{\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^2} + \frac{a\sin(c+dx)}{b^2(a+bx)} + \frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^2} - \frac{\left(ad\cos\left(c-\frac{ad}{b}\right)\right)\int\frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}dx}{b^2} + \frac{\left(ad\sin\left(c-\frac{ad}{b}\right)\right)\int\frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}dx}{b^2}$$

$$= -\frac{ad\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^3} + \frac{\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^2} + \frac{a\sin(c+dx)}{b^2(a+bx)} + \frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^2} + \frac{ad\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^3}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.77

$$\int \frac{x\sin(c+dx)}{(a+bx)^2}\,dx$$

$$= \frac{\text{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)\left(-ad\cos\left(c-\frac{ad}{b}\right)+b\sin\left(c-\frac{ad}{b}\right)\right)+\frac{ab\sin(c+dx)}{a+bx}+\left(b\cos\left(c-\frac{ad}{b}\right)+ad\sin\left(c-\frac{ad}{b}\right)\right.}{b^3}$$

```
[In] Integrate[(x*Sin[c + d*x])/(a + b*x)^2,x]
[Out] (CosIntegral[d*(a/b + x)]*(-(a*d*Cos[c - (a*d)/b]) + b*Sin[c - (a*d)/b]) +
(a*b*Sin[c + d*x])/(a + b*x) + (b*Cos[c - (a*d)/b] + a*d*Sin[c - (a*d)/b])*
SinIntegral[d*(a/b + x)])/b^3
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.52

method	result
risch	$\frac{\left(-2abdx-2a^2d\right)\sin(dx+c)}{2b^2(bx+a)(-dxb-da)}+\frac{\cos\left(\frac{da-cb}{b}\right)\text{Ei}_1\left(\frac{id(bx+a)}{b}\right)ad}{2b^3}+\frac{\cos\left(\frac{da-cb}{b}\right)\text{Ei}_1\left(-\frac{id(bx+a)}{b}\right)ad}{2b^3}-\frac{i\cos\left(\frac{da-cb}{b}\right)\text{Ei}_1}{2b^2}$
derivativedivides	$\frac{-\frac{d^2(da-cb)\left(-\frac{\sin(dx+c)}{(da-cb+b(dx+c))b}+\frac{\frac{\text{Si}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}+\frac{\text{Ci}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}}{b}\right)}{b}+\frac{d^2\left(\frac{\text{Si}\left(dx+c+\frac{da-cb}{b}\right)\text{co}}{b}\right.}{}}{}$
default	$\frac{-\frac{d^2(da-cb)\left(-\frac{\sin(dx+c)}{(da-cb+b(dx+c))b}+\frac{\frac{\text{Si}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}+\frac{\text{Ci}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}}{b}\right)}{b}+\frac{d^2\left(\frac{\text{Si}\left(dx+c+\frac{da-cb}{b}\right)\text{co}}{b}\right.}{}}{}$

[In] int(x*sin(d*x+c)/(b*x+a)^2,x,method=_RETURNVERBOSE)

[Out] 1/2/b^2*(-2*a*b*d*x-2*a^2*d)/(b*x+a)/(-b*d*x-a*d)*sin(d*x+c)+1/2/b^3*cos((a*d-b*c)/b)*Ei(1,I*d*(b*x+a)/b)*a*d+1/2/b^3*cos((a*d-b*c)/b)*Ei(1,-I*d*(b*x+a)/b)*a*d-1/2*I/b^2*cos((a*d-b*c)/b)*Ei(1,I*d*(b*x+a)/b)+1/2*I/b^2*cos((a*d-b*c)/b)*Ei(1,-I*d*(b*x+a)/b)+1/2*I/b^3*sin((a*d-b*c)/b)*Ei(1,I*d*(b*x+a)/b)*a*d-1/2*I/b^3*sin((a*d-b*c)/b)*Ei(1,-I*d*(b*x+a)/b)*a*d+1/2/b^2*sin((a*d-b*c)/b)*Ei(1,I*d*(b*x+a)/b)+1/2/b^2*sin((a*d-b*c)/b)*Ei(1,-I*d*(b*x+a)/b)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.25

$$\int \frac{x\sin(c+dx)}{(a+bx)^2}\,dx = \frac{ab\sin\left(dx+c\right) - \left(\left(abdx+a^2d\right)\operatorname{Ci}\left(\frac{bdx+ad}{b}\right) - \left(b^2x+ab\right)\operatorname{Si}\left(\frac{bdx+ad}{b}\right)\right)\cos\left(-\frac{bc-ad}{b}\right) - \left(\left(b^2x+ab\right)\operatorname{Ci}\left(\frac{bdx+a}{b}\right.\right.}{b^4x+ab^3}$$

[In] integrate(x*sin(d*x+c)/(b*x+a)^2,x, algorithm="fricas")

[Out] (a*b*sin(d*x + c) - ((a*b*d*x + a^2*d)*cos_integral((b*d*x + a*d)/b) - (b^2*x + a*b)*sin_integral((b*d*x + a*d)/b))*cos(-(b*c - a*d)/b) - ((b^2*x + a*b)*cos_integral((b*d*x + a*d)/b) + (a*b*d*x + a^2*d)*sin_integral((b*d*x + a*d)/b))*sin(-(b*c - a*d)/b))/(b^4*x + a*b^3)

Sympy [F]

$$\int \frac{x\sin(c+dx)}{(a+bx)^2}\,dx = \int \frac{x\sin\left(c+dx\right)}{\left(a+bx\right)^2}\,dx$$

[In] integrate(x*sin(d*x+c)/(b*x+a)**2,x)

[Out] Integral(x*sin(c + d*x)/(a + b*x)**2, x)

Maxima [F]

$$\int \frac{x\sin(c+dx)}{(a+bx)^2}\,dx = \int \frac{x\sin\left(dx+c\right)}{\left(bx+a\right)^2}\,dx$$

[In] integrate(x*sin(d*x+c)/(b*x+a)^2,x, algorithm="maxima")

```
[Out] -1/2*((b*cos(c)^2 + b*sin(c)^2)*x*cos(d*x + c) + ((a*(exp_integral_e(3, (I*
b*d*x + I*a*d)/b) + exp_integral_e(3, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a*(
exp_integral_e(3, (I*b*d*x + I*a*d)/b) + exp_integral_e(3, -(I*b*d*x + I*a*
d)/b))*sin(c)^2)*cos(-(b*c - a*d)/b) + (a*(I*exp_integral_e(3, (I*b*d*x + I
*a*d)/b) - I*exp_integral_e(3, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a*(I*exp_i
ntegral_e(3, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(3, -(I*b*d*x + I*a*d)/
b))*sin(c)^2)*sin(-(b*c - a*d)/b))*cos(d*x + c)^2 + ((a*(exp_integral_e(3,
(I*b*d*x + I*a*d)/b) + exp_integral_e(3, -(I*b*d*x + I*a*d)/b))*cos(c)^2 +
a*(exp_integral_e(3, (I*b*d*x + I*a*d)/b) + exp_integral_e(3, -(I*b*d*x + I
*a*d)/b))*sin(c)^2)*cos(-(b*c - a*d)/b) + (a*(I*exp_integral_e(3, (I*b*d*x
+ I*a*d)/b) - I*exp_integral_e(3, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a*(I*ex
p_integral_e(3, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(3, -(I*b*d*x + I*a*
d)/b))*sin(c)^2)*sin(-(b*c - a*d)/b))*sin(d*x + c)^2 + (b*x*cos(d*x + c)^2*
cos(c) + b*x*cos(c)*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b^4*cos(c)^2 + b^
4*sin(c)^2)*d*x^2 + 2*(a*b^3*cos(c)^2 + a*b^3*sin(c)^2)*d*x + (a^2*b^2*cos(
c)^2 + a^2*b^2*sin(c)^2)*d)*cos(d*x + c)^2 + ((b^4*cos(c)^2 + b^4*sin(c)^2)
*d*x^2 + 2*(a*b^3*cos(c)^2 + a*b^3*sin(c)^2)*d*x + (a^2*b^2*cos(c)^2 + a^2*
b^2*sin(c)^2)*d)*sin(d*x + c)^2)*integrate(1/2*x*cos(d*x + c)/(b^3*d*x^3 +
3*a*b^2*d*x^2 + 3*a^2*b*d*x + a^3*d), x) + 2*(((b^4*cos(c)^2 + b^4*sin(c)^2
)*d*x^2 + 2*(a*b^3*cos(c)^2 + a*b^3*sin(c)^2)*d*x + (a^2*b^2*cos(c)^2 + a^2
*b^2*sin(c)^2)*d)*cos(d*x + c)^2 + ((b^4*cos(c)^2 + b^4*sin(c)^2)*d*x^2 + 2
*(a*b^3*cos(c)^2 + a*b^3*sin(c)^2)*d*x + (a^2*b^2*cos(c)^2 + a^2*b^2*sin(c)
^2)*d)*sin(d*x + c)^2)*integrate(1/2*x*cos(d*x + c)/((b^3*d*x^3 + 3*a*b^2*d
*x^2 + 3*a^2*b*d*x + a^3*d)*cos(d*x + c)^2 + (b^3*d*x^3 + 3*a*b^2*d*x^2 + 3
*a^2*b*d*x + a^3*d)*sin(d*x + c)^2), x) + (b*x*cos(d*x + c)^2*sin(c) + b*x*
sin(d*x + c)^2*sin(c))*sin(d*x + 2*c))/(((b^3*cos(c)^2 + b^3*sin(c)^2)*d*x^
2 + 2*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x + (a^2*b*cos(c)^2 + a^2*b*sin(c
)^2)*d)*cos(d*x + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d*x^2 + 2*(a*b^2*co
s(c)^2 + a*b^2*sin(c)^2)*d*x + (a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d)*sin(d*x
 + c)^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 951 vs. $2(130) = 260$.

Time = 0.35 (sec) , antiderivative size = 951, normalized size of antiderivative = 7.67

$$\int \frac{x\sin(c+dx)}{(a+bx)^2}\,dx =$$

$$-\frac{\left((bx+a)a\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)d^2\cos\left(-\frac{bc-ad}{b}\right)\operatorname{Ci}\left(\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-bc+ad}{b}\right)-abcd^2\cos\left(-\frac{bc-ad}{b}\right)\operatorname{Ci}\left(\right.\right.}{}$$

```
[In] integrate(x*sin(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -((b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*cos(-(b*c - a*d)/b)*c
os_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)
- a*b*c*d^2*cos(-(b*c - a*d)/b)*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*
d/(b*x + a) + d) - b*c + a*d)/b) + a^2*d^3*cos(-(b*c - a*d)/b)*cos_integral
(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + (b*x + a)
*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*sin(-(b*c - a*d)/b)*sin_integral
(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) - a*b*c*d^2
*sin(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a)
 + d) - b*c + a*d)/b) + a^2*d^3*sin(-(b*c - a*d)/b)*sin_integral(((b*x + a)
*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + (b*x + a)*b*(b*c/(b*
x + a) - a*d/(b*x + a) + d)*d*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/
(b*x + a) + d) - b*c + a*d)/b)*sin(-(b*c - a*d)/b) - b^2*c*d*cos_integral((
(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*sin(-(b*c - a
*d)/b) + a*b*d^2*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d
) - b*c + a*d)/b)*sin(-(b*c - a*d)/b) - (b*x + a)*b*(b*c/(b*x + a) - a*d/(b
*x + a) + d)*d*cos(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) -
 a*d/(b*x + a) + d) - b*c + a*d)/b) + b^2*c*d*cos(-(b*c - a*d)/b)*sin_integ
ral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) - a*b*d^
2*cos(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a
) + d) - b*c + a*d)/b) + a*b*d^2*sin(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x +
 a) + d)/b))*b/(((b*x + a)*b^4*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b^5*c
+ a*b^4*d)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sin(c+dx)}{(a+bx)^2}\,dx = \int \frac{x\,\sin\left(c+d\,x\right)}{\left(a+b\,x\right)^2}\,dx$$

```
[In] int((x*sin(c + d*x))/(a + b*x)^2,x)
```

```
[Out] int((x*sin(c + d*x))/(a + b*x)^2, x)
```

3.30 $\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 72

$$\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx = \frac{d\cos\left(c-\frac{ad}{b}\right)\operatorname{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^2} - \frac{\sin(c+dx)}{b(a+bx)} - \frac{d\sin\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(\frac{ad}{b}+dx\right)}{b^2}$$

```
[Out] d*Ci(a*d/b+d*x)*cos(-c+a*d/b)/b^2+d*Si(a*d/b+d*x)*sin(-c+a*d/b)/b^2-sin(d*x
+c)/b/(b*x+a)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3378, 3384, 3380, 3383}

$$\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx = \frac{d\cos\left(c-\frac{ad}{b}\right)\operatorname{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b^2} - \frac{d\sin\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(xd+\frac{ad}{b}\right)}{b^2} - \frac{\sin(c+dx)}{b(a+bx)}$$

```
[In] Int[Sin[c + d*x]/(a + b*x)^2,x]
```

```
[Out] (d*Cos[c - (a*d)/b]*CosIntegral[(a*d)/b + d*x])/b^2 - Sin[c + d*x]/(b*(a +
b*x)) - (d*Sin[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\sin(c+dx)}{b(a+bx)}+\frac{d\int \frac{\cos(c+dx)}{a+bx}\,dx}{b}\\ &= -\frac{\sin(c+dx)}{b(a+bx)}+\frac{\left(d\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b}-\frac{\left(d\sin\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b}\\ &= \frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^2}-\frac{\sin(c+dx)}{b(a+bx)}-\frac{d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx = \frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)-\frac{b\sin(c+dx)}{a+bx}-d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(d\left(\frac{a}{b}+x\right)\right)}{b^2}$$

```
[In] Integrate[Sin[c + d*x]/(a + b*x)^2,x]
```

```
[Out] (d*Cos[c - (a*d)/b]*CosIntegral[d*(a/b + x)] - (b*Sin[c + d*x])/(a + b*x) -
 d*Sin[c - (a*d)/b]*SinIntegral[d*(a/b + x)])/b^2
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.49

method	result	size
derivativedivides	$d\left(-\frac{\sin(dx+c)}{(da-cb+b(dx+c))b}+\frac{\frac{\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}+\frac{\mathrm{Ci}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}}{b}\right)$	107
default	$d\left(-\frac{\sin(dx+c)}{(da-cb+b(dx+c))b}+\frac{\frac{\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}+\frac{\mathrm{Ci}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}}{b}\right)$	107
risch	$-\frac{d\,\mathrm{e}^{-\frac{i(da-cb)}{b}}\,\mathrm{Ei}_1\left(-idx-ic-\frac{iad-icb}{b}\right)}{2b^2}-\frac{d\,\mathrm{e}^{\frac{i(da-cb)}{b}}\,\mathrm{Ei}_1\left(idx+ic+\frac{i(da-cb)}{b}\right)}{2b^2}-\frac{(-2dxb-2da)\sin(dx+c)}{2b(bx+a)(-dxb-da)}$	138

```
[In] int(sin(d*x+c)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d*(-sin(d*x+c)/(d*a-c*b+b*(d*x+c))/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b
)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.33

$$\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx = \frac{(bdx+ad)\cos\left(-\frac{bc-ad}{b}\right)\mathrm{Ci}\left(\frac{bdx+ad}{b}\right)+(bdx+ad)\sin\left(-\frac{bc-ad}{b}\right)\mathrm{Si}\left(\frac{bdx+ad}{b}\right)-b\sin\left(dx+c\right)}{b^3x+ab^2}$$

```
[In] integrate(sin(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b*d*x + a*d)*cos(-(b*c - a*d)/b)*cos_integral((b*d*x + a*d)/b) + (b*d*x +
 a*d)*sin(-(b*c - a*d)/b)*sin_integral((b*d*x + a*d)/b) - b*sin(d*x + c))/(
b^3*x + a*b^2)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx = \int \frac{\sin\left(c+dx\right)}{\left(a+bx\right)^2}\,dx$$

```
[In] integrate(sin(d*x+c)/(b*x+a)**2,x)
```

```
[Out] Integral(sin(c + d*x)/(a + b*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.28

$$\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx = \frac{d^2\left(-i\,E_2\left(\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)+i\,E_2\left(-\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)\right)\cos\left(-\frac{bc-ad}{b}\right)+d^2\left(E_2\left(\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)+E_2\left(-\frac{i(\cdots}{}\right.\right.}{2\left((dx+c)b^2-b^2c+abd\right)d}$$

```
[In] integrate(sin(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(d^2*(-I*exp_integral_e(2, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + I*exp_i
ntegral_e(2, -(I*(d*x + c)*b - I*b*c + I*a*d)/b))*cos(-(b*c - a*d)/b) + d^2
*(exp_integral_e(2, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + exp_integral_e(2,
-(I*(d*x + c)*b - I*b*c + I*a*d)/b))*sin(-(b*c - a*d)/b))/(((d*x + c)*b^2 -
 b^2*c + a*b*d)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. 2(73) = 146.

Time = 0.30 (sec) , antiderivative size = 518, normalized size of antiderivative = 7.19

$$\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx = \frac{\left((bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)d^2\cos\left(-\frac{bc-ad}{b}\right)\mathrm{Ci}\left(\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-bc+ad}{b}\right)-bcd^2\cos\left(-\frac{bc-ad}{b}\right)\mathrm{Ci}\left(\frac{(bx+a)(\cdots}{}\right.\right.}{}$$

```
[In] integrate(sin(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*cos(-(b*c - a*d)/b)*cos_
integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) - b
*c*d^2*cos(-(b*c - a*d)/b)*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*
x + a) + d) - b*c + a*d)/b) + a*d^3*cos(-(b*c - a*d)/b)*cos_integral(((b*x
+ a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + (b*x + a)*(b*c/(
b*x + a) - a*d/(b*x + a) + d)*d^2*sin(-(b*c - a*d)/b)*sin_integral(((b*x +
a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) - b*c*d^2*sin(-(b*c
- a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c
 + a*d)/b) + a*d^3*sin(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x +
a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + b*d^2*sin(-(b*x + a)*(b*c/(b*x +
a) - a*d/(b*x + a) + d)/b))*b^2/(((b*x + a)*b^4*(b*c/(b*x + a) - a*d/(b*x +
 a) + d) - b^5*c + a*b^4*d)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx = \int \frac{\sin\left(c+d\,x\right)}{\left(a+b\,x\right)^2}\,dx$$

```
[In] int(sin(c + d*x)/(a + b*x)^2,x)
[Out] int(sin(c + d*x)/(a + b*x)^2, x)
```

3.31 $\int \frac{\sin(c+dx)}{x(a+bx)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 149

$$\int \frac{\sin(c+dx)}{x(a+bx)^2}\,dx = -\frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{ab}+\frac{\text{CosIntegral}(dx)\sin(c)}{a^2} - \frac{\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^2}+\frac{\sin(c+dx)}{a(a+bx)}+\frac{\cos(c)\text{Si}(dx)}{a^2} - \frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^2}+\frac{d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{ab}$$

```
[Out] -d*Ci(a*d/b+d*x)*cos(-c+a*d/b)/a/b+cos(c)*Si(d*x)/a^2-cos(-c+a*d/b)*Si(a*d/
b+d*x)/a^2+Ci(d*x)*sin(c)/a^2+Ci(a*d/b+d*x)*sin(-c+a*d/b)/a^2-d*Si(a*d/b+d*
x)*sin(-c+a*d/b)/a/b+sin(d*x+c)/a/(b*x+a)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6874, 3384, 3380, 3383, 3378}

$$\int \frac{\sin(c+dx)}{x(a+bx)^2}\,dx = -\frac{\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{a^2} - \frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^2}+\frac{\sin(c)\text{CosIntegral}(dx)}{a^2} + \frac{\cos(c)\text{Si}(dx)}{a^2}-\frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{ab} + \frac{d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{ab}+\frac{\sin(c+dx)}{a(a+bx)}$$

```
[In] Int[Sin[c + d*x]/(x*(a + b*x)^2),x]
```

```
[Out] -((d*Cos[c - (a*d)/b]*CosIntegral[(a*d)/b + d*x])/(a*b)) + (CosIntegral[d*x
]*Sin[c])/a^2 - (CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/a^2 + Sin[c +
 d*x]/(a*(a + b*x)) + (Cos[c]*SinIntegral[d*x])/a^2 - (Cos[c - (a*d)/b]*Sin
Integral[(a*d)/b + d*x])/a^2 + (d*Sin[c - (a*d)/b]*SinIntegral[(a*d)/b + d*
x])/(a*b)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\sin(c+dx)}{a^2x} - \frac{b\sin(c+dx)}{a(a+bx)^2} - \frac{b\sin(c+dx)}{a^2(a+bx)}\right) dx$$

$$= \frac{\int \frac{\sin(c+dx)}{x}\,dx}{a^2} - \frac{b\int \frac{\sin(c+dx)}{a+bx}\,dx}{a^2} - \frac{b\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx}{a}$$

$$= \frac{\sin(c+dx)}{a(a+bx)} - \frac{d\int \frac{\cos(c+dx)}{a+bx}\,dx}{a} + \frac{\cos(c)\int \frac{\sin(dx)}{x}\,dx}{a^2} - \frac{\left(b\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^2}$$
$$+ \frac{\sin(c)\int \frac{\cos(dx)}{x}\,dx}{a^2} - \frac{\left(b\sin\left(c-\frac{ad}{b}\right)\right)\int \frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^2}$$
$$= \frac{\text{CosIntegral}(dx)\sin(c)}{a^2} - \frac{\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^2}$$
$$+ \frac{\sin(c+dx)}{a(a+bx)} + \frac{\cos(c)\text{Si}(dx)}{a^2} - \frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^2}$$
$$- \frac{\left(d\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a} + \frac{\left(d\sin\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a}$$
$$= -\frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{ab} + \frac{\text{CosIntegral}(dx)\sin(c)}{a^2}$$
$$- \frac{\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^2} + \frac{\sin(c+dx)}{a(a+bx)} + \frac{\cos(c)\text{Si}(dx)}{a^2}$$
$$- \frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^2} + \frac{d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{ab}$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.93

$$\int \frac{\sin(c+dx)}{x(a+bx)^2}\,dx$$
$$= \frac{\frac{a\cos(dx)\sin(c)}{a+bx} + \text{CosIntegral}(dx)\sin(c) - \frac{\text{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)\left(ad\cos\left(c-\frac{ad}{b}\right)+b\sin\left(c-\frac{ad}{b}\right)\right)}{b} + \frac{a\cos(c)\sin(dx)}{a+bx} + \cos(c)\text{Si}(}{a^2}$$

```
[In] Integrate[Sin[c + d*x]/(x*(a + b*x)^2),x]
```

```
[Out] ((a*Cos[d*x]*Sin[c])/(a + b*x) + CosIntegral[d*x]*Sin[c] - (CosIntegral[d*(
a/b + x)]*(a*d*Cos[c - (a*d)/b] + b*Sin[c - (a*d)/b]))/b + (a*Cos[c]*Sin[d*
x])/(a + b*x) + Cos[c]*SinIntegral[d*x] - Cos[c - (a*d)/b]*SinIntegral[d*(a
/b + x)] + (a*d*Sin[c - (a*d)/b]*SinIntegral[d*(a/b + x)])/b)/a^2
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.41

method	result
derivativedivides	$-\frac{b\left(\frac{\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}-\frac{\mathrm{Ci}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}\right)}{a^2}-\frac{db\left(-\frac{\sin(dx+c)}{(da-cb+b(dx+c))b}+\frac{\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\right.}{b}\right.}{a}$
default	$-\frac{b\left(\frac{\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}-\frac{\mathrm{Ci}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}\right)}{a^2}-\frac{db\left(-\frac{\sin(dx+c)}{(da-cb+b(dx+c))b}+\frac{\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\right.}{b}\right.}{a}$
risch	$\frac{ie^{ic}\,\mathrm{Ei}_1(-idx)}{2a^2}-\frac{ie^{-\frac{i(da-cb)}{b}}\,\mathrm{Ei}_1\left(-idx-ic-\frac{iad-icb}{b}\right)}{2a^2}+\frac{d\,e^{-\frac{i(da-cb)}{b}}\,\mathrm{Ei}_1\left(-idx-ic-\frac{iad-icb}{b}\right)}{2ab}-\frac{e^{-ic}\pi\,\mathrm{csgn}(dx)}{2a^2}$

```
[In] int(sin(d*x+c)/x/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -b/a^2*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin(
(a*d-b*c)/b)/b)-d*b/a*(-sin(d*x+c)/(d*a-c*b+b*(d*x+c))/b+(Si(d*x+c+(a*d-b*c
)/b)*sin((a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)+1/a^2*
(Si(d*x)*cos(c)+Ci(d*x)*sin(c))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.27

$$\int \frac{\sin(c+dx)}{x(a+bx)^2}\,dx = \frac{ab\sin\left(dx+c\right)+\left(b^2x+ab\right)\mathrm{Ci}\left(dx\right)\sin\left(c\right)+\left(b^2x+ab\right)\cos\left(c\right)\mathrm{Si}\left(dx\right)-\left(\left(abdx+a^2d\right)\mathrm{Ci}\left(\frac{bdx+ad}{b}\right)+\left(b^2x\right.\right.}{a^2b^2x+a}$$

```
[In] integrate(sin(d*x+c)/x/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (a*b*sin(d*x + c) + (b^2*x + a*b)*cos_integral(d*x)*sin(c) + (b^2*x + a*b)*
cos(c)*sin_integral(d*x) - ((a*b*d*x + a^2*d)*cos_integral((b*d*x + a*d)/b)
 + (b^2*x + a*b)*sin_integral((b*d*x + a*d)/b))*cos(-(b*c - a*d)/b) + ((b^2
*x + a*b)*cos_integral((b*d*x + a*d)/b) - (a*b*d*x + a^2*d)*sin_integral((b
*d*x + a*d)/b))*sin(-(b*c - a*d)/b))/(a^2*b^2*x + a^3*b)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{x(a+bx)^2}\,dx = \int \frac{\sin(c+dx)}{x(a+bx)^2}\,dx$$

```
[In] integrate(sin(d*x+c)/x/(b*x+a)**2,x)
[Out] Integral(sin(c + d*x)/(x*(a + b*x)**2), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x(a+bx)^2}\,dx = \int \frac{\sin(dx+c)}{(bx+a)^2x}\,dx$$

```
[In] integrate(sin(d*x+c)/x/(b*x+a)^2,x, algorithm="maxima")
[Out] integrate(sin(d*x + c)/((b*x + a)^2*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1281 vs. $2(153) = 306$.

Time = 0.34 (sec) , antiderivative size = 1281, normalized size of antiderivative = 8.60

$$\int \frac{\sin(c+dx)}{x(a+bx)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)/x/(b*x+a)^2,x, algorithm="giac")
[Out] -((b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*cos(-(b*c - a*d)/b)*c
os_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)
- a*b*c*d^2*cos(-(b*c - a*d)/b)*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*
d/(b*x + a) + d) - b*c + a*d)/b) + a^2*d^3*cos(-(b*c - a*d)/b)*cos_integral
(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + (b*x + a)
*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*sin(-(b*c - a*d)/b)*sin_integral
(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) - a*b*c*d^2
*sin(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a)
 + d) - b*c + a*d)/b) + a^2*d^3*sin(-(b*c - a*d)/b)*sin_integral(((b*x + a)
*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) - (b*x + a)*b*(b*c/(b*
x + a) - a*d/(b*x + a) + d)*d*cos_integral((b*x + a)*(b*c/(b*x + a) - a*d/(
b*x + a) + d)/b - c)*sin(c) + b^2*c*d*cos_integral((b*x + a)*(b*c/(b*x + a)
 - a*d/(b*x + a) + d)/b - c)*sin(c) - a*b*d^2*cos_integral((b*x + a)*(b*c/(
b*x + a) - a*d/(b*x + a) + d)/b - c)*sin(c) - (b*x + a)*b*(b*c/(b*x + a) -
```

```
a*d/(b*x + a) + d)*d*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a)
 + d) - b*c + a*d)/b)*sin(-(b*c - a*d)/b) + b^2*c*d*cos_integral(((b*x + a)
*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*sin(-(b*c - a*d)/b) -
a*b*d^2*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c +
 a*d)/b)*sin(-(b*c - a*d)/b) + (b*x + a)*b*(b*c/(b*x + a) - a*d/(b*x + a) +
 d)*d*cos(c)*sin_integral(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b
+ c) - b^2*c*d*cos(c)*sin_integral(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a
) + d)/b + c) + a*b*d^2*cos(c)*sin_integral(-(b*x + a)*(b*c/(b*x + a) - a*d
/(b*x + a) + d)/b + c) + (b*x + a)*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d*
cos(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a)
+ d) - b*c + a*d)/b) - b^2*c*d*cos(-(b*c - a*d)/b)*sin_integral(((b*x + a)*
(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + a*b*d^2*cos(-(b*c - a
*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c +
a*d)/b) + a*b*d^2*sin(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b))*b^
3/(((b*x + a)*a^2*b^4*(b*c/(b*x + a) - a*d/(b*x + a) + d) - a^2*b^5*c + a^3
*b^4*d)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x(a+bx)^2}\,dx = \int \frac{\sin\left(c+d\,x\right)}{x\,{\left(a+b\,x\right)}^2}\,dx$$

```
[In] int(sin(c + d*x)/(x*(a + b*x)^2),x)
```

```
[Out] int(sin(c + d*x)/(x*(a + b*x)^2), x)
```

3.32 $\int \frac{\sin(c+dx)}{x^2(a+bx)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 188

$$
\begin{aligned}
\int \frac{\sin(c+dx)}{x^2(a+bx)^2}\,dx = {} & \frac{d\cos(c)\,\text{CosIntegral}(dx)}{a^2}+\frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^2} \\
& -\frac{2b\,\text{CosIntegral}(dx)\sin(c)}{a^3}+\frac{2b\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^3} \\
& -\frac{\sin(c+dx)}{a^2x}-\frac{b\sin(c+dx)}{a^2(a+bx)}-\frac{2b\cos(c)\text{Si}(dx)}{a^3}-\frac{d\sin(c)\text{Si}(dx)}{a^2} \\
& +\frac{2b\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^3}-\frac{d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^2}
\end{aligned}
$$

```
[Out] d*Ci(d*x)*cos(c)/a^2+d*Ci(a*d/b+d*x)*cos(-c+a*d/b)/a^2-2*b*cos(c)*Si(d*x)/a
^3+2*b*cos(-c+a*d/b)*Si(a*d/b+d*x)/a^3-2*b*Ci(d*x)*sin(c)/a^3-d*Si(d*x)*sin
(c)/a^2-2*b*Ci(a*d/b+d*x)*sin(-c+a*d/b)/a^3+d*Si(a*d/b+d*x)*sin(-c+a*d/b)/a
^2-sin(d*x+c)/a^2/x-b*sin(d*x+c)/a^2/(b*x+a)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used

= {6874, 3378, 3384, 3380, 3383}

$$\int \frac{\sin(c+dx)}{x^2(a+bx)^2}\,dx = -\frac{2b\sin(c)\,\text{CosIntegral}(dx)}{a^3} + \frac{2b\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{a^3} - \frac{2b\cos(c)\text{Si}(dx)}{a^3} + \frac{2b\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^3} + \frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{a^2} - \frac{d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^2} - \frac{b\sin(c+dx)}{a^2(a+bx)} + \frac{d\cos(c)\,\text{CosIntegral}(dx)}{a^2} - \frac{d\sin(c)\text{Si}(dx)}{a^2} - \frac{\sin(c+dx)}{a^2x}$$

```
[In] Int[Sin[c + d*x]/(x^2*(a + b*x)^2),x]
```

```
[Out] (d*Cos[c]*CosIntegral[d*x])/a^2 + (d*Cos[c - (a*d)/b]*CosIntegral[(a*d)/b +
 d*x])/a^2 - (2*b*CosIntegral[d*x]*Sin[c])/a^3 + (2*b*CosIntegral[(a*d)/b +
 d*x]*Sin[c - (a*d)/b])/a^3 - Sin[c + d*x]/(a^2*x) - (b*Sin[c + d*x])/(a^2*
(a + b*x)) - (2*b*Cos[c]*SinIntegral[d*x])/a^3 - (d*Sin[c]*SinIntegral[d*x]
)/a^2 + (2*b*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/a^3 - (d*Sin[c -
(a*d)/b]*SinIntegral[(a*d)/b + d*x])/a^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\sin(c+dx)}{a^2x^2} - \frac{2b\sin(c+dx)}{a^3x} + \frac{b^2\sin(c+dx)}{a^2(a+bx)^2} + \frac{2b^2\sin(c+dx)}{a^3(a+bx)} \right) dx \\
&= \frac{\int \frac{\sin(c+dx)}{x^2}\,dx}{a^2} - \frac{(2b)\int \frac{\sin(c+dx)}{x}\,dx}{a^3} + \frac{(2b^2)\int \frac{\sin(c+dx)}{a+bx}\,dx}{a^3} + \frac{b^2\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx}{a^2} \\
&= -\frac{\sin(c+dx)}{a^2x} - \frac{b\sin(c+dx)}{a^2(a+bx)} + \frac{d\int \frac{\cos(c+dx)}{x}\,dx}{a^2} + \frac{(bd)\int \frac{\cos(c+dx)}{a+bx}\,dx}{a^2} \\
&\quad - \frac{(2b\cos(c))\int \frac{\sin(dx)}{x}\,dx}{a^3} + \frac{\left(2b^2\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^3} \\
&\quad - \frac{(2b\sin(c))\int \frac{\cos(dx)}{x}\,dx}{a^3} + \frac{\left(2b^2\sin\left(c-\frac{ad}{b}\right)\right)\int \frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^3} \\
&= -\frac{2b\,\text{CosIntegral}(dx)\sin(c)}{a^3} + \frac{2b\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^3} \\
&\quad - \frac{\sin(c+dx)}{a^2x} - \frac{b\sin(c+dx)}{a^2(a+bx)} - \frac{2b\cos(c)\text{Si}(dx)}{a^3} + \frac{2b\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^3} \\
&\quad + \frac{(d\cos(c))\int \frac{\cos(dx)}{x}\,dx}{a^2} + \frac{\left(bd\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^2} \\
&\quad - \frac{(d\sin(c))\int \frac{\sin(dx)}{x}\,dx}{a^2} - \frac{\left(bd\sin\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^2} \\
&= \frac{d\cos(c)\,\text{CosIntegral}(dx)}{a^2} + \frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^2} \\
&\quad - \frac{2b\,\text{CosIntegral}(dx)\sin(c)}{a^3} + \frac{2b\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^3} \\
&\quad - \frac{\sin(c+dx)}{a^2x} - \frac{b\sin(c+dx)}{a^2(a+bx)} - \frac{2b\cos(c)\text{Si}(dx)}{a^3} - \frac{d\sin(c)\text{Si}(dx)}{a^2} \\
&\quad + \frac{2b\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^3} - \frac{d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.98

$$\int \frac{\sin(c+dx)}{x^2(a+bx)^2}\,dx =$$
$$-\frac{-ad\cos(c)\,\text{CosIntegral}(dx)-ad\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)+\frac{a(a+2bx)\cos(dx)\sin(c)}{x(a+bx)}+2b\,\text{CosInte}}{}$$

```
[In] Integrate[Sin[c + d*x]/(x^2*(a + b*x)^2),x]
```

```
[Out] -((-(a*d*Cos[c]*CosIntegral[d*x]) - a*d*Cos[c - (a*d)/b]*CosIntegral[d*(a/b
 + x)] + (a*(a + 2*b*x)*Cos[d*x]*Sin[c])/(x*(a + b*x)) + 2*b*CosIntegral[d*
x]*Sin[c] - 2*b*CosIntegral[d*(a/b + x)]*Sin[c - (a*d)/b] + (a*(a + 2*b*x)*
Cos[c]*Sin[d*x])/(x*(a + b*x)) + 2*b*Cos[c]*SinIntegral[d*x] + a*d*Sin[c]*S
inIntegral[d*x] - 2*b*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)] + a*d*Sin[c
 - (a*d)/b]*SinIntegral[d*(a/b + x)])/a^3)
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.36

method	result
derivativedivides	$d\left(\frac{-\frac{\sin(dx+c)}{dx}-\text{Si}(dx)\sin(c)+\text{Ci}(dx)\cos(c)}{a^2}+\frac{b^2\left(-\frac{\sin(dx+c)}{(da-cb+b(dx+c))b}+\frac{\text{Si}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}+\frac{\text{Ci}\left(dx+c+\frac{da-}{b}\right.}{b}\right.}{a^2}\right.$
default	$d\left(\frac{-\frac{\sin(dx+c)}{dx}-\text{Si}(dx)\sin(c)+\text{Ci}(dx)\cos(c)}{a^2}+\frac{b^2\left(-\frac{\sin(dx+c)}{(da-cb+b(dx+c))b}+\frac{\text{Si}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}+\frac{\text{Ci}\left(dx+c+\frac{da-}{b}\right.}{b}\right.}{a^2}\right.$
risch	$-\frac{ib\,e^{ic}\,\text{Ei}_1(-idx)}{a^3}+\frac{ib\,e^{-\frac{i(da-cb)}{b}}\,\text{Ei}_1\left(-idx-ic-\frac{iad-icb}{b}\right)}{a^3}-\frac{d\,e^{ic}\,\text{Ei}_1(-idx)}{2a^2}-\frac{d\,e^{-\frac{i(da-cb)}{b}}\,\text{Ei}_1\left(-idx-ic-\frac{iad}{}\right.}{2a^2}$

```
[In] int(sin(d*x+c)/x^2/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d*(1/a^2*(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c))+b^2/a^2*(-sin(d*x+
c)/(d*a-c*b+b*(d*x+c))/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b+Ci(d*x+c
+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)-2/d/a^3*b*(Si(d*x)*cos(c)+Ci(d*x)*sin(
c))+2/d*b^2/a^3*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c
)/b)*sin((a*d-b*c)/b)/b))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.39

$$\int \frac{\sin(c+dx)}{x^2(a+bx)^2}\,dx$$
$$= \frac{\left(\left(abdx^2+a^2dx\right)\operatorname{Ci}\left(dx\right)-2\left(b^2x^2+abx\right)\operatorname{Si}\left(dx\right)\right)\cos\left(c\right)+\left(\left(abdx^2+a^2dx\right)\operatorname{Ci}\left(\frac{bdx+ad}{b}\right)+2\left(b^2x^2+abx\right)\operatorname{Si}}{}$$

```
[In] integrate(sin(d*x+c)/x^2/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (((a*b*d*x^2 + a^2*d*x)*cos_integral(d*x) - 2*(b^2*x^2 + a*b*x)*sin_integra
l(d*x))*cos(c) + ((a*b*d*x^2 + a^2*d*x)*cos_integral((b*d*x + a*d)/b) + 2*(
b^2*x^2 + a*b*x)*sin_integral((b*d*x + a*d)/b))*cos(-(b*c - a*d)/b) - (2*a*
b*x + a^2)*sin(d*x + c) - (2*(b^2*x^2 + a*b*x)*cos_integral(d*x) + (a*b*d*x
^2 + a^2*d*x)*sin_integral(d*x))*sin(c) - (2*(b^2*x^2 + a*b*x)*cos_integral
((b*d*x + a*d)/b) - (a*b*d*x^2 + a^2*d*x)*sin_integral((b*d*x + a*d)/b))*si
n(-(b*c - a*d)/b))/(a^3*b*x^2 + a^4*x)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{x^2(a+bx)^2}\,dx = \int \frac{\sin\left(c+dx\right)}{x^2\left(a+bx\right)^2}\,dx$$

```
[In] integrate(sin(d*x+c)/x**2/(b*x+a)**2,x)
```

```
[Out] Integral(sin(c + d*x)/(x**2*(a + b*x)**2), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x^2(a+bx)^2}\,dx = \int \frac{\sin\left(dx+c\right)}{\left(bx+a\right)^2x^2}\,dx$$

```
[In] integrate(sin(d*x+c)/x^2/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x + a)^2*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3180 vs. 2(191) = 382.

Time = 0.38 (sec) , antiderivative size = 3180, normalized size of antiderivative = 16.91

$$\int \frac{\sin(c+dx)}{x^2(a+bx)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)/x^2/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^2*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*d^2*cos(c)*cos_integra
l((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)/b - 2*(b*x + a)*a*(b
*c/(b*x + a) - a*d/(b*x + a) + d)*c*d^2*cos(c)*cos_integral((b*x + a)*(b*c/
(b*x + a) - a*d/(b*x + a) + d)/b - c) + a*b*c^2*d^2*cos(c)*cos_integral((b*
x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c) + (b*x + a)*a^2*(b*c/(b*x
 + a) - a*d/(b*x + a) + d)*d^3*cos(c)*cos_integral((b*x + a)*(b*c/(b*x + a)
 - a*d/(b*x + a) + d)/b - c)/b - a^2*c*d^3*cos(c)*cos_integral((b*x + a)*(b
*c/(b*x + a) - a*d/(b*x + a) + d)/b - c) + (b*x + a)^2*a*(b*c/(b*x + a) - a
*d/(b*x + a) + d)^2*d^2*cos(-(b*c - a*d)/b)*cos_integral(((b*x + a)*(b*c/(b
*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)/b - 2*(b*x + a)*a*(b*c/(b*x +
a) - a*d/(b*x + a) + d)*c*d^2*cos(-(b*c - a*d)/b)*cos_integral(((b*x + a)*(
b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + a*b*c^2*d^2*cos(-(b*c
- a*d)/b)*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c
 + a*d)/b) + (b*x + a)*a^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^3*cos(-(b*
c - a*d)/b)*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b
*c + a*d)/b)/b - a^2*c*d^3*cos(-(b*c - a*d)/b)*cos_integral(((b*x + a)*(b*c
/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + (b*x + a)^2*a*(b*c/(b*x +
 a) - a*d/(b*x + a) + d)^2*d^2*sin(c)*sin_integral(-(b*x + a)*(b*c/(b*x + a
) - a*d/(b*x + a) + d)/b + c)/b - 2*(b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x +
 a) + d)*c*d^2*sin(c)*sin_integral(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a
) + d)/b + c) + a*b*c^2*d^2*sin(c)*sin_integral(-(b*x + a)*(b*c/(b*x + a) -
 a*d/(b*x + a) + d)/b + c) + (b*x + a)*a^2*(b*c/(b*x + a) - a*d/(b*x + a) +
 d)*d^3*sin(c)*sin_integral(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/
b + c)/b - a^2*c*d^3*sin(c)*sin_integral(-(b*x + a)*(b*c/(b*x + a) - a*d/(b
*x + a) + d)/b + c) + (b*x + a)^2*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*d
^2*sin(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x +
a) + d) - b*c + a*d)/b)/b - 2*(b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a) +
d)*c*d^2*sin(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(
b*x + a) + d) - b*c + a*d)/b) + a*b*c^2*d^2*sin(-(b*c - a*d)/b)*sin_integra
l(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + (b*x + a
)*a^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^3*sin(-(b*c - a*d)/b)*sin_integ
ral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)/b - a^2*
c*d^3*sin(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x
 + a) + d) - b*c + a*d)/b) - 2*(b*x + a)^2*(b*c/(b*x + a) - a*d/(b*x + a) +
 d)^2*d*cos_integral((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*s
```

```
in(c) + 4*(b*x + a)*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*c*d*cos_integral(
(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*sin(c) - 2*b^2*c^2*d*c
os_integral((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*sin(c) - 2
*(b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*cos_integral((b*x + a)
*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*sin(c) + 2*a*b*c*d^2*cos_integr
al((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*sin(c) - 2*(b*x + a
)^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*d*cos_integral(((b*x + a)*(b*c/(b
*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*sin(-(b*c - a*d)/b) + 4*(b*x +
 a)*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*c*d*cos_integral(((b*x + a)*(b*c/
(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*sin(-(b*c - a*d)/b) - 2*b^2*
c^2*d*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a
*d)/b)*sin(-(b*c - a*d)/b) - 2*(b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a) +
 d)*d^2*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c +
 a*d)/b)*sin(-(b*c - a*d)/b) + 2*a*b*c*d^2*cos_integral(((b*x + a)*(b*c/(b*
x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*sin(-(b*c - a*d)/b) + 2*(b*x +
a)^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*d*cos(c)*sin_integral(-(b*x + a)
*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b + c) - 4*(b*x + a)*b*(b*c/(b*x + a)
- a*d/(b*x + a) + d)*c*d*cos(c)*sin_integral(-(b*x + a)*(b*c/(b*x + a) - a*
d/(b*x + a) + d)/b + c) + 2*b^2*c^2*d*cos(c)*sin_integral(-(b*x + a)*(b*c/(
b*x + a) - a*d/(b*x + a) + d)/b + c) + 2*(b*x + a)*a*(b*c/(b*x + a) - a*d/(
b*x + a) + d)*d^2*cos(c)*sin_integral(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x
+ a) + d)/b + c) - 2*a*b*c*d^2*cos(c)*sin_integral(-(b*x + a)*(b*c/(b*x + a
) - a*d/(b*x + a) + d)/b + c) + 2*(b*x + a)^2*(b*c/(b*x + a) - a*d/(b*x + a
) + d)^2*d*cos(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d
/(b*x + a) + d) - b*c + a*d)/b) - 4*(b*x + a)*b*(b*c/(b*x + a) - a*d/(b*x +
 a) + d)*c*d*cos(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a
*d/(b*x + a) + d) - b*c + a*d)/b) + 2*b^2*c^2*d*cos(-(b*c - a*d)/b)*sin_int
egral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + 2*(b
*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*cos(-(b*c - a*d)/b)*sin_i
ntegral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) - 2*
a*b*c*d^2*cos(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/
(b*x + a) + d) - b*c + a*d)/b) + 2*(b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x +
a) + d)*d^2*sin(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 2*a*b*c
*d^2*sin(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + a^2*d^3*sin(-(
b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b))*b^2/(((b*x + a)^2*a^3*b*(b
*c/(b*x + a) - a*d/(b*x + a) + d)^2 - 2*(b*x + a)*a^3*b^2*(b*c/(b*x + a) -
a*d/(b*x + a) + d)*c + a^3*b^3*c^2 + (b*x + a)*a^4*b*(b*c/(b*x + a) - a*d/(
b*x + a) + d)*d - a^4*b^2*c*d)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x^2(a+bx)^2}\,dx=\int \frac{\sin\left(c+d\,x\right)}{x^2\,{\left(a+b\,x\right)}^2}\,dx$$

```
[In] int(sin(c + d*x)/(x^2*(a + b*x)^2),x)
[Out] int(sin(c + d*x)/(x^2*(a + b*x)^2), x)
```

3.33 $\int \frac{x^3\sin(c+dx)}{(a+bx)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 265

$$
\begin{aligned}
\int \frac{x^3\sin(c+dx)}{(a+bx)^3}\,dx = & -\frac{\cos(c+dx)}{b^3d}+\frac{a^3d\cos(c+dx)}{2b^5(a+bx)} \\
& +\frac{3a^2d\cos\left(c-\frac{ad}{b}\right)\operatorname{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^5} \\
& -\frac{3a\operatorname{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^4} \\
& +\frac{a^3d^2\operatorname{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{2b^6}+\frac{a^3\sin(c+dx)}{2b^4(a+bx)^2} \\
& -\frac{3a^2\sin(c+dx)}{b^4(a+bx)}-\frac{3a\cos\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(\frac{ad}{b}+dx\right)}{b^4} \\
& +\frac{a^3d^2\cos\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(\frac{ad}{b}+dx\right)}{2b^6}-\frac{3a^2d\sin\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(\frac{ad}{b}+dx\right)}{b^5}
\end{aligned}
$$

```
[Out] 3*a^2*d*Ci(a*d/b+d*x)*cos(-c+a*d/b)/b^5-cos(d*x+c)/b^3/d+1/2*a^3*d*cos(d*x+
c)/b^5/(b*x+a)-3*a*cos(-c+a*d/b)*Si(a*d/b+d*x)/b^4+1/2*a^3*d^2*cos(-c+a*d/b
)*Si(a*d/b+d*x)/b^6+3*a*Ci(a*d/b+d*x)*sin(-c+a*d/b)/b^4-1/2*a^3*d^2*Ci(a*d/
b+d*x)*sin(-c+a*d/b)/b^6+3*a^2*d*Si(a*d/b+d*x)*sin(-c+a*d/b)/b^5+1/2*a^3*si
n(d*x+c)/b^4/(b*x+a)^2-3*a^2*sin(d*x+c)/b^4/(b*x+a)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {6874, 2718, 3378, 3384, 3380, 3383}

$$\int \frac{x^3 \sin(c+dx)}{(a+bx)^3} \, dx = \frac{a^3 d^2 \sin\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{2b^6} + \frac{a^3 d^2 \cos\left(c-\frac{ad}{b}\right) \text{Si}\left(xd+\frac{ad}{b}\right)}{2b^6} + \frac{a^3 d \cos(c+dx)}{2b^5(a+bx)} + \frac{a^3 \sin(c+dx)}{2b^4(a+bx)^2} + \frac{3a^2 d \cos\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b^5} - \frac{3a^2 d \sin\left(c-\frac{ad}{b}\right) \text{Si}\left(xd+\frac{ad}{b}\right)}{b^5} - \frac{3a^2 \sin(c+dx)}{b^4(a+bx)} - \frac{3a \sin\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b^4} - \frac{3a \cos\left(c-\frac{ad}{b}\right) \text{Si}\left(xd+\frac{ad}{b}\right)}{b^4} - \frac{\cos(c+dx)}{b^3 d}$$

```
[In] Int[(x^3*Sin[c + d*x])/(a + b*x)^3,x]
```

```
[Out] -(Cos[c + d*x]/(b^3*d)) + (a^3*d*Cos[c + d*x])/(2*b^5*(a + b*x)) + (3*a^2*d
*Cos[c - (a*d)/b]*CosIntegral[(a*d)/b + d*x])/b^5 - (3*a*CosIntegral[(a*d)/
b + d*x]*Sin[c - (a*d)/b])/b^4 + (a^3*d^2*CosIntegral[(a*d)/b + d*x]*Sin[c
- (a*d)/b])/(2*b^6) + (a^3*Sin[c + d*x])/(2*b^4*(a + b*x)^2) - (3*a^2*Sin[c
 + d*x])/(b^4*(a + b*x)) - (3*a*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x]
)/b^4 + (a^3*d^2*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/(2*b^6) - (3*
a^2*d*Sin[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^5
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\sin(c+dx)}{b^3} - \frac{a^3\sin(c+dx)}{b^3(a+bx)^3} + \frac{3a^2\sin(c+dx)}{b^3(a+bx)^2} - \frac{3a\sin(c+dx)}{b^3(a+bx)}\right) dx$$

$$= \frac{\int \sin(c+dx)\,dx}{b^3} - \frac{(3a)\int \frac{\sin(c+dx)}{a+bx}\,dx}{b^3} + \frac{(3a^2)\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx}{b^3} - \frac{a^3\int \frac{\sin(c+dx)}{(a+bx)^3}\,dx}{b^3}$$

$$= -\frac{\cos(c+dx)}{b^3d} + \frac{a^3\sin(c+dx)}{2b^4(a+bx)^2} - \frac{3a^2\sin(c+dx)}{b^4(a+bx)} + \frac{(3a^2d)\int \frac{\cos(c+dx)}{a+bx}\,dx}{b^4} - \frac{(a^3d)\int \frac{\cos(c+dx)}{(a+bx)^2}\,dx}{2b^4} - \frac{\left(3a\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^3} - \frac{\left(3a\sin\left(c-\frac{ad}{b}\right)\right)\int \frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^3}$$

$$= -\frac{\cos(c+dx)}{b^3d} + \frac{a^3d\cos(c+dx)}{2b^5(a+bx)} - \frac{3a\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^4} + \frac{a^3\sin(c+dx)}{2b^4(a+bx)^2} - \frac{3a^2\sin(c+dx)}{b^4(a+bx)} - \frac{3a\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^4} + \frac{(a^3d^2)\int \frac{\sin(c+dx)}{a+bx}\,dx}{2b^5} + \frac{\left(3a^2d\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^4} - \frac{\left(3a^2d\sin\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^4}$$

$$= -\frac{\cos(c+dx)}{b^3d} + \frac{a^3d\cos(c+dx)}{2b^5(a+bx)} + \frac{3a^2d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^5} - \frac{3a\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^4} + \frac{a^3\sin(c+dx)}{2b^4(a+bx)^2} - \frac{3a^2\sin(c+dx)}{b^4(a+bx)} - \frac{3a\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^4} - \frac{3a^2d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^5} + \frac{\left(a^3d^2\cos\left(c-\frac{ad}{b}\right)\right)\int\frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}dx}{2b^5} + \frac{\left(a^3d^2\sin\left(c-\frac{ad}{b}\right)\right)\int\frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}dx}{2b^5}$$

$$= -\frac{\cos(c+dx)}{b^3d} + \frac{a^3d\cos(c+dx)}{2b^5(a+bx)} + \frac{3a^2d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^5} - \frac{3a\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^4} + \frac{a^3d^2\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{2b^6} + \frac{a^3\sin(c+dx)}{2b^4(a+bx)^2} - \frac{3a^2\sin(c+dx)}{b^4(a+bx)} - \frac{3a\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^4} + \frac{a^3d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{2b^6} - \frac{3a^2d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^5}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.89

$$\int \frac{x^3\sin(c+dx)}{(a+bx)^3}\,dx = -\frac{b\cos(dx)\left(-((a+bx)\left(-2ab^2+a^3d^2-2b^3x\right)\cos(c))+a^2bd(5a+6bx)\sin(c)\right)+b\left(a^2bd(5a+6bx)\cos(c\right.}{}$$

```
[In] Integrate[(x^3*Sin[c + d*x])/(a + b*x)^3,x]
```

```
[Out] -1/2*(b*Cos[d*x]*(-((a + b*x)*(-2*a*b^2 + a^3*d^2 - 2*b^3*x)*Cos[c]) + a^2*
b*d*(5*a + 6*b*x)*Sin[c]) + b*(a^2*b*d*(5*a + 6*b*x)*Cos[c] + (a + b*x)*(-2
*a*b^2 + a^3*d^2 - 2*b^3*x)*Sin[c])*Sin[d*x] - a*d*(a + b*x)^2*(CosIntegral
[d*(a/b + x)]*(6*a*b*d*Cos[c - (a*d)/b] + (-6*b^2 + a^2*d^2)*Sin[c - (a*d)/
b]) + ((-6*b^2 + a^2*d^2)*Cos[c - (a*d)/b] - 6*a*b*d*Sin[c - (a*d)/b])*SinI
ntegral[d*(a/b + x)]))/(b^6*d*(a + b*x)^2)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 705, normalized size of antiderivative = 2.66

method	result
risch	$\frac{i(2ia^3b^3d^6x^3+6ia^4b^2d^6x^2-4ib^6d^4x^4+6ia^5bd^6x-16iab^5d^4x^3+2ia^6d^6-24ia^2b^4d^4x^2-16ia^3b^3d^4x-4ia^4b^2d^4)\cos(dx+c)}{4b^5d^3(bx+a)^2(-d^2x^2b^2-2abd^2x-d^2a^2)}$
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int(x^3*sin(d*x+c)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I/b^5/d^3*(2*I*a^3*b^3*d^6*x^3+6*I*a^4*b^2*d^6*x^2+6*I*a^5*b*d^6*x-16*I
*a*b^5*d^4*x^3-4*I*b^6*d^4*x^4-4*I*a^4*b^2*d^4-24*I*a^2*b^4*d^4*x^2-16*I*a^
3*b^3*d^4*x+2*I*a^6*d^6)/(b*x+a)^2/(-b^2*d^2*x^2-2*a*b*d^2*x-a^2*d^2)*cos(d
*x+c)+1/4/b^5/d^3*(12*a^2*b^4*d^5*x^3+34*a^3*b^3*d^5*x^2+32*a^4*b^2*d^5*x+1
0*a^5*b*d^5)/(b*x+a)^2/(-b^2*d^2*x^2-2*a*b*d^2*x-a^2*d^2)*sin(d*x+c)-3/2*d/
b^5*cos((a*d-b*c)/b)*Ei(1,-I*d*(b*x+a)/b)*a^2+1/4*I*d^2/b^6*cos((a*d-b*c)/b
)*Ei(1,-I*d*(b*x+a)/b)*a^3-3/2*d/b^5*cos((a*d-b*c)/b)*Ei(1,I*d*(b*x+a)/b)*a
^2-1/4*I*d^2/b^6*cos((a*d-b*c)/b)*Ei(1,I*d*(b*x+a)/b)*a^3-3/2*I/b^4*cos((a*
d-b*c)/b)*Ei(1,-I*d*(b*x+a)/b)*a+3/2*I/b^4*cos((a*d-b*c)/b)*Ei(1,I*d*(b*x+a
)/b)*a+3/2*I*d/b^5*sin((a*d-b*c)/b)*Ei(1,-I*d*(b*x+a)/b)*a^2+1/4*d^2/b^6*si
n((a*d-b*c)/b)*Ei(1,-I*d*(b*x+a)/b)*a^3-3/2*I*d/b^5*sin((a*d-b*c)/b)*Ei(1,I
*d*(b*x+a)/b)*a^2+1/4*d^2/b^6*sin((a*d-b*c)/b)*Ei(1,I*d*(b*x+a)/b)*a^3-3/2/
b^4*sin((a*d-b*c)/b)*Ei(1,-I*d*(b*x+a)/b)*a-3/2/b^4*sin((a*d-b*c)/b)*Ei(1,I
*d*(b*x+a)/b)*a
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.46

$$\int \frac{x^3\sin(c+dx)}{(a+bx)^3}\,dx$$
$$= \frac{(a^4bd^2-2\,b^5x^2-2\,a^2b^3+(a^3b^2d^2-4\,ab^4)x)\cos(dx+c)+\left(6\,(a^2b^3d^2x^2+2\,a^3b^2d^2x+a^4bd^2)\,\mathrm{Ci}\left(\frac{bdx+ad}{b}\right)-\right.}{}$$

```
[In] integrate(x^3*sin(d*x+c)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*((a^4*b*d^2 - 2*b^5*x^2 - 2*a^2*b^3 + (a^3*b^2*d^2 - 4*a*b^4)*x)*cos(d*
x + c) + (6*(a^2*b^3*d^2*x^2 + 2*a^3*b^2*d^2*x + a^4*b*d^2)*cos_integral((b
*d*x + a*d)/b) + (a^5*d^3 - 6*a^3*b^2*d + (a^3*b^2*d^3 - 6*a*b^4*d)*x^2 + 2
*(a^4*b*d^3 - 6*a^2*b^3*d)*x)*sin_integral((b*d*x + a*d)/b))*cos(-(b*c - a*
```

```
d)/b) - (6*a^2*b^3*d*x + 5*a^3*b^2*d)*sin(d*x + c) - ((a^5*d^3 - 6*a^3*b^2*
d + (a^3*b^2*d^3 - 6*a*b^4*d)*x^2 + 2*(a^4*b*d^3 - 6*a^2*b^3*d)*x)*cos_inte
gral((b*d*x + a*d)/b) - 6*(a^2*b^3*d^2*x^2 + 2*a^3*b^2*d^2*x + a^4*b*d^2)*s
in_integral((b*d*x + a*d)/b))*sin(-(b*c - a*d)/b))/(b^8*d*x^2 + 2*a*b^7*d*x
 + a^2*b^6*d)
```

Sympy [F]

$$\int \frac{x^3\sin(c+dx)}{(a+bx)^3}\,dx=\int \frac{x^3\sin\left(c+dx\right)}{\left(a+bx\right)^3}\,dx$$

```
[In] integrate(x**3*sin(d*x+c)/(b*x+a)**3,x)
```

```
[Out] Integral(x**3*sin(c + d*x)/(a + b*x)**3, x)
```

Maxima [F]

$$\int \frac{x^3\sin(c+dx)}{(a+bx)^3}\,dx=\int \frac{x^3\sin\left(dx+c\right)}{\left(bx+a\right)^3}\,dx$$

```
[In] integrate(x^3*sin(d*x+c)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*((b^2*cos(c)^2 + b^2*sin(c)^2)*d*x^3*cos(d*x + c) + 3*((a^2*(-I*exp_in
tegral_e(4, (I*b*d*x + I*a*d)/b) + I*exp_integral_e(4, -(I*b*d*x + I*a*d)/b
))*cos(c)^2 + a^2*(-I*exp_integral_e(4, (I*b*d*x + I*a*d)/b) + I*exp_integr
al_e(4, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*cos(-(b*c - a*d)/b) + (a^2*(exp_in
tegral_e(4, (I*b*d*x + I*a*d)/b) + exp_integral_e(4, -(I*b*d*x + I*a*d)/b))
*cos(c)^2 + a^2*(exp_integral_e(4, (I*b*d*x + I*a*d)/b) + exp_integral_e(4,
 -(I*b*d*x + I*a*d)/b))*sin(c)^2)*sin(-(b*c - a*d)/b))*cos(d*x + c)^2 - 3*(
a*b*cos(c)^2 + a*b*sin(c)^2)*x*sin(d*x + c) + 3*((a^2*(-I*exp_integral_e(4,
 (I*b*d*x + I*a*d)/b) + I*exp_integral_e(4, -(I*b*d*x + I*a*d)/b))*cos(c)^2
 + a^2*(-I*exp_integral_e(4, (I*b*d*x + I*a*d)/b) + I*exp_integral_e(4, -(I
*b*d*x + I*a*d)/b))*sin(c)^2)*cos(-(b*c - a*d)/b) + (a^2*(exp_integral_e(4,
 (I*b*d*x + I*a*d)/b) + exp_integral_e(4, -(I*b*d*x + I*a*d)/b))*cos(c)^2 +
 a^2*(exp_integral_e(4, (I*b*d*x + I*a*d)/b) + exp_integral_e(4, -(I*b*d*x
+ I*a*d)/b))*sin(c)^2)*sin(-(b*c - a*d)/b))*sin(d*x + c)^2 + ((b^2*d*x^3*co
s(c) + 3*a*b*x*sin(c))*cos(d*x + c)^2 + (b^2*d*x^3*cos(c) + 3*a*b*x*sin(c))
*sin(d*x + c)^2)*cos(d*x + 2*c) + 6*(((a^2*b^5*cos(c)^2 + a^2*b^5*sin(c)^2)
*d^3*x^3 + 3*(a^3*b^4*cos(c)^2 + a^3*b^4*sin(c)^2)*d^3*x^2 + 3*(a^4*b^3*cos
(c)^2 + a^4*b^3*sin(c)^2)*d^3*x + (a^5*b^2*cos(c)^2 + a^5*b^2*sin(c)^2)*d^3
)*cos(d*x + c)^2 + ((a^2*b^5*cos(c)^2 + a^2*b^5*sin(c)^2)*d^3*x^3 + 3*(a^3*
b^4*cos(c)^2 + a^3*b^4*sin(c)^2)*d^3*x^2 + 3*(a^4*b^3*cos(c)^2 + a^4*b^3*si
```

```
n(c)^2)*d^3*x + (a^5*b^2*cos(c)^2 + a^5*b^2*sin(c)^2)*d^3)*sin(d*x + c)^2)*
integrate(1/2*x*cos(d*x + c)/(b^5*d^2*x^4 + 4*a*b^4*d^2*x^3 + 6*a^2*b^3*d^2
*x^2 + 4*a^3*b^2*d^2*x + a^4*b*d^2), x) + 6*(((a^2*b^5*cos(c)^2 + a^2*b^5*s
in(c)^2)*d^3*x^3 + 3*(a^3*b^4*cos(c)^2 + a^3*b^4*sin(c)^2)*d^3*x^2 + 3*(a^4
*b^3*cos(c)^2 + a^4*b^3*sin(c)^2)*d^3*x + (a^5*b^2*cos(c)^2 + a^5*b^2*sin(c
)^2)*d^3)*cos(d*x + c)^2 + ((a^2*b^5*cos(c)^2 + a^2*b^5*sin(c)^2)*d^3*x^3 +
 3*(a^3*b^4*cos(c)^2 + a^3*b^4*sin(c)^2)*d^3*x^2 + 3*(a^4*b^3*cos(c)^2 + a^
4*b^3*sin(c)^2)*d^3*x + (a^5*b^2*cos(c)^2 + a^5*b^2*sin(c)^2)*d^3)*sin(d*x
+ c)^2)*integrate(1/2*x*cos(d*x + c)/((b^5*d^2*x^4 + 4*a*b^4*d^2*x^3 + 6*a^
2*b^3*d^2*x^2 + 4*a^3*b^2*d^2*x + a^4*b*d^2)*cos(d*x + c)^2 + (b^5*d^2*x^4
+ 4*a*b^4*d^2*x^3 + 6*a^2*b^3*d^2*x^2 + 4*a^3*b^2*d^2*x + a^4*b*d^2)*sin(d*
x + c)^2), x) - 12*(((a*b^6*cos(c)^2 + a*b^6*sin(c)^2)*d^2*x^3 + 3*(a^2*b^5
*cos(c)^2 + a^2*b^5*sin(c)^2)*d^2*x^2 + 3*(a^3*b^4*cos(c)^2 + a^3*b^4*sin(c
)^2)*d^2*x + (a^4*b^3*cos(c)^2 + a^4*b^3*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((
a*b^6*cos(c)^2 + a*b^6*sin(c)^2)*d^2*x^3 + 3*(a^2*b^5*cos(c)^2 + a^2*b^5*si
n(c)^2)*d^2*x^2 + 3*(a^3*b^4*cos(c)^2 + a^3*b^4*sin(c)^2)*d^2*x + (a^4*b^3*
cos(c)^2 + a^4*b^3*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(1/2*x*sin(d*x +
 c)/(b^5*d^2*x^4 + 4*a*b^4*d^2*x^3 + 6*a^2*b^3*d^2*x^2 + 4*a^3*b^2*d^2*x +
a^4*b*d^2), x) - 12*(((a*b^6*cos(c)^2 + a*b^6*sin(c)^2)*d^2*x^3 + 3*(a^2*b^
5*cos(c)^2 + a^2*b^5*sin(c)^2)*d^2*x^2 + 3*(a^3*b^4*cos(c)^2 + a^3*b^4*sin(
c)^2)*d^2*x + (a^4*b^3*cos(c)^2 + a^4*b^3*sin(c)^2)*d^2)*cos(d*x + c)^2 + (
(a*b^6*cos(c)^2 + a*b^6*sin(c)^2)*d^2*x^3 + 3*(a^2*b^5*cos(c)^2 + a^2*b^5*s
in(c)^2)*d^2*x^2 + 3*(a^3*b^4*cos(c)^2 + a^3*b^4*sin(c)^2)*d^2*x + (a^4*b^3
*cos(c)^2 + a^4*b^3*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(1/2*x*sin(d*x
+ c)/((b^5*d^2*x^4 + 4*a*b^4*d^2*x^3 + 6*a^2*b^3*d^2*x^2 + 4*a^3*b^2*d^2*x
+ a^4*b*d^2)*cos(d*x + c)^2 + (b^5*d^2*x^4 + 4*a*b^4*d^2*x^3 + 6*a^2*b^3*d^
2*x^2 + 4*a^3*b^2*d^2*x + a^4*b*d^2)*sin(d*x + c)^2), x) + ((b^2*d*x^3*sin(
c) - 3*a*b*x*cos(c))*cos(d*x + c)^2 + (b^2*d*x^3*sin(c) - 3*a*b*x*cos(c))*s
in(d*x + c)^2)*sin(d*x + 2*c))/(((b^5*cos(c)^2 + b^5*sin(c)^2)*d^2*x^3 + 3*
(a*b^4*cos(c)^2 + a*b^4*sin(c)^2)*d^2*x^2 + 3*(a^2*b^3*cos(c)^2 + a^2*b^3*s
in(c)^2)*d^2*x + (a^3*b^2*cos(c)^2 + a^3*b^2*sin(c)^2)*d^2)*cos(d*x + c)^2
+ ((b^5*cos(c)^2 + b^5*sin(c)^2)*d^2*x^3 + 3*(a*b^4*cos(c)^2 + a*b^4*sin(c)
^2)*d^2*x^2 + 3*(a^2*b^3*cos(c)^2 + a^2*b^3*sin(c)^2)*d^2*x + (a^3*b^2*cos(
c)^2 + a^3*b^2*sin(c)^2)*d^2)*sin(d*x + c)^2)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.64 (sec) , antiderivative size = 16724, normalized size of antiderivative = 63.11

$$\int \frac{x^3 \sin(c+dx)}{(a+bx)^3} \, dx = \text{Too large to display}$$

```
[In] integrate(x^3*sin(d*x+c)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*(a^3*b^2*d^3*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*ta
n(1/2*c)^2*tan(1/2*a*d/b)^2 - a^3*b^2*d^3*x^2*imag_part(cos_integral(-d*x -
 a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^3*b^2*d^3*x^2*s
in_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 +
 2*a^3*b^2*d^3*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(
1/2*c)^2*tan(1/2*a*d/b) + 2*a^3*b^2*d^3*x^2*real_part(cos_integral(-d*x - a
*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a^3*b^2*d^3*x^2*real_
part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2
- 2*a^3*b^2*d^3*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*ta
n(1/2*c)*tan(1/2*a*d/b)^2 + 2*a^4*b*d^3*x*imag_part(cos_integral(d*x + a*d/
b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^4*b*d^3*x*imag_part(
cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 +
6*a^2*b^3*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1
/2*c)^2*tan(1/2*a*d/b)^2 + 6*a^2*b^3*d^2*x^2*real_part(cos_integral(-d*x -
a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a^4*b*d^3*x*sin_in
tegral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^3*
b^2*d^3*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^
2 + a^3*b^2*d^3*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*ta
n(1/2*c)^2 - 2*a^3*b^2*d^3*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2
*tan(1/2*c)^2 + 4*a^3*b^2*d^3*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(
1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 4*a^3*b^2*d^3*x^2*imag_part(cos_inte
gral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^3*b^2*d^
3*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b
) - 12*a^2*b^3*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*
tan(1/2*c)^2*tan(1/2*a*d/b) + 12*a^2*b^3*d^2*x^2*imag_part(cos_integral(-d*
x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 4*a^4*b*d^3*x*real
_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)
 + 4*a^4*b*d^3*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1
/2*c)^2*tan(1/2*a*d/b) - 24*a^2*b^3*d^2*x^2*sin_integral((b*d*x + a*d)/b)*t
an(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - a^3*b^2*d^3*x^2*imag_part(cos_i
ntegral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a^3*b^2*d^3*x^2*ima
g_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*a^3*
b^2*d^3*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 +
 12*a^2*b^3*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan
(1/2*c)*tan(1/2*a*d/b)^2 - 12*a^2*b^3*d^2*x^2*imag_part(cos_integral(-d*x -
 a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a^4*b*d^3*x*real_pa
rt(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 -
4*a^4*b*d^3*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*
c)*tan(1/2*a*d/b)^2 + 24*a^2*b^3*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(
1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + a^3*b^2*d^3*x^2*imag_part(cos_inte
gral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^3*b^2*d^3*x^2*imag_par
t(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^3*b^2*d^3
*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^5*d^3*
imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*
d/b)^2 - 6*a*b^4*d*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*
```

```
tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^5*d^3*imag_part(cos_integral(-d*x - a*d/b
))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 6*a*b^4*d*x^2*imag_part(c
os_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 1
2*a^3*b^2*d^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2
*c)^2*tan(1/2*a*d/b)^2 + 12*a^3*b^2*d^2*x*real_part(cos_integral(-d*x - a*d
/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^5*d^3*sin_integral(
(b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 12*a*b^4*d*
x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b
)^2 + 2*a^3*b^2*d^3*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2
*tan(1/2*c) + 2*a^3*b^2*d^3*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1
/2*d*x)^2*tan(1/2*c) - 2*a^4*b*d^3*x*imag_part(cos_integral(d*x + a*d/b))*t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^4*b*d^3*x*imag_part(cos_integral(-d*x - a*
d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 6*a^2*b^3*d^2*x^2*real_part(cos_integra
l(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 6*a^2*b^3*d^2*x^2*real_part(c
os_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a^4*b*d^3*x*sin_
integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^3*b^2*d^3*x^2*r
eal_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 2*a^3*b
^2*d^3*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d
/b) + 8*a^4*b*d^3*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan
(1/2*c)*tan(1/2*a*d/b) - 8*a^4*b*d^3*x*imag_part(cos_integral(-d*x - a*d/b)
)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 24*a^2*b^3*d^2*x^2*real_part(c
os_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 24*a^2
*b^3*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c
)*tan(1/2*a*d/b) + 16*a^4*b*d^3*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x
)^2*tan(1/2*c)*tan(1/2*a*d/b) + 2*a^3*b^2*d^3*x^2*real_part(cos_integral(d*
x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^3*b^2*d^3*x^2*real_part(cos_i
ntegral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 24*a^3*b^2*d^2*x*imag_
part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)
+ 24*a^3*b^2*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan
(1/2*c)^2*tan(1/2*a*d/b) + 2*a^5*d^3*real_part(cos_integral(d*x + a*d/b))*t
an(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 12*a*b^4*d*x^2*real_part(cos_in
tegral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^5*d^3
*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*
a*d/b) - 12*a*b^4*d*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^
2*tan(1/2*c)^2*tan(1/2*a*d/b) - 48*a^3*b^2*d^2*x*sin_integral((b*d*x + a*d)
/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a^4*b*d^3*x*imag_part(co
s_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*a^4*b*d^3*x*im
ag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 6*a^2
*b^3*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*
d/b)^2 - 6*a^2*b^3*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*
x)^2*tan(1/2*a*d/b)^2 - 4*a^4*b*d^3*x*sin_integral((b*d*x + a*d)/b)*tan(1/2
*d*x)^2*tan(1/2*a*d/b)^2 - 2*a^3*b^2*d^3*x^2*real_part(cos_integral(d*x + a
*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^3*b^2*d^3*x^2*real_part(cos_integr
al(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 24*a^3*b^2*d^2*x*imag_part(
cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 24*
```

a^3*b^2*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^5*d^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 12*a*b^4*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^5*d^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 12*a*b^4*d*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 48*a^3*b^2*d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*a^4*b*d^3*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^4*b*d^3*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 6*a^2*b^3*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 6*a^2*b^3*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a^4*b*d^3*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^3*b^2*d^2*x*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 12*a^2*b^3*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 12*a^2*b^3*d*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 6*a^4*b*d^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 6*a^4*b*d^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 24*a^2*b^3*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^3*b^2*d^3*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2 - a^3*b^2*d^3*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 + 2*a^3*b^2*d^3*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2 - 12*a^2*b^3*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 12*a^2*b^3*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^4*b*d^3*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^4*b*d^3*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 24*a^2*b^3*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c) - a^3*b^2*d^3*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 + a^3*b^2*d^3*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 - 2*a^3*b^2*d^3*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2 - a^5*d^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 6*a*b^4*d*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + a^5*d^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 6*a*b^4*d*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 12*a^3*b^2*d^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 12*a^3*b^2*d^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^5*d^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 12*a*b^4*d*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 12*a^2*b^3*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 12*a^2*b^3*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 4*a^4*b*d^3*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 4*a^4*b*d^3*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 24*a^2*b^3*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d

*x)^2*tan(1/2*a*d/b) + 4*a^3*b^2*d^3*x^2*imag_part(cos_integral(d*x + a*d/b
))*tan(1/2*c)*tan(1/2*a*d/b) - 4*a^3*b^2*d^3*x^2*imag_part(cos_integral(-d*
x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^3*b^2*d^3*x^2*sin_integral((b*d
*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) + 4*a^5*d^3*imag_part(cos_integral(d
*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 24*a*b^4*d*x^2*imag
_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) -
 4*a^5*d^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*
tan(1/2*a*d/b) + 24*a*b^4*d*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1
/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 48*a^3*b^2*d^2*x*real_part(cos_integr
al(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 48*a^3*b^2*d^2*
x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a
*d/b) + 8*a^5*d^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*t
an(1/2*a*d/b) - 48*a*b^4*d*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2
*tan(1/2*c)*tan(1/2*a*d/b) - 12*a^2*b^3*d^2*x^2*imag_part(cos_integral(d*x
+ a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 12*a^2*b^3*d^2*x^2*imag_part(cos_in
tegral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 4*a^4*b*d^3*x*real_part
(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 4*a^4*b*d^3*x*rea
l_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 24*a^2*b^3
*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b) - 12*a^4
*b*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan
(1/2*a*d/b) + 12*a^4*b*d^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*
x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 24*a^2*b^3*d*x*real_part(cos_integral(d*
x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 24*a^2*b^3*d*x*rea
l_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/
b) - 24*a^4*b*d^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2
*tan(1/2*a*d/b) - a^3*b^2*d^3*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(
1/2*a*d/b)^2 + a^3*b^2*d^3*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/
2*a*d/b)^2 - 2*a^3*b^2*d^3*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)
^2 - a^5*d^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*
d/b)^2 + 6*a*b^4*d*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*
tan(1/2*a*d/b)^2 + a^5*d^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*
x)^2*tan(1/2*a*d/b)^2 - 6*a*b^4*d*x^2*imag_part(cos_integral(-d*x - a*d/b))
*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 12*a^3*b^2*d^2*x*real_part(cos_integral(
d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 12*a^3*b^2*d^2*x*real_part(
cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*a^5*d^3*sin
_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 12*a*b^4*d*x^2
*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 12*a^2*b^3
*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 -
 12*a^2*b^3*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/
2*a*d/b)^2 - 4*a^4*b*d^3*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*
tan(1/2*a*d/b)^2 - 4*a^4*b*d^3*x*real_part(cos_integral(-d*x - a*d/b))*tan(
1/2*c)*tan(1/2*a*d/b)^2 + 24*a^2*b^3*d^2*x^2*sin_integral((b*d*x + a*d)/b)*
tan(1/2*c)*tan(1/2*a*d/b)^2 + 12*a^4*b*d^2*imag_part(cos_integral(d*x + a*d
/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 12*a^4*b*d^2*imag_part(co
s_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 24*a

```
^2*b^3*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*t
an(1/2*a*d/b)^2 + 24*a^2*b^3*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(
1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 24*a^4*b*d^2*sin_integral((b*d*x +
 a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + a^5*d^3*imag_part(cos
_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 6*a*b^4*d*x^2*imag_
part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^5*d^3*ima
g_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 6*a*b^4*
d*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 +
 12*a^3*b^2*d^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2
*a*d/b)^2 + 12*a^3*b^2*d^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*
c)^2*tan(1/2*a*d/b)^2 + 2*a^5*d^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^
2*tan(1/2*a*d/b)^2 - 12*a*b^4*d*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c
)^2*tan(1/2*a*d/b)^2 + 2*a^4*b*d^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/
b)^2 - 4*b^5*x^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 6*a^3*b^2*d
*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a
*d/b)^2 + 6*a^3*b^2*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*
tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 12*a^3*b^2*d*sin_integral((b*d*x + a*d)/b)*
tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^4*b*d^3*x*imag_part(cos_
integral(d*x + a*d/b))*tan(1/2*d*x)^2 - 2*a^4*b*d^3*x*imag_part(cos_integra
l(-d*x - a*d/b))*tan(1/2*d*x)^2 + 6*a^2*b^3*d^2*x^2*real_part(cos_integral(
d*x + a*d/b))*tan(1/2*d*x)^2 + 6*a^2*b^3*d^2*x^2*real_part(cos_integral(-d*
x - a*d/b))*tan(1/2*d*x)^2 + 4*a^4*b*d^3*x*sin_integral((b*d*x + a*d)/b)*ta
n(1/2*d*x)^2 + 2*a^3*b^2*d^3*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1
/2*c) + 2*a^3*b^2*d^3*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)
- 24*a^3*b^2*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(
1/2*c) + 24*a^3*b^2*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x
)^2*tan(1/2*c) + 2*a^5*d^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x
)^2*tan(1/2*c) - 12*a*b^4*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/
2*d*x)^2*tan(1/2*c) + 2*a^5*d^3*real_part(cos_integral(-d*x - a*d/b))*tan(1
/2*d*x)^2*tan(1/2*c) - 12*a*b^4*d*x^2*real_part(cos_integral(-d*x - a*d/b))
*tan(1/2*d*x)^2*tan(1/2*c) - 48*a^3*b^2*d^2*x*sin_integral((b*d*x + a*d)/b)
*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^4*b*d^3*x*imag_part(cos_integral(d*x + a*d
/b))*tan(1/2*c)^2 + 2*a^4*b*d^3*x*imag_part(cos_integral(-d*x - a*d/b))*tan
(1/2*c)^2 - 6*a^2*b^3*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*
c)^2 - 6*a^2*b^3*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2
 - 4*a^4*b*d^3*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2 + 2*a^3*b^2*d^2
*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + 12*a^2*b^3*d*x*imag_part(cos_integral(d*x
+ a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 12*a^2*b^3*d*x*imag_part(cos_integr
al(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 6*a^4*b*d^2*real_part(cos_i
ntegral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 6*a^4*b*d^2*real_part(c
os_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 24*a^2*b^3*d*x*sin
_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^3*b^2*d^3*x^2*
real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 2*a^3*b^2*d^3*x^2*rea
l_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) + 24*a^3*b^2*d^2*x*imag_p
art(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 24*a^3*b^2*d
```

^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) -
2*a^5*d^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b
) + 12*a*b^4*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(
1/2*a*d/b) - 2*a^5*d^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2
*tan(1/2*a*d/b) + 12*a*b^4*d*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(
1/2*d*x)^2*tan(1/2*a*d/b) + 48*a^3*b^2*d^2*x*sin_integral((b*d*x + a*d)/b)*
tan(1/2*d*x)^2*tan(1/2*a*d/b) + 8*a^4*b*d^3*x*imag_part(cos_integral(d*x +
a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 8*a^4*b*d^3*x*imag_part(cos_integral(-d
*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 24*a^2*b^3*d^2*x^2*real_part(cos_i
ntegral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 24*a^2*b^3*d^2*x^2*real_p
art(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 16*a^4*b*d^3*x*
sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) - 48*a^2*b^3*d*x*im
ag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)
 + 48*a^2*b^3*d*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(
1/2*c)*tan(1/2*a*d/b) + 24*a^4*b*d^2*real_part(cos_integral(d*x + a*d/b))*t
an(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 24*a^4*b*d^2*real_part(cos_integr
al(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 96*a^2*b^3*d*x
*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 2
4*a^3*b^2*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a
*d/b) + 24*a^3*b^2*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2
*tan(1/2*a*d/b) + 2*a^5*d^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)
^2*tan(1/2*a*d/b) - 12*a*b^4*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan
(1/2*c)^2*tan(1/2*a*d/b) + 2*a^5*d^3*real_part(cos_integral(-d*x - a*d/b))*
tan(1/2*c)^2*tan(1/2*a*d/b) - 12*a*b^4*d*x^2*real_part(cos_integral(-d*x -
a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 48*a^3*b^2*d^2*x*sin_integral((b*d*x
+ a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b) - 12*a^3*b^2*d*real_part(cos_integral
(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 12*a^3*b^2*d*re
al_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d
/b) - 2*a^4*b*d^3*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 +
 2*a^4*b*d^3*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - 6*a
^2*b^3*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 - 6*a^
2*b^3*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - 4*a^
4*b*d^3*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 - 2*a^3*b^2*d^2*x*
tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 12*a^2*b^3*d*x*imag_part(cos_integral(d*x
 + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 12*a^2*b^3*d*x*imag_part(cos_i
ntegral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 6*a^4*b*d^2*real_p
art(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 6*a^4*b*d^
2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2
4*a^2*b^3*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2
 + 24*a^3*b^2*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2
*a*d/b)^2 - 24*a^3*b^2*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*
c)*tan(1/2*a*d/b)^2 - 2*a^5*d^3*real_part(cos_integral(d*x + a*d/b))*tan(1/
2*c)*tan(1/2*a*d/b)^2 + 12*a*b^4*d*x^2*real_part(cos_integral(d*x + a*d/b))
*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^5*d^3*real_part(cos_integral(-d*x - a*d/
b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 12*a*b^4*d*x^2*real_part(cos_integral(-d*

x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 48*a^3*b^2*d^2*x*sin_integral((b*
d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b)^2 - 8*a^3*b^2*d^2*x*tan(1/2*d*x)*ta
n(1/2*c)*tan(1/2*a*d/b)^2 + 24*a^2*b^3*d*x*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/
2*a*d/b)^2 + 12*a^3*b^2*d*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)
^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 12*a^3*b^2*d*real_part(cos_integral(-d*x -
 a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^3*b^2*d^2*x*tan(1
/2*c)^2*tan(1/2*a*d/b)^2 - 12*a^2*b^3*d*x*imag_part(cos_integral(d*x + a*d/
b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 12*a^2*b^3*d*x*imag_part(cos_integral(-
d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 6*a^4*b*d^2*real_part(cos_int
egral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 6*a^4*b*d^2*real_part(c
os_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 24*a^2*b^3*d*x*s
in_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 24*a^2*b^3*d*x
*tan(1/2*d*x)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 8*a*b^4*x*tan(1/2*d*x)^2*tan(
1/2*c)^2*tan(1/2*a*d/b)^2 + a^3*b^2*d^3*x^2*imag_part(cos_integral(d*x + a*
d/b)) - a^3*b^2*d^3*x^2*imag_part(cos_integral(-d*x - a*d/b)) + 2*a^3*b^2*d
^3*x^2*sin_integral((b*d*x + a*d)/b) + a^5*d^3*imag_part(cos_integral(d*x +
 a*d/b))*tan(1/2*d*x)^2 - 6*a*b^4*d*x^2*imag_part(cos_integral(d*x + a*d/b)
)*tan(1/2*d*x)^2 - a^5*d^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*
x)^2 + 6*a*b^4*d*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 +
 12*a^3*b^2*d^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2 + 12*
a^3*b^2*d^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 + 2*a^5*
d^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2 - 12*a*b^4*d*x^2*sin_integ
ral((b*d*x + a*d)/b)*tan(1/2*d*x)^2 - 12*a^2*b^3*d^2*x^2*imag_part(cos_inte
gral(d*x + a*d/b))*tan(1/2*c) + 12*a^2*b^3*d^2*x^2*imag_part(cos_integral(-
d*x - a*d/b))*tan(1/2*c) + 4*a^4*b*d^3*x*real_part(cos_integral(d*x + a*d/b
))*tan(1/2*c) + 4*a^4*b*d^3*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2
*c) - 24*a^2*b^3*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c) - 12*a^4*
b*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 12*a
^4*b*d^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) -
24*a^2*b^3*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*
c) - 24*a^2*b^3*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*ta
n(1/2*c) - 24*a^4*b*d^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/
2*c) - a^5*d^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 + 6*a*b^4*
d*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 + a^5*d^3*imag_part
(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 - 6*a*b^4*d*x^2*imag_part(cos_int
egral(-d*x - a*d/b))*tan(1/2*c)^2 - 12*a^3*b^2*d^2*x*real_part(cos_integral
(d*x + a*d/b))*tan(1/2*c)^2 - 12*a^3*b^2*d^2*x*real_part(cos_integral(-d*x
- a*d/b))*tan(1/2*c)^2 - 2*a^5*d^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)
^2 + 12*a*b^4*d*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2 + 2*a^4*b*d^
2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*b^5*x^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 6*a
^3*b^2*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 -
 6*a^3*b^2*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c
)^2 + 12*a^3*b^2*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^
2 + 12*a^2*b^3*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)
- 12*a^2*b^3*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) -

4*a^4*b*d^3*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 4*a^4*
b*d^3*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) + 24*a^2*b^3*d
^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b) + 12*a^4*b*d^2*imag_par
t(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 12*a^4*b*d^2*i
mag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 24*a^2
*b^3*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)
 + 24*a^2*b^3*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(
1/2*a*d/b) + 24*a^4*b*d^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(
1/2*a*d/b) + 4*a^5*d^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(
1/2*a*d/b) - 24*a*b^4*d*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)
*tan(1/2*a*d/b) - 4*a^5*d^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c
)*tan(1/2*a*d/b) + 24*a*b^4*d*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan
(1/2*c)*tan(1/2*a*d/b) + 48*a^3*b^2*d^2*x*real_part(cos_integral(d*x + a*d/
b))*tan(1/2*c)*tan(1/2*a*d/b) + 48*a^3*b^2*d^2*x*real_part(cos_integral(-d*
x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^5*d^3*sin_integral((b*d*x + a*d
)/b)*tan(1/2*c)*tan(1/2*a*d/b) - 48*a*b^4*d*x^2*sin_integral((b*d*x + a*d)/
b)*tan(1/2*c)*tan(1/2*a*d/b) - 24*a^3*b^2*d*imag_part(cos_integral(d*x + a*
d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 24*a^3*b^2*d*imag_part(cos
_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 48*a^3*
b^2*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b
) - 12*a^4*b*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*
a*d/b) + 12*a^4*b*d^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*ta
n(1/2*a*d/b) - 24*a^2*b^3*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*
c)^2*tan(1/2*a*d/b) - 24*a^2*b^3*d*x*real_part(cos_integral(-d*x - a*d/b))*
tan(1/2*c)^2*tan(1/2*a*d/b) - 24*a^4*b*d^2*sin_integral((b*d*x + a*d)/b)*ta
n(1/2*c)^2*tan(1/2*a*d/b) - a^5*d^3*imag_part(cos_integral(d*x + a*d/b))*ta
n(1/2*a*d/b)^2 + 6*a*b^4*d*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2
*a*d/b)^2 + a^5*d^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2
- 6*a*b^4*d*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - 12
*a^3*b^2*d^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 - 12*a
^3*b^2*d^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - 2*a^5
*d^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 + 12*a*b^4*d*x^2*sin_in
tegral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 - 2*a^4*b*d^2*tan(1/2*d*x)^2*tan(1
/2*a*d/b)^2 + 4*b^5*x^2*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 6*a^3*b^2*d*imag_
part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 6*a^3*b^2
*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 +
12*a^3*b^2*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2
+ 12*a^4*b*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/
b)^2 - 12*a^4*b*d^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/
2*a*d/b)^2 + 24*a^2*b^3*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)
*tan(1/2*a*d/b)^2 + 24*a^2*b^3*d*x*real_part(cos_integral(-d*x - a*d/b))*ta
n(1/2*c)*tan(1/2*a*d/b)^2 + 24*a^4*b*d^2*sin_integral((b*d*x + a*d)/b)*tan(
1/2*c)*tan(1/2*a*d/b)^2 - 8*a^4*b*d^2*tan(1/2*d*x)*tan(1/2*c)*tan(1/2*a*d/b
)^2 + 16*b^5*x^2*tan(1/2*d*x)*tan(1/2*c)*tan(1/2*a*d/b)^2 + 20*a^3*b^2*d*ta
n(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^4*b*d^2*tan(1/2*c)^2*tan(1/2

*a*d/b)^2 + 4*b^5*x^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 6*a^3*b^2*d*imag_part
(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 6*a^3*b^2*d*ima
g_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 12*a^3*b
^2*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 20*a^3*b
^2*d*tan(1/2*d*x)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*a^2*b^3*tan(1/2*d*x)^2*
tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^4*b*d^3*x*imag_part(cos_integral(d*x +
a*d/b)) - 2*a^4*b*d^3*x*imag_part(cos_integral(-d*x - a*d/b)) + 6*a^2*b^3*d
^2*x^2*real_part(cos_integral(d*x + a*d/b)) + 6*a^2*b^3*d^2*x^2*real_part(c
os_integral(-d*x - a*d/b)) + 4*a^4*b*d^3*x*sin_integral((b*d*x + a*d)/b) -
2*a^3*b^2*d^2*x*tan(1/2*d*x)^2 - 12*a^2*b^3*d*x*imag_part(cos_integral(d*x
+ a*d/b))*tan(1/2*d*x)^2 + 12*a^2*b^3*d*x*imag_part(cos_integral(-d*x - a*d
/b))*tan(1/2*d*x)^2 + 6*a^4*b*d^2*real_part(cos_integral(d*x + a*d/b))*tan(
1/2*d*x)^2 + 6*a^4*b*d^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)
^2 - 24*a^2*b^3*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2 - 24*a^3*b
^2*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 24*a^3*b^2*d^2*x
*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) + 2*a^5*d^3*real_part(cos
_integral(d*x + a*d/b))*tan(1/2*c) - 12*a*b^4*d*x^2*real_part(cos_integral(
d*x + a*d/b))*tan(1/2*c) + 2*a^5*d^3*real_part(cos_integral(-d*x - a*d/b))*
tan(1/2*c) - 12*a*b^4*d*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c
) - 48*a^3*b^2*d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*c) - 8*a^3*b^2*d
^2*x*tan(1/2*d*x)*tan(1/2*c) + 24*a^2*b^3*d*x*tan(1/2*d*x)^2*tan(1/2*c) - 1
2*a^3*b^2*d*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)
- 12*a^3*b^2*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2
*c) - 2*a^3*b^2*d^2*x*tan(1/2*c)^2 + 12*a^2*b^3*d*x*imag_part(cos_integral(
d*x + a*d/b))*tan(1/2*c)^2 - 12*a^2*b^3*d*x*imag_part(cos_integral(-d*x - a
*d/b))*tan(1/2*c)^2 - 6*a^4*b*d^2*real_part(cos_integral(d*x + a*d/b))*tan(
1/2*c)^2 - 6*a^4*b*d^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 +
 24*a^2*b^3*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2 + 24*a^2*b^3*d*x
*tan(1/2*d*x)*tan(1/2*c)^2 - 8*a*b^4*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + 24*a^3
*b^2*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 24*a^3*b^2
*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) - 2*a^5*d^3*rea
l_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) + 12*a*b^4*d*x^2*real_part
(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 2*a^5*d^3*real_part(cos_integr
al(-d*x - a*d/b))*tan(1/2*a*d/b) + 12*a*b^4*d*x^2*real_part(cos_integral(-d
*x - a*d/b))*tan(1/2*a*d/b) + 48*a^3*b^2*d^2*x*sin_integral((b*d*x + a*d)/b
)*tan(1/2*a*d/b) + 12*a^3*b^2*d*real_part(cos_integral(d*x + a*d/b))*tan(1/
2*d*x)^2*tan(1/2*a*d/b) + 12*a^3*b^2*d*real_part(cos_integral(-d*x - a*d/b)
)*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 48*a^2*b^3*d*x*imag_part(cos_integral(d*x
 + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 48*a^2*b^3*d*x*imag_part(cos_integra
l(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 24*a^4*b*d^2*real_part(cos_int
egral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 24*a^4*b*d^2*real_part(cos_
integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 96*a^2*b^3*d*x*sin_inte
gral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) - 12*a^3*b^2*d*real_part(co
s_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 12*a^3*b^2*d*real_pa
rt(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^3*b^2*d^2*

x*tan(1/2*a*d/b)^2 + 12*a^2*b^3*d*x*imag_part(cos_integral(d*x + a*d/b))*ta
n(1/2*a*d/b)^2 - 12*a^2*b^3*d*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1
/2*a*d/b)^2 - 6*a^4*b*d^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/
b)^2 - 6*a^4*b*d^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 +
 24*a^2*b^3*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 - 24*a^2*b^3
*d*x*tan(1/2*d*x)*tan(1/2*a*d/b)^2 + 8*a*b^4*x*tan(1/2*d*x)^2*tan(1/2*a*d/b
)^2 - 24*a^2*b^3*d*x*tan(1/2*c)*tan(1/2*a*d/b)^2 + 12*a^3*b^2*d*real_part(c
os_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 12*a^3*b^2*d*real_p
art(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 32*a*b^4*x*ta
n(1/2*d*x)*tan(1/2*c)*tan(1/2*a*d/b)^2 + 8*a*b^4*x*tan(1/2*c)^2*tan(1/2*a*d
/b)^2 + a^5*d^3*imag_part(cos_integral(d*x + a*d/b)) - 6*a*b^4*d*x^2*imag_p
art(cos_integral(d*x + a*d/b)) - a^5*d^3*imag_part(cos_integral(-d*x - a*d/
b)) + 6*a*b^4*d*x^2*imag_part(cos_integral(-d*x - a*d/b)) + 12*a^3*b^2*d^2*
x*real_part(cos_integral(d*x + a*d/b)) + 12*a^3*b^2*d^2*x*real_part(cos_int
egral(-d*x - a*d/b)) + 2*a^5*d^3*sin_integral((b*d*x + a*d)/b) - 12*a*b^4*d
*x^2*sin_integral((b*d*x + a*d)/b) - 2*a^4*b*d^2*tan(1/2*d*x)^2 + 4*b^5*x^2
*tan(1/2*d*x)^2 - 6*a^3*b^2*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*
d*x)^2 + 6*a^3*b^2*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 -
 12*a^3*b^2*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2 - 12*a^4*b*d^2*i
mag_part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 12*a^4*b*d^2*imag_part(cos
_integral(-d*x - a*d/b))*tan(1/2*c) - 24*a^2*b^3*d*x*real_part(cos_integral
(d*x + a*d/b))*tan(1/2*c) - 24*a^2*b^3*d*x*real_part(cos_integral(-d*x - a*
d/b))*tan(1/2*c) - 24*a^4*b*d^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c) -
8*a^4*b*d^2*tan(1/2*d*x)*tan(1/2*c) + 16*b^5*x^2*tan(1/2*d*x)*tan(1/2*c) +
20*a^3*b^2*d*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^4*b*d^2*tan(1/2*c)^2 + 4*b^5*x
^2*tan(1/2*c)^2 + 6*a^3*b^2*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*
c)^2 - 6*a^3*b^2*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 + 12*
a^3*b^2*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2 + 20*a^3*b^2*d*tan(1/2
*d*x)*tan(1/2*c)^2 - 4*a^2*b^3*tan(1/2*d*x)^2*tan(1/2*c)^2 + 12*a^4*b*d^2*i
mag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 12*a^4*b*d^2*imag_part
(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) + 24*a^2*b^3*d*x*real_part(cos_
integral(d*x + a*d/b))*tan(1/2*a*d/b) + 24*a^2*b^3*d*x*real_part(cos_integr
al(-d*x - a*d/b))*tan(1/2*a*d/b) + 24*a^4*b*d^2*sin_integral((b*d*x + a*d)/
b)*tan(1/2*a*d/b) - 24*a^3*b^2*d*imag_part(cos_integral(d*x + a*d/b))*tan(1
/2*c)*tan(1/2*a*d/b) + 24*a^3*b^2*d*imag_part(cos_integral(-d*x - a*d/b))*t
an(1/2*c)*tan(1/2*a*d/b) - 48*a^3*b^2*d*sin_integral((b*d*x + a*d)/b)*tan(1
/2*c)*tan(1/2*a*d/b) + 2*a^4*b*d^2*tan(1/2*a*d/b)^2 - 4*b^5*x^2*tan(1/2*a*d
/b)^2 + 6*a^3*b^2*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 -
 6*a^3*b^2*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 + 12*a^
3*b^2*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 - 20*a^3*b^2*d*tan(1
/2*d*x)*tan(1/2*a*d/b)^2 + 4*a^2*b^3*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 20*a
^3*b^2*d*tan(1/2*c)*tan(1/2*a*d/b)^2 + 16*a^2*b^3*tan(1/2*d*x)*tan(1/2*c)*t
an(1/2*a*d/b)^2 + 4*a^2*b^3*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^3*b^2*d^2*x
 - 12*a^2*b^3*d*x*imag_part(cos_integral(d*x + a*d/b)) + 12*a^2*b^3*d*x*ima
g_part(cos_integral(-d*x - a*d/b)) + 6*a^4*b*d^2*real_part(cos_integral(d*x

```
 + a*d/b)) + 6*a^4*b*d^2*real_part(cos_integral(-d*x - a*d/b)) - 24*a^2*b^3
*d*x*sin_integral((b*d*x + a*d)/b) - 24*a^2*b^3*d*x*tan(1/2*d*x) + 8*a*b^4*
x*tan(1/2*d*x)^2 - 24*a^2*b^3*d*x*tan(1/2*c) - 12*a^3*b^2*d*real_part(cos_i
ntegral(d*x + a*d/b))*tan(1/2*c) - 12*a^3*b^2*d*real_part(cos_integral(-d*x
 - a*d/b))*tan(1/2*c) + 32*a*b^4*x*tan(1/2*d*x)*tan(1/2*c) + 8*a*b^4*x*tan(
1/2*c)^2 + 12*a^3*b^2*d*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)
 + 12*a^3*b^2*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) - 8*a*
b^4*x*tan(1/2*a*d/b)^2 + 2*a^4*b*d^2 - 4*b^5*x^2 - 6*a^3*b^2*d*imag_part(co
s_integral(d*x + a*d/b)) + 6*a^3*b^2*d*imag_part(cos_integral(-d*x - a*d/b)
) - 12*a^3*b^2*d*sin_integral((b*d*x + a*d)/b) - 20*a^3*b^2*d*tan(1/2*d*x)
+ 4*a^2*b^3*tan(1/2*d*x)^2 - 20*a^3*b^2*d*tan(1/2*c) + 16*a^2*b^3*tan(1/2*d
*x)*tan(1/2*c) + 4*a^2*b^3*tan(1/2*c)^2 - 4*a^2*b^3*tan(1/2*a*d/b)^2 - 8*a*
b^4*x - 4*a^2*b^3)/(b^8*d*x^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2
+ 2*a*b^7*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + b^8*d*x^2*tan(
1/2*d*x)^2*tan(1/2*c)^2 + b^8*d*x^2*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + b^8*d
*x^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^2*b^6*d*tan(1/2*d*x)^2*tan(1/2*c)^2*
tan(1/2*a*d/b)^2 + 2*a*b^7*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*b^7*d*x*ta
n(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*a*b^7*d*x*tan(1/2*c)^2*tan(1/2*a*d/b)^2 +
 b^8*d*x^2*tan(1/2*d*x)^2 + b^8*d*x^2*tan(1/2*c)^2 + a^2*b^6*d*tan(1/2*d*x)
^2*tan(1/2*c)^2 + b^8*d*x^2*tan(1/2*a*d/b)^2 + a^2*b^6*d*tan(1/2*d*x)^2*tan
(1/2*a*d/b)^2 + a^2*b^6*d*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*b^7*d*x*tan(1
/2*d*x)^2 + 2*a*b^7*d*x*tan(1/2*c)^2 + 2*a*b^7*d*x*tan(1/2*a*d/b)^2 + b^8*d
*x^2 + a^2*b^6*d*tan(1/2*d*x)^2 + a^2*b^6*d*tan(1/2*c)^2 + a^2*b^6*d*tan(1/
2*a*d/b)^2 + 2*a*b^7*d*x + a^2*b^6*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \sin(c+dx)}{(a+bx)^3} \, dx = \int \frac{x^3 \sin(c+d\,x)}{(a+b\,x)^3} \, dx$$

```
[In] int((x^3*sin(c + d*x))/(a + b*x)^3,x)
[Out] int((x^3*sin(c + d*x))/(a + b*x)^3, x)
```

3.34 $\int \frac{x^2\sin(c+dx)}{(a+bx)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 241

$$\begin{aligned}\int \frac{x^2\sin(c+dx)}{(a+bx)^3}\,dx = &-\frac{a^2d\cos(c+dx)}{2b^4(a+bx)}-\frac{2ad\cos\left(c-\frac{ad}{b}\right)\operatorname{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^4}\\&+\frac{\operatorname{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^3}\\&-\frac{a^2d^2\operatorname{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{2b^5}-\frac{a^2\sin(c+dx)}{2b^3(a+bx)^2}\\&+\frac{2a\sin(c+dx)}{b^3(a+bx)}+\frac{\cos\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(\frac{ad}{b}+dx\right)}{b^3}\\&-\frac{a^2d^2\cos\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(\frac{ad}{b}+dx\right)}{2b^5}+\frac{2ad\sin\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(\frac{ad}{b}+dx\right)}{b^4}\end{aligned}$$

```
[Out] -2*a*d*Ci(a*d/b+d*x)*cos(-c+a*d/b)/b^4-1/2*a^2*d*cos(d*x+c)/b^4/(b*x+a)+cos
(-c+a*d/b)*Si(a*d/b+d*x)/b^3-1/2*a^2*d^2*cos(-c+a*d/b)*Si(a*d/b+d*x)/b^5-Ci
(a*d/b+d*x)*sin(-c+a*d/b)/b^3+1/2*a^2*d^2*Ci(a*d/b+d*x)*sin(-c+a*d/b)/b^5-2
*a*d*Si(a*d/b+d*x)*sin(-c+a*d/b)/b^4-1/2*a^2*sin(d*x+c)/b^3/(b*x+a)^2+2*a*s
in(d*x+c)/b^3/(b*x+a)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6874, 3378, 3384, 3380, 3383}

$$\int \frac{x^2\sin(c+dx)}{(a+bx)^3}\,dx = -\frac{a^2d^2\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{2b^5} - \frac{a^2d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{2b^5} - \frac{a^2d\cos(c+dx)}{2b^4(a+bx)} - \frac{a^2\sin(c+dx)}{2b^3(a+bx)^2} - \frac{2ad\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b^4} + \frac{2ad\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b^4} + \frac{\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b^3} + \frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b^3} + \frac{2a\sin(c+dx)}{b^3(a+bx)}$$

```
[In] Int[(x^2*Sin[c + d*x])/(a + b*x)^3,x]
```

```
[Out] -1/2*(a^2*d*Cos[c + d*x])/(b^4*(a + b*x)) - (2*a*d*Cos[c - (a*d)/b]*CosInte
gral[(a*d)/b + d*x])/b^4 + (CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/b^
3 - (a^2*d^2*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/(2*b^5) - (a^2*Si
n[c + d*x])/(2*b^3*(a + b*x)^2) + (2*a*Sin[c + d*x])/(b^3*(a + b*x)) + (Cos
[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^3 - (a^2*d^2*Cos[c - (a*d)/b]*S
inIntegral[(a*d)/b + d*x])/(2*b^5) + (2*a*d*Sin[c - (a*d)/b]*SinIntegral[(a
*d)/b + d*x])/b^4
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a^2\sin(c+dx)}{b^2(a+bx)^3}-\frac{2a\sin(c+dx)}{b^2(a+bx)^2}+\frac{\sin(c+dx)}{b^2(a+bx)}\right)dx \\
&= \frac{\int \frac{\sin(c+dx)}{a+bx}\,dx}{b^2}-\frac{(2a)\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx}{b^2}+\frac{a^2\int \frac{\sin(c+dx)}{(a+bx)^3}\,dx}{b^2} \\
&= -\frac{a^2\sin(c+dx)}{2b^3(a+bx)^2}+\frac{2a\sin(c+dx)}{b^3(a+bx)}-\frac{(2ad)\int \frac{\cos(c+dx)}{a+bx}\,dx}{b^3}+\frac{(a^2d)\int \frac{\cos(c+dx)}{(a+bx)^2}\,dx}{2b^3} \\
&\quad+\frac{\cos\left(c-\frac{ad}{b}\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^2}+\frac{\sin\left(c-\frac{ad}{b}\right)\int \frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^2} \\
&= -\frac{a^2d\cos(c+dx)}{2b^4(a+bx)}+\frac{\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^3}-\frac{a^2\sin(c+dx)}{2b^3(a+bx)^2} \\
&\quad+\frac{2a\sin(c+dx)}{b^3(a+bx)}+\frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^3}-\frac{(a^2d^2)\int \frac{\sin(c+dx)}{a+bx}\,dx}{2b^4} \\
&\quad-\frac{\left(2ad\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^3}+\frac{\left(2ad\sin\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^3} \\
&= -\frac{a^2d\cos(c+dx)}{2b^4(a+bx)}-\frac{2ad\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^4} \\
&\quad+\frac{\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^3}-\frac{a^2\sin(c+dx)}{2b^3(a+bx)^2}+\frac{2a\sin(c+dx)}{b^3(a+bx)} \\
&\quad+\frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^3}+\frac{2ad\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^4} \\
&\quad-\frac{\left(a^2d^2\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{2b^4}-\frac{\left(a^2d^2\sin\left(c-\frac{ad}{b}\right)\right)\int \frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{2b^4}
\end{aligned}
$$

$$= -\frac{a^2 d\cos(c+dx)}{2b^4(a+bx)} - \frac{2ad\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^4} + \frac{\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^3} - \frac{a^2d^2\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{2b^5} - \frac{a^2\sin(c+dx)}{2b^3(a+bx)^2} + \frac{2a\sin(c+dx)}{b^3(a+bx)} + \frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^3} - \frac{a^2d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{2b^5} + \frac{2ad\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^4}$$

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.64

$$\int \frac{x^2\sin(c+dx)}{(a+bx)^3}\,dx = -\frac{-\text{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)\left(-4abd\cos\left(c-\frac{ad}{b}\right)+(2b^2-a^2d^2)\sin\left(c-\frac{ad}{b}\right)\right)+\frac{ab(ad(a+bx)\cos(c+dx)-b(3a+4b}{(a+bx)^2}}{2b^5}$$

```
[In] Integrate[(x^2*Sin[c + d*x])/(a + b*x)^3,x]
```

```
[Out] -1/2*(-(CosIntegral[d*(a/b + x)]*(-4*a*b*d*Cos[c - (a*d)/b] + (2*b^2 - a^2*
d^2)*Sin[c - (a*d)/b])) + (a*b*(a*d*(a + b*x)*Cos[c + d*x] - b*(3*a + 4*b*x
)*Sin[c + d*x]))/(a + b*x)^2 + ((-2*b^2 + a^2*d^2)*Cos[c - (a*d)/b] - 4*a*b
*d*Sin[c - (a*d)/b])*SinIntegral[d*(a/b + x)])/b^5
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 621, normalized size of antiderivative = 2.58

method	result
risch	$-\frac{i\left(2ia^2b^3d^4x^3+6ia^3b^2d^4x^2+6ia^4b\,d^4x+2ia^5d^4\right)\cos(dx+c)}{4b^4d(bx+a)^2\left(-d^2x^2b^2-2ab\,d^2x-d^2a^2\right)}-\frac{\left(8a\,b^3d^3x^3+22a^2b^2d^3x^2+20a^3b\,d^3x+6a^4d^3\right)\sin(dx+c)}{4b^3d(bx+a)^2\left(-d^2x^2b^2-2ab\,d^2x-d^2a^2\right)}$
derivativedivides	$\frac{d^3c^2\left(-\frac{\sin(dx+c)}{2(da-cb+b(dx+c))^2b}+\frac{-\frac{\cos(dx+c)}{(da-cb+b(dx+c))b}-\frac{\frac{\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}-\frac{\mathrm{Ci}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}}{b}}{2b}\right)+\frac{2d^3(d}{}}{}$
default	$\frac{d^3c^2\left(-\frac{\sin(dx+c)}{2(da-cb+b(dx+c))^2b}+\frac{-\frac{\cos(dx+c)}{(da-cb+b(dx+c))b}-\frac{\frac{\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}-\frac{\mathrm{Ci}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}}{b}}{2b}\right)+\frac{2d^3(d}{}}{}$

```
[In] int(x^2*sin(d*x+c)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I/b^4/d*(2*I*a^2*b^3*d^4*x^3+6*I*a^3*b^2*d^4*x^2+6*I*a^4*b*d^4*x+2*I*a
^5*d^4)/(b*x+a)^2/(-b^2*d^2*x^2-2*a*b*d^2*x-a^2*d^2)*cos(d*x+c)-1/4/b^3/d*(
8*a*b^3*d^3*x^3+22*a^2*b^2*d^3*x^2+20*a^3*b*d^3*x+6*a^4*d^3)/(b*x+a)^2/(-b^
2*d^2*x^2-2*a*b*d^2*x-a^2*d^2)*sin(d*x+c)+1/4*I/b^5*cos((a*d-b*c)/b)*Ei(1,I
*d*(b*x+a)/b)*a^2*d^2-1/4*I/b^5*cos((a*d-b*c)/b)*Ei(1,-I*d*(b*x+a)/b)*a^2*d
^2-1/2*I/b^3*cos((a*d-b*c)/b)*Ei(1,I*d*(b*x+a)/b)+1/b^4*cos((a*d-b*c)/b)*Ei
(1,I*d*(b*x+a)/b)*a*d+1/2*I/b^3*cos((a*d-b*c)/b)*Ei(1,-I*d*(b*x+a)/b)+1/b^4
*cos((a*d-b*c)/b)*Ei(1,-I*d*(b*x+a)/b)*a*d-1/4/b^5*sin((a*d-b*c)/b)*Ei(1,I*
d*(b*x+a)/b)*a^2*d^2-1/4/b^5*sin((a*d-b*c)/b)*Ei(1,-I*d*(b*x+a)/b)*a^2*d^2+
1/2/b^3*sin((a*d-b*c)/b)*Ei(1,I*d*(b*x+a)/b)+I/b^4*sin((a*d-b*c)/b)*Ei(1,I*
d*(b*x+a)/b)*a*d+1/2/b^3*sin((a*d-b*c)/b)*Ei(1,-I*d*(b*x+a)/b)-I/b^4*sin((a
*d-b*c)/b)*Ei(1,-I*d*(b*x+a)/b)*a*d
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.35

$$\int \frac{x^2\sin(c+dx)}{(a+bx)^3}\,dx =$$
$$-\frac{\left(a^2b^2dx+a^3bd\right)\cos\left(dx+c\right)+\left(4\left(ab^3dx^2+2\,a^2b^2dx+a^3bd\right)\mathrm{Ci}\left(\frac{bdx+ad}{b}\right)+\left(a^4d^2-2\,a^2b^2+\left(a^2b^2d^2-2\right.\right.\right.}{}$$

```
[In] integrate(x^2*sin(d*x+c)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*((a^2*b^2*d*x + a^3*b*d)*cos(d*x + c) + (4*(a*b^3*d*x^2 + 2*a^2*b^2*d*
x + a^3*b*d)*cos_integral((b*d*x + a*d)/b) + (a^4*d^2 - 2*a^2*b^2 + (a^2*b^
2*d^2 - 2*b^4)*x^2 + 2*(a^3*b*d^2 - 2*a*b^3)*x)*sin_integral((b*d*x + a*d)/
b))*cos(-(b*c - a*d)/b) - (4*a*b^3*x + 3*a^2*b^2)*sin(d*x + c) - ((a^4*d^2
- 2*a^2*b^2 + (a^2*b^2*d^2 - 2*b^4)*x^2 + 2*(a^3*b*d^2 - 2*a*b^3)*x)*cos_in
tegral((b*d*x + a*d)/b) - 4*(a*b^3*d*x^2 + 2*a^2*b^2*d*x + a^3*b*d)*sin_int
egral((b*d*x + a*d)/b))*sin(-(b*c - a*d)/b))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5
)
```

Sympy [F]

$$\int \frac{x^2 \sin(c+dx)}{(a+bx)^3} dx = \int \frac{x^2 \sin{(c+dx)}}{(a+bx)^3} dx$$

```
[In] integrate(x**2*sin(d*x+c)/(b*x+a)**3,x)
```

```
[Out] Integral(x**2*sin(c + d*x)/(a + b*x)**3, x)
```

Maxima [F]

$$\int \frac{x^2 \sin(c+dx)}{(a+bx)^3} dx = \int \frac{x^2 \sin{(dx+c)}}{(bx+a)^3} dx$$

```
[In] integrate(x^2*sin(d*x+c)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*((b*cos(c)^2 + b*sin(c)^2)*d*x^2*cos(d*x + c) + ((a*(I*exp_integral_e(
4, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(4, -(I*b*d*x + I*a*d)/b))*cos(c)
^2 + a*(I*exp_integral_e(4, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(4, -(I*
b*d*x + I*a*d)/b))*sin(c)^2)*cos(-(b*c - a*d)/b) - (a*(exp_integral_e(4, (I
*b*d*x + I*a*d)/b) + exp_integral_e(4, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a*
(exp_integral_e(4, (I*b*d*x + I*a*d)/b) + exp_integral_e(4, -(I*b*d*x + I*a
*d)/b))*sin(c)^2)*sin(-(b*c - a*d)/b))*cos(d*x + c)^2 + (b*cos(c)^2 + b*sin
(c)^2)*x*sin(d*x + c) + ((a*(I*exp_integral_e(4, (I*b*d*x + I*a*d)/b) - I*e
xp_integral_e(4, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a*(I*exp_integral_e(4, (
I*b*d*x + I*a*d)/b) - I*exp_integral_e(4, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*
cos(-(b*c - a*d)/b) - (a*(exp_integral_e(4, (I*b*d*x + I*a*d)/b) + exp_inte
gral_e(4, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a*(exp_integral_e(4, (I*b*d*x +
 I*a*d)/b) + exp_integral_e(4, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*sin(-(b*c -
 a*d)/b))*sin(d*x + c)^2 + ((b*d*x^2*cos(c) - b*x*sin(c))*cos(d*x + c)^2 +
(b*d*x^2*cos(c) - b*x*sin(c))*sin(d*x + c)^2)*cos(d*x + 2*c) - 6*(((a*b^4*c
os(c)^2 + a*b^4*sin(c)^2)*d^3*x^3 + 3*(a^2*b^3*cos(c)^2 + a^2*b^3*sin(c)^2)
```

```
*d^3*x^2 + 3*(a^3*b^2*cos(c)^2 + a^3*b^2*sin(c)^2)*d^3*x + (a^4*b*cos(c)^2
+ a^4*b*sin(c)^2)*d^3)*cos(d*x + c)^2 + ((a*b^4*cos(c)^2 + a*b^4*sin(c)^2)*
d^3*x^3 + 3*(a^2*b^3*cos(c)^2 + a^2*b^3*sin(c)^2)*d^3*x^2 + 3*(a^3*b^2*cos(
c)^2 + a^3*b^2*sin(c)^2)*d^3*x + (a^4*b*cos(c)^2 + a^4*b*sin(c)^2)*d^3)*sin
(d*x + c)^2)*integrate(1/2*x*cos(d*x + c)/(b^4*d^2*x^4 + 4*a*b^3*d^2*x^3 +
6*a^2*b^2*d^2*x^2 + 4*a^3*b*d^2*x + a^4*d^2), x) - 6*(((a*b^4*cos(c)^2 + a*
b^4*sin(c)^2)*d^3*x^3 + 3*(a^2*b^3*cos(c)^2 + a^2*b^3*sin(c)^2)*d^3*x^2 + 3
*(a^3*b^2*cos(c)^2 + a^3*b^2*sin(c)^2)*d^3*x + (a^4*b*cos(c)^2 + a^4*b*sin(
c)^2)*d^3)*cos(d*x + c)^2 + ((a*b^4*cos(c)^2 + a*b^4*sin(c)^2)*d^3*x^3 + 3*
(a^2*b^3*cos(c)^2 + a^2*b^3*sin(c)^2)*d^3*x^2 + 3*(a^3*b^2*cos(c)^2 + a^3*b
^2*sin(c)^2)*d^3*x + (a^4*b*cos(c)^2 + a^4*b*sin(c)^2)*d^3)*sin(d*x + c)^2)
*integrate(1/2*x*cos(d*x + c)/((b^4*d^2*x^4 + 4*a*b^3*d^2*x^3 + 6*a^2*b^2*d
^2*x^2 + 4*a^3*b*d^2*x + a^4*d^2)*cos(d*x + c)^2 + (b^4*d^2*x^4 + 4*a*b^3*d
^2*x^3 + 6*a^2*b^2*d^2*x^2 + 4*a^3*b*d^2*x + a^4*d^2)*sin(d*x + c)^2), x) +
 4*(((b^5*cos(c)^2 + b^5*sin(c)^2)*d^2*x^3 + 3*(a*b^4*cos(c)^2 + a*b^4*sin(
c)^2)*d^2*x^2 + 3*(a^2*b^3*cos(c)^2 + a^2*b^3*sin(c)^2)*d^2*x + (a^3*b^2*co
s(c)^2 + a^3*b^2*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b^5*cos(c)^2 + b^5*sin(c
)^2)*d^2*x^3 + 3*(a*b^4*cos(c)^2 + a*b^4*sin(c)^2)*d^2*x^2 + 3*(a^2*b^3*cos
(c)^2 + a^2*b^3*sin(c)^2)*d^2*x + (a^3*b^2*cos(c)^2 + a^3*b^2*sin(c)^2)*d^2
)*sin(d*x + c)^2)*integrate(1/2*x*sin(d*x + c)/(b^4*d^2*x^4 + 4*a*b^3*d^2*x
^3 + 6*a^2*b^2*d^2*x^2 + 4*a^3*b*d^2*x + a^4*d^2), x) + 4*(((b^5*cos(c)^2 +
 b^5*sin(c)^2)*d^2*x^3 + 3*(a*b^4*cos(c)^2 + a*b^4*sin(c)^2)*d^2*x^2 + 3*(a
^2*b^3*cos(c)^2 + a^2*b^3*sin(c)^2)*d^2*x + (a^3*b^2*cos(c)^2 + a^3*b^2*sin
(c)^2)*d^2)*cos(d*x + c)^2 + ((b^5*cos(c)^2 + b^5*sin(c)^2)*d^2*x^3 + 3*(a*
b^4*cos(c)^2 + a*b^4*sin(c)^2)*d^2*x^2 + 3*(a^2*b^3*cos(c)^2 + a^2*b^3*sin(
c)^2)*d^2*x + (a^3*b^2*cos(c)^2 + a^3*b^2*sin(c)^2)*d^2)*sin(d*x + c)^2)*in
tegrate(1/2*x*sin(d*x + c)/((b^4*d^2*x^4 + 4*a*b^3*d^2*x^3 + 6*a^2*b^2*d^2*
x^2 + 4*a^3*b*d^2*x + a^4*d^2)*cos(d*x + c)^2 + (b^4*d^2*x^4 + 4*a*b^3*d^2*
x^3 + 6*a^2*b^2*d^2*x^2 + 4*a^3*b*d^2*x + a^4*d^2)*sin(d*x + c)^2), x) + ((
b*d*x^2*sin(c) + b*x*cos(c))*cos(d*x + c)^2 + (b*d*x^2*sin(c) + b*x*cos(c))
*sin(d*x + c)^2)*sin(d*x + 2*c))/(((b^4*cos(c)^2 + b^4*sin(c)^2)*d^2*x^3 +
3*(a*b^3*cos(c)^2 + a*b^3*sin(c)^2)*d^2*x^2 + 3*(a^2*b^2*cos(c)^2 + a^2*b^2
*sin(c)^2)*d^2*x + (a^3*b*cos(c)^2 + a^3*b*sin(c)^2)*d^2)*cos(d*x + c)^2 +
((b^4*cos(c)^2 + b^4*sin(c)^2)*d^2*x^3 + 3*(a*b^3*cos(c)^2 + a*b^3*sin(c)^2
)*d^2*x^2 + 3*(a^2*b^2*cos(c)^2 + a^2*b^2*sin(c)^2)*d^2*x + (a^3*b*cos(c)^2
 + a^3*b*sin(c)^2)*d^2)*sin(d*x + c)^2)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.62 (sec) , antiderivative size = 15410, normalized size of antiderivative = 63.94

$$\int \frac{x^2 \sin(c+dx)}{(a+bx)^3} \, dx = \text{Too large to display}$$

```
[In] integrate(x^2*sin(d*x+c)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(a^2*b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*t
an(1/2*c)^2*tan(1/2*a*d/b)^2 - a^2*b^2*d^2*x^2*imag_part(cos_integral(-d*x
- a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d^2*x^2*
sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2
+ 2*a^2*b^2*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan
(1/2*c)^2*tan(1/2*a*d/b) + 2*a^2*b^2*d^2*x^2*real_part(cos_integral(-d*x -
a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a^2*b^2*d^2*x^2*real
_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2
 - 2*a^2*b^2*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*t
an(1/2*c)*tan(1/2*a*d/b)^2 + 2*a^3*b*d^2*x*imag_part(cos_integral(d*x + a*d
/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^3*b*d^2*x*imag_part
(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 +
 4*a*b^3*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*
c)^2*tan(1/2*a*d/b)^2 + 4*a*b^3*d*x^2*real_part(cos_integral(-d*x - a*d/b))
*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a^3*b*d^2*x*sin_integral(
(b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^2*b^2*d^2
*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + a^2
*b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c
)^2 - 2*a^2*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/
2*c)^2 + 4*a^2*b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x
)^2*tan(1/2*c)*tan(1/2*a*d/b) - 4*a^2*b^2*d^2*x^2*imag_part(cos_integral(-d
*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^2*b^2*d^2*x^2*s
in_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 8*a
*b^3*d*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2
*tan(1/2*a*d/b) + 8*a*b^3*d*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1
/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 4*a^3*b*d^2*x*real_part(cos_integra
l(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 4*a^3*b*d^2*x*
real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a
*d/b) - 16*a*b^3*d*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2
*c)^2*tan(1/2*a*d/b) - a^2*b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))
*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a^2*b^2*d^2*x^2*imag_part(cos_integral(-
d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*a^2*b^2*d^2*x^2*sin_integ
ral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 8*a*b^3*d*x^2*imag_p
art(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 -
 8*a*b^3*d*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2
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*c)*tan(1/2*a*d/b)^2 - 4*a^3*b*d^2*x*real_part(cos_integral(d*x + a*d/b))*t
an(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a^3*b*d^2*x*real_part(cos_int
egral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 16*a*b^3*
d*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b
)^2 + a^2*b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan
(1/2*a*d/b)^2 - a^2*b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1
/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b)*
tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^4*d^2*imag_part(cos_integral(d*x + a*d/b)
)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*b^4*x^2*imag_part(cos_in
tegral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^4*d^2
*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*
a*d/b)^2 + 2*b^4*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*t
an(1/2*c)^2*tan(1/2*a*d/b)^2 + 8*a^2*b^2*d*x*real_part(cos_integral(d*x + a
*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 8*a^2*b^2*d*x*real_pa
rt(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2
 + 2*a^4*d^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(
1/2*a*d/b)^2 - 4*b^4*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1
/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d^2*x^2*real_part(cos_integral(d*x + a
*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 2*a^2*b^2*d^2*x^2*real_part(cos_integral
(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^3*b*d^2*x*imag_part(cos_int
egral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^3*b*d^2*x*imag_part(c
os_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a*b^3*d*x^2*real
_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a*b^3*d*x^
2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a^3
*b*d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*
b^2*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d
/b) - 2*a^2*b^2*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^
2*tan(1/2*a*d/b) + 8*a^3*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1
/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 8*a^3*b*d^2*x*imag_part(cos_integral(
-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 16*a*b^3*d*x^2*re
al_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)
 + 16*a*b^3*d*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(
1/2*c)*tan(1/2*a*d/b) + 16*a^3*b*d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/
2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 2*a^2*b^2*d^2*x^2*real_part(cos_integr
al(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^2*b^2*d^2*x^2*real_part(
cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 16*a^2*b^2*d*x*im
ag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/
b) + 16*a^2*b^2*d*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*ta
n(1/2*c)^2*tan(1/2*a*d/b) + 2*a^4*d^2*real_part(cos_integral(d*x + a*d/b))*
tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 4*b^4*x^2*real_part(cos_integr
al(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^4*d^2*rea
l_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/
b) - 4*b^4*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2
*c)^2*tan(1/2*a*d/b) - 32*a^2*b^2*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2
*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a^3*b*d^2*x*imag_part(cos_integral(

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d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*a^3*b*d^2*x*imag_part(cos
_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 4*a*b^3*d*x^2*re
al_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 4*a*b^
3*d*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)
^2 - 4*a^3*b*d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d
/b)^2 - 2*a^2*b^2*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*t
an(1/2*a*d/b)^2 - 2*a^2*b^2*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*t
an(1/2*c)*tan(1/2*a*d/b)^2 + 16*a^2*b^2*d*x*imag_part(cos_integral(d*x + a*
d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 16*a^2*b^2*d*x*imag_part
(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2
*a^4*d^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan
(1/2*a*d/b)^2 + 4*b^4*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)
^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^4*d^2*real_part(cos_integral(-d*x - a*
d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*b^4*x^2*real_part(cos_
integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 32*a^2
*b^2*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*
d/b)^2 + 2*a^3*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*ta
n(1/2*a*d/b)^2 - 2*a^3*b*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/
2*c)^2*tan(1/2*a*d/b)^2 + 4*a*b^3*d*x^2*real_part(cos_integral(d*x + a*d/b)
)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a*b^3*d*x^2*real_part(cos_integral(-d*x
 - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a^3*b*d^2*x*sin_integral((b*d*
x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d*x*tan(1/2*d*x)^2*ta
n(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*a*b^3*x*imag_part(cos_integral(d*x + a*d/b)
)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a*b^3*x*imag_part(cos_in
tegral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a^3*
b*d*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/
2*a*d/b)^2 + 4*a^3*b*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2
*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 8*a*b^3*x*sin_integral((b*d*x + a*d)/b)*ta
n(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^2*b^2*d^2*x^2*imag_part(cos_
integral(d*x + a*d/b))*tan(1/2*d*x)^2 - a^2*b^2*d^2*x^2*imag_part(cos_integ
ral(-d*x - a*d/b))*tan(1/2*d*x)^2 + 2*a^2*b^2*d^2*x^2*sin_integral((b*d*x +
 a*d)/b)*tan(1/2*d*x)^2 - 8*a*b^3*d*x^2*imag_part(cos_integral(d*x + a*d/b)
)*tan(1/2*d*x)^2*tan(1/2*c) + 8*a*b^3*d*x^2*imag_part(cos_integral(-d*x - a
*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^3*b*d^2*x*real_part(cos_integral(d*x
 + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^3*b*d^2*x*real_part(cos_integral
(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 16*a*b^3*d*x^2*sin_integral((b*
d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c) - a^2*b^2*d^2*x^2*imag_part(cos_int
egral(d*x + a*d/b))*tan(1/2*c)^2 + a^2*b^2*d^2*x^2*imag_part(cos_integral(-
d*x - a*d/b))*tan(1/2*c)^2 - 2*a^2*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b
)*tan(1/2*c)^2 - a^4*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^
2*tan(1/2*c)^2 + 2*b^4*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x
)^2*tan(1/2*c)^2 + a^4*d^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*b^4*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2
*d*x)^2*tan(1/2*c)^2 - 8*a^2*b^2*d*x*real_part(cos_integral(d*x + a*d/b))*t
an(1/2*d*x)^2*tan(1/2*c)^2 - 8*a^2*b^2*d*x*real_part(cos_integral(-d*x - a*
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d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^4*d^2*sin_integral((b*d*x + a*d)/b)
*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*b^4*x^2*sin_integral((b*d*x + a*d)/b)*tan(
1/2*d*x)^2*tan(1/2*c)^2 + 8*a*b^3*d*x^2*imag_part(cos_integral(d*x + a*d/b)
)*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 8*a*b^3*d*x^2*imag_part(cos_integral(-d*x
 - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 4*a^3*b*d^2*x*real_part(cos_inte
gral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 4*a^3*b*d^2*x*real_part(
cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 16*a*b^3*d*x^2*
sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4*a^2*b^2*d^2
*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 4*a^2
*b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b
) + 8*a^2*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/
b) + 4*a^4*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*
c)*tan(1/2*a*d/b) - 8*b^4*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*
d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 4*a^4*d^2*imag_part(cos_integral(-d*x -
a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 8*b^4*x^2*imag_part(cos_
integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 32*a^2*b
^2*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1
/2*a*d/b) + 32*a^2*b^2*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*
x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^4*d^2*sin_integral((b*d*x + a*d)/b)*ta
n(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 16*b^4*x^2*sin_integral((b*d*x + a
*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 8*a*b^3*d*x^2*imag_part(c
os_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 8*a*b^3*d*x^2*imag_
part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 4*a^3*b*d^2*
x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 4*a^3*
b*d^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) -
 16*a*b^3*d*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b) -
 8*a^3*b*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2
*tan(1/2*a*d/b) + 8*a^3*b*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d
*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 8*a*b^3*x*real_part(cos_integral(d*x +
a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 8*a*b^3*x*real_part(co
s_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 16*a
^3*b*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*
d/b) - a^2*b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^
2 + a^2*b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2
- 2*a^2*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 - a^4*d^
2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*
b^4*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^
2 + a^4*d^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*
d/b)^2 - 2*b^4*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan
(1/2*a*d/b)^2 - 8*a^2*b^2*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*
d*x)^2*tan(1/2*a*d/b)^2 - 8*a^2*b^2*d*x*real_part(cos_integral(-d*x - a*d/b
))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*a^4*d^2*sin_integral((b*d*x + a*d)/b
)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 4*b^4*x^2*sin_integral((b*d*x + a*d)/b)
*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 8*a*b^3*d*x^2*imag_part(cos_integral(d*x
 + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 8*a*b^3*d*x^2*imag_part(cos_integr

al(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a^3*b*d^2*x*real_part(cos
_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a^3*b*d^2*x*real_pa
rt(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 16*a*b^3*d*x^2
*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b)^2 + 8*a^3*b*d*imag
_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2
 - 8*a^3*b*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c
)*tan(1/2*a*d/b)^2 + 8*a*b^3*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2
*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 8*a*b^3*x*real_part(cos_integral(-d*x
 - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 16*a^3*b*d*sin_inte
gral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + a^4*d^2*
imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*b^4*
x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^
4*d^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 +
 2*b^4*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b
)^2 + 8*a^2*b^2*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1
/2*a*d/b)^2 + 8*a^2*b^2*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c
)^2*tan(1/2*a*d/b)^2 + 2*a^4*d^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2
*tan(1/2*a*d/b)^2 - 4*b^4*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*ta
n(1/2*a*d/b)^2 + 2*a^3*b*d*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2
*a^2*b^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*t
an(1/2*a*d/b)^2 + 2*a^2*b^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d
*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*a^2*b^2*sin_integral((b*d*x + a*d)/
b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^3*b*d^2*x*imag_part(c
os_integral(d*x + a*d/b))*tan(1/2*d*x)^2 - 2*a^3*b*d^2*x*imag_part(cos_inte
gral(-d*x - a*d/b))*tan(1/2*d*x)^2 + 4*a*b^3*d*x^2*real_part(cos_integral(d
*x + a*d/b))*tan(1/2*d*x)^2 + 4*a*b^3*d*x^2*real_part(cos_integral(-d*x - a
*d/b))*tan(1/2*d*x)^2 + 4*a^3*b*d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2
*d*x)^2 + 2*a^2*b^2*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)
 + 2*a^2*b^2*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) - 16*
a^2*b^2*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)
+ 16*a^2*b^2*d*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1
/2*c) + 2*a^4*d^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1
/2*c) - 4*b^4*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1
/2*c) + 2*a^4*d^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(
1/2*c) - 4*b^4*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan
(1/2*c) - 32*a^2*b^2*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1
/2*c) - 2*a^3*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 + 2
*a^3*b*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 - 4*a*b^3*d
*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 - 4*a*b^3*d*x^2*real
_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 - 4*a^3*b*d^2*x*sin_integral
((b*d*x + a*d)/b)*tan(1/2*c)^2 + 2*a^2*b^2*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2
+ 4*a*b^3*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^
2 - 4*a*b^3*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*
c)^2 - 4*a^3*b*d*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/
2*c)^2 - 4*a^3*b*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan

(1/2*c)^2 + 8*a*b^3*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*b^2*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 2*a^2*b^2*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) + 16*a^2*b^2*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 16*a^2*b^2*d*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 2*a^4*d^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4*b^4*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 2*a^4*d^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4*b^4*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 32*a^2*b^2*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 8*a^3*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 8*a^3*b*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 16*a*b^3*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 16*a*b^3*d*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 16*a^3*b*d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) - 16*a*b^3*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 16*a*b^3*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 16*a^3*b*d*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 16*a^3*b*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 32*a*b^3*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 16*a^2*b^2*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 16*a^2*b^2*d*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^4*d^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 4*b^4*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^4*d^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 4*b^4*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 32*a^2*b^2*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b) - 4*a^2*b^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 4*a^2*b^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a^3*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 + 2*a^3*b*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - 4*a*b^3*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 - 4*a*b^3*d*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - 4*a^3*b*d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 - 2*a^2*b^2*d*x*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 4*a*b^3*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 4*a*b^3*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 4*a^3*b*d*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 4*a^3*b*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 8*a*b^3*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 16*a^2*b^2*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 16*a^2*b^2*d*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^4*d^2*real_part(cos_integral(d*x + a*d/b))*t an(1/2*c)*tan(1/2*a*d/b)^2 + 4*b^4*x^2*real_part(cos_integral(d*x + a*d/b)) *tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^4*d^2*real_part(cos_integral(-d*x - a*d/ b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*b^4*x^2*real_part(cos_integral(-d*x - a *d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 32*a^2*b^2*d*x*sin_integral((b*d*x + a *d)/b)*tan(1/2*c)*tan(1/2*a*d/b)^2 - 8*a^2*b^2*d*x*tan(1/2*d*x)*tan(1/2*c)* tan(1/2*a*d/b)^2 + 16*a*b^3*x*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a^2*b^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*ta n(1/2*a*d/b)^2 + 4*a^2*b^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d* x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^2*b^2*d*x*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*a*b^3*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a *d/b)^2 + 4*a*b^3*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a^3*b*d*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2* tan(1/2*a*d/b)^2 + 4*a^3*b*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2* c)^2*tan(1/2*a*d/b)^2 - 8*a*b^3*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^ 2*tan(1/2*a*d/b)^2 + 16*a*b^3*x*tan(1/2*d*x)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^2*b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b)) - a^2*b^2*d^2*x^2*im ag_part(cos_integral(-d*x - a*d/b)) + 2*a^2*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b) + a^4*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2 - 2*b^4*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2 - a^4*d^2*ima g_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 + 2*b^4*x^2*imag_part(cos _integral(-d*x - a*d/b))*tan(1/2*d*x)^2 + 8*a^2*b^2*d*x*real_part(cos_integ ral(d*x + a*d/b))*tan(1/2*d*x)^2 + 8*a^2*b^2*d*x*real_part(cos_integral(-d* x - a*d/b))*tan(1/2*d*x)^2 + 2*a^4*d^2*sin_integral((b*d*x + a*d)/b)*tan(1/ 2*d*x)^2 - 4*b^4*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2 - 8*a*b^3 *d*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 8*a*b^3*d*x^2*imag _part(cos_integral(-d*x - a*d/b))*tan(1/2*c) + 4*a^3*b*d^2*x*real_part(cos_ integral(d*x + a*d/b))*tan(1/2*c) + 4*a^3*b*d^2*x*real_part(cos_integral(-d *x - a*d/b))*tan(1/2*c) - 16*a*b^3*d*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c) - 8*a^3*b*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 8*a^3*b*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan (1/2*c) - 8*a*b^3*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan (1/2*c) - 8*a*b^3*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*ta n(1/2*c) - 16*a^3*b*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2* c) - a^4*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 + 2*b^4*x^2* imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 + a^4*d^2*imag_part(cos_i ntegral(-d*x - a*d/b))*tan(1/2*c)^2 - 2*b^4*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 - 8*a^2*b^2*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 - 8*a^2*b^2*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/ 2*c)^2 - 2*a^4*d^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2 + 4*b^4*x^2*s in_integral((b*d*x + a*d)/b)*tan(1/2*c)^2 + 2*a^3*b*d*tan(1/2*d*x)^2*tan(1/ 2*c)^2 + 2*a^2*b^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*b^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*t an(1/2*c)^2 + 4*a^2*b^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/ 2*c)^2 + 8*a*b^3*d*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 8*a*b^3*d*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) - 4*a^

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3*b*d^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 4*a^3*b*d^2
*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) + 16*a*b^3*d*x^2*si
n_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b) + 8*a^3*b*d*imag_part(cos_integr
al(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 8*a^3*b*d*imag_part(cos_in
tegral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 8*a*b^3*x*real_part(c
os_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 8*a*b^3*x*real_pa
rt(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 16*a^3*b*d*s
in_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4*a^4*d^2*imag
_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 8*b^4*x^2*imag
_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 4*a^4*d^2*imag
_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 8*b^4*x^2*ima
g_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 32*a^2*b^2*d
*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 32*a^2*
b^2*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 8
*a^4*d^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) - 16*b^4*x
^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) - 8*a^2*b^2*imag
_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) +
 8*a^2*b^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*
tan(1/2*a*d/b) - 16*a^2*b^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*ta
n(1/2*c)*tan(1/2*a*d/b) - 8*a^3*b*d*imag_part(cos_integral(d*x + a*d/b))*ta
n(1/2*c)^2*tan(1/2*a*d/b) + 8*a^3*b*d*imag_part(cos_integral(-d*x - a*d/b))
*tan(1/2*c)^2*tan(1/2*a*d/b) - 8*a*b^3*x*real_part(cos_integral(d*x + a*d/b
))*tan(1/2*c)^2*tan(1/2*a*d/b) - 8*a*b^3*x*real_part(cos_integral(-d*x - a*
d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 16*a^3*b*d*sin_integral((b*d*x + a*d)/b
)*tan(1/2*c)^2*tan(1/2*a*d/b) - a^4*d^2*imag_part(cos_integral(d*x + a*d/b)
)*tan(1/2*a*d/b)^2 + 2*b^4*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2
*a*d/b)^2 + a^4*d^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2
 - 2*b^4*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - 8*a^2*
b^2*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 - 8*a^2*b^2*d
*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - 2*a^4*d^2*sin_i
ntegral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 + 4*b^4*x^2*sin_integral((b*d*x +
 a*d)/b)*tan(1/2*a*d/b)^2 - 2*a^3*b*d*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*a
^2*b^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2
 - 2*a^2*b^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a
*d/b)^2 + 4*a^2*b^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*
d/b)^2 + 8*a^3*b*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*
a*d/b)^2 - 8*a^3*b*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1
/2*a*d/b)^2 + 8*a*b^3*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan
(1/2*a*d/b)^2 + 8*a*b^3*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*
tan(1/2*a*d/b)^2 + 16*a^3*b*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(
1/2*a*d/b)^2 - 8*a^3*b*d*tan(1/2*d*x)*tan(1/2*c)*tan(1/2*a*d/b)^2 + 12*a^2*
b^2*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^3*b*d*tan(1/2*c)^2*tan
(1/2*a*d/b)^2 - 2*a^2*b^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2
*tan(1/2*a*d/b)^2 + 2*a^2*b^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2
*c)^2*tan(1/2*a*d/b)^2 - 4*a^2*b^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)
```

^2*tan(1/2*a*d/b)^2 + 12*a^2*b^2*tan(1/2*d*x)*tan(1/2*c)^2*tan(1/2*a*d/b)^2
 + 2*a^3*b*d^2*x*imag_part(cos_integral(d*x + a*d/b)) - 2*a^3*b*d^2*x*imag_
part(cos_integral(-d*x - a*d/b)) + 4*a*b^3*d*x^2*real_part(cos_integral(d*x
 + a*d/b)) + 4*a*b^3*d*x^2*real_part(cos_integral(-d*x - a*d/b)) + 4*a^3*b*
d^2*x*sin_integral((b*d*x + a*d)/b) - 2*a^2*b^2*d*x*tan(1/2*d*x)^2 - 4*a*b^
3*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2 + 4*a*b^3*x*imag_pa
rt(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 + 4*a^3*b*d*real_part(cos_int
egral(d*x + a*d/b))*tan(1/2*d*x)^2 + 4*a^3*b*d*real_part(cos_integral(-d*x
- a*d/b))*tan(1/2*d*x)^2 - 8*a*b^3*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*
d*x)^2 - 16*a^2*b^2*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 1
6*a^2*b^2*d*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) + 2*a^4*d^2*
real_part(cos_integral(d*x + a*d/b))*tan(1/2*c) - 4*b^4*x^2*real_part(cos_i
ntegral(d*x + a*d/b))*tan(1/2*c) + 2*a^4*d^2*real_part(cos_integral(-d*x -
a*d/b))*tan(1/2*c) - 4*b^4*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/
2*c) - 32*a^2*b^2*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*c) - 8*a^2*b^2*
d*x*tan(1/2*d*x)*tan(1/2*c) + 16*a*b^3*x*tan(1/2*d*x)^2*tan(1/2*c) - 4*a^2*
b^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 4*a^2*
b^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2
*b^2*d*x*tan(1/2*c)^2 + 4*a*b^3*x*imag_part(cos_integral(d*x + a*d/b))*tan(
1/2*c)^2 - 4*a*b^3*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 - 4
*a^3*b*d*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 - 4*a^3*b*d*real
_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 + 8*a*b^3*x*sin_integral((b*
d*x + a*d)/b)*tan(1/2*c)^2 + 16*a*b^3*x*tan(1/2*d*x)*tan(1/2*c)^2 + 16*a^2*
b^2*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 16*a^2*b^2*d*
x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) - 2*a^4*d^2*real_par
t(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) + 4*b^4*x^2*real_part(cos_integ
ral(d*x + a*d/b))*tan(1/2*a*d/b) - 2*a^4*d^2*real_part(cos_integral(-d*x -
a*d/b))*tan(1/2*a*d/b) + 4*b^4*x^2*real_part(cos_integral(-d*x - a*d/b))*ta
n(1/2*a*d/b) + 32*a^2*b^2*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)
+ 4*a^2*b^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d
/b) + 4*a^2*b^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/
2*a*d/b) - 16*a*b^3*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1
/2*a*d/b) + 16*a*b^3*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan
(1/2*a*d/b) + 16*a^3*b*d*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*ta
n(1/2*a*d/b) + 16*a^3*b*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*
tan(1/2*a*d/b) - 32*a*b^3*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/
2*a*d/b) - 4*a^2*b^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(
1/2*a*d/b) - 4*a^2*b^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*t
an(1/2*a*d/b) + 2*a^2*b^2*d*x*tan(1/2*a*d/b)^2 + 4*a*b^3*x*imag_part(cos_in
tegral(d*x + a*d/b))*tan(1/2*a*d/b)^2 - 4*a*b^3*x*imag_part(cos_integral(-d
*x - a*d/b))*tan(1/2*a*d/b)^2 - 4*a^3*b*d*real_part(cos_integral(d*x + a*d/
b))*tan(1/2*a*d/b)^2 - 4*a^3*b*d*real_part(cos_integral(-d*x - a*d/b))*tan(
1/2*a*d/b)^2 + 8*a*b^3*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 - 1
6*a*b^3*x*tan(1/2*d*x)*tan(1/2*a*d/b)^2 - 16*a*b^3*x*tan(1/2*c)*tan(1/2*a*d
/b)^2 + 4*a^2*b^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a

```
*d/b)^2 + 4*a^2*b^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/
2*a*d/b)^2 + a^4*d^2*imag_part(cos_integral(d*x + a*d/b)) - 2*b^4*x^2*imag_
part(cos_integral(d*x + a*d/b)) - a^4*d^2*imag_part(cos_integral(-d*x - a*d
/b)) + 2*b^4*x^2*imag_part(cos_integral(-d*x - a*d/b)) + 8*a^2*b^2*d*x*real
_part(cos_integral(d*x + a*d/b)) + 8*a^2*b^2*d*x*real_part(cos_integral(-d*
x - a*d/b)) + 2*a^4*d^2*sin_integral((b*d*x + a*d)/b) - 4*b^4*x^2*sin_integ
ral((b*d*x + a*d)/b) - 2*a^3*b*d*tan(1/2*d*x)^2 - 2*a^2*b^2*imag_part(cos_i
ntegral(d*x + a*d/b))*tan(1/2*d*x)^2 + 2*a^2*b^2*imag_part(cos_integral(-d*
x - a*d/b))*tan(1/2*d*x)^2 - 4*a^2*b^2*sin_integral((b*d*x + a*d)/b)*tan(1/
2*d*x)^2 - 8*a^3*b*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 8*a^
3*b*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) - 8*a*b^3*x*real_par
t(cos_integral(d*x + a*d/b))*tan(1/2*c) - 8*a*b^3*x*real_part(cos_integral(
-d*x - a*d/b))*tan(1/2*c) - 16*a^3*b*d*sin_integral((b*d*x + a*d)/b)*tan(1/
2*c) - 8*a^3*b*d*tan(1/2*d*x)*tan(1/2*c) + 12*a^2*b^2*tan(1/2*d*x)^2*tan(1/
2*c) - 2*a^3*b*d*tan(1/2*c)^2 + 2*a^2*b^2*imag_part(cos_integral(d*x + a*d/
b))*tan(1/2*c)^2 - 2*a^2*b^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*
c)^2 + 4*a^2*b^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2 + 12*a^2*b^2*ta
n(1/2*d*x)*tan(1/2*c)^2 + 8*a^3*b*d*imag_part(cos_integral(d*x + a*d/b))*ta
n(1/2*a*d/b) - 8*a^3*b*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/
b) + 8*a*b^3*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) + 8*a*b^
3*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) + 16*a^3*b*d*sin_i
ntegral((b*d*x + a*d)/b)*tan(1/2*a*d/b) - 8*a^2*b^2*imag_part(cos_integral(
d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^2*b^2*imag_part(cos_integral(
-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 16*a^2*b^2*sin_integral((b*d*x +
 a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) + 2*a^3*b*d*tan(1/2*a*d/b)^2 + 2*a^2*b^2
*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 - 2*a^2*b^2*imag_par
t(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 + 4*a^2*b^2*sin_integral((b*
d*x + a*d)/b)*tan(1/2*a*d/b)^2 - 12*a^2*b^2*tan(1/2*d*x)*tan(1/2*a*d/b)^2 -
 12*a^2*b^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d*x - 4*a*b^3*x*imag_pa
rt(cos_integral(d*x + a*d/b)) + 4*a*b^3*x*imag_part(cos_integral(-d*x - a*d
/b)) + 4*a^3*b*d*real_part(cos_integral(d*x + a*d/b)) + 4*a^3*b*d*real_part
(cos_integral(-d*x - a*d/b)) - 8*a*b^3*x*sin_integral((b*d*x + a*d)/b) - 16
*a*b^3*x*tan(1/2*d*x) - 16*a*b^3*x*tan(1/2*c) - 4*a^2*b^2*real_part(cos_int
egral(d*x + a*d/b))*tan(1/2*c) - 4*a^2*b^2*real_part(cos_integral(-d*x - a*
d/b))*tan(1/2*c) + 4*a^2*b^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a
*d/b) + 4*a^2*b^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) + 2*
a^3*b*d - 2*a^2*b^2*imag_part(cos_integral(d*x + a*d/b)) + 2*a^2*b^2*imag_p
art(cos_integral(-d*x - a*d/b)) - 4*a^2*b^2*sin_integral((b*d*x + a*d)/b) -
 12*a^2*b^2*tan(1/2*d*x) - 12*a^2*b^2*tan(1/2*c))/(b^7*x^2*tan(1/2*d*x)^2*t
an(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*b^6*x*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/
2*a*d/b)^2 + b^7*x^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + b^7*x^2*tan(1/2*d*x)^2*t
an(1/2*a*d/b)^2 + b^7*x^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^2*b^5*tan(1/2*d
*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*b^6*x*tan(1/2*d*x)^2*tan(1/2*c)^2
 + 2*a*b^6*x*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*a*b^6*x*tan(1/2*c)^2*tan(1
/2*a*d/b)^2 + b^7*x^2*tan(1/2*d*x)^2 + b^7*x^2*tan(1/2*c)^2 + a^2*b^5*tan(1
```

```
/2*d*x)^2*tan(1/2*c)^2 + b^7*x^2*tan(1/2*a*d/b)^2 + a^2*b^5*tan(1/2*d*x)^2*
tan(1/2*a*d/b)^2 + a^2*b^5*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*b^6*x*tan(1/
2*d*x)^2 + 2*a*b^6*x*tan(1/2*c)^2 + 2*a*b^6*x*tan(1/2*a*d/b)^2 + b^7*x^2 +
a^2*b^5*tan(1/2*d*x)^2 + a^2*b^5*tan(1/2*c)^2 + a^2*b^5*tan(1/2*a*d/b)^2 +
2*a*b^6*x + a^2*b^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \sin(c+dx)}{(a+bx)^3} \, dx = \int \frac{x^2 \, \sin\left(c+d\,x\right)}{\left(a+b\,x\right)^3} \, dx$$

```
[In] int((x^2*sin(c + d*x))/(a + b*x)^3,x)
```

```
[Out] int((x^2*sin(c + d*x))/(a + b*x)^3, x)
```

3.35 $\int \frac{x\sin(c+dx)}{(a+bx)^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 179

$$\int \frac{x\sin(c+dx)}{(a+bx)^3}\,dx=\frac{ad\cos(c+dx)}{2b^3(a+bx)}+\frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^3}+\frac{ad^2\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{2b^4}+\frac{a\sin(c+dx)}{2b^2(a+bx)^2}-\frac{\sin(c+dx)}{b^2(a+bx)}+\frac{ad^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{2b^4}-\frac{d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^3}$$

```
[Out] d*Ci(a*d/b+d*x)*cos(-c+a*d/b)/b^3+1/2*a*d*cos(d*x+c)/b^3/(b*x+a)+1/2*a*d^2*
cos(-c+a*d/b)*Si(a*d/b+d*x)/b^4-1/2*a*d^2*Ci(a*d/b+d*x)*sin(-c+a*d/b)/b^4+d
*Si(a*d/b+d*x)*sin(-c+a*d/b)/b^3+1/2*a*sin(d*x+c)/b^2/(b*x+a)^2-sin(d*x+c)/
b^2/(b*x+a)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6874, 3378, 3384, 3380, 3383}

$$\int \frac{x\sin(c+dx)}{(a+bx)^3}\,dx=\frac{ad^2\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{2b^4}+\frac{ad^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{2b^4}+\frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{b^3}-\frac{d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b^3}+\frac{ad\cos(c+dx)}{2b^3(a+bx)}-\frac{\sin(c+dx)}{b^2(a+bx)}+\frac{a\sin(c+dx)}{2b^2(a+bx)^2}$$

```
[In] Int[(x*Sin[c + d*x])/(a + b*x)^3,x]
```

```
[Out] (a*d*Cos[c + d*x])/(2*b^3*(a + b*x)) + (d*Cos[c - (a*d)/b]*CosIntegral[(a*d
)/b + d*x])/b^3 + (a*d^2*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/(2*b^
4) + (a*Sin[c + d*x])/(2*b^2*(a + b*x)^2) - Sin[c + d*x]/(b^2*(a + b*x)) +
(a*d^2*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/(2*b^4) - (d*Sin[c - (a
*d)/b]*SinIntegral[(a*d)/b + d*x])/b^3
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{a\sin(c+dx)}{b(a+bx)^3}+\frac{\sin(c+dx)}{b(a+bx)^2}\right)dx \\
&= \frac{\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx}{b}-\frac{a\int \frac{\sin(c+dx)}{(a+bx)^3}\,dx}{b} \\
&= \frac{a\sin(c+dx)}{2b^2(a+bx)^2}-\frac{\sin(c+dx)}{b^2(a+bx)}+\frac{d\int \frac{\cos(c+dx)}{a+bx}\,dx}{b^2}-\frac{(ad)\int \frac{\cos(c+dx)}{(a+bx)^2}\,dx}{2b^2}
\end{aligned}$$

$$= \frac{ad\cos(c+dx)}{2b^3(a+bx)} + \frac{a\sin(c+dx)}{2b^2(a+bx)^2} - \frac{\sin(c+dx)}{b^2(a+bx)} + \frac{(ad^2)\int \frac{\sin(c+dx)}{a+bx}\,dx}{2b^3} + \frac{\left(d\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^2} - \frac{\left(d\sin\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^2}$$

$$= \frac{ad\cos(c+dx)}{2b^3(a+bx)} + \frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^3} + \frac{a\sin(c+dx)}{2b^2(a+bx)^2} - \frac{\sin(c+dx)}{b^2(a+bx)} - \frac{d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^3} + \frac{\left(ad^2\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{2b^3} + \frac{\left(ad^2\sin\left(c-\frac{ad}{b}\right)\right)\int \frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{2b^3}$$

$$= \frac{ad\cos(c+dx)}{2b^3(a+bx)} + \frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^3} + \frac{ad^2\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{2b^4} + \frac{a\sin(c+dx)}{2b^2(a+bx)^2} - \frac{\sin(c+dx)}{b^2(a+bx)} + \frac{ad^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{2b^4} - \frac{d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^3}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.88

$$\int \frac{x\sin(c+dx)}{(a+bx)^3}\,dx = \frac{b\cos(dx)(ad(a+bx)\cos(c)-b(a+2bx)\sin(c)) - b(b(a+2bx)\cos(c)+ad(a+bx)\sin(c))\sin(dx)+d(a+}{2}$$

```
[In] Integrate[(x*Sin[c + d*x])/(a + b*x)^3,x]
```

```
[Out] (b*Cos[d*x]*(a*d*(a + b*x)*Cos[c] - b*(a + 2*b*x)*Sin[c]) - b*(b*(a + 2*b*x
)*Cos[c] + a*d*(a + b*x)*Sin[c])*Sin[d*x] + d*(a + b*x)^2*(CosIntegral[d*(a
/b + x)]*(2*b*Cos[c - (a*d)/b] + a*d*Sin[c - (a*d)/b]) + (a*d*Cos[c - (a*d)
/b] - 2*b*Sin[c - (a*d)/b])*SinIntegral[d*(a/b + x)]))/(2*b^4*(a + b*x)^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(174) = 348$.

Time = 0.35 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.34

method	result
derivativedivides	$\frac{\frac{d^3\left(-\frac{\sin(dx+c)}{(da-cb+b(dx+c))b}+\frac{\frac{\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}+\frac{\mathrm{Ci}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}}{b}\right)}{b}-\frac{(da-cb)d^3\left(-\frac{\sin(dx+c)}{2(da-cb+b(dx+c))}\right.}{}}{}$
default	$\frac{\frac{d^3\left(-\frac{\sin(dx+c)}{(da-cb+b(dx+c))b}+\frac{\frac{\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}+\frac{\mathrm{Ci}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}}{b}\right)}{b}-\frac{(da-cb)d^3\left(-\frac{\sin(dx+c)}{2(da-cb+b(dx+c))}\right.}{}}{}$
risch	$\frac{i(2ia\,b^3d^3x^3+6ia^2b^2d^3x^2+6ia^3b\,d^3x+2ia^4d^3)\cos(dx+c)}{4b^3(bx+a)^2(-d^2x^2b^2-2ab\,d^2x-d^2a^2)}+\frac{(4b^4d^2x^3+10a\,b^3d^2x^2+8a^2b^2d^2x+2a^3b\,d^2)\sin(dx+c)}{4b^3(bx+a)^2(-d^2x^2b^2-2ab\,d^2x-d^2a^2)}-$

```
[In] int(x*sin(d*x+c)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(d^3/b*(-sin(d*x+c)/(d*a-c*b+b*(d*x+c))/b+(Si(d*x+c+(a*d-b*c)/b)*sin(
(a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)-(a*d-b*c)/b*d^3
*(-1/2*sin(d*x+c)/(d*a-c*b+b*(d*x+c))^2/b+1/2*(-cos(d*x+c)/(d*a-c*b+b*(d*x+
c))/b-(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((
a*d-b*c)/b)/b)/b)/b)-d^3*c*(-1/2*sin(d*x+c)/(d*a-c*b+b*(d*x+c))^2/b+1/2*(-c
os(d*x+c)/(d*a-c*b+b*(d*x+c))/b-(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-C
i(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)/b)/b))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.47

$$\int \frac{x\sin(c+dx)}{(a+bx)^3}\,dx = \frac{(ab^2dx+a^2bd)\cos(dx+c)+\left(2\left(b^3dx^2+2\,ab^2dx+a^2bd\right)\mathrm{Ci}\left(\frac{bdx+ad}{b}\right)+\left(ab^2d^2x^2+2\,a^2bd^2x+a^3d^2\right)\mathrm{Si}\left(\frac{b}{}\right.\right.}{}$$

```
[In] integrate(x*sin(d*x+c)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*((a*b^2*d*x + a^2*b*d)*cos(d*x + c) + (2*(b^3*d*x^2 + 2*a*b^2*d*x + a^2
*b*d)*cos_integral((b*d*x + a*d)/b) + (a*b^2*d^2*x^2 + 2*a^2*b*d^2*x + a^3*
```

```
d^2)*sin_integral((b*d*x + a*d)/b))*cos(-(b*c - a*d)/b) - (2*b^3*x + a*b^2)
*sin(d*x + c) - ((a*b^2*d^2*x^2 + 2*a^2*b*d^2*x + a^3*d^2)*cos_integral((b*
d*x + a*d)/b) - 2*(b^3*d*x^2 + 2*a*b^2*d*x + a^2*b*d)*sin_integral((b*d*x +
 a*d)/b))*sin(-(b*c - a*d)/b))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

Sympy [F]

$$\int \frac{x\sin(c+dx)}{(a+bx)^3}\,dx = \int \frac{x\sin{(c+dx)}}{(a+bx)^3}\,dx$$

```
[In] integrate(x*sin(d*x+c)/(b*x+a)**3,x)
```

```
[Out] Integral(x*sin(c + d*x)/(a + b*x)**3, x)
```

Maxima [F]

$$\int \frac{x\sin(c+dx)}{(a+bx)^3}\,dx = \int \frac{x\sin{(dx+c)}}{(bx+a)^3}\,dx$$

```
[In] integrate(x*sin(d*x+c)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*((b*cos(c)^2 + b*sin(c)^2)*x*cos(d*x + c) + ((a*(exp_integral_e(4, (I*
b*d*x + I*a*d)/b) + exp_integral_e(4, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a*(
exp_integral_e(4, (I*b*d*x + I*a*d)/b) + exp_integral_e(4, -(I*b*d*x + I*a*
d)/b))*sin(c)^2)*cos(-(b*c - a*d)/b) + (a*(I*exp_integral_e(4, (I*b*d*x + I
*a*d)/b) - I*exp_integral_e(4, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a*(I*exp_i
ntegral_e(4, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(4, -(I*b*d*x + I*a*d)/
b))*sin(c)^2)*sin(-(b*c - a*d)/b))*cos(d*x + c)^2 + ((a*(exp_integral_e(4,
(I*b*d*x + I*a*d)/b) + exp_integral_e(4, -(I*b*d*x + I*a*d)/b))*cos(c)^2 +
a*(exp_integral_e(4, (I*b*d*x + I*a*d)/b) + exp_integral_e(4, -(I*b*d*x + I
*a*d)/b))*sin(c)^2)*cos(-(b*c - a*d)/b) + (a*(I*exp_integral_e(4, (I*b*d*x
+ I*a*d)/b) - I*exp_integral_e(4, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a*(I*ex
p_integral_e(4, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(4, -(I*b*d*x + I*a*
d)/b))*sin(c)^2)*sin(-(b*c - a*d)/b))*sin(d*x + c)^2 + (b*x*cos(d*x + c)^2*
cos(c) + b*x*cos(c)*sin(d*x + c)^2)*cos(d*x + 2*c) + 4*(((b^5*cos(c)^2 + b^
5*sin(c)^2)*d*x^3 + 3*(a*b^4*cos(c)^2 + a*b^4*sin(c)^2)*d*x^2 + 3*(a^2*b^3*
cos(c)^2 + a^2*b^3*sin(c)^2)*d*x + (a^3*b^2*cos(c)^2 + a^3*b^2*sin(c)^2)*d)
*cos(d*x + c)^2 + ((b^5*cos(c)^2 + b^5*sin(c)^2)*d*x^3 + 3*(a*b^4*cos(c)^2
+ a*b^4*sin(c)^2)*d*x^2 + 3*(a^2*b^3*cos(c)^2 + a^2*b^3*sin(c)^2)*d*x + (a^
3*b^2*cos(c)^2 + a^3*b^2*sin(c)^2)*d)*sin(d*x + c)^2)*integrate(1/2*x*cos(d
*x + c)/(b^4*d*x^4 + 4*a*b^3*d*x^3 + 6*a^2*b^2*d*x^2 + 4*a^3*b*d*x + a^4*d)
, x) + 4*(((b^5*cos(c)^2 + b^5*sin(c)^2)*d*x^3 + 3*(a*b^4*cos(c)^2 + a*b^4*
```

```
sin(c)^2)*d*x^2 + 3*(a^2*b^3*cos(c)^2 + a^2*b^3*sin(c)^2)*d*x + (a^3*b^2*co
s(c)^2 + a^3*b^2*sin(c)^2)*d)*cos(d*x + c)^2 + ((b^5*cos(c)^2 + b^5*sin(c)^
2)*d*x^3 + 3*(a*b^4*cos(c)^2 + a*b^4*sin(c)^2)*d*x^2 + 3*(a^2*b^3*cos(c)^2
+ a^2*b^3*sin(c)^2)*d*x + (a^3*b^2*cos(c)^2 + a^3*b^2*sin(c)^2)*d)*sin(d*x
+ c)^2)*integrate(1/2*x*cos(d*x + c)/((b^4*d*x^4 + 4*a*b^3*d*x^3 + 6*a^2*b^
2*d*x^2 + 4*a^3*b*d*x + a^4*d)*cos(d*x + c)^2 + (b^4*d*x^4 + 4*a*b^3*d*x^3
+ 6*a^2*b^2*d*x^2 + 4*a^3*b*d*x + a^4*d)*sin(d*x + c)^2), x) + (b*x*cos(d*x
 + c)^2*sin(c) + b*x*sin(d*x + c)^2*sin(c))*sin(d*x + 2*c))/(((b^4*cos(c)^2
 + b^4*sin(c)^2)*d*x^3 + 3*(a*b^3*cos(c)^2 + a*b^3*sin(c)^2)*d*x^2 + 3*(a^2
*b^2*cos(c)^2 + a^2*b^2*sin(c)^2)*d*x + (a^3*b*cos(c)^2 + a^3*b*sin(c)^2)*d
)*cos(d*x + c)^2 + ((b^4*cos(c)^2 + b^4*sin(c)^2)*d*x^3 + 3*(a*b^3*cos(c)^2
 + a*b^3*sin(c)^2)*d*x^2 + 3*(a^2*b^2*cos(c)^2 + a^2*b^2*sin(c)^2)*d*x + (a
^3*b*cos(c)^2 + a^3*b*sin(c)^2)*d)*sin(d*x + c)^2)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.54 (sec) , antiderivative size = 10535, normalized size of antiderivative = 58.85

$$\int \frac{x\sin(c+dx)}{(a+bx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x*sin(d*x+c)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*(a*b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(
1/2*c)^2*tan(1/2*a*d/b)^2 - a*b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d
/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*b^2*d^2*x^2*sin_int
egral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*b
^2*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2
*tan(1/2*a*d/b) + 2*a*b^2*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan
(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a*b^2*d^2*x^2*real_part(cos_int
egral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a*b^2*d^
2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1
/2*a*d/b)^2 + 2*a^2*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*
x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^2*b*d^2*x*imag_part(cos_integral(-
d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^3*d*x^2*re
al_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/
b)^2 + 2*b^3*d*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan
(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a^2*b*d^2*x*sin_integral((b*d*x + a*d)/b)*ta
n(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a*b^2*d^2*x^2*imag_part(cos_in
tegral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + a*b^2*d^2*x^2*imag_part(
cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*b^2*d^2*x^2*s
in_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a*b^2*d^2*x^2*
imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/
b) - 4*a*b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*t
```

```
an(1/2*c)*tan(1/2*a*d/b) + 8*a*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b)*ta
n(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 4*b^3*d*x^2*imag_part(cos_integral
(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 4*b^3*d*x^2*ima
g_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/
b) + 4*a^2*b*d^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(
1/2*c)^2*tan(1/2*a*d/b) + 4*a^2*b*d^2*x*real_part(cos_integral(-d*x - a*d/b
))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 8*b^3*d*x^2*sin_integral((b
*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - a*b^2*d^2*x^2*i
mag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a*b^2
*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b
)^2 - 2*a*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*
a*d/b)^2 + 4*b^3*d*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*
tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*b^3*d*x^2*imag_part(cos_integral(-d*x - a*d
/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a^2*b*d^2*x*real_part(c
os_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a^
2*b*d^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*t
an(1/2*a*d/b)^2 + 8*b^3*d*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*
tan(1/2*c)*tan(1/2*a*d/b)^2 + a*b^2*d^2*x^2*imag_part(cos_integral(d*x + a*
d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a*b^2*d^2*x^2*imag_part(cos_integral(
-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*b^2*d^2*x^2*sin_integral
((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^3*d^2*imag_part(cos_int
egral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^3*d^2*
imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a
*d/b)^2 + 4*a*b^2*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*t
an(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a*b^2*d*x*real_part(cos_integral(-d*x - a*
d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^3*d^2*sin_integral
((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*b^2*d^
2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 2*a*
b^2*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)
 - 2*a^2*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/
2*c)^2 + 2*a^2*b*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2
*tan(1/2*c)^2 - 2*b^3*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*b^3*d*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1
/2*d*x)^2*tan(1/2*c)^2 - 4*a^2*b*d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/
2*d*x)^2*tan(1/2*c)^2 - 2*a*b^2*d^2*x^2*real_part(cos_integral(d*x + a*d/b)
)*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 2*a*b^2*d^2*x^2*real_part(cos_integral(-d
*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 8*a^2*b*d^2*x*imag_part(cos_in
tegral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 8*a^2*b*d^2
*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*
a*d/b) + 8*b^3*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*ta
n(1/2*c)*tan(1/2*a*d/b) + 8*b^3*d*x^2*real_part(cos_integral(-d*x - a*d/b))
*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 16*a^2*b*d^2*x*sin_integral((b*
d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 2*a*b^2*d^2*x^2*re
al_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a*b^2*d^
2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 8
```

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*a*b^2*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2
*tan(1/2*a*d/b) + 8*a*b^2*d*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2
*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^3*d^2*real_part(cos_integral(d*x
+ a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^3*d^2*real_part(
cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 16
*a*b^2*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/
2*a*d/b) - 2*a^2*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^
2*tan(1/2*a*d/b)^2 + 2*a^2*b*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*ta
n(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*b^3*d*x^2*real_part(cos_integral(d*x + a*
d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*b^3*d*x^2*real_part(cos_integral(
-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 4*a^2*b*d^2*x*sin_integral
((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*a*b^2*d^2*x^2*real_pa
rt(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a*b^2*d^2*x^2
*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 8*a*b^
2*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/
2*a*d/b)^2 - 8*a*b^2*d*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)
^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^3*d^2*real_part(cos_integral(d*x + a*d
/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^3*d^2*real_part(cos_i
ntegral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 16*a*b^
2*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b
)^2 + 2*a^2*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1
/2*a*d/b)^2 - 2*a^2*b*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c
)^2*tan(1/2*a*d/b)^2 + 2*b^3*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan
(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^3*d*x^2*real_part(cos_integral(-d*x - a*d/
b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a^2*b*d^2*x*sin_integral((b*d*x + a*d
)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*b^2*d*x*tan(1/2*d*x)^2*tan(1/2*c)^
2*tan(1/2*a*d/b)^2 + 2*a^2*b*d*real_part(cos_integral(d*x + a*d/b))*tan(1/2
*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b*d*real_part(cos_integral(-d
*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a*b^2*d^2*x^2*i
mag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2 - a*b^2*d^2*x^2*imag_par
t(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 + 2*a*b^2*d^2*x^2*sin_integral
((b*d*x + a*d)/b)*tan(1/2*d*x)^2 - 4*b^3*d*x^2*imag_part(cos_integral(d*x +
 a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 4*b^3*d*x^2*imag_part(cos_integral(-d*
x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^2*b*d^2*x*real_part(cos_integra
l(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^2*b*d^2*x*real_part(cos_int
egral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 8*b^3*d*x^2*sin_integral((
b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c) - a*b^2*d^2*x^2*imag_part(cos_int
egral(d*x + a*d/b))*tan(1/2*c)^2 + a*b^2*d^2*x^2*imag_part(cos_integral(-d*
x - a*d/b))*tan(1/2*c)^2 - 2*a*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b)*ta
n(1/2*c)^2 - a^3*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*ta
n(1/2*c)^2 + a^3*d^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*t
an(1/2*c)^2 - 4*a*b^2*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)
^2*tan(1/2*c)^2 - 4*a*b^2*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*a^3*d^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)
^2*tan(1/2*c)^2 + 4*b^3*d*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*
```

d*x)^2*tan(1/2*a*d/b) - 4*b^3*d*x^2*imag_part(cos_integral(-d*x - a*d/b))*t
an(1/2*d*x)^2*tan(1/2*a*d/b) - 4*a^2*b*d^2*x*real_part(cos_integral(d*x + a
*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 4*a^2*b*d^2*x*real_part(cos_integral
(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 8*b^3*d*x^2*sin_integral((b
*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4*a*b^2*d^2*x^2*imag_part(co
s_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 4*a*b^2*d^2*x^2*imag_p
art(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 8*a*b^2*d^2*x^2
*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) + 4*a^3*d^2*imag_p
art(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 4
*a^3*d^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*ta
n(1/2*a*d/b) + 16*a*b^2*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*
x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 16*a*b^2*d*x*real_part(cos_integral(-d*x -
 a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^3*d^2*sin_integral(
(b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 4*b^3*d*x^2*ima
g_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 4*b^3*d*x^2
*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 4*a^2*
b*d^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) +
4*a^2*b*d^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*
d/b) - 8*b^3*d*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b
) - 4*a^2*b*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c
)^2*tan(1/2*a*d/b) + 4*a^2*b*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/
2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 8*a^2*b*d*sin_integral((b*d*x + a*d)
/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - a*b^2*d^2*x^2*imag_part(co
s_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 + a*b^2*d^2*x^2*imag_part(cos_int
egral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - 2*a*b^2*d^2*x^2*sin_integral((b*d*x
 + a*d)/b)*tan(1/2*a*d/b)^2 - a^3*d^2*imag_part(cos_integral(d*x + a*d/b))*
tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a^3*d^2*imag_part(cos_integral(-d*x - a*d
/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 4*a*b^2*d*x*real_part(cos_integral(d
*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 4*a*b^2*d*x*real_part(cos_in
tegral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*a^3*d^2*sin_integ
ral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 4*b^3*d*x^2*imag_par
t(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*b^3*d*x^2*imag
_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a^2*b*d^2
*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a^2
*b*d^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2
 + 8*b^3*d*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4
*a^2*b*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan
(1/2*a*d/b)^2 - 4*a^2*b*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x
)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 8*a^2*b*d*sin_integral((b*d*x + a*d)/b)*t
an(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + a^3*d^2*imag_part(cos_integral(
d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^3*d^2*imag_part(cos_integra
l(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a*b^2*d*x*real_part(cos_
integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a*b^2*d*x*real_par
t(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^3*d^2*sin
_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b*d*tan(1/

2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b*d^2*x*imag_part(cos_integr
al(d*x + a*d/b))*tan(1/2*d*x)^2 - 2*a^2*b*d^2*x*imag_part(cos_integral(-d*x
 - a*d/b))*tan(1/2*d*x)^2 + 2*b^3*d*x^2*real_part(cos_integral(d*x + a*d/b)
)*tan(1/2*d*x)^2 + 2*b^3*d*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/
2*d*x)^2 + 4*a^2*b*d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2 + 2*a
*b^2*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 2*a*b^2*d^2*
x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) - 8*a*b^2*d*x*imag_par
t(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 8*a*b^2*d*x*imag_p
art(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 2*a^3*d^2*real_
part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 2*a^3*d^2*real_
part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 16*a*b^2*d*x*s
in_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*b*d^2*x*imag
_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 + 2*a^2*b*d^2*x*imag_part(cos
_integral(-d*x - a*d/b))*tan(1/2*c)^2 - 2*b^3*d*x^2*real_part(cos_integral(
d*x + a*d/b))*tan(1/2*c)^2 - 2*b^3*d*x^2*real_part(cos_integral(-d*x - a*d/
b))*tan(1/2*c)^2 - 4*a^2*b*d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2
 + 2*a*b^2*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*b*d*real_part(cos_integr
al(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*b*d*real_part(cos_inte
gral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*b^2*d^2*x^2*real_part
(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 2*a*b^2*d^2*x^2*real_part(cos_
integral(-d*x - a*d/b))*tan(1/2*a*d/b) + 8*a*b^2*d*x*imag_part(cos_integral
(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 8*a*b^2*d*x*imag_part(cos_in
tegral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 2*a^3*d^2*real_part(c
os_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 2*a^3*d^2*real_pa
rt(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 16*a*b^2*d*x
*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 8*a^2*b*d^2*
x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 8*a^2*b*
d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 8*b
^3*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 8
*b^3*d*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)
 + 16*a^2*b*d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) +
8*a^2*b*d*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*ta
n(1/2*a*d/b) + 8*a^2*b*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)
^2*tan(1/2*c)*tan(1/2*a*d/b) - 8*a*b^2*d*x*imag_part(cos_integral(d*x + a*d
/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 8*a*b^2*d*x*imag_part(cos_integral(-d*x
 - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^3*d^2*real_part(cos_integral(d*
x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^3*d^2*real_part(cos_integral(
-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 16*a*b^2*d*x*sin_integral((b*d
*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a^2*b*d^2*x*imag_part(cos_inte
gral(d*x + a*d/b))*tan(1/2*a*d/b)^2 + 2*a^2*b*d^2*x*imag_part(cos_integral(
-d*x - a*d/b))*tan(1/2*a*d/b)^2 - 2*b^3*d*x^2*real_part(cos_integral(d*x +
a*d/b))*tan(1/2*a*d/b)^2 - 2*b^3*d*x^2*real_part(cos_integral(-d*x - a*d/b)
)*tan(1/2*a*d/b)^2 - 4*a^2*b*d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*
d/b)^2 - 2*a*b^2*d*x*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*a^2*b*d*real_part(
cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*a^2*b*d*real

_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 8*a*b^2
*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 8*a
*b^2*d*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2
- 2*a^3*d^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^
2 - 2*a^3*d^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/
b)^2 + 16*a*b^2*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b)
^2 - 8*a*b^2*d*x*tan(1/2*d*x)*tan(1/2*c)*tan(1/2*a*d/b)^2 + 8*b^3*x*tan(1/2
*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a*b^2*d*x*tan(1/2*c)^2*tan(1/2*a*d/
b)^2 + 2*a^2*b*d*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*
a*d/b)^2 + 2*a^2*b*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan
(1/2*a*d/b)^2 + 8*b^3*x*tan(1/2*d*x)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a*b^2*
d^2*x^2*imag_part(cos_integral(d*x + a*d/b)) - a*b^2*d^2*x^2*imag_part(cos_
integral(-d*x - a*d/b)) + 2*a*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b) + a
^3*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2 - a^3*d^2*imag_p
art(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 + 4*a*b^2*d*x*real_part(cos_
integral(d*x + a*d/b))*tan(1/2*d*x)^2 + 4*a*b^2*d*x*real_part(cos_integral(
-d*x - a*d/b))*tan(1/2*d*x)^2 + 2*a^3*d^2*sin_integral((b*d*x + a*d)/b)*tan
(1/2*d*x)^2 - 4*b^3*d*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c) +
 4*b^3*d*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) + 4*a^2*b*d^2
*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 4*a^2*b*d^2*x*real_par
t(cos_integral(-d*x - a*d/b))*tan(1/2*c) - 8*b^3*d*x^2*sin_integral((b*d*x
+ a*d)/b)*tan(1/2*c) - 4*a^2*b*d*imag_part(cos_integral(d*x + a*d/b))*tan(1
/2*d*x)^2*tan(1/2*c) + 4*a^2*b*d*imag_part(cos_integral(-d*x - a*d/b))*tan(
1/2*d*x)^2*tan(1/2*c) - 8*a^2*b*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x
)^2*tan(1/2*c) - a^3*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2
+ a^3*d^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 - 4*a*b^2*d*x*
real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 - 4*a*b^2*d*x*real_part(c
os_integral(-d*x - a*d/b))*tan(1/2*c)^2 - 2*a^3*d^2*sin_integral((b*d*x + a
*d)/b)*tan(1/2*c)^2 + 2*a^2*b*d*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*b^3*d*x^2*i
mag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 4*b^3*d*x^2*imag_part(
cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) - 4*a^2*b*d^2*x*real_part(cos_in
tegral(d*x + a*d/b))*tan(1/2*a*d/b) - 4*a^2*b*d^2*x*real_part(cos_integral(
-d*x - a*d/b))*tan(1/2*a*d/b) + 8*b^3*d*x^2*sin_integral((b*d*x + a*d)/b)*t
an(1/2*a*d/b) + 4*a^2*b*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)
^2*tan(1/2*a*d/b) - 4*a^2*b*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2
*d*x)^2*tan(1/2*a*d/b) + 8*a^2*b*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*
x)^2*tan(1/2*a*d/b) + 4*a^3*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/
2*c)*tan(1/2*a*d/b) - 4*a^3*d^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1
/2*c)*tan(1/2*a*d/b) + 16*a*b^2*d*x*real_part(cos_integral(d*x + a*d/b))*ta
n(1/2*c)*tan(1/2*a*d/b) + 16*a*b^2*d*x*real_part(cos_integral(-d*x - a*d/b)
)*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^3*d^2*sin_integral((b*d*x + a*d)/b)*tan(1
/2*c)*tan(1/2*a*d/b) - 4*a^2*b*d*imag_part(cos_integral(d*x + a*d/b))*tan(1
/2*c)^2*tan(1/2*a*d/b) + 4*a^2*b*d*imag_part(cos_integral(-d*x - a*d/b))*ta
n(1/2*c)^2*tan(1/2*a*d/b) - 8*a^2*b*d*sin_integral((b*d*x + a*d)/b)*tan(1/2
*c)^2*tan(1/2*a*d/b) - a^3*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2

*a*d/b)^2 + a^3*d^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2
- 4*a*b^2*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 - 4*a*b
^2*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - 2*a^3*d^2*s
in_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 - 2*a^2*b*d*tan(1/2*d*x)^2*ta
n(1/2*a*d/b)^2 + 4*a^2*b*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*
tan(1/2*a*d/b)^2 - 4*a^2*b*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*
c)*tan(1/2*a*d/b)^2 + 8*a^2*b*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*ta
n(1/2*a*d/b)^2 - 8*a^2*b*d*tan(1/2*d*x)*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a*b
^2*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^2*b*d*tan(1/2*c)^2*tan(
1/2*a*d/b)^2 + 4*a*b^2*tan(1/2*d*x)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b
*d^2*x*imag_part(cos_integral(d*x + a*d/b)) - 2*a^2*b*d^2*x*imag_part(cos_i
ntegral(-d*x - a*d/b)) + 2*b^3*d*x^2*real_part(cos_integral(d*x + a*d/b)) +
 2*b^3*d*x^2*real_part(cos_integral(-d*x - a*d/b)) + 4*a^2*b*d^2*x*sin_inte
gral((b*d*x + a*d)/b) - 2*a*b^2*d*x*tan(1/2*d*x)^2 + 2*a^2*b*d*real_part(co
s_integral(d*x + a*d/b))*tan(1/2*d*x)^2 + 2*a^2*b*d*real_part(cos_integral(
-d*x - a*d/b))*tan(1/2*d*x)^2 - 8*a*b^2*d*x*imag_part(cos_integral(d*x + a*
d/b))*tan(1/2*c) + 8*a*b^2*d*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/
2*c) + 2*a^3*d^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 2*a^3*d^
2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) - 16*a*b^2*d*x*sin_integ
ral((b*d*x + a*d)/b)*tan(1/2*c) - 8*a*b^2*d*x*tan(1/2*d*x)*tan(1/2*c) + 8*b
^3*x*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*b^2*d*x*tan(1/2*c)^2 - 2*a^2*b*d*real_
part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 - 2*a^2*b*d*real_part(cos_inte
gral(-d*x - a*d/b))*tan(1/2*c)^2 + 8*b^3*x*tan(1/2*d*x)*tan(1/2*c)^2 + 8*a*
b^2*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 8*a*b^2*d*x*i
mag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) - 2*a^3*d^2*real_part(c
os_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 2*a^3*d^2*real_part(cos_integral
(-d*x - a*d/b))*tan(1/2*a*d/b) + 16*a*b^2*d*x*sin_integral((b*d*x + a*d)/b)
*tan(1/2*a*d/b) + 8*a^2*b*d*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)
*tan(1/2*a*d/b) + 8*a^2*b*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c
)*tan(1/2*a*d/b) + 2*a*b^2*d*x*tan(1/2*a*d/b)^2 - 2*a^2*b*d*real_part(cos_i
ntegral(d*x + a*d/b))*tan(1/2*a*d/b)^2 - 2*a^2*b*d*real_part(cos_integral(-
d*x - a*d/b))*tan(1/2*a*d/b)^2 - 8*b^3*x*tan(1/2*d*x)*tan(1/2*a*d/b)^2 - 8*
b^3*x*tan(1/2*c)*tan(1/2*a*d/b)^2 + a^3*d^2*imag_part(cos_integral(d*x + a*
d/b)) - a^3*d^2*imag_part(cos_integral(-d*x - a*d/b)) + 4*a*b^2*d*x*real_pa
rt(cos_integral(d*x + a*d/b)) + 4*a*b^2*d*x*real_part(cos_integral(-d*x - a
*d/b)) + 2*a^3*d^2*sin_integral((b*d*x + a*d)/b) - 2*a^2*b*d*tan(1/2*d*x)^2
 - 4*a^2*b*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 4*a^2*b*d*im
ag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) - 8*a^2*b*d*sin_integral((b*
d*x + a*d)/b)*tan(1/2*c) - 8*a^2*b*d*tan(1/2*d*x)*tan(1/2*c) + 4*a*b^2*tan(
1/2*d*x)^2*tan(1/2*c) - 2*a^2*b*d*tan(1/2*c)^2 + 4*a*b^2*tan(1/2*d*x)*tan(1
/2*c)^2 + 4*a^2*b*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 4
*a^2*b*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) + 8*a^2*b*d*s
in_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b) + 2*a^2*b*d*tan(1/2*a*d/b)^2 -
4*a*b^2*tan(1/2*d*x)*tan(1/2*a*d/b)^2 - 4*a*b^2*tan(1/2*c)*tan(1/2*a*d/b)^2
 + 2*a*b^2*d*x + 2*a^2*b*d*real_part(cos_integral(d*x + a*d/b)) + 2*a^2*b*d

```
*real_part(cos_integral(-d*x - a*d/b)) - 8*b^3*x*tan(1/2*d*x) - 8*b^3*x*tan
(1/2*c) + 2*a^2*b*d - 4*a*b^2*tan(1/2*d*x) - 4*a*b^2*tan(1/2*c))/(b^6*x^2*t
an(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*b^5*x*tan(1/2*d*x)^2*tan(
1/2*c)^2*tan(1/2*a*d/b)^2 + b^6*x^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + b^6*x^2*t
an(1/2*d*x)^2*tan(1/2*a*d/b)^2 + b^6*x^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^
2*b^4*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*b^5*x*tan(1/2*d*x)
^2*tan(1/2*c)^2 + 2*a*b^5*x*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*a*b^5*x*tan
(1/2*c)^2*tan(1/2*a*d/b)^2 + b^6*x^2*tan(1/2*d*x)^2 + b^6*x^2*tan(1/2*c)^2
+ a^2*b^4*tan(1/2*d*x)^2*tan(1/2*c)^2 + b^6*x^2*tan(1/2*a*d/b)^2 + a^2*b^4*
tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a^2*b^4*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2
*a*b^5*x*tan(1/2*d*x)^2 + 2*a*b^5*x*tan(1/2*c)^2 + 2*a*b^5*x*tan(1/2*a*d/b)
^2 + b^6*x^2 + a^2*b^4*tan(1/2*d*x)^2 + a^2*b^4*tan(1/2*c)^2 + a^2*b^4*tan(
1/2*a*d/b)^2 + 2*a*b^5*x + a^2*b^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sin(c+dx)}{(a+bx)^3}\,dx = \int \frac{x\,\sin\left(c+d\,x\right)}{{\left(a+b\,x\right)}^3}\,dx$$

```
[In] int((x*sin(c + d*x))/(a + b*x)^3,x)
[Out] int((x*sin(c + d*x))/(a + b*x)^3, x)
```

3.36 $\int \frac{\sin(c+dx)}{(a+bx)^3} dx$

Optimal result

Integrand size = 14, antiderivative size = 104

$$\int \frac{\sin(c+dx)}{(a+bx)^3} dx = -\frac{d\cos(c+dx)}{2b^2(a+bx)} - \frac{d^2\operatorname{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{2b^3} - \frac{\sin(c+dx)}{2b(a+bx)^2} - \frac{d^2\cos\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(\frac{ad}{b}+dx\right)}{2b^3}$$

```
[Out] -1/2*d*cos(d*x+c)/b^2/(b*x+a)-1/2*d^2*cos(-c+a*d/b)*Si(a*d/b+d*x)/b^3+1/2*d
^2*Ci(a*d/b+d*x)*sin(-c+a*d/b)/b^3-1/2*sin(d*x+c)/b/(b*x+a)^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3378, 3384, 3380, 3383}

$$\int \frac{\sin(c+dx)}{(a+bx)^3} dx = -\frac{d^2\sin\left(c-\frac{ad}{b}\right)\operatorname{CosIntegral}\left(xd+\frac{ad}{b}\right)}{2b^3} - \frac{d^2\cos\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(xd+\frac{ad}{b}\right)}{2b^3} - \frac{d\cos(c+dx)}{2b^2(a+bx)} - \frac{\sin(c+dx)}{2b(a+bx)^2}$$

```
[In] Int[Sin[c + d*x]/(a + b*x)^3,x]
```

```
[Out] -1/2*(d*Cos[c + d*x])/(b^2*(a + b*x)) - (d^2*CosIntegral[(a*d)/b + d*x]*Sin
[c - (a*d)/b])/(2*b^3) - Sin[c + d*x]/(2*b*(a + b*x)^2) - (d^2*Cos[c - (a*d
)/b]*SinIntegral[(a*d)/b + d*x])/(2*b^3)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sin(c+dx)}{2b(a+bx)^2}+\frac{d\int\frac{\cos(c+dx)}{(a+bx)^2}\,dx}{2b}\\
&= -\frac{d\cos(c+dx)}{2b^2(a+bx)}-\frac{\sin(c+dx)}{2b(a+bx)^2}-\frac{d^2\int\frac{\sin(c+dx)}{a+bx}\,dx}{2b^2}\\
&= -\frac{d\cos(c+dx)}{2b^2(a+bx)}-\frac{\sin(c+dx)}{2b(a+bx)^2}-\frac{\left(d^2\cos\left(c-\frac{ad}{b}\right)\right)\int\frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{2b^2}\\
&\quad-\frac{\left(d^2\sin\left(c-\frac{ad}{b}\right)\right)\int\frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{2b^2}\\
&= -\frac{d\cos(c+dx)}{2b^2(a+bx)}-\frac{d^2\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{2b^3}\\
&\quad-\frac{\sin(c+dx)}{2b(a+bx)^2}-\frac{d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{2b^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int \frac{\sin(c+dx)}{(a+bx)^3}\,dx = -\frac{d^2\,\text{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)\sin\left(c-\frac{ad}{b}\right)+\frac{b(d(a+bx)\cos(c+dx)+b\sin(c+dx))}{(a+bx)^2}+d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(d\left(\frac{a}{b}+x\right)\right)}{2b^3}$$

```
[In] Integrate[Sin[c + d*x]/(a + b*x)^3,x]
```

```
[Out] -1/2*(d^2*CosIntegral[d*(a/b + x)]*Sin[c - (a*d)/b] + (b*(d*(a + b*x)*Cos[c
 + d*x] + b*Sin[c + d*x]))/(a + b*x)^2 + d^2*Cos[c - (a*d)/b]*SinIntegral[d
*(a/b + x)])/b^3
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.39

method	result
derivativedivides	$d^2\left(-\frac{\sin(dx+c)}{2(da-cb+b(dx+c))^2 b}+\frac{-\frac{\cos(dx+c)}{(da-cb+b(dx+c))b}-\frac{\frac{\text{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}-\frac{\text{Ci}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}}{b}}{2b}\right)$
default	$d^2\left(-\frac{\sin(dx+c)}{2(da-cb+b(dx+c))^2 b}+\frac{-\frac{\cos(dx+c)}{(da-cb+b(dx+c))b}-\frac{\frac{\text{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}-\frac{\text{Ci}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}}{b}}{2b}\right)$
risch	$-\frac{i d^2 e^{-\frac{i(da-cb)}{b}}\,\text{Ei}_1\left(-idx-ic-\frac{iad-icb}{b}\right)}{4b^3}+\frac{i d^2 e^{\frac{i(da-cb)}{b}}\,\text{Ei}_1\left(idx+ic+\frac{i(da-cb)}{b}\right)}{4b^3}+\frac{i(-2ib^3d^3x^3-6iab^2d^3x^2-6ia\cdots}{4b^2(bx+a)^2(-d^2x^2b^2\cdots}$

```
[In] int(sin(d*x+c)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^2*(-1/2*sin(d*x+c)/(d*a-c*b+b*(d*x+c))^2/b+1/2*(-cos(d*x+c)/(d*a-c*b+b*(d
*x+c))/b-(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*si
n((a*d-b*c)/b)/b)/b)/b)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.58

$$\int \frac{\sin(c+dx)}{(a+bx)^3}\,dx = -\frac{b^2\sin(dx+c) - (b^2d^2x^2 + 2\,abd^2x + a^2d^2)\,\mathrm{Ci}\left(\frac{bdx+ad}{b}\right)\sin\left(-\frac{bc-ad}{b}\right) + (b^2d^2x^2 + 2\,abd^2x + a^2d^2)\cos\left(-\frac{b}{}\right.}{2\,(b^5x^2 + 2\,ab^4x + a^2b^3)}$$

[In] integrate(sin(d*x+c)/(b*x+a)^3,x, algorithm="fricas")

[Out] -1/2*(b^2*sin(d*x + c) - (b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*cos_integral((b*d*x + a*d)/b)*sin(-(b*c - a*d)/b) + (b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*cos(-(b*c - a*d)/b)*sin_integral((b*d*x + a*d)/b) + (b^2*d*x + a*b*d)*cos(d*x + c))/(b^5*x^2 + 2*a*b^4*x + a^2*b^3)

Sympy [F]

$$\int \frac{\sin(c+dx)}{(a+bx)^3}\,dx = \int \frac{\sin(c+dx)}{(a+bx)^3}\,dx$$

[In] integrate(sin(d*x+c)/(b*x+a)**3,x)

[Out] Integral(sin(c + d*x)/(a + b*x)**3, x)

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.91

$$\int \frac{\sin(c+dx)}{(a+bx)^3}\,dx = \frac{d^3\left(-i\,E_3\left(\frac{i\,(dx+c)b - i\,bc + i\,ad}{b}\right) + i\,E_3\left(-\frac{i\,(dx+c)b - i\,bc + i\,ad}{b}\right)\right)\cos\left(-\frac{bc-ad}{b}\right) + d^3\left(E_3\left(\frac{i\,(dx+c)b - i\,bc + i\,ad}{b}\right) + E_3\left(-\frac{i\,(}{}\right.\right.}{2\left((dx+c)^2b^3 + b^3c^2 - 2\,ab^2cd + a^2bd^2 - 2\,(b^3c - ab^2d)(dx+c)\right)d}$$

[In] integrate(sin(d*x+c)/(b*x+a)^3,x, algorithm="maxima")

[Out] 1/2*(d^3*(-I*exp_integral_e(3, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + I*exp_integral_e(3, -(I*(d*x + c)*b - I*b*c + I*a*d)/b))*cos(-(b*c - a*d)/b) + d^3*(exp_integral_e(3, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + exp_integral_e(3, -(I*(d*x + c)*b - I*b*c + I*a*d)/b))*sin(-(b*c - a*d)/b))/(((d*x + c)^2*b^3 + b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2 - 2*(b^3*c - a*b^2*d)*(d*x + c))*d)

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.41 (sec) , antiderivative size = 5727, normalized size of antiderivative = 55.07

$$\int \frac{\sin(c+dx)}{(a+bx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)/(b*x+a)^3,x, algorithm="giac")
[Out] -1/4*(b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1
/2*c)^2*tan(1/2*a*d/b)^2 - b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b)
)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^2*d^2*x^2*sin_integral
((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^2*d^2*
x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/
2*a*d/b) + 2*b^2*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)
^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*b^2*d^2*x^2*real_part(cos_integral(d*x +
 a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*b^2*d^2*x^2*real_pa
rt(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 +
 2*a*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)
^2*tan(1/2*a*d/b)^2 - 2*a*b*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan
(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a*b*d^2*x*sin_integral((b*d*x
 + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - b^2*d^2*x^2*imag_
part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + b^2*d^2*x^2*i
mag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*b^2*d^
2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*b^2*d^2
*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2
*a*d/b) - 4*b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^
2*tan(1/2*c)*tan(1/2*a*d/b) + 8*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b)*t
an(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 4*a*b*d^2*x*real_part(cos_integra
l(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 4*a*b*d^2*x*re
al_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d
/b) - b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1
/2*a*d/b)^2 + b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x
)^2*tan(1/2*a*d/b)^2 - 2*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*
d*x)^2*tan(1/2*a*d/b)^2 - 4*a*b*d^2*x*real_part(cos_integral(d*x + a*d/b))*
tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a*b*d^2*x*real_part(cos_inte
gral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + b^2*d^2*x^
2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - b^2*
d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2
 + 2*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^
2 + a^2*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^
2*tan(1/2*a*d/b)^2 - a^2*d^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*
d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*d^2*sin_integral((b*d*x + a*d)
/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^2*d^2*x^2*real_part(
```

cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 2*b^2*d^2*x^2*real_p
art(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*b*d^2*x*ima
g_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*b*d^2*x
*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a*b*
d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*b^2*d^2
*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 2
*b^2*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a
*d/b) + 8*a*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan
(1/2*c)*tan(1/2*a*d/b) - 8*a*b*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*
tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 16*a*b*d^2*x*sin_integral((b*d*x
 + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 2*b^2*d^2*x^2*real_pa
rt(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*b^2*d^2*x^2*r
eal_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^2*d^
2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*
a*d/b) + 2*a^2*d^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan
(1/2*c)^2*tan(1/2*a*d/b) - 2*a*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))
*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*a*b*d^2*x*imag_part(cos_integral(-d*x
- a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 4*a*b*d^2*x*sin_integral((b*d*x
 + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*b^2*d^2*x^2*real_part(cos_in
tegral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*b^2*d^2*x^2*real_part(
cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^2*d^2*real_pa
rt(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 -
2*a^2*d^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*t
an(1/2*a*d/b)^2 + 2*a*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*
c)^2*tan(1/2*a*d/b)^2 - 2*a*b*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*t
an(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a*b*d^2*x*sin_integral((b*d*x + a*d)/b)*ta
n(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^2*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2
*a*d/b)^2 + b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2
 - b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 + 2*b^2
*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2 + 4*a*b*d^2*x*real_pa
rt(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a*b*d^2*x*real_
part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - b^2*d^2*x^2*im
ag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 + b^2*d^2*x^2*imag_part(cos
_integral(-d*x - a*d/b))*tan(1/2*c)^2 - 2*b^2*d^2*x^2*sin_integral((b*d*x +
 a*d)/b)*tan(1/2*c)^2 - a^2*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/
2*d*x)^2*tan(1/2*c)^2 + a^2*d^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1
/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*d^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*
x)^2*tan(1/2*c)^2 - 4*a*b*d^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/
2*d*x)^2*tan(1/2*a*d/b) - 4*a*b*d^2*x*real_part(cos_integral(-d*x - a*d/b))
*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4*b^2*d^2*x^2*imag_part(cos_integral(d*x +
 a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 4*b^2*d^2*x^2*imag_part(cos_integral(-
d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 8*b^2*d^2*x^2*sin_integral((b*d*x
 + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) + 4*a^2*d^2*imag_part(cos_integral(d*x
 + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 4*a^2*d^2*imag_part(c
os_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^2

*d^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)
 + 4*a*b*d^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*
d/b) + 4*a*b*d^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1
/2*a*d/b) - b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)
^2 + b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - 2
*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 - a^2*d^2*imag_
part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a^2*d^2*i
mag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*a^
2*d^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 4*a*b
*d^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4
*a*b*d^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^
2 + a^2*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b
)^2 - a^2*d^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*
d/b)^2 + 2*a^2*d^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b
)^2 + 2*a*b*d*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*b*d^2*x*im
ag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2 - 2*a*b*d^2*x*imag_part(c
os_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 + 4*a*b*d^2*x*sin_integral((b*d*x
 + a*d)/b)*tan(1/2*d*x)^2 + 2*b^2*d^2*x^2*real_part(cos_integral(d*x + a*d/
b))*tan(1/2*c) + 2*b^2*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/
2*c) + 2*a^2*d^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/
2*c) + 2*a^2*d^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1
/2*c) - 2*a*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 + 2*a
*b*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 - 4*a*b*d^2*x*s
in_integral((b*d*x + a*d)/b)*tan(1/2*c)^2 + 2*b^2*d*x*tan(1/2*d*x)^2*tan(1/
2*c)^2 - 2*b^2*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)
 - 2*b^2*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) - 2*a^
2*d^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) -
2*a^2*d^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/
b) + 8*a*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*
d/b) - 8*a*b*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2
*a*d/b) + 16*a*b*d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d
/b) + 2*a^2*d^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a
*d/b) + 2*a^2*d^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/
2*a*d/b) - 2*a*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^
2 + 2*a*b*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - 4*
a*b*d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 - 2*b^2*d*x*tan(1/
2*d*x)^2*tan(1/2*a*d/b)^2 - 2*a^2*d^2*real_part(cos_integral(d*x + a*d/b))*
tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^2*d^2*real_part(cos_integral(-d*x - a*d/b
))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 8*b^2*d*x*tan(1/2*d*x)*tan(1/2*c)*tan(1/2*
a*d/b)^2 - 2*b^2*d*x*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + b^2*d^2*x^2*imag_part(
cos_integral(d*x + a*d/b)) - b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d/
b)) + 2*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b) + a^2*d^2*imag_part(cos_i
ntegral(d*x + a*d/b))*tan(1/2*d*x)^2 - a^2*d^2*imag_part(cos_integral(-d*x
 - a*d/b))*tan(1/2*d*x)^2 + 2*a^2*d^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*
d*x)^2 + 4*a*b*d^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 4*a*

```
b*d^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) - a^2*d^2*imag_par
t(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 + a^2*d^2*imag_part(cos_integral(
-d*x - a*d/b))*tan(1/2*c)^2 - 2*a^2*d^2*sin_integral((b*d*x + a*d)/b)*tan(1
/2*c)^2 + 2*a*b*d*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a*b*d^2*x*real_part(cos_i
ntegral(d*x + a*d/b))*tan(1/2*a*d/b) - 4*a*b*d^2*x*real_part(cos_integral(-
d*x - a*d/b))*tan(1/2*a*d/b) + 4*a^2*d^2*imag_part(cos_integral(d*x + a*d/b
))*tan(1/2*c)*tan(1/2*a*d/b) - 4*a^2*d^2*imag_part(cos_integral(-d*x - a*d/
b))*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^2*d^2*sin_integral((b*d*x + a*d)/b)*tan
(1/2*c)*tan(1/2*a*d/b) - a^2*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1
/2*a*d/b)^2 + a^2*d^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^
2 - 2*a^2*d^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 - 2*a*b*d*tan(
1/2*d*x)^2*tan(1/2*a*d/b)^2 - 8*a*b*d*tan(1/2*d*x)*tan(1/2*c)*tan(1/2*a*d/b
)^2 - 4*b^2*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a*b*d*tan(1/2*c)
^2*tan(1/2*a*d/b)^2 - 4*b^2*tan(1/2*d*x)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*
a*b*d^2*x*imag_part(cos_integral(d*x + a*d/b)) - 2*a*b*d^2*x*imag_part(cos_
integral(-d*x - a*d/b)) + 4*a*b*d^2*x*sin_integral((b*d*x + a*d)/b) - 2*b^2
*d*x*tan(1/2*d*x)^2 + 2*a^2*d^2*real_part(cos_integral(d*x + a*d/b))*tan(1/
2*c) + 2*a^2*d^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) - 8*b^2*d
*x*tan(1/2*d*x)*tan(1/2*c) - 2*b^2*d*x*tan(1/2*c)^2 - 2*a^2*d^2*real_part(c
os_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 2*a^2*d^2*real_part(cos_integral
(-d*x - a*d/b))*tan(1/2*a*d/b) + 2*b^2*d*x*tan(1/2*a*d/b)^2 + a^2*d^2*imag_
part(cos_integral(d*x + a*d/b)) - a^2*d^2*imag_part(cos_integral(-d*x - a*d
/b)) + 2*a^2*d^2*sin_integral((b*d*x + a*d)/b) - 2*a*b*d*tan(1/2*d*x)^2 - 8
*a*b*d*tan(1/2*d*x)*tan(1/2*c) - 4*b^2*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*b*d*
tan(1/2*c)^2 - 4*b^2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*a*b*d*tan(1/2*a*d/b)^2 +
 4*b^2*tan(1/2*d*x)*tan(1/2*a*d/b)^2 + 4*b^2*tan(1/2*c)*tan(1/2*a*d/b)^2 +
2*b^2*d*x + 2*a*b*d + 4*b^2*tan(1/2*d*x) + 4*b^2*tan(1/2*c))/(b^5*x^2*tan(1
/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*b^4*x*tan(1/2*d*x)^2*tan(1/2*
c)^2*tan(1/2*a*d/b)^2 + b^5*x^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + b^5*x^2*tan(1
/2*d*x)^2*tan(1/2*a*d/b)^2 + b^5*x^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^2*b^
3*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*b^4*x*tan(1/2*d*x)^2*t
an(1/2*c)^2 + 2*a*b^4*x*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*a*b^4*x*tan(1/2
*c)^2*tan(1/2*a*d/b)^2 + b^5*x^2*tan(1/2*d*x)^2 + b^5*x^2*tan(1/2*c)^2 + a^
2*b^3*tan(1/2*d*x)^2*tan(1/2*c)^2 + b^5*x^2*tan(1/2*a*d/b)^2 + a^2*b^3*tan(
1/2*d*x)^2*tan(1/2*a*d/b)^2 + a^2*b^3*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*b
^4*x*tan(1/2*d*x)^2 + 2*a*b^4*x*tan(1/2*c)^2 + 2*a*b^4*x*tan(1/2*a*d/b)^2 +
 b^5*x^2 + a^2*b^3*tan(1/2*d*x)^2 + a^2*b^3*tan(1/2*c)^2 + a^2*b^3*tan(1/2*
a*d/b)^2 + 2*a*b^4*x + a^2*b^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{(a+bx)^3}\,dx = \int \frac{\sin\left(c+d\,x\right)}{\left(a+b\,x\right)^3}\,dx$$

```
[In] int(sin(c + d*x)/(a + b*x)^3,x)
```

```
[Out] int(sin(c + d*x)/(a + b*x)^3, x)
```

3.37 $\int \frac{\sin(c+dx)}{x(a+bx)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 261

$$\begin{aligned}\int \frac{\sin(c+dx)}{x(a+bx)^3}\,dx &= \frac{d\cos(c+dx)}{2ab(a+bx)} - \frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^2b} \\ &\quad + \frac{\text{CosIntegral}(dx)\sin(c)}{a^3} - \frac{\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^3} \\ &\quad + \frac{d^2\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{2ab^2} + \frac{\sin(c+dx)}{2a(a+bx)^2} \\ &\quad + \frac{\sin(c+dx)}{a^2(a+bx)} + \frac{\cos(c)\text{Si}(dx)}{a^3} - \frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^3} \\ &\quad + \frac{d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{2ab^2} + \frac{d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^2b}\end{aligned}$$

```
[Out] -d*Ci(a*d/b+d*x)*cos(-c+a*d/b)/a^2/b+1/2*d*cos(d*x+c)/a/b/(b*x+a)+cos(c)*Si
(d*x)/a^3-cos(-c+a*d/b)*Si(a*d/b+d*x)/a^3+1/2*d^2*cos(-c+a*d/b)*Si(a*d/b+d*
x)/a/b^2+Ci(d*x)*sin(c)/a^3+Ci(a*d/b+d*x)*sin(-c+a*d/b)/a^3-1/2*d^2*Ci(a*d/
b+d*x)*sin(-c+a*d/b)/a/b^2-d*Si(a*d/b+d*x)*sin(-c+a*d/b)/a^2/b+1/2*sin(d*x+
c)/a/(b*x+a)^2+sin(d*x+c)/a^2/(b*x+a)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6874, 3384, 3380, 3383, 3378}

$$\int \frac{\sin(c+dx)}{x(a+bx)^3}\,dx = -\frac{\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{a^3} - \frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^3} + \frac{\sin(c)\,\text{CosIntegral}(dx)}{a^3} + \frac{\cos(c)\text{Si}(dx)}{a^3} - \frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{a^2b} + \frac{d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^2b} + \frac{\sin(c+dx)}{a^2(a+bx)} + \frac{d^2\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{2ab^2} + \frac{d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{2ab^2} + \frac{\sin(c+dx)}{2a(a+bx)^2} + \frac{d\cos(c+dx)}{2ab(a+bx)}$$

```
[In] Int[Sin[c + d*x]/(x*(a + b*x)^3),x]
```

```
[Out] (d*Cos[c + d*x])/(2*a*b*(a + b*x)) - (d*Cos[c - (a*d)/b]*CosIntegral[(a*d)/
b + d*x])/(a^2*b) + (CosIntegral[d*x]*Sin[c])/a^3 - (CosIntegral[(a*d)/b +
d*x]*Sin[c - (a*d)/b])/a^3 + (d^2*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/
b])/(2*a*b^2) + Sin[c + d*x]/(2*a*(a + b*x)^2) + Sin[c + d*x]/(a^2*(a + b*x
)) + (Cos[c]*SinIntegral[d*x])/a^3 - (Cos[c - (a*d)/b]*SinIntegral[(a*d)/b
+ d*x])/a^3 + (d^2*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/(2*a*b^2) +
 (d*Sin[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/(a^2*b)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\sin(c+dx)}{a^3x}-\frac{b\sin(c+dx)}{a(a+bx)^3}-\frac{b\sin(c+dx)}{a^2(a+bx)^2}-\frac{b\sin(c+dx)}{a^3(a+bx)}\right)dx \\
&= \frac{\int \frac{\sin(c+dx)}{x}\,dx}{a^3}-\frac{b\int \frac{\sin(c+dx)}{a+bx}\,dx}{a^3}-\frac{b\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx}{a^2}-\frac{b\int \frac{\sin(c+dx)}{(a+bx)^3}\,dx}{a} \\
&= \frac{\sin(c+dx)}{2a(a+bx)^2}+\frac{\sin(c+dx)}{a^2(a+bx)}-\frac{d\int \frac{\cos(c+dx)}{a+bx}\,dx}{a^2}-\frac{d\int \frac{\cos(c+dx)}{(a+bx)^2}\,dx}{2a}+\frac{\cos(c)\int \frac{\sin(dx)}{x}\,dx}{a^3} \\
&\quad -\frac{\left(b\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^3}+\frac{\sin(c)\int \frac{\cos(dx)}{x}\,dx}{a^3}-\frac{\left(b\sin\left(c-\frac{ad}{b}\right)\right)\int \frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^3} \\
&= \frac{d\cos(c+dx)}{2ab(a+bx)}+\frac{\text{CosIntegral}(dx)\sin(c)}{a^3}-\frac{\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^3} \\
&\quad +\frac{\sin(c+dx)}{2a(a+bx)^2}+\frac{\sin(c+dx)}{a^2(a+bx)}+\frac{\cos(c)\text{Si}(dx)}{a^3} \\
&\quad -\frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^3}+\frac{d^2\int \frac{\sin(c+dx)}{a+bx}\,dx}{2ab} \\
&\quad -\frac{\left(d\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^2}+\frac{\left(d\sin\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^2} \\
&= \frac{d\cos(c+dx)}{2ab(a+bx)}-\frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^2b}+\frac{\text{CosIntegral}(dx)\sin(c)}{a^3} \\
&\quad -\frac{\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^3}+\frac{\sin(c+dx)}{2a(a+bx)^2}+\frac{\sin(c+dx)}{a^2(a+bx)} \\
&\quad +\frac{\cos(c)\text{Si}(dx)}{a^3}-\frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^3}+\frac{d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^2b} \\
&\quad +\frac{\left(d^2\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{2ab}+\frac{\left(d^2\sin\left(c-\frac{ad}{b}\right)\right)\int \frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{2ab}
\end{aligned}$$

$$= \frac{d\cos(c+dx)}{2ab(a+bx)} - \frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^2b} + \frac{\text{CosIntegral}(dx)\sin(c)}{a^3} - \frac{\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^3} + \frac{d^2\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{2ab^2} + \frac{\sin(c+dx)}{2a(a+bx)^2} + \frac{\sin(c+dx)}{a^2(a+bx)} + \frac{\cos(c)\text{Si}(dx)}{a^3} - \frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^3} + \frac{d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{2ab^2} + \frac{d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^2b}$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.72

$$\int \frac{\sin(c+dx)}{x(a+bx)^3}\,dx = \frac{a^3bd\cos(c+dx) + a^2b^2dx\cos(c+dx) + 2b^2(a+bx)^2\,\text{CosIntegral}(dx)\sin(c) + (a+bx)^2\,\text{CosIntegral}\left(d\left(\frac{a}{b}\right.\right.}{}$$

```
[In] Integrate[Sin[c + d*x]/(x*(a + b*x)^3),x]
```

```
[Out] (a^3*b*d*Cos[c + d*x] + a^2*b^2*d*x*Cos[c + d*x] + 2*b^2*(a + b*x)^2*CosInt
egral[d*x]*Sin[c] + (a + b*x)^2*CosIntegral[d*(a/b + x)]*(-2*a*b*d*Cos[c -
(a*d)/b] + (-2*b^2 + a^2*d^2)*Sin[c - (a*d)/b]) + 3*a^2*b^2*Sin[c + d*x] +
2*a*b^3*x*Sin[c + d*x] + 2*a^2*b^2*Cos[c]*SinIntegral[d*x] + 4*a*b^3*x*Cos[
c]*SinIntegral[d*x] + 2*b^4*x^2*Cos[c]*SinIntegral[d*x] - 2*a^2*b^2*Cos[c -
 (a*d)/b]*SinIntegral[d*(a/b + x)] + a^4*d^2*Cos[c - (a*d)/b]*SinIntegral[d
*(a/b + x)] - 4*a*b^3*x*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)] + 2*a^3*b
*d^2*x*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)] - 2*b^4*x^2*Cos[c - (a*d)/
b]*SinIntegral[d*(a/b + x)] + a^2*b^2*d^2*x^2*Cos[c - (a*d)/b]*SinIntegral[
d*(a/b + x)] + 2*a^3*b*d*Sin[c - (a*d)/b]*SinIntegral[d*(a/b + x)] + 4*a^2*
b^2*d*x*Sin[c - (a*d)/b]*SinIntegral[d*(a/b + x)] + 2*a*b^3*d*x^2*Sin[c - (
a*d)/b]*SinIntegral[d*(a/b + x)])/(2*a^3*b^2*(a + b*x)^2)
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.38

method	result
derivativedivides	$\frac{\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin(c)}{a^3} - \frac{db\left(-\frac{\sin(dx+c)}{(da-cb+b(dx+c))b} + \frac{\frac{\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b} + \frac{\mathrm{Ci}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}}{b}\right)}{a^2}$
default	$\frac{\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin(c)}{a^3} - \frac{db\left(-\frac{\sin(dx+c)}{(da-cb+b(dx+c))b} + \frac{\frac{\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b} + \frac{\mathrm{Ci}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}}{b}\right)}{a^2}$
risch	$-\frac{i e^{-ic}\,\mathrm{Ei}_1(-idx)}{2a^3} + \frac{d\,e^{-\frac{i(da-cb)}{b}}\,\mathrm{Ei}_1\left(-idx-ic-\frac{iad-icb}{b}\right)}{2b\,a^2} - \frac{i e^{-\frac{i(da-cb)}{b}}\,\mathrm{Ei}_1\left(-idx-ic-\frac{iad-icb}{b}\right)}{2a^3} - \frac{i e^{\frac{i(da-cb)}{b}}\,\mathrm{E}}{}$

```
[In] int(sin(d*x+c)/x/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))-d*b/a^2*(-sin(d*x+c)/(d*a-c*b+b*(d*x+
c))/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((
a*d-b*c)/b)/b)/b)-d^2*b/a*(-1/2*sin(d*x+c)/(d*a-c*b+b*(d*x+c))^2/b+1/2*(-co
s(d*x+c)/(d*a-c*b+b*(d*x+c))/b-(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci
(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)/b)/b)-b/a^3*(Si(d*x+c+(a*d-b*c)/b)*
cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.50

$$\int \frac{\sin(c+dx)}{x(a+bx)^3}\,dx$$
$$= \frac{2\left(b^4x^2+2\,ab^3x+a^2b^2\right)\mathrm{Ci}\left(dx\right)\sin\left(c\right)+2\left(b^4x^2+2\,ab^3x+a^2b^2\right)\cos\left(c\right)\mathrm{Si}\left(dx\right)+\left(a^2b^2dx+a^3bd\right)\cos\left(dx+\right.}{}$$

```
[In] integrate(sin(d*x+c)/x/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(b^4*x^2 + 2*a*b^3*x + a^2*b^2)*cos_integral(d*x)*sin(c) + 2*(b^4*x^
2 + 2*a*b^3*x + a^2*b^2)*cos(c)*sin_integral(d*x) + (a^2*b^2*d*x + a^3*b*d)
*cos(d*x + c) - (2*(a*b^3*d*x^2 + 2*a^2*b^2*d*x + a^3*b*d)*cos_integral((b*
d*x + a*d)/b) - (a^4*d^2 - 2*a^2*b^2 + (a^2*b^2*d^2 - 2*b^4)*x^2 + 2*(a^3*b
*d^2 - 2*a*b^3)*x)*sin_integral((b*d*x + a*d)/b))*cos(-(b*c - a*d)/b) + (2*
a*b^3*x + 3*a^2*b^2)*sin(d*x + c) - ((a^4*d^2 - 2*a^2*b^2 + (a^2*b^2*d^2 -
2*b^4)*x^2 + 2*(a^3*b*d^2 - 2*a*b^3)*x)*cos_integral((b*d*x + a*d)/b) + 2*(
a*b^3*d*x^2 + 2*a^2*b^2*d*x + a^3*b*d)*sin_integral((b*d*x + a*d)/b))*sin(-
(b*c - a*d)/b))/(a^3*b^4*x^2 + 2*a^4*b^3*x + a^5*b^2)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{x(a+bx)^3}\,dx = \int \frac{\sin(c+dx)}{x(a+bx)^3}\,dx$$

```
[In] integrate(sin(d*x+c)/x/(b*x+a)**3,x)
[Out] Integral(sin(c + d*x)/(x*(a + b*x)**3), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x(a+bx)^3}\,dx = \int \frac{\sin(dx+c)}{(bx+a)^3 x}\,dx$$

```
[In] integrate(sin(d*x+c)/x/(b*x+a)^3,x, algorithm="maxima")
[Out] integrate(sin(d*x + c)/((b*x + a)^3*x), x)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.67 (sec) , antiderivative size = 17806, normalized size of antiderivative = 68.22

$$\int \frac{\sin(c+dx)}{x(a+bx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)/x/(b*x+a)^3,x, algorithm="giac")
[Out] 1/4*(a^2*b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*ta
n(1/2*c)^2*tan(1/2*a*d/b)^2 - a^2*b^2*d^2*x^2*imag_part(cos_integral(-d*x -
 a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d^2*x^2*s
in_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 +
 2*a^2*b^2*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(
1/2*c)^2*tan(1/2*a*d/b) + 2*a^2*b^2*d^2*x^2*real_part(cos_integral(-d*x - a
*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a^2*b^2*d^2*x^2*real_
part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2
- 2*a^2*b^2*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*ta
n(1/2*c)*tan(1/2*a*d/b)^2 + 2*a^3*b*d^2*x*imag_part(cos_integral(d*x + a*d/
b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^3*b*d^2*x*imag_part(
cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 -
2*a*b^3*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c
)^2*tan(1/2*a*d/b)^2 - 2*a*b^3*d*x^2*real_part(cos_integral(-d*x - a*d/b))*
```

tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a^3*b*d^2*x*sin_integral((
b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^2*b^2*d^2*
x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + a^2*
b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)
^2 - 2*a^2*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2
*c)^2 + 4*a^2*b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)
^2*tan(1/2*c)*tan(1/2*a*d/b) - 4*a^2*b^2*d^2*x^2*imag_part(cos_integral(-d*
x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^2*b^2*d^2*x^2*si
n_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 4*a*
b^3*d*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*
tan(1/2*a*d/b) - 4*a*b^3*d*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/
2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 4*a^3*b*d^2*x*real_part(cos_integral
(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 4*a^3*b*d^2*x*r
eal_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*
d/b) + 8*a*b^3*d*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c
)^2*tan(1/2*a*d/b) - a^2*b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*t
an(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a^2*b^2*d^2*x^2*imag_part(cos_integral(-d*
x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*a^2*b^2*d^2*x^2*sin_integra
l((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 4*a*b^3*d*x^2*imag_par
t(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4
*a*b^3*d*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c
)*tan(1/2*a*d/b)^2 - 4*a^3*b*d^2*x*real_part(cos_integral(d*x + a*d/b))*tan
(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a^3*b*d^2*x*real_part(cos_integ
ral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 8*a*b^3*d*x
^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2
 + a^2*b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/
2*a*d/b)^2 - a^2*b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*
c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan
(1/2*c)^2*tan(1/2*a*d/b)^2 + a^4*d^2*imag_part(cos_integral(d*x + a*d/b))*t
an(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*b^4*x^2*imag_part(cos_integ
ral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*b^4*x^2*
imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 -
 a^4*d^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*
tan(1/2*a*d/b)^2 + 2*b^4*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*
d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^4*x^2*imag_part(cos_integral(-d*
x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*a^2*b^2*d*x*real_part(
cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4
*a^2*b^2*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c
)^2*tan(1/2*a*d/b)^2 - 4*b^4*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c
)^2*tan(1/2*a*d/b)^2 + 2*a^4*d^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)
^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*b^4*x^2*sin_integral((b*d*x + a*d)/b)*
tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d^2*x^2*real_part(
cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 2*a^2*b^2*d^2*x^2*re
al_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^3*b*d^2
*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^3

*b*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*b^3*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*b^3*d*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a^3*b*d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*b^2*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 2*a^2*b^2*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 8*a^3*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 8*a^3*b*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 8*a*b^3*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 8*a*b^3*d*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 16*a^3*b*d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 2*a^2*b^2*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^2*b^2*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 8*a^2*b^2*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 8*a^2*b^2*d*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^4*d^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 4*b^4*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^4*d^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 4*b^4*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 16*a^2*b^2*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a^3*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*a^3*b*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*a*b^3*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*a*b^3*d*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 4*a^3*b*d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*a^2*b^2*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^2*b^2*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 8*a^2*b^2*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 8*a^2*b^2*d*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^4*d^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*b^4*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*b^4*x^2*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^4*d^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*b^4*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*b^4*x^2*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 16*a^2*b^2*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*a^3*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^3*b*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a*b^3*d*x^2*re

al_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a*b^3*
d*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 +
4*a^3*b*d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2
+ 2*a^2*b^2*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*a*b^3*x*im
ag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/
b)^2 - 4*a*b^3*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*t
an(1/2*a*d/b)^2 + 4*a*b^3*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d
*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a*b^3*x*imag_part(cos_integral(-d*x
))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^3*b*d*real_part(cos_i
ntegral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^3*
b*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1
/2*a*d/b)^2 - 8*a*b^3*x*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1
/2*a*d/b)^2 - 8*a*b^3*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/
2*c)^2*tan(1/2*a*d/b)^2 + a^2*b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/
b))*tan(1/2*d*x)^2 - a^2*b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*
tan(1/2*d*x)^2 + 2*a^2*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*
x)^2 + 4*a*b^3*d*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*ta
n(1/2*c) - 4*a*b^3*d*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)
^2*tan(1/2*c) + 4*a^3*b*d^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*
d*x)^2*tan(1/2*c) + 4*a^3*b*d^2*x*real_part(cos_integral(-d*x - a*d/b))*tan
(1/2*d*x)^2*tan(1/2*c) + 8*a*b^3*d*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/
2*d*x)^2*tan(1/2*c) - a^2*b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*
tan(1/2*c)^2 + a^2*b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/
2*c)^2 - 2*a^2*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2 - a^4
*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*b
^4*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2
*b^4*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + a^4*d^2
*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*b^4*
x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*b
^4*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a^2*b^
2*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*
a^2*b^2*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)
^2 - 4*b^4*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^4*d^2*si
n_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*b^4*x^2*sin_int
egral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a*b^3*d*x^2*imag_par
t(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4*a*b^3*d*x^2*
imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 4*a^3
*b*d^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)
- 4*a^3*b*d^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1
/2*a*d/b) - 8*a*b^3*d*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(
1/2*a*d/b) + 4*a^2*b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2
*c)*tan(1/2*a*d/b) - 4*a^2*b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b)
)*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^2*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/
b)*tan(1/2*c)*tan(1/2*a*d/b) + 4*a^4*d^2*imag_part(cos_integral(d*x + a*d/b
))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 8*b^4*x^2*imag_part(cos_integ

```
ral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 4*a^4*d^2*imag
_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)
+ 8*b^4*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)
*tan(1/2*a*d/b) - 16*a^2*b^2*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1
/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 16*a^2*b^2*d*x*real_part(cos_integral
(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^4*d^2*sin_in
tegral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 16*b^4*x
^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) +
 4*a*b^3*d*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*
d/b) - 4*a*b^3*d*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan
(1/2*a*d/b) + 4*a^3*b*d^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)
^2*tan(1/2*a*d/b) + 4*a^3*b*d^2*x*real_part(cos_integral(-d*x - a*d/b))*tan
(1/2*c)^2*tan(1/2*a*d/b) + 8*a*b^3*d*x^2*sin_integral((b*d*x + a*d)/b)*tan(
1/2*c)^2*tan(1/2*a*d/b) + 4*a^3*b*d*imag_part(cos_integral(d*x + a*d/b))*ta
n(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 4*a^3*b*d*imag_part(cos_integral
(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 8*a*b^3*x*real
_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)
 - 8*a*b^3*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c
)^2*tan(1/2*a*d/b) + 8*a^3*b*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2
*tan(1/2*c)^2*tan(1/2*a*d/b) - a^2*b^2*d^2*x^2*imag_part(cos_integral(d*x +
 a*d/b))*tan(1/2*a*d/b)^2 + a^2*b^2*d^2*x^2*imag_part(cos_integral(-d*x - a
*d/b))*tan(1/2*a*d/b)^2 - 2*a^2*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b)*t
an(1/2*a*d/b)^2 - a^4*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)
^2*tan(1/2*a*d/b)^2 + 2*b^4*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/
2*d*x)^2*tan(1/2*a*d/b)^2 + 2*b^4*x^2*imag_part(cos_integral(d*x))*tan(1/2*
d*x)^2*tan(1/2*a*d/b)^2 + a^4*d^2*imag_part(cos_integral(-d*x - a*d/b))*tan
(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*b^4*x^2*imag_part(cos_integral(-d*x - a*d/
b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*b^4*x^2*imag_part(cos_integral(-d*x
))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 4*a^2*b^2*d*x*real_part(cos_integral(d
*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 4*a^2*b^2*d*x*real_part(cos_
integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 4*b^4*x^2*sin_int
egral(d*x)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*a^4*d^2*sin_integral((b*d*x
+ a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 4*b^4*x^2*sin_integral((b*d*x +
 a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 4*a*b^3*d*x^2*imag_part(cos_inte
gral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a*b^3*d*x^2*imag_part(co
s_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a^3*b*d^2*x*real_
part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a^3*b*d^2*x
*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 8*a*b^
3*d*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a^3*b
*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a
*d/b)^2 + 4*a^3*b*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*ta
n(1/2*c)*tan(1/2*a*d/b)^2 + 8*a*b^3*x*real_part(cos_integral(d*x + a*d/b))*
tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 8*a*b^3*x*real_part(cos_integr
al(d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 8*a*b^3*x*real_part(c
os_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 8*a
```

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*b^3*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/
b)^2 - 8*a^3*b*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*ta
n(1/2*a*d/b)^2 + a^4*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*
tan(1/2*a*d/b)^2 - 2*b^4*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c
)^2*tan(1/2*a*d/b)^2 - 2*b^4*x^2*imag_part(cos_integral(d*x))*tan(1/2*c)^2*
tan(1/2*a*d/b)^2 - a^4*d^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)
^2*tan(1/2*a*d/b)^2 + 2*b^4*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1
/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^4*x^2*imag_part(cos_integral(-d*x))*tan(1/2*
c)^2*tan(1/2*a*d/b)^2 - 4*a^2*b^2*d*x*real_part(cos_integral(d*x + a*d/b))*
tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*a^2*b^2*d*x*real_part(cos_integral(-d*x -
 a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*b^4*x^2*sin_integral(d*x)*tan(1/
2*c)^2*tan(1/2*a*d/b)^2 + 2*a^4*d^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c
)^2*tan(1/2*a*d/b)^2 - 4*b^4*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2
*tan(1/2*a*d/b)^2 + 2*a^3*b*d*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2
- 2*a^2*b^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^
2*tan(1/2*a*d/b)^2 - 2*a^2*b^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*
tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*imag_part(cos_integral(-d*x - a*d
/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*imag_part(cos
_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*a^2*b^2*s
in_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*a^2*b^2*s
in_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 +
 2*a^3*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2 - 2*a^3*
b*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 - 2*a*b^3*d*x^
2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2 - 2*a*b^3*d*x^2*real_
part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 + 4*a^3*b*d^2*x*sin_integra
l((b*d*x + a*d)/b)*tan(1/2*d*x)^2 + 2*a^2*b^2*d^2*x^2*real_part(cos_integra
l(d*x + a*d/b))*tan(1/2*c) + 2*a^2*b^2*d^2*x^2*real_part(cos_integral(-d*x
- a*d/b))*tan(1/2*c) + 8*a^2*b^2*d*x*imag_part(cos_integral(d*x + a*d/b))*t
an(1/2*d*x)^2*tan(1/2*c) - 8*a^2*b^2*d*x*imag_part(cos_integral(-d*x - a*d/
b))*tan(1/2*d*x)^2*tan(1/2*c) + 2*a^4*d^2*real_part(cos_integral(d*x + a*d/
b))*tan(1/2*d*x)^2*tan(1/2*c) - 4*b^4*x^2*real_part(cos_integral(d*x + a*d/
b))*tan(1/2*d*x)^2*tan(1/2*c) + 4*b^4*x^2*real_part(cos_integral(d*x))*tan(
1/2*d*x)^2*tan(1/2*c) + 2*a^4*d^2*real_part(cos_integral(-d*x - a*d/b))*tan
(1/2*d*x)^2*tan(1/2*c) - 4*b^4*x^2*real_part(cos_integral(-d*x - a*d/b))*ta
n(1/2*d*x)^2*tan(1/2*c) + 4*b^4*x^2*real_part(cos_integral(-d*x))*tan(1/2*d
*x)^2*tan(1/2*c) + 16*a^2*b^2*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x
)^2*tan(1/2*c) - 2*a^3*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2
*c)^2 + 2*a^3*b*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 +
2*a*b^3*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 + 2*a*b^3*d
*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 - 4*a^3*b*d^2*x*sin
_integral((b*d*x + a*d)/b)*tan(1/2*c)^2 + 2*a^2*b^2*d*x*tan(1/2*d*x)^2*tan(
1/2*c)^2 + 4*a*b^3*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*ta
n(1/2*c)^2 - 4*a*b^3*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*
c)^2 - 4*a*b^3*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1
/2*c)^2 + 4*a*b^3*x*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)
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^2 + 2*a^3*b*d*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^3*b*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 8*a*b^3*x*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 8*a*b^3*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*b^2*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 2*a^2*b^2*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) - 8*a^2*b^2*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 8*a^2*b^2*d*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 2*a^4*d^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4*b^4*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 2*a^4*d^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4*b^4*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 16*a^2*b^2*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 8*a^3*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 8*a^3*b*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 8*a*b^3*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 8*a*b^3*d*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 16*a^3*b*d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) - 16*a*b^3*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 16*a*b^3*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 8*a^3*b*d*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 8*a^3*b*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 32*a*b^3*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^2*b^2*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 8*a^2*b^2*d*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^4*d^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 4*b^4*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^4*d^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 4*b^4*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 16*a^2*b^2*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b) - 4*a^2*b^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 4*a^2*b^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a^3*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 + 2*a^3*b*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 + 2*a*b^3*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 + 2*a*b^3*d*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - 4*a^3*b*d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 - 2*a^2*b^2*d*x*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 4*a*b^3*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 4*a*b^3*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 4*a*b^3*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 4*a*b^3*x*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*a^3*b*d*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*a^3*b*d*real_part(cos_

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integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 8*a*b^3*x*sin_int
egral(d*x)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 8*a*b^3*x*sin_integral((b*d*x
+ a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 8*a^2*b^2*d*x*imag_part(cos_int
egral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 8*a^2*b^2*d*x*imag_part(c
os_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^4*d^2*real_par
t(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*b^4*x^2*real_p
art(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*b^4*x^2*real
_part(cos_integral(d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^4*d^2*real_part(
cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*b^4*x^2*real_pa
rt(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*b^4*x^2*real
_part(cos_integral(-d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 16*a^2*b^2*d*x*sin_
integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b)^2 - 8*a^2*b^2*d*x*tan(1
/2*d*x)*tan(1/2*c)*tan(1/2*a*d/b)^2 - 8*a*b^3*x*tan(1/2*d*x)^2*tan(1/2*c)*t
an(1/2*a*d/b)^2 + 4*a^2*b^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*
x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a^2*b^2*real_part(cos_integral(d*x))*t
an(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a^2*b^2*real_part(cos_integra
l(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a^2*b^2*rea
l_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a
^2*b^2*d*x*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*a*b^3*x*imag_part(cos_integral
(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*a*b^3*x*imag_part(cos_inte
gral(d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a*b^3*x*imag_part(cos_integral
(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a*b^3*x*imag_part(cos_int
egral(-d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^3*b*d*real_part(cos_integr
al(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^3*b*d*real_part(cos_in
tegral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 8*a*b^3*x*sin_integra
l(d*x)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 8*a*b^3*x*sin_integral((b*d*x + a*d)
/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 8*a*b^3*x*tan(1/2*d*x)*tan(1/2*c)^2*tan
(1/2*a*d/b)^2 + a^2*b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b)) - a^2*
b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b)) + 2*a^2*b^2*d^2*x^2*sin_i
ntegral((b*d*x + a*d)/b) + a^4*d^2*imag_part(cos_integral(d*x + a*d/b))*tan
(1/2*d*x)^2 - 2*b^4*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2
 + 2*b^4*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 - a^4*d^2*imag_par
t(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 + 2*b^4*x^2*imag_part(cos_inte
gral(-d*x - a*d/b))*tan(1/2*d*x)^2 - 2*b^4*x^2*imag_part(cos_integral(-d*x)
)*tan(1/2*d*x)^2 - 4*a^2*b^2*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1
/2*d*x)^2 - 4*a^2*b^2*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x
)^2 + 4*b^4*x^2*sin_integral(d*x)*tan(1/2*d*x)^2 + 2*a^4*d^2*sin_integral((
b*d*x + a*d)/b)*tan(1/2*d*x)^2 - 4*b^4*x^2*sin_integral((b*d*x + a*d)/b)*ta
n(1/2*d*x)^2 + 4*a*b^3*d*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c
) - 4*a*b^3*d*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) + 4*a^3*
b*d^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 4*a^3*b*d^2*x*rea
l_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) + 8*a*b^3*d*x^2*sin_integral(
(b*d*x + a*d)/b)*tan(1/2*c) + 4*a^3*b*d*imag_part(cos_integral(d*x + a*d/b)
)*tan(1/2*d*x)^2*tan(1/2*c) - 4*a^3*b*d*imag_part(cos_integral(-d*x - a*d/b
))*tan(1/2*d*x)^2*tan(1/2*c) - 8*a*b^3*x*real_part(cos_integral(d*x + a*d/b
```

))*tan(1/2*d*x)^2*tan(1/2*c) + 8*a*b^3*x*real_part(cos_integral(d*x))*tan(1
/2*d*x)^2*tan(1/2*c) - 8*a*b^3*x*real_part(cos_integral(-d*x - a*d/b))*tan(
1/2*d*x)^2*tan(1/2*c) + 8*a*b^3*x*real_part(cos_integral(-d*x))*tan(1/2*d*x
)^2*tan(1/2*c) + 8*a^3*b*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan
(1/2*c) - a^4*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 + 2*b^4
*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 - 2*b^4*x^2*imag_par
t(cos_integral(d*x))*tan(1/2*c)^2 + a^4*d^2*imag_part(cos_integral(-d*x - a
*d/b))*tan(1/2*c)^2 - 2*b^4*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1
/2*c)^2 + 2*b^4*x^2*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 + 4*a^2*b^2*
d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 + 4*a^2*b^2*d*x*real_
part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 - 4*b^4*x^2*sin_integral(d*x)
*tan(1/2*c)^2 - 2*a^4*d^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2 + 4*b^
4*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2 + 2*a^3*b*d*tan(1/2*d*x)^2
*tan(1/2*c)^2 + 2*a^2*b^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)
^2*tan(1/2*c)^2 - 2*a^2*b^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan
(1/2*c)^2 - 2*a^2*b^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*
tan(1/2*c)^2 + 2*a^2*b^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1
/2*c)^2 - 4*a^2*b^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a^2*b
^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a*b^3*d*x^
2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) + 4*a*b^3*d*x^2*imag_
part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) - 4*a^3*b*d^2*x*real_part(c
os_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 4*a^3*b*d^2*x*real_part(cos_inte
gral(-d*x - a*d/b))*tan(1/2*a*d/b) - 8*a*b^3*d*x^2*sin_integral((b*d*x + a*
d)/b)*tan(1/2*a*d/b) - 4*a^3*b*d*imag_part(cos_integral(d*x + a*d/b))*tan(1
/2*d*x)^2*tan(1/2*a*d/b) + 4*a^3*b*d*imag_part(cos_integral(-d*x - a*d/b))*
tan(1/2*d*x)^2*tan(1/2*a*d/b) + 8*a*b^3*x*real_part(cos_integral(d*x + a*d/
b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 8*a*b^3*x*real_part(cos_integral(-d*x -
 a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 8*a^3*b*d*sin_integral((b*d*x + a*
d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4*a^4*d^2*imag_part(cos_integral(d*x
+ a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 8*b^4*x^2*imag_part(cos_integral(d*x
+ a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 4*a^4*d^2*imag_part(cos_integral(-d*x
 - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 8*b^4*x^2*imag_part(cos_integral(-d*
x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 16*a^2*b^2*d*x*real_part(cos_integr
al(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 16*a^2*b^2*d*x*real_part(cos_i
ntegral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^4*d^2*sin_integral((
b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) - 16*b^4*x^2*sin_integral((b*d*x
+ a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) - 8*a^2*b^2*imag_part(cos_integral(d*x
+ a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^2*b^2*imag_part(co
s_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 16*a^2
*b^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)
 + 4*a^3*b*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/
b) - 4*a^3*b*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a
*d/b) - 8*a*b^3*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2
*a*d/b) - 8*a*b^3*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(
1/2*a*d/b) + 8*a^3*b*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a

*d/b) - a^4*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 + 2*b
^4*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 + 2*b^4*x^2*im
ag_part(cos_integral(d*x))*tan(1/2*a*d/b)^2 + a^4*d^2*imag_part(cos_integra
l(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - 2*b^4*x^2*imag_part(cos_integral(-d*x -
a*d/b))*tan(1/2*a*d/b)^2 - 2*b^4*x^2*imag_part(cos_integral(-d*x))*tan(1/2
*a*d/b)^2 + 4*a^2*b^2*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/
b)^2 + 4*a^2*b^2*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2
+ 4*b^4*x^2*sin_integral(d*x)*tan(1/2*a*d/b)^2 - 2*a^4*d^2*sin_integral((b
*d*x + a*d)/b)*tan(1/2*a*d/b)^2 + 4*b^4*x^2*sin_integral((b*d*x + a*d)/b)*t
an(1/2*a*d/b)^2 - 2*a^3*b*d*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*ima
g_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*a^2*b
^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*a^2*b^2
*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*
a^2*b^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 4*a
^2*b^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 4*a^2*b^2*sin_in
tegral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 4*a^3*b*d*imag_pa
rt(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a^3*b*d*imag_
part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 8*a*b^3*x*re
al_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 8*a*b^3*x*
real_part(cos_integral(d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 8*a*b^3*x*real_p
art(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 8*a*b^3*x*rea
l_part(cos_integral(-d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 8*a^3*b*d*sin_inte
gral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b)^2 - 8*a^3*b*d*tan(1/2*d*x)*
tan(1/2*c)*tan(1/2*a*d/b)^2 - 12*a^2*b^2*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*
a*d/b)^2 - 2*a^3*b*d*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^2*b^2*imag_part(co
s_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^2*b^2*imag_par
t(cos_integral(d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*imag_part(co
s_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*imag_pa
rt(cos_integral(-d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*a^2*b^2*sin_integr
al(d*x)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*a^2*b^2*sin_integral((b*d*x + a*d
)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 12*a^2*b^2*tan(1/2*d*x)*tan(1/2*c)^2*t
an(1/2*a*d/b)^2 + 2*a^3*b*d^2*x*imag_part(cos_integral(d*x + a*d/b)) - 2*a^
3*b*d^2*x*imag_part(cos_integral(-d*x - a*d/b)) - 2*a*b^3*d*x^2*real_part(c
os_integral(d*x + a*d/b)) - 2*a*b^3*d*x^2*real_part(cos_integral(-d*x - a*d
/b)) + 4*a^3*b*d^2*x*sin_integral((b*d*x + a*d)/b) - 2*a^2*b^2*d*x*tan(1/2*
d*x)^2 - 4*a*b^3*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2 + 4*
a*b^3*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 + 4*a*b^3*x*imag_part(c
os_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 - 4*a*b^3*x*imag_part(cos_integra
l(-d*x))*tan(1/2*d*x)^2 - 2*a^3*b*d*real_part(cos_integral(d*x + a*d/b))*ta
n(1/2*d*x)^2 - 2*a^3*b*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)
^2 + 8*a*b^3*x*sin_integral(d*x)*tan(1/2*d*x)^2 - 8*a*b^3*x*sin_integral((b
*d*x + a*d)/b)*tan(1/2*d*x)^2 + 8*a^2*b^2*d*x*imag_part(cos_integral(d*x +
a*d/b))*tan(1/2*c) - 8*a^2*b^2*d*x*imag_part(cos_integral(-d*x - a*d/b))*ta
n(1/2*c) + 2*a^4*d^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c) - 4*b^
4*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 4*b^4*x^2*real_part

```
(cos_integral(d*x))*tan(1/2*c) + 2*a^4*d^2*real_part(cos_integral(-d*x - a*
d/b))*tan(1/2*c) - 4*b^4*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*
c) + 4*b^4*x^2*real_part(cos_integral(-d*x))*tan(1/2*c) + 16*a^2*b^2*d*x*si
n_integral((b*d*x + a*d)/b)*tan(1/2*c) - 8*a^2*b^2*d*x*tan(1/2*d*x)*tan(1/2
*c) - 8*a*b^3*x*tan(1/2*d*x)^2*tan(1/2*c) - 4*a^2*b^2*real_part(cos_integra
l(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^2*b^2*real_part(cos_integra
l(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 4*a^2*b^2*real_part(cos_integral(-d*x -
 a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^2*b^2*real_part(cos_integral(-d*x)
)*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*b^2*d*x*tan(1/2*c)^2 + 4*a*b^3*x*imag_p
art(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 - 4*a*b^3*x*imag_part(cos_integ
ral(d*x))*tan(1/2*c)^2 - 4*a*b^3*x*imag_part(cos_integral(-d*x - a*d/b))*ta
n(1/2*c)^2 + 4*a*b^3*x*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 + 2*a^3*b
*d*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 + 2*a^3*b*d*real_part(
cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 - 8*a*b^3*x*sin_integral(d*x)*tan(
1/2*c)^2 + 8*a*b^3*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2 - 8*a*b^3*x
*tan(1/2*d*x)*tan(1/2*c)^2 - 8*a^2*b^2*d*x*imag_part(cos_integral(d*x + a*d
/b))*tan(1/2*a*d/b) + 8*a^2*b^2*d*x*imag_part(cos_integral(-d*x - a*d/b))*t
an(1/2*a*d/b) - 2*a^4*d^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/
b) + 4*b^4*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 2*a^4*
d^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) + 4*b^4*x^2*real_p
art(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) - 16*a^2*b^2*d*x*sin_integra
l((b*d*x + a*d)/b)*tan(1/2*a*d/b) + 4*a^2*b^2*real_part(cos_integral(d*x +
a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4*a^2*b^2*real_part(cos_integral(-d
*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 16*a*b^3*x*imag_part(cos_integ
ral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 16*a*b^3*x*imag_part(cos_inte
gral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 8*a^3*b*d*real_part(cos_int
egral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 8*a^3*b*d*real_part(cos_int
egral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 32*a*b^3*x*sin_integral((b
*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) - 4*a^2*b^2*real_part(cos_integral
(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 4*a^2*b^2*real_part(cos_integr
al(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^2*b^2*d*x*tan(1/2*a*d/b
)^2 + 4*a*b^3*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 + 4*a
*b^3*x*imag_part(cos_integral(d*x))*tan(1/2*a*d/b)^2 - 4*a*b^3*x*imag_part(
cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - 4*a*b^3*x*imag_part(cos_inte
gral(-d*x))*tan(1/2*a*d/b)^2 + 2*a^3*b*d*real_part(cos_integral(d*x + a*d/b
))*tan(1/2*a*d/b)^2 + 2*a^3*b*d*real_part(cos_integral(-d*x - a*d/b))*tan(1
/2*a*d/b)^2 + 8*a*b^3*x*sin_integral(d*x)*tan(1/2*a*d/b)^2 + 8*a*b^3*x*sin_
integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 + 8*a*b^3*x*tan(1/2*d*x)*tan(1/2
*a*d/b)^2 + 8*a*b^3*x*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a^2*b^2*real_part(cos
_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a^2*b^2*real_part(c
os_integral(d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a^2*b^2*real_part(cos_int
egral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a^2*b^2*real_part(cos_
integral(-d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 + a^4*d^2*imag_part(cos_integra
l(d*x + a*d/b)) - 2*b^4*x^2*imag_part(cos_integral(d*x + a*d/b)) + 2*b^4*x^
2*imag_part(cos_integral(d*x)) - a^4*d^2*imag_part(cos_integral(-d*x - a*d/
```

```
b)) + 2*b^4*x^2*imag_part(cos_integral(-d*x - a*d/b)) - 2*b^4*x^2*imag_part
(cos_integral(-d*x)) - 4*a^2*b^2*d*x*real_part(cos_integral(d*x + a*d/b)) -
 4*a^2*b^2*d*x*real_part(cos_integral(-d*x - a*d/b)) + 4*b^4*x^2*sin_integr
al(d*x) + 2*a^4*d^2*sin_integral((b*d*x + a*d)/b) - 4*b^4*x^2*sin_integral(
(b*d*x + a*d)/b) - 2*a^3*b*d*tan(1/2*d*x)^2 - 2*a^2*b^2*imag_part(cos_integ
ral(d*x + a*d/b))*tan(1/2*d*x)^2 + 2*a^2*b^2*imag_part(cos_integral(d*x))*t
an(1/2*d*x)^2 + 2*a^2*b^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x
)^2 - 2*a^2*b^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2 + 4*a^2*b^2*si
n_integral(d*x)*tan(1/2*d*x)^2 - 4*a^2*b^2*sin_integral((b*d*x + a*d)/b)*ta
n(1/2*d*x)^2 + 4*a^3*b*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c) -
4*a^3*b*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) - 8*a*b^3*x*real
_part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 8*a*b^3*x*real_part(cos_integ
ral(d*x))*tan(1/2*c) - 8*a*b^3*x*real_part(cos_integral(-d*x - a*d/b))*tan(
1/2*c) + 8*a*b^3*x*real_part(cos_integral(-d*x))*tan(1/2*c) + 8*a^3*b*d*sin
_integral((b*d*x + a*d)/b)*tan(1/2*c) - 8*a^3*b*d*tan(1/2*d*x)*tan(1/2*c) -
 12*a^2*b^2*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^3*b*d*tan(1/2*c)^2 + 2*a^2*b^2*
imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 - 2*a^2*b^2*imag_part(cos
_integral(d*x))*tan(1/2*c)^2 - 2*a^2*b^2*imag_part(cos_integral(-d*x - a*d/
b))*tan(1/2*c)^2 + 2*a^2*b^2*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 - 4
*a^2*b^2*sin_integral(d*x)*tan(1/2*c)^2 + 4*a^2*b^2*sin_integral((b*d*x + a
*d)/b)*tan(1/2*c)^2 - 12*a^2*b^2*tan(1/2*d*x)*tan(1/2*c)^2 - 4*a^3*b*d*imag
_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) + 4*a^3*b*d*imag_part(cos_i
ntegral(-d*x - a*d/b))*tan(1/2*a*d/b) + 8*a*b^3*x*real_part(cos_integral(d*
x + a*d/b))*tan(1/2*a*d/b) + 8*a*b^3*x*real_part(cos_integral(-d*x - a*d/b)
)*tan(1/2*a*d/b) - 8*a^3*b*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b) -
 8*a^2*b^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) +
 8*a^2*b^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)
- 16*a^2*b^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) + 2*a^
3*b*d*tan(1/2*a*d/b)^2 + 2*a^2*b^2*imag_part(cos_integral(d*x + a*d/b))*tan
(1/2*a*d/b)^2 + 2*a^2*b^2*imag_part(cos_integral(d*x))*tan(1/2*a*d/b)^2 - 2
*a^2*b^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - 2*a^2*b^2
*imag_part(cos_integral(-d*x))*tan(1/2*a*d/b)^2 + 4*a^2*b^2*sin_integral(d*
x)*tan(1/2*a*d/b)^2 + 4*a^2*b^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b
)^2 + 12*a^2*b^2*tan(1/2*d*x)*tan(1/2*a*d/b)^2 + 12*a^2*b^2*tan(1/2*c)*tan(
1/2*a*d/b)^2 + 2*a^2*b^2*d*x - 4*a*b^3*x*imag_part(cos_integral(d*x + a*d/b
)) + 4*a*b^3*x*imag_part(cos_integral(d*x)) + 4*a*b^3*x*imag_part(cos_integ
ral(-d*x - a*d/b)) - 4*a*b^3*x*imag_part(cos_integral(-d*x)) - 2*a^3*b*d*re
al_part(cos_integral(d*x + a*d/b)) - 2*a^3*b*d*real_part(cos_integral(-d*x
- a*d/b)) + 8*a*b^3*x*sin_integral(d*x) - 8*a*b^3*x*sin_integral((b*d*x + a
*d)/b) + 8*a*b^3*x*tan(1/2*d*x) + 8*a*b^3*x*tan(1/2*c) - 4*a^2*b^2*real_par
t(cos_integral(d*x + a*d/b))*tan(1/2*c) + 4*a^2*b^2*real_part(cos_integral(
d*x))*tan(1/2*c) - 4*a^2*b^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*
c) + 4*a^2*b^2*real_part(cos_integral(-d*x))*tan(1/2*c) + 4*a^2*b^2*real_pa
rt(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) + 4*a^2*b^2*real_part(cos_inte
gral(-d*x - a*d/b))*tan(1/2*a*d/b) + 2*a^3*b*d - 2*a^2*b^2*imag_part(cos_in
```

```
tegral(d*x + a*d/b)) + 2*a^2*b^2*imag_part(cos_integral(d*x)) + 2*a^2*b^2*i
mag_part(cos_integral(-d*x - a*d/b)) - 2*a^2*b^2*imag_part(cos_integral(-d*
x)) + 4*a^2*b^2*sin_integral(d*x) - 4*a^2*b^2*sin_integral((b*d*x + a*d)/b)
 + 12*a^2*b^2*tan(1/2*d*x) + 12*a^2*b^2*tan(1/2*c))/(a^3*b^4*x^2*tan(1/2*d*
x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^4*b^3*x*tan(1/2*d*x)^2*tan(1/2*c)^
2*tan(1/2*a*d/b)^2 + a^3*b^4*x^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + a^3*b^4*x^2*
tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a^3*b^4*x^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2
 + a^5*b^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^4*b^3*x*tan(1
/2*d*x)^2*tan(1/2*c)^2 + 2*a^4*b^3*x*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*a^
4*b^3*x*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^3*b^4*x^2*tan(1/2*d*x)^2 + a^3*b^
4*x^2*tan(1/2*c)^2 + a^5*b^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + a^3*b^4*x^2*tan(
1/2*a*d/b)^2 + a^5*b^2*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a^5*b^2*tan(1/2*c)
^2*tan(1/2*a*d/b)^2 + 2*a^4*b^3*x*tan(1/2*d*x)^2 + 2*a^4*b^3*x*tan(1/2*c)^2
 + 2*a^4*b^3*x*tan(1/2*a*d/b)^2 + a^3*b^4*x^2 + a^5*b^2*tan(1/2*d*x)^2 + a^
5*b^2*tan(1/2*c)^2 + a^5*b^2*tan(1/2*a*d/b)^2 + 2*a^4*b^3*x + a^5*b^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x(a+bx)^3}\,dx = \int \frac{\sin\left(c+d\,x\right)}{x\,\left(a+b\,x\right)^3}\,dx$$

```
[In] int(sin(c + d*x)/(x*(a + b*x)^3),x)
[Out] int(sin(c + d*x)/(x*(a + b*x)^3), x)
```

3.38 $\int \frac{\sin(c+dx)}{x^2(a+bx)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 299

$$\begin{aligned}\int \frac{\sin(c+dx)}{x^2(a+bx)^3}\,dx = & -\frac{d\cos(c+dx)}{2a^2(a+bx)}+\frac{d\cos(c)\,\text{CosIntegral}(dx)}{a^3} \\ & +\frac{2d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^3} \\ & -\frac{3b\,\text{CosIntegral}(dx)\sin(c)}{a^4}+\frac{3b\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^4} \\ & -\frac{d^2\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{2a^2b}-\frac{\sin(c+dx)}{a^3x} \\ & -\frac{b\sin(c+dx)}{2a^2(a+bx)^2}-\frac{2b\sin(c+dx)}{a^3(a+bx)}-\frac{3b\cos(c)\text{Si}(dx)}{a^4} \\ & -\frac{d\sin(c)\text{Si}(dx)}{a^3}+\frac{3b\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^4} \\ & -\frac{d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{2a^2b}-\frac{2d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^3}\end{aligned}$$

```
[Out] d*Ci(d*x)*cos(c)/a^3+2*d*Ci(a*d/b+d*x)*cos(-c+a*d/b)/a^3-1/2*d*cos(d*x+c)/a
^2/(b*x+a)-3*b*cos(c)*Si(d*x)/a^4+3*b*cos(-c+a*d/b)*Si(a*d/b+d*x)/a^4-1/2*d
^2*cos(-c+a*d/b)*Si(a*d/b+d*x)/a^2/b-3*b*Ci(d*x)*sin(c)/a^4-d*Si(d*x)*sin(c
)/a^3-3*b*Ci(a*d/b+d*x)*sin(-c+a*d/b)/a^4+1/2*d^2*Ci(a*d/b+d*x)*sin(-c+a*d/
b)/a^2/b+2*d*Si(a*d/b+d*x)*sin(-c+a*d/b)/a^3-sin(d*x+c)/a^3/x-1/2*b*sin(d*x
+c)/a^2/(b*x+a)^2-2*b*sin(d*x+c)/a^3/(b*x+a)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6874, 3378, 3384, 3380, 3383}

$$\int \frac{\sin(c+dx)}{x^2(a+bx)^3}\,dx = -\frac{3b\sin(c)\,\text{CosIntegral}(dx)}{a^4}+\frac{3b\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{a^4} -\frac{3b\cos(c)\text{Si}(dx)}{a^4}+\frac{3b\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^4} +\frac{2d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{a^3}-\frac{2d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^3} -\frac{2b\sin(c+dx)}{a^3(a+bx)}+\frac{d\cos(c)\,\text{CosIntegral}(dx)}{a^3}-\frac{d\sin(c)\text{Si}(dx)}{a^3} -\frac{\sin(c+dx)}{a^3x}-\frac{d^2\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{2a^2b} -\frac{d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{2a^2b}-\frac{b\sin(c+dx)}{2a^2(a+bx)^2}-\frac{d\cos(c+dx)}{2a^2(a+bx)}$$

```
[In] Int[Sin[c + d*x]/(x^2*(a + b*x)^3),x]
```

```
[Out] -1/2*(d*Cos[c + d*x])/(a^2*(a + b*x)) + (d*Cos[c]*CosIntegral[d*x])/a^3 + (
2*d*Cos[c - (a*d)/b]*CosIntegral[(a*d)/b + d*x])/a^3 - (3*b*CosIntegral[d*x
]*Sin[c])/a^4 + (3*b*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/a^4 - (d^
2*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/(2*a^2*b) - Sin[c + d*x]/(a^
3*x) - (b*Sin[c + d*x])/(2*a^2*(a + b*x)^2) - (2*b*Sin[c + d*x])/(a^3*(a +
b*x)) - (3*b*Cos[c]*SinIntegral[d*x])/a^4 - (d*Sin[c]*SinIntegral[d*x])/a^3
 + (3*b*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/a^4 - (d^2*Cos[c - (a*
d)/b]*SinIntegral[(a*d)/b + d*x])/(2*a^2*b) - (2*d*Sin[c - (a*d)/b]*SinInte
gral[(a*d)/b + d*x])/a^3
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\sin(c+dx)}{a^3x^2} - \frac{3b\sin(c+dx)}{a^4x} + \frac{b^2\sin(c+dx)}{a^2(a+bx)^3} + \frac{2b^2\sin(c+dx)}{a^3(a+bx)^2} + \frac{3b^2\sin(c+dx)}{a^4(a+bx)}\right) dx$$

$$= \frac{\int \frac{\sin(c+dx)}{x^2}\,dx}{a^3} - \frac{(3b)\int \frac{\sin(c+dx)}{x}\,dx}{a^4} + \frac{(3b^2)\int \frac{\sin(c+dx)}{a+bx}\,dx}{a^4} + \frac{(2b^2)\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx}{a^3} + \frac{b^2\int \frac{\sin(c+dx)}{(a+bx)^3}\,dx}{a^2}$$

$$= -\frac{\sin(c+dx)}{a^3x} - \frac{b\sin(c+dx)}{2a^2(a+bx)^2} - \frac{2b\sin(c+dx)}{a^3(a+bx)} + \frac{d\int \frac{\cos(c+dx)}{x}\,dx}{a^3} + \frac{(2bd)\int \frac{\cos(c+dx)}{a+bx}\,dx}{a^3} + \frac{(bd)\int \frac{\cos(c+dx)}{(a+bx)^2}\,dx}{2a^2} - \frac{(3b\cos(c))\int \frac{\sin(dx)}{x}\,dx}{a^4} + \frac{\left(3b^2\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^4} - \frac{(3b\sin(c))\int \frac{\cos(dx)}{x}\,dx}{a^4} + \frac{\left(3b^2\sin\left(c-\frac{ad}{b}\right)\right)\int \frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^4}$$

$$= -\frac{d\cos(c+dx)}{2a^2(a+bx)} - \frac{3b\,\text{CosIntegral}(dx)\sin(c)}{a^4} + \frac{3b\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^4} - \frac{\sin(c+dx)}{a^3x} - \frac{b\sin(c+dx)}{2a^2(a+bx)^2} - \frac{2b\sin(c+dx)}{a^3(a+bx)} - \frac{3b\cos(c)\text{Si}(dx)}{a^4} + \frac{3b\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^4} - \frac{d^2\int \frac{\sin(c+dx)}{a+bx}\,dx}{2a^2} + \frac{(d\cos(c))\int \frac{\cos(dx)}{x}\,dx}{a^3} + \frac{\left(2bd\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^3} - \frac{(d\sin(c))\int \frac{\sin(dx)}{x}\,dx}{a^3} - \frac{\left(2bd\sin\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^3}$$

$$= -\frac{d\cos(c+dx)}{2a^2(a+bx)} + \frac{d\cos(c)\operatorname{CosIntegral}(dx)}{a^3} + \frac{2d\cos\left(c-\frac{ad}{b}\right)\operatorname{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^3} - \frac{3b\operatorname{CosIntegral}(dx)\sin(c)}{a^4} + \frac{3b\operatorname{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^4} - \frac{\sin(c+dx)}{a^3x} - \frac{b\sin(c+dx)}{2a^2(a+bx)^2} - \frac{2b\sin(c+dx)}{a^3(a+bx)} - \frac{3b\cos(c)\text{Si}(dx)}{a^4} - \frac{d\sin(c)\text{Si}(dx)}{a^3} + \frac{3b\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^4} - \frac{2d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^3} - \frac{\left(d^2\cos\left(c-\frac{ad}{b}\right)\right)\int\frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}dx}{2a^2} - \frac{\left(d^2\sin\left(c-\frac{ad}{b}\right)\right)\int\frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}dx}{2a^2}$$

$$= -\frac{d\cos(c+dx)}{2a^2(a+bx)} + \frac{d\cos(c)\operatorname{CosIntegral}(dx)}{a^3} + \frac{2d\cos\left(c-\frac{ad}{b}\right)\operatorname{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^3} - \frac{3b\operatorname{CosIntegral}(dx)\sin(c)}{a^4} + \frac{3b\operatorname{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^4} - \frac{d^2\operatorname{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{2a^2b} - \frac{\sin(c+dx)}{a^3x} - \frac{b\sin(c+dx)}{2a^2(a+bx)^2} - \frac{2b\sin(c+dx)}{a^3(a+bx)} - \frac{3b\cos(c)\text{Si}(dx)}{a^4} - \frac{d\sin(c)\text{Si}(dx)}{a^3} + \frac{3b\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^4} - \frac{d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{2a^2b} - \frac{2d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^3}$$

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.81

$$\int \frac{\sin(c+dx)}{x^2(a+bx)^3}\,dx = -\frac{a^3bdx\cos(c+dx) + a^2b^2dx^2\cos(c+dx) + 2bx(a+bx)^2\operatorname{CosIntegral}(dx)(-ad\cos(c)+3b\sin(c)) + x(a$$

```
[In] Integrate[Sin[c + d*x]/(x^2*(a + b*x)^3),x]
```

```
[Out] -1/2*(a^3*b*d*x*Cos[c + d*x] + a^2*b^2*d*x^2*Cos[c + d*x] + 2*b*x*(a + b*x)
^2*CosIntegral[d*x]*(-(a*d*Cos[c]) + 3*b*Sin[c]) + x*(a + b*x)^2*CosIntegra
l[d*(a/b + x)]*(-4*a*b*d*Cos[c - (a*d)/b] + (-6*b^2 + a^2*d^2)*Sin[c - (a*d
)/b]) + 2*a^3*b*Sin[c + d*x] + 9*a^2*b^2*x*Sin[c + d*x] + 6*a*b^3*x^2*Sin[c
 + d*x] + 6*a^2*b^2*x*Cos[c]*SinIntegral[d*x] + 12*a*b^3*x^2*Cos[c]*SinInte
gral[d*x] + 6*b^4*x^3*Cos[c]*SinIntegral[d*x] + 2*a^3*b*d*x*Sin[c]*SinInteg
ral[d*x] + 4*a^2*b^2*d*x^2*Sin[c]*SinIntegral[d*x] + 2*a*b^3*d*x^3*Sin[c]*S
inIntegral[d*x] - 6*a^2*b^2*x*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)] + a
^4*d^2*x*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)] - 12*a*b^3*x^2*Cos[c - (
a*d)/b]*SinIntegral[d*(a/b + x)] + 2*a^3*b*d^2*x^2*Cos[c - (a*d)/b]*SinInte
```

```
gral[d*(a/b + x)] - 6*b^4*x^3*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)] + a
^2*b^2*d^2*x^3*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)] + 4*a^3*b*d*x*Sin[
c - (a*d)/b]*SinIntegral[d*(a/b + x)] + 8*a^2*b^2*d*x^2*Sin[c - (a*d)/b]*Si
nIntegral[d*(a/b + x)] + 4*a*b^3*d*x^3*Sin[c - (a*d)/b]*SinIntegral[d*(a/b
+ x)])/(a^4*b*x*(a + b*x)^2)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.35

method	result
derivativedivides	$d\left(-\frac{3b(\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin(c))}{a^4d}+\frac{2b^2\left(-\frac{\sin(dx+c)}{(da-cb+b(dx+c))b}+\frac{\frac{\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}+\frac{\mathrm{Ci}\left(dx+c+\frac{da-cb}{b}\right)\cos}{b}}{b}\right.}{a^3}\right.$
default	$d\left(-\frac{3b(\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin(c))}{a^4d}+\frac{2b^2\left(-\frac{\sin(dx+c)}{(da-cb+b(dx+c))b}+\frac{\frac{\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}+\frac{\mathrm{Ci}\left(dx+c+\frac{da-cb}{b}\right)\cos}{b}}{b}\right.}{a^3}\right.$
risch	$-\frac{d\,\mathrm{e}^{-\frac{i(da-cb)}{b}}\,\mathrm{Ei}_1\left(-idx-ic-\frac{iad-icb}{b}\right)}{a^3}-\frac{3ib\,\mathrm{e}^{ic}\,\mathrm{Ei}_1(-idx)}{2a^4}+\frac{3ib\,\mathrm{e}^{-\frac{i(da-cb)}{b}}\,\mathrm{Ei}_1\left(-idx-ic-\frac{iad-icb}{b}\right)}{2a^4}-\frac{d\,\mathrm{e}^{ic}\,\mathrm{Ei}_1(}{2a^3}$

```
[In] int(sin(d*x+c)/x^2/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] d*(-3/a^4/d*b*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))+2*b^2/a^3*(-sin(d*x+c)/(d*a-c
*b+b*(d*x+c))/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c
)/b)*cos((a*d-b*c)/b)/b)/b)+d*b^2/a^2*(-1/2*sin(d*x+c)/(d*a-c*b+b*(d*x+c))^
2/b+1/2*(-cos(d*x+c)/(d*a-c*b+b*(d*x+c))/b-(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-
b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)/b)/b)+1/a^3*(-sin(d*x+c
)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c))+3/d*b^2/a^4*(Si(d*x+c+(a*d-b*c)/b)*cos
((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.67

$$\int \frac{\sin(c+dx)}{x^2(a+bx)^3}\,dx = -\frac{(a^2b^2dx^2+a^3bdx)\cos(dx+c)-2\left((ab^3dx^3+2\,a^2b^2dx^2+a^3bdx)\,\mathrm{Ci}\,(dx)-3\,(b^4x^3+2\,ab^3x^2+a^2b^2x)\right.}{}$$

[In] integrate(sin(d*x+c)/x^2/(b*x+a)^3,x, algorithm="fricas")

[Out] -1/2*((a^2*b^2*d*x^2 + a^3*b*d*x)*cos(d*x + c) - 2*((a*b^3*d*x^3 + 2*a^2*b^2*d*x^2 + a^3*b*d*x)*cos_integral(d*x) - 3*(b^4*x^3 + 2*a*b^3*x^2 + a^2*b^2*x)*sin_integral(d*x))*cos(c) - (4*(a*b^3*d*x^3 + 2*a^2*b^2*d*x^2 + a^3*b*d*x)*cos_integral((b*d*x + a*d)/b) - ((a^2*b^2*d^2 - 6*b^4)*x^3 + 2*(a^3*b*d^2 - 6*a*b^3)*x^2 + (a^4*d^2 - 6*a^2*b^2)*x)*sin_integral((b*d*x + a*d)/b))*cos(-(b*c - a*d)/b) + (6*a*b^3*x^2 + 9*a^2*b^2*x + 2*a^3*b)*sin(d*x + c) + 2*(3*(b^4*x^3 + 2*a*b^3*x^2 + a^2*b^2*x)*cos_integral(d*x) + (a*b^3*d*x^3 + 2*a^2*b^2*d*x^2 + a^3*b*d*x)*sin_integral(d*x))*sin(c) - (((a^2*b^2*d^2 - 6*b^4)*x^3 + 2*(a^3*b*d^2 - 6*a*b^3)*x^2 + (a^4*d^2 - 6*a^2*b^2)*x)*cos_integral((b*d*x + a*d)/b) + 4*(a*b^3*d*x^3 + 2*a^2*b^2*d*x^2 + a^3*b*d*x)*sin_integral((b*d*x + a*d)/b))*sin(-(b*c - a*d)/b))/(a^4*b^3*x^3 + 2*a^5*b^2*x^2 + a^6*b*x)

Sympy [F]

$$\int \frac{\sin(c+dx)}{x^2(a+bx)^3}\,dx = \int \frac{\sin(c+dx)}{x^2\,(a+bx)^3}\,dx$$

[In] integrate(sin(d*x+c)/x**2/(b*x+a)**3,x)

[Out] Integral(sin(c + d*x)/(x**2*(a + b*x)**3), x)

Maxima [F]

$$\int \frac{\sin(c+dx)}{x^2(a+bx)^3}\,dx = \int \frac{\sin(dx+c)}{(bx+a)^3x^2}\,dx$$

[In] integrate(sin(d*x+c)/x^2/(b*x+a)^3,x, algorithm="maxima")

[Out] integrate(sin(d*x + c)/((b*x + a)^3*x^2), x)

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.71 (sec) , antiderivative size = 20808, normalized size of antiderivative = 69.59

$$\int \frac{\sin(c+dx)}{x^2(a+bx)^3}\,dx = \text{Too large to display}$$

[In] integrate(sin(d*x+c)/x^2/(b*x+a)^3,x, algorithm="giac")

[Out] -1/4*(a^2*b^2*d^2*x^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^2*b^2*d^2*x^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d^2*x^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d^2*x^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^2*b^2*d^2*x^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a^2*b^2*d^2*x^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^2*b^2*d^2*x^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*a^3*b*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^3*b*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*a*b^3*d*x^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*b^3*d*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*a*b^3*d*x^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*b^3*d*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a^3*b*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^2*b^2*d^2*x^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + a^2*b^2*d^2*x^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*b^2*d^2*x^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a^2*b^2*d^2*x^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 4*a^2*b^2*d^2*x^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^2*b^2*d^2*x^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 8*a*b^3*d*x^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 8*a*b^3*d*x^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 4*a^3*b*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 4*a^3*b*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 16*a*b^3*d*x^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - a^2*b^2*d^2*x^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a^2*b^2*d^2*x^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*a^2*b^2*d^2*x^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^

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2*tan(1/2*a*d/b)^2 - 8*a*b^3*d*x^3*imag_part(cos_integral(d*x + a*d/b))*tan
(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a*b^3*d*x^3*imag_part(cos_integ
ral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 8*a*b^3*d*x^3*imag_p
art(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2
 - 4*a*b^3*d*x^3*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan
(1/2*a*d/b)^2 - 4*a^3*b*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/
2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a^3*b*d^2*x^2*real_part(cos_integr
al(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 8*a*b^3*d*x^
3*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 16*a*b^3*d
*x^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)
^2 + a^2*b^2*d^2*x^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(
1/2*a*d/b)^2 - a^2*b^2*d^2*x^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/
2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d^2*x^3*sin_integral((b*d*x + a*d)/b)*t
an(1/2*c)^2*tan(1/2*a*d/b)^2 + a^4*d^2*x*imag_part(cos_integral(d*x + a*d/b
))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 6*b^4*x^3*imag_part(cos_i
ntegral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 6*b^4*
x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)
^2 - a^4*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2
*c)^2*tan(1/2*a*d/b)^2 + 6*b^4*x^3*imag_part(cos_integral(-d*x - a*d/b))*ta
n(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 6*b^4*x^3*imag_part(cos_integr
al(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 8*a^2*b^2*d*x^2*re
al_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/
b)^2 + 4*a^2*b^2*d*x^2*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*
c)^2*tan(1/2*a*d/b)^2 - 8*a^2*b^2*d*x^2*real_part(cos_integral(-d*x - a*d/b
))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a^2*b^2*d*x^2*real_part
(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 12*b^4*
x^3*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^4*
d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d
/b)^2 - 12*b^4*x^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^
2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d^2*x^3*real_part(cos_integral(d*x + a*d/b))
*tan(1/2*d*x)^2*tan(1/2*c) + 2*a^2*b^2*d^2*x^3*real_part(cos_integral(-d*x
 - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^3*b*d^2*x^2*imag_part(cos_integra
l(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^3*b*d^2*x^2*imag_part(cos
_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a*b^3*d*x^3*real_p
art(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*b^3*d*x^3*
real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a*b^3*d*x^3*re
al_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*b^3*d
*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a^3*b*d^
2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*b^2
*d^2*x^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)
 - 2*a^2*b^2*d^2*x^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*t
an(1/2*a*d/b) + 8*a^3*b*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/
2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 8*a^3*b*d^2*x^2*imag_part(cos_integral
(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 16*a*b^3*d*x^3*r
eal_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b
```

) - 16*a*b^3*d*x^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan (1/2*c)*tan(1/2*a*d/b) + 16*a^3*b*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan (1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 2*a^2*b^2*d^2*x^3*real_part(cos_int egral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^2*b^2*d^2*x^3*real_pa rt(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 16*a^2*b^2*d*x ^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2 *a*d/b) - 16*a^2*b^2*d*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d* x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^4*d^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 12*b^4*x^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a ^4*d^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2* tan(1/2*a*d/b) - 12*b^4*x^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d *x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 32*a^2*b^2*d*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a^3*b*d^2*x^2*imag_p art(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*a^3*b*d^ 2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 4*a*b^3*d*x^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/ 2*a*d/b)^2 - 2*a*b^3*d*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 4*a*b^3*d*x^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2* d*x)^2*tan(1/2*a*d/b)^2 - 2*a*b^3*d*x^3*real_part(cos_integral(-d*x))*tan(1 /2*d*x)^2*tan(1/2*a*d/b)^2 - 4*a^3*b*d^2*x^2*sin_integral((b*d*x + a*d)/b)* tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*a^2*b^2*d^2*x^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^2*b^2*d^2*x^3*real_part(cos _integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 16*a^2*b^2*d*x^2*ima g_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^ 2 + 8*a^2*b^2*d*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)* tan(1/2*a*d/b)^2 + 16*a^2*b^2*d*x^2*imag_part(cos_integral(-d*x - a*d/b))*t an(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 8*a^2*b^2*d*x^2*imag_part(cos_i ntegral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^4*d^2*x*rea l_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^ 2 + 12*b^4*x^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2* c)*tan(1/2*a*d/b)^2 + 12*b^4*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^ 2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^4*d^2*x*real_part(cos_integral(-d*x - a *d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 12*b^4*x^3*real_part(co s_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 12*b ^4*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/ b)^2 + 16*a^2*b^2*d*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2 *a*d/b)^2 - 32*a^2*b^2*d*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*t an(1/2*c)*tan(1/2*a*d/b)^2 + 2*a^3*b*d^2*x^2*imag_part(cos_integral(d*x + a *d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^3*b*d^2*x^2*imag_part(cos_integr al(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*a*b^3*d*x^3*real_part(c os_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*b^3*d*x^3*rea l_part(cos_integral(d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*a*b^3*d*x^3*rea l_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*b^3* d*x^3*real_part(cos_integral(-d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a^3*b

*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^
2*b^2*d*x^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 12*a*b^3*x^2*ima
g_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b
)^2 - 12*a*b^3*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2
*tan(1/2*a*d/b)^2 + 12*a*b^3*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(
1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 12*a*b^3*x^2*imag_part(cos_integ
ral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*a^3*b*d*x*real_
part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^
2 + 2*a^3*b*d*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*ta
n(1/2*a*d/b)^2 - 4*a^3*b*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*
d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^3*b*d*x*real_part(cos_integral(-
d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 24*a*b^3*x^2*sin_integ
ral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 24*a*b^3*x^2*sin_in
tegral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^2*
b^2*d^2*x^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2 - a^2*b^2*d
^2*x^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 + 2*a^2*b^2*d^2
*x^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2 + 8*a*b^3*d*x^3*imag_part
(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a*b^3*d*x^3*imag_
part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 8*a*b^3*d*x^3*imag_part
(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 4*a*b^3*d*x^3*imag
_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^3*b*d^2*x^2*real_
part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^3*b*d^2*x^2
*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 8*a*b^3*
d*x^3*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) + 16*a*b^3*d*x^3*sin_inte
gral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c) - a^2*b^2*d^2*x^3*imag_part
(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 + a^2*b^2*d^2*x^3*imag_part(cos_in
tegral(-d*x - a*d/b))*tan(1/2*c)^2 - 2*a^2*b^2*d^2*x^3*sin_integral((b*d*x
+ a*d)/b)*tan(1/2*c)^2 - a^4*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan
(1/2*d*x)^2*tan(1/2*c)^2 + 6*b^4*x^3*imag_part(cos_integral(d*x + a*d/b))*t
an(1/2*d*x)^2*tan(1/2*c)^2 - 6*b^4*x^3*imag_part(cos_integral(d*x))*tan(1/2
*d*x)^2*tan(1/2*c)^2 + a^4*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(
1/2*d*x)^2*tan(1/2*c)^2 - 6*b^4*x^3*imag_part(cos_integral(-d*x - a*d/b))*t
an(1/2*d*x)^2*tan(1/2*c)^2 + 6*b^4*x^3*imag_part(cos_integral(-d*x))*tan(1/
2*d*x)^2*tan(1/2*c)^2 + 8*a^2*b^2*d*x^2*real_part(cos_integral(d*x + a*d/b)
)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a^2*b^2*d*x^2*real_part(cos_integral(d*x)
)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 8*a^2*b^2*d*x^2*real_part(cos_integral(-d*x
 - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a^2*b^2*d*x^2*real_part(cos_inte
gral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 12*b^4*x^3*sin_integral(d*x)*tan(
1/2*d*x)^2*tan(1/2*c)^2 - 2*a^4*d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2
*d*x)^2*tan(1/2*c)^2 + 12*b^4*x^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x
)^2*tan(1/2*c)^2 - 8*a*b^3*d*x^3*imag_part(cos_integral(d*x + a*d/b))*tan(1
/2*d*x)^2*tan(1/2*a*d/b) + 8*a*b^3*d*x^3*imag_part(cos_integral(-d*x - a*d/
b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 4*a^3*b*d^2*x^2*real_part(cos_integral(
d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 4*a^3*b*d^2*x^2*real_part(cos
_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 16*a*b^3*d*x^3*sin

_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4*a^2*b^2*d^2*x^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 4*a^2*b^2*d^2*x^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^2*b^2*d^2*x^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) + 4*a^4*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 24*b^4*x^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 4*a^4*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 24*b^4*x^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 32*a^2*b^2*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 32*a^2*b^2*d*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^4*d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 48*b^4*x^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 8*a*b^3*d*x^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 8*a*b^3*d*x^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 4*a^3*b*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 4*a^3*b*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 16*a*b^3*d*x^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b) + 8*a^3*b*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 8*a^3*b*d*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 24*a*b^3*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 24*a*b^3*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 16*a^3*b*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - a^2*b^2*d^2*x^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 + a^2*b^2*d^2*x^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - 2*a^2*b^2*d^2*x^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 - a^4*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 6*b^4*x^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 6*b^4*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a^4*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 6*b^4*x^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 6*b^4*x^3*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 8*a^2*b^2*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 4*a^2*b^2*d*x^2*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 8*a^2*b^2*d*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 4*a^2*b^2*d*x^2*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 12*b^4*x^3*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*a^4*d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 12*b^4*x^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 8*a*b^3*d*x^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a*b^3*d*x^3*imag_part(cos_integral(d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 8*a*b^3*d*x^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 -

4*a*b^3*d*x^3*imag_part(cos_integral(-d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a^3*b*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a^3*b*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 8*a*b^3*d*x^3*sin_integral(d*x)*tan(1/2*c)*tan(1/2*a*d/b)^2 - 16*a*b^3*d*x^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b)^2 - 8*a^3*b*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a^3*b*d*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 8*a^3*b*d*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a^3*b*d*x*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 24*a*b^3*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 24*a*b^3*x^2*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 24*a*b^3*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 24*a*b^3*x^2*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 8*a^3*b*d*x*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 16*a^3*b*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + a^4*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 6*b^4*x^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 6*b^4*x^3*imag_part(cos_integral(d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^4*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 6*b^4*x^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 6*b^4*x^3*imag_part(cos_integral(-d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 8*a^2*b^2*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a^2*b^2*d*x^2*real_part(cos_integral(d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 8*a^2*b^2*d*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a^2*b^2*d*x^2*real_part(cos_integral(-d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 12*b^4*x^3*sin_integral(d*x)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^4*d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 12*b^4*x^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^3*b*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 6*a^2*b^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 6*a^2*b^2*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 6*a^2*b^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 6*a^2*b^2*x*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 12*a^2*b^2*x*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 12*a^2*b^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^3*b*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2 - 2*a^3*b*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 - 4*a*b^3*d*x^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2 - 2*a*b^3*d*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2 - 4*a*b^3*d*x^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 - 2*a*b^3*d*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2 + 4*a^3*b*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2 + 2*a^2*b^2*d^2*x^3*real_part(cos_integral(d*x + a*d/

b))*tan(1/2*c) + 2*a^2*b^2*d^2*x^3*real_part(cos_integral(-d*x - a*d/b))*ta
n(1/2*c) + 16*a^2*b^2*d*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*
x)^2*tan(1/2*c) + 8*a^2*b^2*d*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)
^2*tan(1/2*c) - 16*a^2*b^2*d*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(
1/2*d*x)^2*tan(1/2*c) - 8*a^2*b^2*d*x^2*imag_part(cos_integral(-d*x))*tan(1
/2*d*x)^2*tan(1/2*c) + 2*a^4*d^2*x*real_part(cos_integral(d*x + a*d/b))*tan
(1/2*d*x)^2*tan(1/2*c) - 12*b^4*x^3*real_part(cos_integral(d*x + a*d/b))*ta
n(1/2*d*x)^2*tan(1/2*c) + 12*b^4*x^3*real_part(cos_integral(d*x))*tan(1/2*d
*x)^2*tan(1/2*c) + 2*a^4*d^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/
2*d*x)^2*tan(1/2*c) - 12*b^4*x^3*real_part(cos_integral(-d*x - a*d/b))*tan(
1/2*d*x)^2*tan(1/2*c) + 12*b^4*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*
x)^2*tan(1/2*c) + 16*a^2*b^2*d*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2
*c) + 32*a^2*b^2*d*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2
*c) - 2*a^3*b*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 + 2
*a^3*b*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 + 4*a*b^3
*d*x^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 + 2*a*b^3*d*x^3*re
al_part(cos_integral(d*x))*tan(1/2*c)^2 + 4*a*b^3*d*x^3*real_part(cos_integ
ral(-d*x - a*d/b))*tan(1/2*c)^2 + 2*a*b^3*d*x^3*real_part(cos_integral(-d*x
))*tan(1/2*c)^2 - 4*a^3*b*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^
2 + 2*a^2*b^2*d*x^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 12*a*b^3*x^2*imag_part(co
s_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 12*a*b^3*x^2*imag_pa
rt(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 12*a*b^3*x^2*imag_part(
cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 12*a*b^3*x^2*imag
_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a^3*b*d*x*real_pa
rt(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^3*b*d*x*rea
l_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a^3*b*d*x*real_pa
rt(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^3*b*d*x*re
al_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 24*a*b^3*x^2*sin_
integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 24*a*b^3*x^2*sin_integral((b*d*
x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*b^2*d^2*x^3*real_part(cos_i
ntegral(d*x + a*d/b))*tan(1/2*a*d/b) - 2*a^2*b^2*d^2*x^3*real_part(cos_inte
gral(-d*x - a*d/b))*tan(1/2*a*d/b) - 16*a^2*b^2*d*x^2*imag_part(cos_integra
l(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 16*a^2*b^2*d*x^2*imag_part(
cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 2*a^4*d^2*x*rea
l_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 12*b^4*x^
3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 2*a^
4*d^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)
 + 12*b^4*x^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*
a*d/b) - 32*a^2*b^2*d*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(
1/2*a*d/b) + 8*a^3*b*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c
)*tan(1/2*a*d/b) - 8*a^3*b*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*ta
n(1/2*c)*tan(1/2*a*d/b) - 16*a*b^3*d*x^3*real_part(cos_integral(d*x + a*d/b
))*tan(1/2*c)*tan(1/2*a*d/b) - 16*a*b^3*d*x^3*real_part(cos_integral(-d*x -
 a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 16*a^3*b*d^2*x^2*sin_integral((b*d*x +
 a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) - 48*a*b^3*x^2*imag_part(cos_integral(d*

x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 48*a*b^3*x^2*imag_pa
rt(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 1
6*a^3*b*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*
tan(1/2*a*d/b) - 16*a^3*b*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2
*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 96*a*b^3*x^2*sin_integral((b*d*x + a*d)
/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 16*a^2*b^2*d*x^2*imag_part(c
os_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 16*a^2*b^2*d*x^2*im
ag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^4*d^2
*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 12*b^
4*x^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*
a^4*d^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)
 - 12*b^4*x^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*
d/b) + 32*a^2*b^2*d*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*
a*d/b) - 12*a^2*b^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*t
an(1/2*c)^2*tan(1/2*a*d/b) - 12*a^2*b^2*x*real_part(cos_integral(-d*x - a*d
/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a^3*b*d^2*x^2*imag_part
(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 + 2*a^3*b*d^2*x^2*imag_part(co
s_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 + 4*a*b^3*d*x^3*real_part(cos_in
tegral(d*x + a*d/b))*tan(1/2*a*d/b)^2 - 2*a*b^3*d*x^3*real_part(cos_integra
l(d*x))*tan(1/2*a*d/b)^2 + 4*a*b^3*d*x^3*real_part(cos_integral(-d*x - a*d/
b))*tan(1/2*a*d/b)^2 - 2*a*b^3*d*x^3*real_part(cos_integral(-d*x))*tan(1/2*
a*d/b)^2 - 4*a^3*b*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 -
 2*a^2*b^2*d*x^2*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 12*a*b^3*x^2*imag_part(c
os_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 12*a*b^3*x^2*im
ag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 12*a*b^3*x^2*i
mag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 12*a
*b^3*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 4*
a^3*b*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b
)^2 - 2*a^3*b*d*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b
)^2 + 4*a^3*b*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(
1/2*a*d/b)^2 - 2*a^3*b*d*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan
(1/2*a*d/b)^2 + 24*a*b^3*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*a*d/b
)^2 + 24*a*b^3*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d
/b)^2 - 16*a^2*b^2*d*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*ta
n(1/2*a*d/b)^2 + 8*a^2*b^2*d*x^2*imag_part(cos_integral(d*x))*tan(1/2*c)*ta
n(1/2*a*d/b)^2 + 16*a^2*b^2*d*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan
(1/2*c)*tan(1/2*a*d/b)^2 - 8*a^2*b^2*d*x^2*imag_part(cos_integral(-d*x))*ta
n(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^4*d^2*x*real_part(cos_integral(d*x + a*d/b)
)*tan(1/2*c)*tan(1/2*a*d/b)^2 + 12*b^4*x^3*real_part(cos_integral(d*x + a*d
/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 12*b^4*x^3*real_part(cos_integral(d*x))*
tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^4*d^2*x*real_part(cos_integral(-d*x - a*d
/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 12*b^4*x^3*real_part(cos_integral(-d*x -
 a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 12*b^4*x^3*real_part(cos_integral(-d
*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 16*a^2*b^2*d*x^2*sin_integral(d*x)*tan(1
/2*c)*tan(1/2*a*d/b)^2 - 32*a^2*b^2*d*x^2*sin_integral((b*d*x + a*d)/b)*tan

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(1/2*c)*tan(1/2*a*d/b)^2 - 8*a^2*b^2*d*x^2*tan(1/2*d*x)*tan(1/2*c)*tan(1/2*
a*d/b)^2 - 24*a*b^3*x^2*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 12*a^2
*b^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1
/2*a*d/b)^2 + 12*a^2*b^2*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(
1/2*c)*tan(1/2*a*d/b)^2 + 12*a^2*b^2*x*real_part(cos_integral(-d*x - a*d/b)
)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 12*a^2*b^2*x*real_part(cos_i
ntegral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^2*b^2*d*x^2
*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 12*a*b^3*x^2*imag_part(cos_integral(d*x +
a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 12*a*b^3*x^2*imag_part(cos_integral
(d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 12*a*b^3*x^2*imag_part(cos_integral(
-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 12*a*b^3*x^2*imag_part(cos_i
ntegral(-d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*a^3*b*d*x*real_part(cos_in
tegral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^3*b*d*x*real_part(
cos_integral(d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*a^3*b*d*x*real_part(co
s_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^3*b*d*x*real_
part(cos_integral(-d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 24*a*b^3*x^2*sin_i
ntegral(d*x)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 24*a*b^3*x^2*sin_integral((b*d
*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 24*a*b^3*x^2*tan(1/2*d*x)*tan(
1/2*c)^2*tan(1/2*a*d/b)^2 + a^2*b^2*d^2*x^3*imag_part(cos_integral(d*x + a*
d/b)) - a^2*b^2*d^2*x^3*imag_part(cos_integral(-d*x - a*d/b)) + 2*a^2*b^2*d
^2*x^3*sin_integral((b*d*x + a*d)/b) + a^4*d^2*x*imag_part(cos_integral(d*x
 + a*d/b))*tan(1/2*d*x)^2 - 6*b^4*x^3*imag_part(cos_integral(d*x + a*d/b))*
tan(1/2*d*x)^2 + 6*b^4*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 - a^
4*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 + 6*b^4*x^3*im
ag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 - 6*b^4*x^3*imag_part(co
s_integral(-d*x))*tan(1/2*d*x)^2 - 8*a^2*b^2*d*x^2*real_part(cos_integral(d
*x + a*d/b))*tan(1/2*d*x)^2 - 4*a^2*b^2*d*x^2*real_part(cos_integral(d*x))*
tan(1/2*d*x)^2 - 8*a^2*b^2*d*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(
1/2*d*x)^2 - 4*a^2*b^2*d*x^2*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2 +
 12*b^4*x^3*sin_integral(d*x)*tan(1/2*d*x)^2 + 2*a^4*d^2*x*sin_integral((b*
d*x + a*d)/b)*tan(1/2*d*x)^2 - 12*b^4*x^3*sin_integral((b*d*x + a*d)/b)*tan
(1/2*d*x)^2 + 8*a*b^3*d*x^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)
 + 4*a*b^3*d*x^3*imag_part(cos_integral(d*x))*tan(1/2*c) - 8*a*b^3*d*x^3*im
ag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) - 4*a*b^3*d*x^3*imag_part(co
s_integral(-d*x))*tan(1/2*c) + 4*a^3*b*d^2*x^2*real_part(cos_integral(d*x +
 a*d/b))*tan(1/2*c) + 4*a^3*b*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))
*tan(1/2*c) + 8*a*b^3*d*x^3*sin_integral(d*x)*tan(1/2*c) + 16*a*b^3*d*x^3*s
in_integral((b*d*x + a*d)/b)*tan(1/2*c) + 8*a^3*b*d*x*imag_part(cos_integra
l(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^3*b*d*x*imag_part(cos_integ
ral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 8*a^3*b*d*x*imag_part(cos_integral(-d
*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 4*a^3*b*d*x*imag_part(cos_integral
(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 24*a*b^3*x^2*real_part(cos_integral(d*x
 + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 24*a*b^3*x^2*real_part(cos_integral(
d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 24*a*b^3*x^2*real_part(cos_integral(-d*x
- a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 24*a*b^3*x^2*real_part(cos_integral(-
```

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d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 8*a^3*b*d*x*sin_integral(d*x)*tan(1/2*d*x
)^2*tan(1/2*c) + 16*a^3*b*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*
tan(1/2*c) - a^4*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 +
6*b^4*x^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 - 6*b^4*x^3*ima
g_part(cos_integral(d*x))*tan(1/2*c)^2 + a^4*d^2*x*imag_part(cos_integral(-
d*x - a*d/b))*tan(1/2*c)^2 - 6*b^4*x^3*imag_part(cos_integral(-d*x - a*d/b)
)*tan(1/2*c)^2 + 6*b^4*x^3*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 + 8*a
^2*b^2*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 + 4*a^2*b^2*
d*x^2*real_part(cos_integral(d*x))*tan(1/2*c)^2 + 8*a^2*b^2*d*x^2*real_part
(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 + 4*a^2*b^2*d*x^2*real_part(cos_i
ntegral(-d*x))*tan(1/2*c)^2 - 12*b^4*x^3*sin_integral(d*x)*tan(1/2*c)^2 - 2
*a^4*d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2 + 12*b^4*x^3*sin_inte
gral((b*d*x + a*d)/b)*tan(1/2*c)^2 + 2*a^3*b*d*x*tan(1/2*d*x)^2*tan(1/2*c)^
2 + 6*a^2*b^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2
*c)^2 - 6*a^2*b^2*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^
2 - 6*a^2*b^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/
2*c)^2 + 6*a^2*b^2*x*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c
)^2 - 12*a^2*b^2*x*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 12*a^2*b
^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 8*a*b^3*d*
x^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) + 8*a*b^3*d*x^3*ima
g_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) - 4*a^3*b*d^2*x^2*real_pa
rt(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 4*a^3*b*d^2*x^2*real_part(co
s_integral(-d*x - a*d/b))*tan(1/2*a*d/b) - 16*a*b^3*d*x^3*sin_integral((b*d
*x + a*d)/b)*tan(1/2*a*d/b) - 8*a^3*b*d*x*imag_part(cos_integral(d*x + a*d/
b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 8*a^3*b*d*x*imag_part(cos_integral(-d*x
 - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 24*a*b^3*x^2*real_part(cos_integ
ral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 24*a*b^3*x^2*real_part(co
s_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 16*a^3*b*d*x*sin_
integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4*a^4*d^2*x*imag_
part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 24*b^4*x^3*imag
_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 4*a^4*d^2*x*im
ag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 24*b^4*x^3*
imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 32*a^2*b^
2*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 32
*a^2*b^2*d*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d
/b) + 8*a^4*d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) -
 48*b^4*x^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) - 24*a^
2*b^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(
1/2*a*d/b) + 24*a^2*b^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x
)^2*tan(1/2*c)*tan(1/2*a*d/b) - 48*a^2*b^2*x*sin_integral((b*d*x + a*d)/b)*
tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^3*b*d*x*imag_part(cos_integr
al(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 8*a^3*b*d*x*imag_part(cos_in
tegral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 24*a*b^3*x^2*real_part(
cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 24*a*b^3*x^2*real_
part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 16*a^3*b*d*x
```

*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b) - a^4*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 + 6*b^4*x^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 + 6*b^4*x^3*imag_part(cos_integral(d*x))*tan(1/2*a*d/b)^2 + a^4*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - 6*b^4*x^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - 6*b^4*x^3*imag_part(cos_integral(-d*x))*tan(1/2*a*d/b)^2 + 8*a^2*b^2*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 - 4*a^2*b^2*d*x^2*real_part(cos_integral(d*x))*tan(1/2*a*d/b)^2 + 8*a^2*b^2*d*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - 4*a^2*b^2*d*x^2*real_part(cos_integral(-d*x))*tan(1/2*a*d/b)^2 + 12*b^4*x^3*sin_integral(d*x)*tan(1/2*a*d/b)^2 - 2*a^4*d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 + 12*b^4*x^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 - 2*a^3*b*d*x*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 6*a^2*b^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 6*a^2*b^2*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 6*a^2*b^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 6*a^2*b^2*x*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 12*a^2*b^2*x*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 12*a^2*b^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 8*a^3*b*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a^3*b*d*x*imag_part(cos_integral(d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 8*a^3*b*d*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a^3*b*d*x*imag_part(cos_integral(-d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 24*a*b^3*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 24*a*b^3*x^2*real_part(cos_integral(d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 24*a*b^3*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 24*a*b^3*x^2*real_part(cos_integral(-d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 8*a^3*b*d*x*sin_integral(d*x)*tan(1/2*c)*tan(1/2*a*d/b)^2 - 16*a^3*b*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b)^2 - 8*a^3*b*d*x*tan(1/2*d*x)*tan(1/2*c)*tan(1/2*a*d/b)^2 - 36*a^2*b^2*x*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^3*b*d*x*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 6*a^2*b^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 6*a^2*b^2*x*imag_part(cos_integral(d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 6*a^2*b^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 6*a^2*b^2*x*imag_part(cos_integral(-d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 12*a^2*b^2*x*sin_integral(d*x)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 12*a^2*b^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 36*a^2*b^2*x*tan(1/2*d*x)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^3*b*d^2*x^2*imag_part(cos_integral(d*x + a*d/b)) - 2*a^3*b*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b)) - 4*a*b^3*d*x^3*real_part(cos_integral(d*x + a*d/b)) - 2*a*b^3*d*x^3*real_part(cos_integral(d*x)) - 4*a*b^3*d*x^3*real_part(cos_integral(-d*x - a*d/b)) - 2*a*b^3*d*x^3*real_part(cos_integral(-d*x)) + 4*a^3*b*d^2*x^2*sin_integral((b*d*x + a*d)/b) - 2*a^2*b^2*d*x^2*tan(1/2*d*x)^2 - 12*a*b^3*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2 + 12*a*b^3*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 + 12*a*b^3*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x

)^2 - 12*a*b^3*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 4*a^3*b*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2 - 2*a^3*b*d*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2 - 4*a^3*b*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 - 2*a^3*b*d*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2 + 24*a*b^3*x^2*sin_integral(d*x)*tan(1/2*d*x)^2 - 24*a*b^3*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2 + 16*a^2*b^2*d*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 8*a^2*b^2*d*x^2*imag_part(cos_integral(d*x))*tan(1/2*c) - 16*a^2*b^2*d*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) - 8*a^2*b^2*d*x^2*imag_part(cos_integral(-d*x))*tan(1/2*c) + 2*a^4*d^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c) - 12*b^4*x^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 12*b^4*x^3*real_part(cos_integral(d*x))*tan(1/2*c) + 2*a^4*d^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) - 12*b^4*x^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) + 12*b^4*x^3*real_part(cos_integral(-d*x))*tan(1/2*c) + 16*a^2*b^2*d*x^2*sin_integral(d*x)*tan(1/2*c) + 32*a^2*b^2*d*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c) - 8*a^2*b^2*d*x^2*tan(1/2*d*x)*tan(1/2*c) - 24*a*b^3*x^2*tan(1/2*d*x)^2*tan(1/2*c) - 12*a^2*b^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 12*a^2*b^2*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 12*a^2*b^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 12*a^2*b^2*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*b^2*d*x^2*tan(1/2*c)^2 + 12*a*b^3*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 - 12*a*b^3*x^2*imag_part(cos_integral(d*x))*tan(1/2*c)^2 - 12*a*b^3*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 + 12*a*b^3*x^2*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 + 4*a^3*b*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 + 2*a^3*b*d*x*real_part(cos_integral(d*x))*tan(1/2*c)^2 + 4*a^3*b*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 + 2*a^3*b*d*x*real_part(cos_integral(-d*x))*tan(1/2*c)^2 - 24*a*b^3*x^2*sin_integral(d*x)*tan(1/2*c)^2 + 24*a*b^3*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2 - 24*a*b^3*x^2*tan(1/2*d*x)*tan(1/2*c)^2 - 16*a^2*b^2*d*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) + 16*a^2*b^2*d*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) - 2*a^4*d^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) + 12*b^4*x^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 2*a^4*d^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) + 12*b^4*x^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) - 32*a^2*b^2*d*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b) + 12*a^2*b^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 12*a^2*b^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 48*a*b^3*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 48*a*b^3*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 16*a^3*b*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 16*a^3*b*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 96*a*b^3*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) - 12*a^2*b^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 12*a^2*b^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^2*b^2

*d*x^2*tan(1/2*a*d/b)^2 + 12*a*b^3*x^2*imag_part(cos_integral(d*x + a*d/b))
*tan(1/2*a*d/b)^2 + 12*a*b^3*x^2*imag_part(cos_integral(d*x))*tan(1/2*a*d/b
)^2 - 12*a*b^3*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 -
12*a*b^3*x^2*imag_part(cos_integral(-d*x))*tan(1/2*a*d/b)^2 + 4*a^3*b*d*x*
real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 - 2*a^3*b*d*x*real_pa
rt(cos_integral(d*x))*tan(1/2*a*d/b)^2 + 4*a^3*b*d*x*real_part(cos_integral
(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - 2*a^3*b*d*x*real_part(cos_integral(-d*x)
)*tan(1/2*a*d/b)^2 + 24*a*b^3*x^2*sin_integral(d*x)*tan(1/2*a*d/b)^2 + 24*a
*b^3*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 + 24*a*b^3*x^2*tan(
1/2*d*x)*tan(1/2*a*d/b)^2 + 24*a*b^3*x^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 12*a
^2*b^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 +
12*a^2*b^2*x*real_part(cos_integral(d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 12
*a^2*b^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^
2 + 12*a^2*b^2*x*real_part(cos_integral(-d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2
- 8*a^3*b*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 8*a^3*b*tan(1/2*d*x)
*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^4*d^2*x*imag_part(cos_integral(d*x + a*d
/b)) - 6*b^4*x^3*imag_part(cos_integral(d*x + a*d/b)) + 6*b^4*x^3*imag_part
(cos_integral(d*x)) - a^4*d^2*x*imag_part(cos_integral(-d*x - a*d/b)) + 6*b
^4*x^3*imag_part(cos_integral(-d*x - a*d/b)) - 6*b^4*x^3*imag_part(cos_inte
gral(-d*x)) - 8*a^2*b^2*d*x^2*real_part(cos_integral(d*x + a*d/b)) - 4*a^2*
b^2*d*x^2*real_part(cos_integral(d*x)) - 8*a^2*b^2*d*x^2*real_part(cos_inte
gral(-d*x - a*d/b)) - 4*a^2*b^2*d*x^2*real_part(cos_integral(-d*x)) + 12*b^
4*x^3*sin_integral(d*x) + 2*a^4*d^2*x*sin_integral((b*d*x + a*d)/b) - 12*b^
4*x^3*sin_integral((b*d*x + a*d)/b) - 2*a^3*b*d*x*tan(1/2*d*x)^2 - 6*a^2*b^
2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2 + 6*a^2*b^2*x*imag_
part(cos_integral(d*x))*tan(1/2*d*x)^2 + 6*a^2*b^2*x*imag_part(cos_integral
(-d*x - a*d/b))*tan(1/2*d*x)^2 - 6*a^2*b^2*x*imag_part(cos_integral(-d*x))*
tan(1/2*d*x)^2 + 12*a^2*b^2*x*sin_integral(d*x)*tan(1/2*d*x)^2 - 12*a^2*b^2
*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2 + 8*a^3*b*d*x*imag_part(cos
_integral(d*x + a*d/b))*tan(1/2*c) + 4*a^3*b*d*x*imag_part(cos_integral(d*x
))*tan(1/2*c) - 8*a^3*b*d*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c
) - 4*a^3*b*d*x*imag_part(cos_integral(-d*x))*tan(1/2*c) - 24*a*b^3*x^2*rea
l_part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 24*a*b^3*x^2*real_part(cos_i
ntegral(d*x))*tan(1/2*c) - 24*a*b^3*x^2*real_part(cos_integral(-d*x - a*d/b
))*tan(1/2*c) + 24*a*b^3*x^2*real_part(cos_integral(-d*x))*tan(1/2*c) + 8*a
^3*b*d*x*sin_integral(d*x)*tan(1/2*c) + 16*a^3*b*d*x*sin_integral((b*d*x +
a*d)/b)*tan(1/2*c) - 8*a^3*b*d*x*tan(1/2*d*x)*tan(1/2*c) - 36*a^2*b^2*x*tan
(1/2*d*x)^2*tan(1/2*c) - 2*a^3*b*d*x*tan(1/2*c)^2 + 6*a^2*b^2*x*imag_part(c
os_integral(d*x + a*d/b))*tan(1/2*c)^2 - 6*a^2*b^2*x*imag_part(cos_integral
(d*x))*tan(1/2*c)^2 - 6*a^2*b^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan
(1/2*c)^2 + 6*a^2*b^2*x*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 - 12*a^2
*b^2*x*sin_integral(d*x)*tan(1/2*c)^2 + 12*a^2*b^2*x*sin_integral((b*d*x +
a*d)/b)*tan(1/2*c)^2 - 36*a^2*b^2*x*tan(1/2*d*x)*tan(1/2*c)^2 - 8*a^3*b*d*x
*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) + 8*a^3*b*d*x*imag_par
t(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) + 24*a*b^3*x^2*real_part(cos_i

ntegral(d*x + a*d/b))*tan(1/2*a*d/b) + 24*a*b^3*x^2*real_part(cos_integral(
-d*x - a*d/b))*tan(1/2*a*d/b) - 16*a^3*b*d*x*sin_integral((b*d*x + a*d)/b)*
tan(1/2*a*d/b) - 24*a^2*b^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*
c)*tan(1/2*a*d/b) + 24*a^2*b^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(
1/2*c)*tan(1/2*a*d/b) - 48*a^2*b^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*
c)*tan(1/2*a*d/b) + 2*a^3*b*d*x*tan(1/2*a*d/b)^2 + 6*a^2*b^2*x*imag_part(co
s_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 + 6*a^2*b^2*x*imag_part(cos_integ
ral(d*x))*tan(1/2*a*d/b)^2 - 6*a^2*b^2*x*imag_part(cos_integral(-d*x - a*d/
b))*tan(1/2*a*d/b)^2 - 6*a^2*b^2*x*imag_part(cos_integral(-d*x))*tan(1/2*a*
d/b)^2 + 12*a^2*b^2*x*sin_integral(d*x)*tan(1/2*a*d/b)^2 + 12*a^2*b^2*x*sin
_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 + 36*a^2*b^2*x*tan(1/2*d*x)*tan
(1/2*a*d/b)^2 + 36*a^2*b^2*x*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d*x^2
- 12*a*b^3*x^2*imag_part(cos_integral(d*x + a*d/b)) + 12*a*b^3*x^2*imag_par
t(cos_integral(d*x)) + 12*a*b^3*x^2*imag_part(cos_integral(-d*x - a*d/b)) -
12*a*b^3*x^2*imag_part(cos_integral(-d*x)) - 4*a^3*b*d*x*real_part(cos_int
egral(d*x + a*d/b)) - 2*a^3*b*d*x*real_part(cos_integral(d*x)) - 4*a^3*b*d*
x*real_part(cos_integral(-d*x - a*d/b)) - 2*a^3*b*d*x*real_part(cos_integra
l(-d*x)) + 24*a*b^3*x^2*sin_integral(d*x) - 24*a*b^3*x^2*sin_integral((b*d*
x + a*d)/b) + 24*a*b^3*x^2*tan(1/2*d*x) + 24*a*b^3*x^2*tan(1/2*c) - 12*a^2*
b^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 12*a^2*b^2*x*real_p
art(cos_integral(d*x))*tan(1/2*c) - 12*a^2*b^2*x*real_part(cos_integral(-d*
x - a*d/b))*tan(1/2*c) + 12*a^2*b^2*x*real_part(cos_integral(-d*x))*tan(1/2
*c) - 8*a^3*b*tan(1/2*d*x)^2*tan(1/2*c) - 8*a^3*b*tan(1/2*d*x)*tan(1/2*c)^2
+ 12*a^2*b^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) + 12*a^
2*b^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) + 8*a^3*b*tan(
1/2*d*x)*tan(1/2*a*d/b)^2 + 8*a^3*b*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*a^3*b*d
*x - 6*a^2*b^2*x*imag_part(cos_integral(d*x + a*d/b)) + 6*a^2*b^2*x*imag_pa
rt(cos_integral(d*x)) + 6*a^2*b^2*x*imag_part(cos_integral(-d*x - a*d/b)) -
6*a^2*b^2*x*imag_part(cos_integral(-d*x)) + 12*a^2*b^2*x*sin_integral(d*x)
- 12*a^2*b^2*x*sin_integral((b*d*x + a*d)/b) + 36*a^2*b^2*x*tan(1/2*d*x) +
36*a^2*b^2*x*tan(1/2*c) + 8*a^3*b*tan(1/2*d*x) + 8*a^3*b*tan(1/2*c))/(a^4*
b^3*x^3*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^5*b^2*x^2*tan(1/
2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^4*b^3*x^3*tan(1/2*d*x)^2*tan(1/2
*c)^2 + a^4*b^3*x^3*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a^4*b^3*x^3*tan(1/2*c
)^2*tan(1/2*a*d/b)^2 + a^6*b*x*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2
+ 2*a^5*b^2*x^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^5*b^2*x^2*tan(1/2*d*x)^2
*tan(1/2*a*d/b)^2 + 2*a^5*b^2*x^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^4*b^3*x
^3*tan(1/2*d*x)^2 + a^4*b^3*x^3*tan(1/2*c)^2 + a^6*b*x*tan(1/2*d*x)^2*tan(1
/2*c)^2 + a^4*b^3*x^3*tan(1/2*a*d/b)^2 + a^6*b*x*tan(1/2*d*x)^2*tan(1/2*a*d
/b)^2 + a^6*b*x*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^5*b^2*x^2*tan(1/2*d*x)^
2 + 2*a^5*b^2*x^2*tan(1/2*c)^2 + 2*a^5*b^2*x^2*tan(1/2*a*d/b)^2 + a^4*b^3*x
^3 + a^6*b*x*tan(1/2*d*x)^2 + a^6*b*x*tan(1/2*c)^2 + a^6*b*x*tan(1/2*a*d/b)
^2 + 2*a^5*b^2*x^2 + a^6*b*x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x^2(a+bx)^3}\,dx = \int \frac{\sin\left(c+d\,x\right)}{x^2\,{\left(a+b\,x\right)}^3}\,dx$$

```
[In] int(sin(c + d*x)/(x^2*(a + b*x)^3),x)
[Out] int(sin(c + d*x)/(x^2*(a + b*x)^3), x)
```

3.39 $\int \frac{\sin(c+dx)}{x^3(a+bx)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 377

$$\begin{aligned}\int \frac{\sin(c+dx)}{x^3(a+bx)^3}\,dx =& -\frac{d\cos(c+dx)}{2a^3x}+\frac{bd\cos(c+dx)}{2a^3(a+bx)}-\frac{3bd\cos(c)\,\text{CosIntegral}(dx)}{a^4}\\&-\frac{3bd\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^4}+\frac{6b^2\,\text{CosIntegral}(dx)\sin(c)}{a^5}\\&-\frac{d^2\,\text{CosIntegral}(dx)\sin(c)}{2a^3}-\frac{6b^2\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^5}\\&+\frac{d^2\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{2a^3}-\frac{\sin(c+dx)}{2a^3x^2}\\&+\frac{3b\sin(c+dx)}{a^4x}+\frac{b^2\sin(c+dx)}{2a^3(a+bx)^2}+\frac{3b^2\sin(c+dx)}{a^4(a+bx)}+\frac{6b^2\cos(c)\text{Si}(dx)}{a^5}\\&-\frac{d^2\cos(c)\text{Si}(dx)}{2a^3}+\frac{3bd\sin(c)\text{Si}(dx)}{a^4}-\frac{6b^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^5}\\&+\frac{d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{2a^3}+\frac{3bd\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^4}\end{aligned}$$

```
[Out] -3*b*d*Ci(d*x)*cos(c)/a^4-3*b*d*Ci(a*d/b+d*x)*cos(-c+a*d/b)/a^4-1/2*d*cos(d
*x+c)/a^3/x+1/2*b*d*cos(d*x+c)/a^3/(b*x+a)+6*b^2*cos(c)*Si(d*x)/a^5-1/2*d^2
*cos(c)*Si(d*x)/a^3-6*b^2*cos(-c+a*d/b)*Si(a*d/b+d*x)/a^5+1/2*d^2*cos(-c+a*
d/b)*Si(a*d/b+d*x)/a^3+6*b^2*Ci(d*x)*sin(c)/a^5-1/2*d^2*Ci(d*x)*sin(c)/a^3+
3*b*d*Si(d*x)*sin(c)/a^4+6*b^2*Ci(a*d/b+d*x)*sin(-c+a*d/b)/a^5-1/2*d^2*Ci(a
*d/b+d*x)*sin(-c+a*d/b)/a^3-3*b*d*Si(a*d/b+d*x)*sin(-c+a*d/b)/a^4-1/2*sin(d
*x+c)/a^3/x^2+3*b*sin(d*x+c)/a^4/x+1/2*b^2*sin(d*x+c)/a^3/(b*x+a)^2+3*b^2*s
in(d*x+c)/a^4/(b*x+a)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6874, 3378, 3384, 3380, 3383}

$$\begin{aligned}\int \frac{\sin(c+dx)}{x^3(a+bx)^3}\,dx =& \frac{6b^2\sin(c)\,\text{CosIntegral}(dx)}{a^5} - \frac{6b^2\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{a^5} \\ &+ \frac{6b^2\cos(c)\text{Si}(dx)}{a^5} - \frac{6b^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^5} \\ &+ \frac{3b^2\sin(c+dx)}{a^4(a+bx)} - \frac{3bd\cos(c)\,\text{CosIntegral}(dx)}{a^4} \\ &- \frac{3bd\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{a^4} + \frac{3bd\sin(c)\text{Si}(dx)}{a^4} \\ &+ \frac{3bd\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^4} + \frac{3b\sin(c+dx)}{a^4x} + \frac{b^2\sin(c+dx)}{2a^3(a+bx)^2} \\ &+ \frac{d^2\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(xd+\frac{ad}{b}\right)}{2a^3} + \frac{d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{2a^3} \\ &+ \frac{bd\cos(c+dx)}{2a^3(a+bx)} - \frac{d^2\sin(c)\,\text{CosIntegral}(dx)}{2a^3} \\ &- \frac{d^2\cos(c)\text{Si}(dx)}{2a^3} - \frac{\sin(c+dx)}{2a^3x^2} - \frac{d\cos(c+dx)}{2a^3x}\end{aligned}$$

```
[In] Int[Sin[c + d*x]/(x^3*(a + b*x)^3),x]
```

```
[Out] -1/2*(d*Cos[c + d*x])/(a^3*x) + (b*d*Cos[c + d*x])/(2*a^3*(a + b*x)) - (3*b
*d*Cos[c]*CosIntegral[d*x])/a^4 - (3*b*d*Cos[c - (a*d)/b]*CosIntegral[(a*d)
/b + d*x])/a^4 + (6*b^2*CosIntegral[d*x]*Sin[c])/a^5 - (d^2*CosIntegral[d*x
]*Sin[c])/(2*a^3) - (6*b^2*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/a^5
 + (d^2*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/(2*a^3) - Sin[c + d*x]
/(2*a^3*x^2) + (3*b*Sin[c + d*x])/(a^4*x) + (b^2*Sin[c + d*x])/(2*a^3*(a +
b*x)^2) + (3*b^2*Sin[c + d*x])/(a^4*(a + b*x)) + (6*b^2*Cos[c]*SinIntegral[
d*x])/a^5 - (d^2*Cos[c]*SinIntegral[d*x])/(2*a^3) + (3*b*d*Sin[c]*SinIntegr
al[d*x])/a^4 - (6*b^2*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/a^5 + (d
^2*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/(2*a^3) + (3*b*d*Sin[c - (a
*d)/b]*SinIntegral[(a*d)/b + d*x])/a^4
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\sin(c+dx)}{a^3x^3} - \frac{3b\sin(c+dx)}{a^4x^2} + \frac{6b^2\sin(c+dx)}{a^5x} - \frac{b^3\sin(c+dx)}{a^3(a+bx)^3} - \frac{3b^3\sin(c+dx)}{a^4(a+bx)^2} - \frac{6b^3\sin(c+dx)}{a^5(a+bx)}\right) dx$$

$$= \frac{\int \frac{\sin(c+dx)}{x^3}\,dx}{a^3} - \frac{(3b)\int \frac{\sin(c+dx)}{x^2}\,dx}{a^4} + \frac{(6b^2)\int \frac{\sin(c+dx)}{x}\,dx}{a^5} - \frac{(6b^3)\int \frac{\sin(c+dx)}{a+bx}\,dx}{a^5} - \frac{(3b^3)\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx}{a^4} - \frac{b^3\int \frac{\sin(c+dx)}{(a+bx)^3}\,dx}{a^3}$$

$$= -\frac{\sin(c+dx)}{2a^3x^2} + \frac{3b\sin(c+dx)}{a^4x} + \frac{b^2\sin(c+dx)}{2a^3(a+bx)^2} + \frac{3b^2\sin(c+dx)}{a^4(a+bx)} + \frac{d\int \frac{\cos(c+dx)}{x^2}\,dx}{2a^3} - \frac{(3bd)\int \frac{\cos(c+dx)}{x}\,dx}{a^4} - \frac{(3b^2d)\int \frac{\cos(c+dx)}{a+bx}\,dx}{a^4} - \frac{(b^2d)\int \frac{\cos(c+dx)}{(a+bx)^2}\,dx}{2a^3} + \frac{(6b^2\cos(c))\int \frac{\sin(dx)}{x}\,dx}{a^5} - \frac{\left(6b^3\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^5} + \frac{(6b^2\sin(c))\int \frac{\cos(dx)}{x}\,dx}{a^5} - \frac{\left(6b^3\sin\left(c-\frac{ad}{b}\right)\right)\int \frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^5}$$

$$
\begin{aligned}
&= -\frac{d\cos(c+dx)}{2a^3x} + \frac{bd\cos(c+dx)}{2a^3(a+bx)} + \frac{6b^2\,\text{CosIntegral}(dx)\sin(c)}{a^5} \\
&\quad - \frac{6b^2\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^5} - \frac{\sin(c+dx)}{2a^3x^2} \\
&\quad + \frac{3b\sin(c+dx)}{a^4x} + \frac{b^2\sin(c+dx)}{2a^3(a+bx)^2} + \frac{3b^2\sin(c+dx)}{a^4(a+bx)} + \frac{6b^2\cos(c)\text{Si}(dx)}{a^5} \\
&\quad - \frac{6b^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^5} - \frac{d^2\int\frac{\sin(c+dx)}{x}\,dx}{2a^3} + \frac{(bd^2)\int\frac{\sin(c+dx)}{a+bx}\,dx}{2a^3} \\
&\quad - \frac{(3bd\cos(c))\int\frac{\cos(dx)}{x}\,dx}{a^4} - \frac{\left(3b^2d\cos\left(c-\frac{ad}{b}\right)\right)\int\frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^4} \\
&\quad + \frac{(3bd\sin(c))\int\frac{\sin(dx)}{x}\,dx}{a^4} + \frac{\left(3b^2d\sin\left(c-\frac{ad}{b}\right)\right)\int\frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^4} \\
&= -\frac{d\cos(c+dx)}{2a^3x} + \frac{bd\cos(c+dx)}{2a^3(a+bx)} - \frac{3bd\cos(c)\,\text{CosIntegral}(dx)}{a^4} \\
&\quad - \frac{3bd\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^4} + \frac{6b^2\,\text{CosIntegral}(dx)\sin(c)}{a^5} \\
&\quad - \frac{6b^2\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^5} - \frac{\sin(c+dx)}{2a^3x^2} + \frac{3b\sin(c+dx)}{a^4x} \\
&\quad + \frac{b^2\sin(c+dx)}{2a^3(a+bx)^2} + \frac{3b^2\sin(c+dx)}{a^4(a+bx)} + \frac{6b^2\cos(c)\text{Si}(dx)}{a^5} + \frac{3bd\sin(c)\text{Si}(dx)}{a^4} \\
&\quad - \frac{6b^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^5} + \frac{3bd\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^4} \\
&\quad - \frac{(d^2\cos(c))\int\frac{\sin(dx)}{x}\,dx}{2a^3} + \frac{\left(bd^2\cos\left(c-\frac{ad}{b}\right)\right)\int\frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{2a^3} \\
&\quad - \frac{(d^2\sin(c))\int\frac{\cos(dx)}{x}\,dx}{2a^3} + \frac{\left(bd^2\sin\left(c-\frac{ad}{b}\right)\right)\int\frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{2a^3}
\end{aligned}
$$

$$= -\frac{d\cos(c+dx)}{2a^3x} + \frac{bd\cos(c+dx)}{2a^3(a+bx)} - \frac{3bd\cos(c)\,\text{CosIntegral}(dx)}{a^4} - \frac{3bd\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^4} + \frac{6b^2\,\text{CosIntegral}(dx)\sin(c)}{a^5} - \frac{d^2\,\text{CosIntegral}(dx)\sin(c)}{2a^3} - \frac{6b^2\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^5} + \frac{d^2\,\text{CosIntegral}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{2a^3} - \frac{\sin(c+dx)}{2a^3x^2} + \frac{3b\sin(c+dx)}{a^4x} + \frac{b^2\sin(c+dx)}{2a^3(a+bx)^2} + \frac{3b^2\sin(c+dx)}{a^4(a+bx)} + \frac{6b^2\cos(c)\text{Si}(dx)}{a^5} - \frac{d^2\cos(c)\text{Si}(dx)}{2a^3} + \frac{3bd\sin(c)\text{Si}(dx)}{a^4} - \frac{6b^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^5} + \frac{d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{2a^3} + \frac{3bd\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^4}$$

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 630, normalized size of antiderivative = 1.67

$$\int \frac{\sin(c+dx)}{x^3(a+bx)^3}\,dx = \frac{-a^4dx\cos(c+dx) - a^3bdx^2\cos(c+dx) - x^2(a+bx)^2\,\text{CosIntegral}(dx)\left(6abd\cos(c) + (-12b^2+a^2d^2)\sin\right.}{}$$

```
[In] Integrate[Sin[c + d*x]/(x^3*(a + b*x)^3),x]
```

```
[Out] (-(a^4*d*x*Cos[c + d*x]) - a^3*b*d*x^2*Cos[c + d*x] - x^2*(a + b*x)^2*CosIn
tegral[d*x]*(6*a*b*d*Cos[c] + (-12*b^2 + a^2*d^2)*Sin[c]) + x^2*(a + b*x)^2
*CosIntegral[d*(a/b + x)]*(-6*a*b*d*Cos[c - (a*d)/b] + (-12*b^2 + a^2*d^2)*
Sin[c - (a*d)/b]) - a^4*Sin[c + d*x] + 4*a^3*b*x*Sin[c + d*x] + 18*a^2*b^2*
x^2*Sin[c + d*x] + 12*a*b^3*x^3*Sin[c + d*x] + 12*a^2*b^2*x^2*Cos[c]*SinInt
egral[d*x] - a^4*d^2*x^2*Cos[c]*SinIntegral[d*x] + 24*a*b^3*x^3*Cos[c]*SinI
ntegral[d*x] - 2*a^3*b*d^2*x^3*Cos[c]*SinIntegral[d*x] + 12*b^4*x^4*Cos[c]*
SinIntegral[d*x] - a^2*b^2*d^2*x^4*Cos[c]*SinIntegral[d*x] + 6*a^3*b*d*x^2*
Sin[c]*SinIntegral[d*x] + 12*a^2*b^2*d*x^3*Sin[c]*SinIntegral[d*x] + 6*a*b^
3*d*x^4*Sin[c]*SinIntegral[d*x] - 12*a^2*b^2*x^2*Cos[c - (a*d)/b]*SinIntegr
al[d*(a/b + x)] + a^4*d^2*x^2*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)] - 2
4*a*b^3*x^3*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)] + 2*a^3*b*d^2*x^3*Cos
[c - (a*d)/b]*SinIntegral[d*(a/b + x)] - 12*b^4*x^4*Cos[c - (a*d)/b]*SinInt
egral[d*(a/b + x)] + a^2*b^2*d^2*x^4*Cos[c - (a*d)/b]*SinIntegral[d*(a/b +
x)] + 6*a^3*b*d*x^2*Sin[c - (a*d)/b]*SinIntegral[d*(a/b + x)] + 12*a^2*b^2*
d*x^3*Sin[c - (a*d)/b]*SinIntegral[d*(a/b + x)] + 6*a*b^3*d*x^4*Sin[c - (a*
d)/b]*SinIntegral[d*(a/b + x)])/(2*a^5*x^2*(a + b*x)^2)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.24

method	result
derivativedivides	$d^2\left(-\frac{3b^3\left(-\frac{\sin(dx+c)}{(da-cb+b(dx+c))b}+\frac{\frac{\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}+\frac{\mathrm{Ci}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}}{b}\right)}{d\,a^4}-\frac{6b^3\left(\frac{\mathrm{Si}\left(dx+c+\frac{da-}{b}\right.}{}\right.}{}\right.$
default	$d^2\left(-\frac{3b^3\left(-\frac{\sin(dx+c)}{(da-cb+b(dx+c))b}+\frac{\frac{\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}+\frac{\mathrm{Ci}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}}{b}\right)}{d\,a^4}-\frac{6b^3\left(\frac{\mathrm{Si}\left(dx+c+\frac{da-}{b}\right.}{}\right.}{}\right.$
risch	$\frac{id^2\mathrm{e}^{-\frac{i(da-cb)}{b}}\mathrm{Ei}_1\left(-idx-ic-\frac{iad-icb}{b}\right)}{4a^3}-\frac{id^2\mathrm{e}^{ic}\mathrm{Ei}_1(-idx)}{4a^3}+\frac{3db\,\mathrm{e}^{-\frac{i(da-cb)}{b}}\mathrm{Ei}_1\left(-idx-ic-\frac{iad-icb}{b}\right)}{2a^4}+\frac{3ib^2\mathrm{e}^{ic}\mathrm{Ei}_1}{a^5}$

```
[In] int(sin(d*x+c)/x^3/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^2*(-3/d*b^3/a^4*(-sin(d*x+c)/(d*a-c*b+b*(d*x+c))/b+(Si(d*x+c+(a*d-b*c)/b)
*sin((a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)-6/d^2*b^3/
a^5*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*
d-b*c)/b)/b)-b^3/a^3*(-1/2*sin(d*x+c)/(d*a-c*b+b*(d*x+c))^2/b+1/2*(-cos(d*x
+c)/(d*a-c*b+b*(d*x+c))/b-(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+
c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)/b)/b)+1/a^3*(-1/2*sin(d*x+c)/d^2/x^2-1/2
*cos(d*x+c)/d/x-1/2*Si(d*x)*cos(c)-1/2*Ci(d*x)*sin(c))-3/d/a^4*b*(-sin(d*x+
c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c))+6/d^2/a^5*b^2*(Si(d*x)*cos(c)+Ci(d*x)
*sin(c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 585, normalized size of antiderivative = 1.55

$$\int \frac{\sin(c+dx)}{x^3(a+bx)^3}\,dx =$$
$$-\frac{(a^3bdx^2+a^4dx)\cos(dx+c)+(6(ab^3dx^4+2a^2b^2dx^3+a^3bdx^2)\,\mathrm{Ci}(dx)+((a^2b^2d^2-12b^4)x^4+2(a^3bd^2}{}$$

```
[In] integrate(sin(d*x+c)/x^3/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*((a^3*b*d*x^2 + a^4*d*x)*cos(d*x + c) + (6*(a*b^3*d*x^4 + 2*a^2*b^2*d*
x^3 + a^3*b*d*x^2)*cos_integral(d*x) + ((a^2*b^2*d^2 - 12*b^4)*x^4 + 2*(a^3
*b*d^2 - 12*a*b^3)*x^3 + (a^4*d^2 - 12*a^2*b^2)*x^2)*sin_integral(d*x))*cos
(c) + (6*(a*b^3*d*x^4 + 2*a^2*b^2*d*x^3 + a^3*b*d*x^2)*cos_integral((b*d*x
+ a*d)/b) - ((a^2*b^2*d^2 - 12*b^4)*x^4 + 2*(a^3*b*d^2 - 12*a*b^3)*x^3 + (a
^4*d^2 - 12*a^2*b^2)*x^2)*sin_integral((b*d*x + a*d)/b))*cos(-(b*c - a*d)/b
) - (12*a*b^3*x^3 + 18*a^2*b^2*x^2 + 4*a^3*b*x - a^4)*sin(d*x + c) + (((a^2
*b^2*d^2 - 12*b^4)*x^4 + 2*(a^3*b*d^2 - 12*a*b^3)*x^3 + (a^4*d^2 - 12*a^2*b
^2)*x^2)*cos_integral(d*x) - 6*(a*b^3*d*x^4 + 2*a^2*b^2*d*x^3 + a^3*b*d*x^2
)*sin_integral(d*x))*sin(c) + (((a^2*b^2*d^2 - 12*b^4)*x^4 + 2*(a^3*b*d^2 -
 12*a*b^3)*x^3 + (a^4*d^2 - 12*a^2*b^2)*x^2)*cos_integral((b*d*x + a*d)/b)
+ 6*(a*b^3*d*x^4 + 2*a^2*b^2*d*x^3 + a^3*b*d*x^2)*sin_integral((b*d*x + a*d
)/b))*sin(-(b*c - a*d)/b))/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{x^3(a+bx)^3}\,dx = \int \frac{\sin{(c+dx)}}{x^3\,(a+bx)^3}\,dx$$

```
[In] integrate(sin(d*x+c)/x**3/(b*x+a)**3,x)
[Out] Integral(sin(c + d*x)/(x**3*(a + b*x)**3), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x^3(a+bx)^3}\,dx = \int \frac{\sin{(dx+c)}}{(bx+a)^3x^3}\,dx$$

```
[In] integrate(sin(d*x+c)/x^3/(b*x+a)^3,x, algorithm="maxima")
[Out] integrate(sin(d*x + c)/((b*x + a)^3*x^3), x)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.76 (sec) , antiderivative size = 24116, normalized size of antiderivative = 63.97

$$\int \frac{\sin(c+dx)}{x^3(a+bx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)/x^3/(b*x+a)^3,x, algorithm="giac")
```

[Out] 1/4*(a^2*b^2*d^2*x^4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*ta
n(1/2*c)^2*tan(1/2*a*d/b)^2 + a^2*b^2*d^2*x^4*imag_part(cos_integral(d*x))*
tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^2*b^2*d^2*x^4*imag_part(co
s_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^
2*b^2*d^2*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan
(1/2*a*d/b)^2 + 2*a^2*b^2*d^2*x^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*
c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d^2*x^4*sin_integral((b*d*x + a*d)/b)*tan
(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d^2*x^4*real_part(cos
_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^2*
b^2*d^2*x^4*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)
^2*tan(1/2*a*d/b) - 2*a^2*b^2*d^2*x^4*real_part(cos_integral(d*x + a*d/b))*
tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^2*b^2*d^2*x^4*real_part(co
s_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^2*b^2*d^2
*x^4*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/
2*a*d/b)^2 - 2*a^2*b^2*d^2*x^4*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2
*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*a^3*b*d^2*x^3*imag_part(cos_integral(d*x +
 a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^3*b*d^2*x^3*ima
g_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*
a^3*b*d^2*x^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*
c)^2*tan(1/2*a*d/b)^2 - 2*a^3*b*d^2*x^3*imag_part(cos_integral(-d*x))*tan(1
/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 6*a*b^3*d*x^4*real_part(cos_integ
ral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 6*a*b^3*d*
x^4*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)
^2 - 6*a*b^3*d*x^4*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan
(1/2*c)^2*tan(1/2*a*d/b)^2 + 6*a*b^3*d*x^4*real_part(cos_integral(-d*x))*ta
n(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a^3*b*d^2*x^3*sin_integral(d
*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a^3*b*d^2*x^3*sin_inte
gral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^2*b^
2*d^2*x^4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2
+ a^2*b^2*d^2*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2
+ a^2*b^2*d^2*x^4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(
1/2*c)^2 - a^2*b^2*d^2*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan
(1/2*c)^2 + 2*a^2*b^2*d^2*x^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2
 - 2*a^2*b^2*d^2*x^4*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c
)^2 + 4*a^2*b^2*d^2*x^4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2
*tan(1/2*c)*tan(1/2*a*d/b) - 4*a^2*b^2*d^2*x^4*imag_part(cos_integral(-d*x
- a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^2*b^2*d^2*x^4*sin_
integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 12*a*b
^3*d*x^4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*t
an(1/2*a*d/b) - 12*a*b^3*d*x^4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/
2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 4*a^3*b*d^2*x^3*real_part(cos_integr
al(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 4*a^3*b*d^2*x
^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/
2*a*d/b) + 24*a*b^3*d*x^4*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(
1/2*c)^2*tan(1/2*a*d/b) - a^2*b^2*d^2*x^4*imag_part(cos_integral(d*x + a*d/

b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - a^2*b^2*d^2*x^4*imag_part(cos_integra
l(d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a^2*b^2*d^2*x^4*imag_part(cos_int
egral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a^2*b^2*d^2*x^4*imag
_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*a^2*b^2*d^2*x
^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*a^2*b^2*d^2*x^4*si
n_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 12*a*b^3*d*x^
4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*
d/b)^2 + 12*a*b^3*d*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2
*c)*tan(1/2*a*d/b)^2 + 12*a*b^3*d*x^4*imag_part(cos_integral(-d*x - a*d/b))
*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 12*a*b^3*d*x^4*imag_part(cos_
integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a^3*b*d^2*x^
3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*
d/b)^2 - 4*a^3*b*d^2*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/
2*c)*tan(1/2*a*d/b)^2 - 4*a^3*b*d^2*x^3*real_part(cos_integral(-d*x - a*d/b
))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a^3*b*d^2*x^3*real_part(c
os_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 24*a*b^3*d*
x^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 24*a*b^3
*d*x^4*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/
b)^2 + a^2*b^2*d^2*x^4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*ta
n(1/2*a*d/b)^2 + a^2*b^2*d^2*x^4*imag_part(cos_integral(d*x))*tan(1/2*c)^2*
tan(1/2*a*d/b)^2 - a^2*b^2*d^2*x^4*imag_part(cos_integral(-d*x - a*d/b))*ta
n(1/2*c)^2*tan(1/2*a*d/b)^2 - a^2*b^2*d^2*x^4*imag_part(cos_integral(-d*x))
*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d^2*x^4*sin_integral(d*x)*tan(1/
2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d^2*x^4*sin_integral((b*d*x + a*d)/b)*t
an(1/2*c)^2*tan(1/2*a*d/b)^2 + a^4*d^2*x^2*imag_part(cos_integral(d*x + a*d
/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 12*b^4*x^4*imag_part(co
s_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^4
*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a
*d/b)^2 - 12*b^4*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)
^2*tan(1/2*a*d/b)^2 - a^4*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan
(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 12*b^4*x^4*imag_part(cos_integr
al(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^4*d^2*x^
2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^
2 + 12*b^4*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*ta
n(1/2*a*d/b)^2 - 12*a^2*b^2*d*x^3*real_part(cos_integral(d*x + a*d/b))*tan(
1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 12*a^2*b^2*d*x^3*real_part(cos_i
ntegral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 12*a^2*b^2*d*x
^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/
2*a*d/b)^2 + 12*a^2*b^2*d*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*
tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^4*d^2*x^2*sin_integral(d*x)*tan(1/2*d*x
)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 24*b^4*x^4*sin_integral(d*x)*tan(1/2*d*
x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^4*d^2*x^2*sin_integral((b*d*x + a*
d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 24*b^4*x^4*sin_integra
l((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2
*d^2*x^4*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 2

*a^2*b^2*d^2*x^4*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 2
*a^2*b^2*d^2*x^4*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1
/2*c) - 2*a^2*b^2*d^2*x^4*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(
1/2*c) - 2*a^3*b*d^2*x^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^
2*tan(1/2*c)^2 + 2*a^3*b*d^2*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^
2*tan(1/2*c)^2 + 2*a^3*b*d^2*x^3*imag_part(cos_integral(-d*x - a*d/b))*tan(
1/2*d*x)^2*tan(1/2*c)^2 - 2*a^3*b*d^2*x^3*imag_part(cos_integral(-d*x))*tan
(1/2*d*x)^2*tan(1/2*c)^2 + 6*a*b^3*d*x^4*real_part(cos_integral(d*x + a*d/b
))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 6*a*b^3*d*x^4*real_part(cos_integral(d*x))
*tan(1/2*d*x)^2*tan(1/2*c)^2 + 6*a*b^3*d*x^4*real_part(cos_integral(-d*x -
a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 6*a*b^3*d*x^4*real_part(cos_integral(
-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a^3*b*d^2*x^3*sin_integral(d*x)*tan(
1/2*d*x)^2*tan(1/2*c)^2 - 4*a^3*b*d^2*x^3*sin_integral((b*d*x + a*d)/b)*tan
(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*b^2*d^2*x^4*real_part(cos_integral(d*x + a
*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 2*a^2*b^2*d^2*x^4*real_part(cos_inte
gral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 8*a^3*b*d^2*x^3*imag_pa
rt(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 8*
a^3*b*d^2*x^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*
c)*tan(1/2*a*d/b) - 24*a*b^3*d*x^4*real_part(cos_integral(d*x + a*d/b))*tan
(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 24*a*b^3*d*x^4*real_part(cos_integr
al(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 16*a^3*b*d^2*x
^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) +
 2*a^2*b^2*d^2*x^4*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/
2*a*d/b) + 2*a^2*b^2*d^2*x^4*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*
c)^2*tan(1/2*a*d/b) + 24*a^2*b^2*d*x^3*imag_part(cos_integral(d*x + a*d/b))
*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 24*a^2*b^2*d*x^3*imag_part(co
s_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^
4*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*
tan(1/2*a*d/b) - 24*b^4*x^4*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*
x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^4*d^2*x^2*real_part(cos_integral(-d*
x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 24*b^4*x^4*real_pa
rt(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) +
 48*a^2*b^2*d*x^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2
*tan(1/2*a*d/b) - 2*a^3*b*d^2*x^3*imag_part(cos_integral(d*x + a*d/b))*tan(
1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*a^3*b*d^2*x^3*imag_part(cos_integral(d*x))*
tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*a^3*b*d^2*x^3*imag_part(cos_integral(-d
*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*a^3*b*d^2*x^3*imag_part(co
s_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 6*a*b^3*d*x^4*real_part
(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 6*a*b^3*d*x^4
*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 6*a*b^3*d*x
^4*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 -
6*a*b^3*d*x^4*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2
 - 4*a^3*b*d^2*x^3*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 4*a^
3*b*d^2*x^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 -
 2*a^2*b^2*d^2*x^4*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*

a*d/b)^2 - 2*a^2*b^2*d^2*x^4*real_part(cos_integral(d*x))*tan(1/2*c)*tan(1/
2*a*d/b)^2 - 2*a^2*b^2*d^2*x^4*real_part(cos_integral(-d*x - a*d/b))*tan(1/
2*c)*tan(1/2*a*d/b)^2 - 2*a^2*b^2*d^2*x^4*real_part(cos_integral(-d*x))*tan
(1/2*c)*tan(1/2*a*d/b)^2 - 24*a^2*b^2*d*x^3*imag_part(cos_integral(d*x + a*
d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 24*a^2*b^2*d*x^3*imag_pa
rt(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 24*a^2*b
^2*d*x^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*ta
n(1/2*a*d/b)^2 - 24*a^2*b^2*d*x^3*imag_part(cos_integral(-d*x))*tan(1/2*d*x
)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^4*d^2*x^2*real_part(cos_integral(d*x
+ a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 24*b^4*x^4*real_part
(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*
a^4*d^2*x^2*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*
a*d/b)^2 + 24*b^4*x^4*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c
)*tan(1/2*a*d/b)^2 - 2*a^4*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*ta
n(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 24*b^4*x^4*real_part(cos_integra
l(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^4*d^2*x^2
*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 +
 24*b^4*x^4*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2
*a*d/b)^2 + 48*a^2*b^2*d*x^3*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)*ta
n(1/2*a*d/b)^2 - 48*a^2*b^2*d*x^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x
)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*a^3*b*d^2*x^3*imag_part(cos_integral(d*
x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^3*b*d^2*x^3*imag_part(cos_i
ntegral(d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^3*b*d^2*x^3*imag_part(cos
_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^3*b*d^2*x^3*im
ag_part(cos_integral(-d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 6*a*b^3*d*x^4*r
eal_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 6*a*b^3
*d*x^4*real_part(cos_integral(d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 6*a*b^3
*d*x^4*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2
+ 6*a*b^3*d*x^4*real_part(cos_integral(-d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2
 + 4*a^3*b*d^2*x^3*sin_integral(d*x)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a^3*
b*d^2*x^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 24*
a*b^3*x^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*
tan(1/2*a*d/b)^2 - 24*a*b^3*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2
*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 24*a*b^3*x^3*imag_part(cos_integral(-d*x -
 a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 24*a*b^3*x^3*imag_p
art(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 6*a^
3*b*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*
tan(1/2*a*d/b)^2 + 6*a^3*b*d*x^2*real_part(cos_integral(d*x))*tan(1/2*d*x)^
2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 6*a^3*b*d*x^2*real_part(cos_integral(-d*x
 - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 6*a^3*b*d*x^2*rea
l_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4
8*a*b^3*x^3*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2
- 48*a*b^3*x^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*ta
n(1/2*a*d/b)^2 + a^2*b^2*d^2*x^4*imag_part(cos_integral(d*x + a*d/b))*tan(1
/2*d*x)^2 - a^2*b^2*d^2*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 - a

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^2*b^2*d^2*x^4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 + a^2*b
^2*d^2*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 2*a^2*b^2*d^2*x^4
*sin_integral(d*x)*tan(1/2*d*x)^2 + 2*a^2*b^2*d^2*x^4*sin_integral((b*d*x +
 a*d)/b)*tan(1/2*d*x)^2 + 12*a*b^3*d*x^4*imag_part(cos_integral(d*x + a*d/b
))*tan(1/2*d*x)^2*tan(1/2*c) + 12*a*b^3*d*x^4*imag_part(cos_integral(d*x))*
tan(1/2*d*x)^2*tan(1/2*c) - 12*a*b^3*d*x^4*imag_part(cos_integral(-d*x - a*
d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 12*a*b^3*d*x^4*imag_part(cos_integral(-d*
x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^3*b*d^2*x^3*real_part(cos_integral(d*x
+ a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 4*a^3*b*d^2*x^3*real_part(cos_integra
l(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^3*b*d^2*x^3*real_part(cos_integral(
-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 4*a^3*b*d^2*x^3*real_part(cos_in
tegral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 24*a*b^3*d*x^4*sin_integral(d*x)*
tan(1/2*d*x)^2*tan(1/2*c) + 24*a*b^3*d*x^4*sin_integral((b*d*x + a*d)/b)*ta
n(1/2*d*x)^2*tan(1/2*c) - a^2*b^2*d^2*x^4*imag_part(cos_integral(d*x + a*d/
b))*tan(1/2*c)^2 + a^2*b^2*d^2*x^4*imag_part(cos_integral(d*x))*tan(1/2*c)^
2 + a^2*b^2*d^2*x^4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 - a^
2*b^2*d^2*x^4*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 + 2*a^2*b^2*d^2*x^
4*sin_integral(d*x)*tan(1/2*c)^2 - 2*a^2*b^2*d^2*x^4*sin_integral((b*d*x +
a*d)/b)*tan(1/2*c)^2 - a^4*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan
(1/2*d*x)^2*tan(1/2*c)^2 + 12*b^4*x^4*imag_part(cos_integral(d*x + a*d/b))*
tan(1/2*d*x)^2*tan(1/2*c)^2 + a^4*d^2*x^2*imag_part(cos_integral(d*x))*tan(
1/2*d*x)^2*tan(1/2*c)^2 - 12*b^4*x^4*imag_part(cos_integral(d*x))*tan(1/2*d
*x)^2*tan(1/2*c)^2 + a^4*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(
1/2*d*x)^2*tan(1/2*c)^2 - 12*b^4*x^4*imag_part(cos_integral(-d*x - a*d/b))*
tan(1/2*d*x)^2*tan(1/2*c)^2 - a^4*d^2*x^2*imag_part(cos_integral(-d*x))*tan
(1/2*d*x)^2*tan(1/2*c)^2 + 12*b^4*x^4*imag_part(cos_integral(-d*x))*tan(1/2
*d*x)^2*tan(1/2*c)^2 + 12*a^2*b^2*d*x^3*real_part(cos_integral(d*x + a*d/b)
)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 12*a^2*b^2*d*x^3*real_part(cos_integral(d*x
))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 12*a^2*b^2*d*x^3*real_part(cos_integral(-d
*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 12*a^2*b^2*d*x^3*real_part(cos_i
ntegral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^4*d^2*x^2*sin_integral(d*x
)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 24*b^4*x^4*sin_integral(d*x)*tan(1/2*d*x)^2
*tan(1/2*c)^2 - 2*a^4*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*
tan(1/2*c)^2 + 24*b^4*x^4*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(
1/2*c)^2 - 12*a*b^3*d*x^4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)
^2*tan(1/2*a*d/b) + 12*a*b^3*d*x^4*imag_part(cos_integral(-d*x - a*d/b))*ta
n(1/2*d*x)^2*tan(1/2*a*d/b) - 4*a^3*b*d^2*x^3*real_part(cos_integral(d*x +
a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 4*a^3*b*d^2*x^3*real_part(cos_integ
ral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 24*a*b^3*d*x^4*sin_integ
ral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4*a^2*b^2*d^2*x^4*imag
_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 4*a^2*b^2*d^2*
x^4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^2
*b^2*d^2*x^4*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) + 4*a^
4*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*ta
n(1/2*a*d/b) - 48*b^4*x^4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)
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^2*tan(1/2*c)*tan(1/2*a*d/b) - 4*a^4*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 48*b^4*x^4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 48*a^2*b^2*d*x^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 48*a^2*b^2*d*x^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^4*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 96*b^4*x^4*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 12*a*b^3*d*x^4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 12*a*b^3*d*x^4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 4*a^3*b*d^2*x^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 4*a^3*b*d^2*x^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 24*a*b^3*d*x^4*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b) + 12*a^3*b*d*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 12*a^3*b*d*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 48*a*b^3*x^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 48*a*b^3*x^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 24*a^3*b*d*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - a^2*b^2*d^2*x^4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 - a^2*b^2*d^2*x^4*imag_part(cos_integral(d*x))*tan(1/2*a*d/b)^2 + a^2*b^2*d^2*x^4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 + a^2*b^2*d^2*x^4*imag_part(cos_integral(-d*x))*tan(1/2*a*d/b)^2 - 2*a^2*b^2*d^2*x^4*sin_integral(d*x)*tan(1/2*a*d/b)^2 - 2*a^2*b^2*d^2*x^4*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 - a^4*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 12*b^4*x^4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - a^4*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 12*b^4*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a^4*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 12*b^4*x^4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a^4*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 12*b^4*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 12*a^2*b^2*d*x^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 12*a^2*b^2*d*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 12*a^2*b^2*d*x^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 12*a^2*b^2*d*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*a^4*d^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 24*b^4*x^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*a^4*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 24*b^4*x^4*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 12*a*b^3*d*x^4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 12*a*b^3*d*x^4*imag_part(cos_integral(d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 12*a*b^3*d*x^4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 12*a*b^3*d*x^4*imag_p art(cos_integral(-d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a^3*b*d^2*x^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a^3*b*d^2*x^3*real_part(cos_integral(d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a^3*b*d^2*x^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a^3*b*d^2*x^3*real_part(cos_integral(-d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 24*a*b^3*d*x^4*sin_integral(d*x)*tan(1/2*c)*tan(1/2*a*d/b)^2 - 24*a*b^3*d*x^4*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b)^2 - 12*a^3*b*d*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 12*a^3*b*d*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 12*a^3*b*d*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 12*a^3*b*d*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 48*a*b^3*x^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 48*a*b^3*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 48*a*b^3*x^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 48*a*b^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 24*a^3*b*d*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 24*a^3*b*d*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + a^4*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 12*b^4*x^4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^4*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 12*b^4*x^4*imag_part(cos_integral(d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^4*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 12*b^4*x^4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^4*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 12*b^4*x^4*imag_part(cos_integral(-d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 12*a^2*b^2*d*x^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 12*a^2*b^2*d*x^3*real_part(cos_integral(d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 12*a^2*b^2*d*x^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 12*a^2*b^2*d*x^3*real_part(cos_integral(-d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^4*d^2*x^2*sin_integral(d*x)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 24*b^4*x^4*sin_integral(d*x)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^4*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 24*b^4*x^4*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^3*b*d*x^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 12*a^2*b^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 12*a^2*b^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 12*a^2*b^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 12*a^2*b^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 24*a^2*b^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 24*a^2*b^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^3*b*d^2*x^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2 - 2*a^3*b*d^2*x^3*imag_part(cos_integral(d*x))*tan(1/2

*d*x)^2 - 2*a^3*b*d^2*x^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x
)^2 + 2*a^3*b*d^2*x^3*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 6*a*b^
3*d*x^4*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2 - 6*a*b^3*d*x^4
*real_part(cos_integral(d*x))*tan(1/2*d*x)^2 - 6*a*b^3*d*x^4*real_part(cos_
integral(-d*x - a*d/b))*tan(1/2*d*x)^2 - 6*a*b^3*d*x^4*real_part(cos_integr
al(-d*x))*tan(1/2*d*x)^2 - 4*a^3*b*d^2*x^3*sin_integral(d*x)*tan(1/2*d*x)^2
 + 4*a^3*b*d^2*x^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2 + 2*a^2*b^2
*d^2*x^4*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c) - 2*a^2*b^2*d^2*x^
4*real_part(cos_integral(d*x))*tan(1/2*c) + 2*a^2*b^2*d^2*x^4*real_part(cos
_integral(-d*x - a*d/b))*tan(1/2*c) - 2*a^2*b^2*d^2*x^4*real_part(cos_integ
ral(-d*x))*tan(1/2*c) + 24*a^2*b^2*d*x^3*imag_part(cos_integral(d*x + a*d/b
))*tan(1/2*d*x)^2*tan(1/2*c) + 24*a^2*b^2*d*x^3*imag_part(cos_integral(d*x)
)*tan(1/2*d*x)^2*tan(1/2*c) - 24*a^2*b^2*d*x^3*imag_part(cos_integral(-d*x
 - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 24*a^2*b^2*d*x^3*imag_part(cos_integr
al(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 2*a^4*d^2*x^2*real_part(cos_integral(
d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 24*b^4*x^4*real_part(cos_integral
(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^4*d^2*x^2*real_part(cos_inte
gral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 24*b^4*x^4*real_part(cos_integral(d*
x))*tan(1/2*d*x)^2*tan(1/2*c) + 2*a^4*d^2*x^2*real_part(cos_integral(-d*x -
 a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 24*b^4*x^4*real_part(cos_integral(-d*x
 - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^4*d^2*x^2*real_part(cos_integral
(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 24*b^4*x^4*real_part(cos_integral(-d*x)
)*tan(1/2*d*x)^2*tan(1/2*c) + 48*a^2*b^2*d*x^3*sin_integral(d*x)*tan(1/2*d*
x)^2*tan(1/2*c) + 48*a^2*b^2*d*x^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*
x)^2*tan(1/2*c) - 2*a^3*b*d^2*x^3*imag_part(cos_integral(d*x + a*d/b))*tan(
1/2*c)^2 + 2*a^3*b*d^2*x^3*imag_part(cos_integral(d*x))*tan(1/2*c)^2 + 2*a^
3*b*d^2*x^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 - 2*a^3*b*d^
2*x^3*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 + 6*a*b^3*d*x^4*real_part(
cos_integral(d*x + a*d/b))*tan(1/2*c)^2 + 6*a*b^3*d*x^4*real_part(cos_integ
ral(d*x))*tan(1/2*c)^2 + 6*a*b^3*d*x^4*real_part(cos_integral(-d*x - a*d/b)
)*tan(1/2*c)^2 + 6*a*b^3*d*x^4*real_part(cos_integral(-d*x))*tan(1/2*c)^2 +
 4*a^3*b*d^2*x^3*sin_integral(d*x)*tan(1/2*c)^2 - 4*a^3*b*d^2*x^3*sin_integ
ral((b*d*x + a*d)/b)*tan(1/2*c)^2 + 24*a*b^3*x^3*imag_part(cos_integral(d*x
 + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 24*a*b^3*x^3*imag_part(cos_integra
l(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 24*a*b^3*x^3*imag_part(cos_integral(-
d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 24*a*b^3*x^3*imag_part(cos_inte
gral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 6*a^3*b*d*x^2*real_part(cos_integ
ral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 6*a^3*b*d*x^2*real_part(cos
_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 6*a^3*b*d*x^2*real_part(cos_i
ntegral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 6*a^3*b*d*x^2*real_par
t(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 48*a*b^3*x^3*sin_integr
al(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 48*a*b^3*x^3*sin_integral((b*d*x + a*
d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*b^2*d^2*x^4*real_part(cos_integra
l(d*x + a*d/b))*tan(1/2*a*d/b) - 2*a^2*b^2*d^2*x^4*real_part(cos_integral(-
d*x - a*d/b))*tan(1/2*a*d/b) - 24*a^2*b^2*d*x^3*imag_part(cos_integral(d*x

+ a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 24*a^2*b^2*d*x^3*imag_part(cos_in
tegral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 2*a^4*d^2*x^2*real_pa
rt(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 24*b^4*x^4*re
al_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 2*a^4*d^
2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) +
24*b^4*x^4*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*
d/b) - 48*a^2*b^2*d*x^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/
2*a*d/b) + 8*a^3*b*d^2*x^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*
tan(1/2*a*d/b) - 8*a^3*b*d^2*x^3*imag_part(cos_integral(-d*x - a*d/b))*tan(
1/2*c)*tan(1/2*a*d/b) - 24*a*b^3*d*x^4*real_part(cos_integral(d*x + a*d/b))
*tan(1/2*c)*tan(1/2*a*d/b) - 24*a*b^3*d*x^4*real_part(cos_integral(-d*x - a
*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 16*a^3*b*d^2*x^3*sin_integral((b*d*x + a
*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) - 96*a*b^3*x^3*imag_part(cos_integral(d*x
+ a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 96*a*b^3*x^3*imag_part
(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 24*
a^3*b*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*
tan(1/2*a*d/b) - 24*a^3*b*d*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1
/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 192*a*b^3*x^3*sin_integral((b*d*x + a
*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 24*a^2*b^2*d*x^3*imag_par
t(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 24*a^2*b^2*d*x^3
*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^4*
d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) -
24*b^4*x^4*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)
+ 2*a^4*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2
*a*d/b) - 24*b^4*x^4*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan
(1/2*a*d/b) + 48*a^2*b^2*d*x^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*t
an(1/2*a*d/b) - 24*a^2*b^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2
*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 24*a^2*b^2*x^2*real_part(cos_integral
(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a^3*b*d^2*x^
3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 - 2*a^3*b*d^2*x^3*i
mag_part(cos_integral(d*x))*tan(1/2*a*d/b)^2 + 2*a^3*b*d^2*x^3*imag_part(co
s_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 + 2*a^3*b*d^2*x^3*imag_part(cos_
integral(-d*x))*tan(1/2*a*d/b)^2 + 6*a*b^3*d*x^4*real_part(cos_integral(d*x
+ a*d/b))*tan(1/2*a*d/b)^2 - 6*a*b^3*d*x^4*real_part(cos_integral(d*x))*ta
n(1/2*a*d/b)^2 + 6*a*b^3*d*x^4*real_part(cos_integral(-d*x - a*d/b))*tan(1/
2*a*d/b)^2 - 6*a*b^3*d*x^4*real_part(cos_integral(-d*x))*tan(1/2*a*d/b)^2 -
4*a^3*b*d^2*x^3*sin_integral(d*x)*tan(1/2*a*d/b)^2 - 4*a^3*b*d^2*x^3*sin_i
ntegral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 + 24*a*b^3*x^3*imag_part(cos_inte
gral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 24*a*b^3*x^3*imag_part
(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 24*a*b^3*x^3*imag_par
t(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 24*a*b^3*x^
3*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 6*a^3*b*d
*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 -
6*a^3*b*d*x^2*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2
+ 6*a^3*b*d*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1

/2*a*d/b)^2 - 6*a^3*b*d*x^2*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*ta
n(1/2*a*d/b)^2 + 48*a*b^3*x^3*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*a*d/
b)^2 + 48*a*b^3*x^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*
d/b)^2 - 24*a^2*b^2*d*x^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*t
an(1/2*a*d/b)^2 + 24*a^2*b^2*d*x^3*imag_part(cos_integral(d*x))*tan(1/2*c)*
tan(1/2*a*d/b)^2 + 24*a^2*b^2*d*x^3*imag_part(cos_integral(-d*x - a*d/b))*t
an(1/2*c)*tan(1/2*a*d/b)^2 - 24*a^2*b^2*d*x^3*imag_part(cos_integral(-d*x))
*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^4*d^2*x^2*real_part(cos_integral(d*x + a
*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 24*b^4*x^4*real_part(cos_integral(d*x
+ a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^4*d^2*x^2*real_part(cos_integra
l(d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 24*b^4*x^4*real_part(cos_integral(d*x
))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^4*d^2*x^2*real_part(cos_integral(-d*x
- a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 24*b^4*x^4*real_part(cos_integral(-
d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^4*d^2*x^2*real_part(cos_int
egral(-d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 24*b^4*x^4*real_part(cos_integra
l(-d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 48*a^2*b^2*d*x^3*sin_integral(d*x)*t
an(1/2*c)*tan(1/2*a*d/b)^2 - 48*a^2*b^2*d*x^3*sin_integral((b*d*x + a*d)/b)
*tan(1/2*c)*tan(1/2*a*d/b)^2 - 48*a*b^3*x^3*tan(1/2*d*x)^2*tan(1/2*c)*tan(1
/2*a*d/b)^2 + 24*a^2*b^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d
*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 24*a^2*b^2*x^2*real_part(cos_integral(d
*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 24*a^2*b^2*x^2*real_part(
cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 24
*a^2*b^2*x^2*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/
2*a*d/b)^2 - 24*a*b^3*x^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2
*tan(1/2*a*d/b)^2 - 24*a*b^3*x^3*imag_part(cos_integral(d*x))*tan(1/2*c)^2*
tan(1/2*a*d/b)^2 + 24*a*b^3*x^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1
/2*c)^2*tan(1/2*a*d/b)^2 + 24*a*b^3*x^3*imag_part(cos_integral(-d*x))*tan(1
/2*c)^2*tan(1/2*a*d/b)^2 - 6*a^3*b*d*x^2*real_part(cos_integral(d*x + a*d/b
))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 6*a^3*b*d*x^2*real_part(cos_integral(d*x
))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 6*a^3*b*d*x^2*real_part(cos_integral(-d*
x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 6*a^3*b*d*x^2*real_part(cos_int
egral(-d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 48*a*b^3*x^3*sin_integral(d*x)
*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 48*a*b^3*x^3*sin_integral((b*d*x + a*d)/b)
*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 48*a*b^3*x^3*tan(1/2*d*x)*tan(1/2*c)^2*tan
(1/2*a*d/b)^2 - 2*a^4*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^
2*b^2*d^2*x^4*imag_part(cos_integral(d*x + a*d/b)) - a^2*b^2*d^2*x^4*imag_p
art(cos_integral(d*x)) - a^2*b^2*d^2*x^4*imag_part(cos_integral(-d*x - a*d/
b)) + a^2*b^2*d^2*x^4*imag_part(cos_integral(-d*x)) - 2*a^2*b^2*d^2*x^4*sin
_integral(d*x) + 2*a^2*b^2*d^2*x^4*sin_integral((b*d*x + a*d)/b) + a^4*d^2*
x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2 - 12*b^4*x^4*imag_p
art(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2 - a^4*d^2*x^2*imag_part(cos_i
ntegral(d*x))*tan(1/2*d*x)^2 + 12*b^4*x^4*imag_part(cos_integral(d*x))*tan(
1/2*d*x)^2 - a^4*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)
^2 + 12*b^4*x^4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 + a^4*
d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 12*b^4*x^4*imag_part

(cos_integral(-d*x))*tan(1/2*d*x)^2 - 12*a^2*b^2*d*x^3*real_part(cos_integr
al(d*x + a*d/b))*tan(1/2*d*x)^2 - 12*a^2*b^2*d*x^3*real_part(cos_integral(d
*x))*tan(1/2*d*x)^2 - 12*a^2*b^2*d*x^3*real_part(cos_integral(-d*x - a*d/b)
)*tan(1/2*d*x)^2 - 12*a^2*b^2*d*x^3*real_part(cos_integral(-d*x))*tan(1/2*d
*x)^2 - 2*a^4*d^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2 + 24*b^4*x^4*sin_int
egral(d*x)*tan(1/2*d*x)^2 + 2*a^4*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan
(1/2*d*x)^2 - 24*b^4*x^4*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2 + 12*
a*b^3*d*x^4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 12*a*b^3*d*x^
4*imag_part(cos_integral(d*x))*tan(1/2*c) - 12*a*b^3*d*x^4*imag_part(cos_in
tegral(-d*x - a*d/b))*tan(1/2*c) - 12*a*b^3*d*x^4*imag_part(cos_integral(-d
*x))*tan(1/2*c) + 4*a^3*b*d^2*x^3*real_part(cos_integral(d*x + a*d/b))*tan(
1/2*c) - 4*a^3*b*d^2*x^3*real_part(cos_integral(d*x))*tan(1/2*c) + 4*a^3*b*
d^2*x^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) - 4*a^3*b*d^2*x^3*
real_part(cos_integral(-d*x))*tan(1/2*c) + 24*a*b^3*d*x^4*sin_integral(d*x)
*tan(1/2*c) + 24*a*b^3*d*x^4*sin_integral((b*d*x + a*d)/b)*tan(1/2*c) + 12*
a^3*b*d*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)
+ 12*a^3*b*d*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 1
2*a^3*b*d*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*
c) - 12*a^3*b*d*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)
 - 48*a*b^3*x^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2
*c) + 48*a*b^3*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) -
 48*a*b^3*x^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*
c) + 48*a*b^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) +
 24*a^3*b*d*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) + 24*a^3*b*d*x^
2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c) - a^4*d^2*x^2*ima
g_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 + 12*b^4*x^4*imag_part(cos_i
ntegral(d*x + a*d/b))*tan(1/2*c)^2 + a^4*d^2*x^2*imag_part(cos_integral(d*x
))*tan(1/2*c)^2 - 12*b^4*x^4*imag_part(cos_integral(d*x))*tan(1/2*c)^2 + a^
4*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 - 12*b^4*x^4*i
mag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 - a^4*d^2*x^2*imag_part(c
os_integral(-d*x))*tan(1/2*c)^2 + 12*b^4*x^4*imag_part(cos_integral(-d*x))*
tan(1/2*c)^2 + 12*a^2*b^2*d*x^3*real_part(cos_integral(d*x + a*d/b))*tan(1/
2*c)^2 + 12*a^2*b^2*d*x^3*real_part(cos_integral(d*x))*tan(1/2*c)^2 + 12*a^
2*b^2*d*x^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 + 12*a^2*b^2
*d*x^3*real_part(cos_integral(-d*x))*tan(1/2*c)^2 + 2*a^4*d^2*x^2*sin_integ
ral(d*x)*tan(1/2*c)^2 - 24*b^4*x^4*sin_integral(d*x)*tan(1/2*c)^2 - 2*a^4*d
^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2 + 24*b^4*x^4*sin_integral
((b*d*x + a*d)/b)*tan(1/2*c)^2 - 2*a^3*b*d*x^2*tan(1/2*d*x)^2*tan(1/2*c)^2
+ 12*a^2*b^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/
2*c)^2 - 12*a^2*b^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2
*c)^2 - 12*a^2*b^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2
*tan(1/2*c)^2 + 12*a^2*b^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2
*tan(1/2*c)^2 - 24*a^2*b^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^
2 + 24*a^2*b^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^
2 - 12*a*b^3*d*x^4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) + 12

```
*a*b^3*d*x^4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) - 4*a^3*b
*d^2*x^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 4*a^3*b*d^2*
x^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) - 24*a*b^3*d*x^4*s
in_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b) - 12*a^3*b*d*x^2*imag_part(cos_
integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 12*a^3*b*d*x^2*imag_
part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 48*a*b^3*x
^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 48*
a*b^3*x^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/
b) - 24*a^3*b*d*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*
d/b) + 4*a^4*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/
2*a*d/b) - 48*b^4*x^4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1
/2*a*d/b) - 4*a^4*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*
tan(1/2*a*d/b) + 48*b^4*x^4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c
)*tan(1/2*a*d/b) - 48*a^2*b^2*d*x^3*real_part(cos_integral(d*x + a*d/b))*ta
n(1/2*c)*tan(1/2*a*d/b) - 48*a^2*b^2*d*x^3*real_part(cos_integral(-d*x - a*
d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^4*d^2*x^2*sin_integral((b*d*x + a*d)/
b)*tan(1/2*c)*tan(1/2*a*d/b) - 96*b^4*x^4*sin_integral((b*d*x + a*d)/b)*tan
(1/2*c)*tan(1/2*a*d/b) - 48*a^2*b^2*x^2*imag_part(cos_integral(d*x + a*d/b)
)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 48*a^2*b^2*x^2*imag_part(cos_i
ntegral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 96*a^2*b^
2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b
) + 12*a^3*b*d*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/
2*a*d/b) - 12*a^3*b*d*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^
2*tan(1/2*a*d/b) - 48*a*b^3*x^3*real_part(cos_integral(d*x + a*d/b))*tan(1/
2*c)^2*tan(1/2*a*d/b) - 48*a*b^3*x^3*real_part(cos_integral(-d*x - a*d/b))*
tan(1/2*c)^2*tan(1/2*a*d/b) + 24*a^3*b*d*x^2*sin_integral((b*d*x + a*d)/b)*
tan(1/2*c)^2*tan(1/2*a*d/b) - a^4*d^2*x^2*imag_part(cos_integral(d*x + a*d/
b))*tan(1/2*a*d/b)^2 + 12*b^4*x^4*imag_part(cos_integral(d*x + a*d/b))*tan(
1/2*a*d/b)^2 - a^4*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*a*d/b)^2 +
12*b^4*x^4*imag_part(cos_integral(d*x))*tan(1/2*a*d/b)^2 + a^4*d^2*x^2*imag
_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - 12*b^4*x^4*imag_part(c
os_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 + a^4*d^2*x^2*imag_part(cos_int
egral(-d*x))*tan(1/2*a*d/b)^2 - 12*b^4*x^4*imag_part(cos_integral(-d*x))*ta
n(1/2*a*d/b)^2 + 12*a^2*b^2*d*x^3*real_part(cos_integral(d*x + a*d/b))*tan(
1/2*a*d/b)^2 - 12*a^2*b^2*d*x^3*real_part(cos_integral(d*x))*tan(1/2*a*d/b)
^2 + 12*a^2*b^2*d*x^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^
2 - 12*a^2*b^2*d*x^3*real_part(cos_integral(-d*x))*tan(1/2*a*d/b)^2 - 2*a^4
*d^2*x^2*sin_integral(d*x)*tan(1/2*a*d/b)^2 + 24*b^4*x^4*sin_integral(d*x)*
tan(1/2*a*d/b)^2 - 2*a^4*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/
b)^2 + 24*b^4*x^4*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 + 2*a^3*b*
d*x^2*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 12*a^2*b^2*x^2*imag_part(cos_integr
al(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 12*a^2*b^2*x^2*imag_part
(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 12*a^2*b^2*x^2*imag_p
art(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 12*a^2*b^
2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 24*a^
```

2*b^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 24*a^2*b^2*x^
2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 12*a^3*b*
d*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 12
*a^3*b*d*x^2*imag_part(cos_integral(d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 12*
a^3*b*d*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)
^2 - 12*a^3*b*d*x^2*imag_part(cos_integral(-d*x))*tan(1/2*c)*tan(1/2*a*d/b)
^2 + 48*a*b^3*x^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a
*d/b)^2 + 48*a*b^3*x^3*real_part(cos_integral(d*x))*tan(1/2*c)*tan(1/2*a*d/
b)^2 + 48*a*b^3*x^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/
2*a*d/b)^2 + 48*a*b^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*c)*tan(1/2*
a*d/b)^2 + 24*a^3*b*d*x^2*sin_integral(d*x)*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2
4*a^3*b*d*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b)^2 + 8
*a^3*b*d*x^2*tan(1/2*d*x)*tan(1/2*c)*tan(1/2*a*d/b)^2 - 72*a^2*b^2*x^2*tan(
1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*a^3*b*d*x^2*tan(1/2*c)^2*tan(1/2
*a*d/b)^2 - 12*a^2*b^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^
2*tan(1/2*a*d/b)^2 - 12*a^2*b^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*c)
^2*tan(1/2*a*d/b)^2 + 12*a^2*b^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*
tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 12*a^2*b^2*x^2*imag_part(cos_integral(-d*x)
)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 24*a^2*b^2*x^2*sin_integral(d*x)*tan(1/2*
c)^2*tan(1/2*a*d/b)^2 - 24*a^2*b^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/
2*c)^2*tan(1/2*a*d/b)^2 - 72*a^2*b^2*x^2*tan(1/2*d*x)*tan(1/2*c)^2*tan(1/2*
a*d/b)^2 + 2*a^3*b*d^2*x^3*imag_part(cos_integral(d*x + a*d/b)) - 2*a^3*b*d
^2*x^3*imag_part(cos_integral(d*x)) - 2*a^3*b*d^2*x^3*imag_part(cos_integra
l(-d*x - a*d/b)) + 2*a^3*b*d^2*x^3*imag_part(cos_integral(-d*x)) - 6*a*b^3*
d*x^4*real_part(cos_integral(d*x + a*d/b)) - 6*a*b^3*d*x^4*real_part(cos_in
tegral(d*x)) - 6*a*b^3*d*x^4*real_part(cos_integral(-d*x - a*d/b)) - 6*a*b^
3*d*x^4*real_part(cos_integral(-d*x)) - 4*a^3*b*d^2*x^3*sin_integral(d*x) +
 4*a^3*b*d^2*x^3*sin_integral((b*d*x + a*d)/b) - 24*a*b^3*x^3*imag_part(cos
_integral(d*x + a*d/b))*tan(1/2*d*x)^2 + 24*a*b^3*x^3*imag_part(cos_integra
l(d*x))*tan(1/2*d*x)^2 + 24*a*b^3*x^3*imag_part(cos_integral(-d*x - a*d/b))
*tan(1/2*d*x)^2 - 24*a*b^3*x^3*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2
 - 6*a^3*b*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2 - 6*a^
3*b*d*x^2*real_part(cos_integral(d*x))*tan(1/2*d*x)^2 - 6*a^3*b*d*x^2*real_
part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 - 6*a^3*b*d*x^2*real_part(c
os_integral(-d*x))*tan(1/2*d*x)^2 + 48*a*b^3*x^3*sin_integral(d*x)*tan(1/2*
d*x)^2 - 48*a*b^3*x^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2 + 24*a^2
*b^2*d*x^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 24*a^2*b^2*d*x
^3*imag_part(cos_integral(d*x))*tan(1/2*c) - 24*a^2*b^2*d*x^3*imag_part(cos
_integral(-d*x - a*d/b))*tan(1/2*c) - 24*a^2*b^2*d*x^3*imag_part(cos_integr
al(-d*x))*tan(1/2*c) + 2*a^4*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*t
an(1/2*c) - 24*b^4*x^4*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c) - 2*
a^4*d^2*x^2*real_part(cos_integral(d*x))*tan(1/2*c) + 24*b^4*x^4*real_part(
cos_integral(d*x))*tan(1/2*c) + 2*a^4*d^2*x^2*real_part(cos_integral(-d*x -
 a*d/b))*tan(1/2*c) - 24*b^4*x^4*real_part(cos_integral(-d*x - a*d/b))*tan(
1/2*c) - 2*a^4*d^2*x^2*real_part(cos_integral(-d*x))*tan(1/2*c) + 24*b^4*x^

```
4*real_part(cos_integral(-d*x))*tan(1/2*c) + 48*a^2*b^2*d*x^3*sin_integral(
d*x)*tan(1/2*c) + 48*a^2*b^2*d*x^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)
 - 48*a*b^3*x^3*tan(1/2*d*x)^2*tan(1/2*c) - 24*a^2*b^2*x^2*real_part(cos_in
tegral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 24*a^2*b^2*x^2*real_part(c
os_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 24*a^2*b^2*x^2*real_part(cos_
integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 24*a^2*b^2*x^2*real_par
t(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 24*a*b^3*x^3*imag_part(co
s_integral(d*x + a*d/b))*tan(1/2*c)^2 - 24*a*b^3*x^3*imag_part(cos_integral
(d*x))*tan(1/2*c)^2 - 24*a*b^3*x^3*imag_part(cos_integral(-d*x - a*d/b))*ta
n(1/2*c)^2 + 24*a*b^3*x^3*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 + 6*a^
3*b*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 + 6*a^3*b*d*x^2
*real_part(cos_integral(d*x))*tan(1/2*c)^2 + 6*a^3*b*d*x^2*real_part(cos_in
tegral(-d*x - a*d/b))*tan(1/2*c)^2 + 6*a^3*b*d*x^2*real_part(cos_integral(-
d*x))*tan(1/2*c)^2 - 48*a*b^3*x^3*sin_integral(d*x)*tan(1/2*c)^2 + 48*a*b^3
*x^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2 - 48*a*b^3*x^3*tan(1/2*d*x)
*tan(1/2*c)^2 - 2*a^4*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 - 24*a^2*b^2*d*x^3*im
ag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) + 24*a^2*b^2*d*x^3*imag_p
art(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) - 2*a^4*d^2*x^2*real_part(co
s_integral(d*x + a*d/b))*tan(1/2*a*d/b) + 24*b^4*x^4*real_part(cos_integral
(d*x + a*d/b))*tan(1/2*a*d/b) - 2*a^4*d^2*x^2*real_part(cos_integral(-d*x -
 a*d/b))*tan(1/2*a*d/b) + 24*b^4*x^4*real_part(cos_integral(-d*x - a*d/b))*
tan(1/2*a*d/b) - 48*a^2*b^2*d*x^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d
/b) + 24*a^2*b^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*ta
n(1/2*a*d/b) + 24*a^2*b^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2
*d*x)^2*tan(1/2*a*d/b) - 96*a*b^3*x^3*imag_part(cos_integral(d*x + a*d/b))*
tan(1/2*c)*tan(1/2*a*d/b) + 96*a*b^3*x^3*imag_part(cos_integral(-d*x - a*d/
b))*tan(1/2*c)*tan(1/2*a*d/b) - 24*a^3*b*d*x^2*real_part(cos_integral(d*x +
 a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 24*a^3*b*d*x^2*real_part(cos_integral(
-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 192*a*b^3*x^3*sin_integral((b*d*
x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) - 24*a^2*b^2*x^2*real_part(cos_integr
al(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 24*a^2*b^2*x^2*real_part(cos
_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 24*a*b^3*x^3*imag_pa
rt(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 + 24*a*b^3*x^3*imag_part(cos
_integral(d*x))*tan(1/2*a*d/b)^2 - 24*a*b^3*x^3*imag_part(cos_integral(-d*x
 - a*d/b))*tan(1/2*a*d/b)^2 - 24*a*b^3*x^3*imag_part(cos_integral(-d*x))*ta
n(1/2*a*d/b)^2 + 6*a^3*b*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2
*a*d/b)^2 - 6*a^3*b*d*x^2*real_part(cos_integral(d*x))*tan(1/2*a*d/b)^2 + 6
*a^3*b*d*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - 6*a^3
*b*d*x^2*real_part(cos_integral(-d*x))*tan(1/2*a*d/b)^2 + 48*a*b^3*x^3*sin_
integral(d*x)*tan(1/2*a*d/b)^2 + 48*a*b^3*x^3*sin_integral((b*d*x + a*d)/b)
*tan(1/2*a*d/b)^2 + 48*a*b^3*x^3*tan(1/2*d*x)*tan(1/2*a*d/b)^2 + 2*a^4*d*x*
tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 48*a*b^3*x^3*tan(1/2*c)*tan(1/2*a*d/b)^2
+ 24*a^2*b^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*
d/b)^2 + 24*a^2*b^2*x^2*real_part(cos_integral(d*x))*tan(1/2*c)*tan(1/2*a*d
/b)^2 + 24*a^2*b^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan
```

(1/2*a*d/b)^2 + 24*a^2*b^2*x^2*real_part(cos_integral(-d*x))*tan(1/2*c)*tan
(1/2*a*d/b)^2 + 8*a^4*d*x*tan(1/2*d*x)*tan(1/2*c)*tan(1/2*a*d/b)^2 - 16*a^3
*b*x*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*a^4*d*x*tan(1/2*c)^2*ta
n(1/2*a*d/b)^2 - 16*a^3*b*x*tan(1/2*d*x)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^
4*d^2*x^2*imag_part(cos_integral(d*x + a*d/b)) - 12*b^4*x^4*imag_part(cos_i
ntegral(d*x + a*d/b)) - a^4*d^2*x^2*imag_part(cos_integral(d*x)) + 12*b^4*x
^4*imag_part(cos_integral(d*x)) - a^4*d^2*x^2*imag_part(cos_integral(-d*x -
 a*d/b)) + 12*b^4*x^4*imag_part(cos_integral(-d*x - a*d/b)) + a^4*d^2*x^2*i
mag_part(cos_integral(-d*x)) - 12*b^4*x^4*imag_part(cos_integral(-d*x)) - 1
2*a^2*b^2*d*x^3*real_part(cos_integral(d*x + a*d/b)) - 12*a^2*b^2*d*x^3*rea
l_part(cos_integral(d*x)) - 12*a^2*b^2*d*x^3*real_part(cos_integral(-d*x -
a*d/b)) - 12*a^2*b^2*d*x^3*real_part(cos_integral(-d*x)) - 2*a^4*d^2*x^2*si
n_integral(d*x) + 24*b^4*x^4*sin_integral(d*x) + 2*a^4*d^2*x^2*sin_integral
((b*d*x + a*d)/b) - 24*b^4*x^4*sin_integral((b*d*x + a*d)/b) + 2*a^3*b*d*x^
2*tan(1/2*d*x)^2 - 12*a^2*b^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(
1/2*d*x)^2 + 12*a^2*b^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 + 1
2*a^2*b^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 - 12*a^2
*b^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2 + 24*a^2*b^2*x^2*sin_
integral(d*x)*tan(1/2*d*x)^2 - 24*a^2*b^2*x^2*sin_integral((b*d*x + a*d)/b)
*tan(1/2*d*x)^2 + 12*a^3*b*d*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1
/2*c) + 12*a^3*b*d*x^2*imag_part(cos_integral(d*x))*tan(1/2*c) - 12*a^3*b*d
*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) - 12*a^3*b*d*x^2*imag
_part(cos_integral(-d*x))*tan(1/2*c) - 48*a*b^3*x^3*real_part(cos_integral(
d*x + a*d/b))*tan(1/2*c) + 48*a*b^3*x^3*real_part(cos_integral(d*x))*tan(1/
2*c) - 48*a*b^3*x^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) + 48*a
*b^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*c) + 24*a^3*b*d*x^2*sin_inte
gral(d*x)*tan(1/2*c) + 24*a^3*b*d*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2
*c) + 8*a^3*b*d*x^2*tan(1/2*d*x)*tan(1/2*c) - 72*a^2*b^2*x^2*tan(1/2*d*x)^2
*tan(1/2*c) + 2*a^3*b*d*x^2*tan(1/2*c)^2 + 12*a^2*b^2*x^2*imag_part(cos_int
egral(d*x + a*d/b))*tan(1/2*c)^2 - 12*a^2*b^2*x^2*imag_part(cos_integral(d*
x))*tan(1/2*c)^2 - 12*a^2*b^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan
(1/2*c)^2 + 12*a^2*b^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 - 24*
a^2*b^2*x^2*sin_integral(d*x)*tan(1/2*c)^2 + 24*a^2*b^2*x^2*sin_integral((b
*d*x + a*d)/b)*tan(1/2*c)^2 - 72*a^2*b^2*x^2*tan(1/2*d*x)*tan(1/2*c)^2 - 12
*a^3*b*d*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) + 12*a^3*b
*d*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) + 48*a*b^3*x^3*
real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) + 48*a*b^3*x^3*real_par
t(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) - 24*a^3*b*d*x^2*sin_integral(
(b*d*x + a*d)/b)*tan(1/2*a*d/b) - 48*a^2*b^2*x^2*imag_part(cos_integral(d*x
 + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 48*a^2*b^2*x^2*imag_part(cos_integra
l(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 96*a^2*b^2*x^2*sin_integral((b
*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) - 2*a^3*b*d*x^2*tan(1/2*a*d/b)^2 +
 12*a^2*b^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 + 12*
a^2*b^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*a*d/b)^2 - 12*a^2*b^2*x^2*
imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - 12*a^2*b^2*x^2*ima

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g_part(cos_integral(-d*x))*tan(1/2*a*d/b)^2 + 24*a^2*b^2*x^2*sin_integral(d
*x)*tan(1/2*a*d/b)^2 + 24*a^2*b^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2
*a*d/b)^2 + 72*a^2*b^2*x^2*tan(1/2*d*x)*tan(1/2*a*d/b)^2 + 72*a^2*b^2*x^2*t
an(1/2*c)*tan(1/2*a*d/b)^2 + 4*a^4*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)
^2 + 4*a^4*tan(1/2*d*x)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 24*a*b^3*x^3*imag_p
art(cos_integral(d*x + a*d/b)) + 24*a*b^3*x^3*imag_part(cos_integral(d*x))
+ 24*a*b^3*x^3*imag_part(cos_integral(-d*x - a*d/b)) - 24*a*b^3*x^3*imag_pa
rt(cos_integral(-d*x)) - 6*a^3*b*d*x^2*real_part(cos_integral(d*x + a*d/b))
 - 6*a^3*b*d*x^2*real_part(cos_integral(d*x)) - 6*a^3*b*d*x^2*real_part(cos
_integral(-d*x - a*d/b)) - 6*a^3*b*d*x^2*real_part(cos_integral(-d*x)) + 48
*a*b^3*x^3*sin_integral(d*x) - 48*a*b^3*x^3*sin_integral((b*d*x + a*d)/b) +
 48*a*b^3*x^3*tan(1/2*d*x) + 2*a^4*d*x*tan(1/2*d*x)^2 + 48*a*b^3*x^3*tan(1/
2*c) - 24*a^2*b^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 24*
a^2*b^2*x^2*real_part(cos_integral(d*x))*tan(1/2*c) - 24*a^2*b^2*x^2*real_p
art(cos_integral(-d*x - a*d/b))*tan(1/2*c) + 24*a^2*b^2*x^2*real_part(cos_i
ntegral(-d*x))*tan(1/2*c) + 8*a^4*d*x*tan(1/2*d*x)*tan(1/2*c) - 16*a^3*b*x*
tan(1/2*d*x)^2*tan(1/2*c) + 2*a^4*d*x*tan(1/2*c)^2 - 16*a^3*b*x*tan(1/2*d*x
)*tan(1/2*c)^2 + 24*a^2*b^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/
2*a*d/b) + 24*a^2*b^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d
/b) - 2*a^4*d*x*tan(1/2*a*d/b)^2 + 16*a^3*b*x*tan(1/2*d*x)*tan(1/2*a*d/b)^2
 + 16*a^3*b*x*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^3*b*d*x^2 - 12*a^2*b^2*x^2*
imag_part(cos_integral(d*x + a*d/b)) + 12*a^2*b^2*x^2*imag_part(cos_integra
l(d*x)) + 12*a^2*b^2*x^2*imag_part(cos_integral(-d*x - a*d/b)) - 12*a^2*b^2
*x^2*imag_part(cos_integral(-d*x)) + 24*a^2*b^2*x^2*sin_integral(d*x) - 24*
a^2*b^2*x^2*sin_integral((b*d*x + a*d)/b) + 72*a^2*b^2*x^2*tan(1/2*d*x) + 7
2*a^2*b^2*x^2*tan(1/2*c) + 4*a^4*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^4*tan(1/2*
d*x)*tan(1/2*c)^2 - 4*a^4*tan(1/2*d*x)*tan(1/2*a*d/b)^2 - 4*a^4*tan(1/2*c)*
tan(1/2*a*d/b)^2 - 2*a^4*d*x + 16*a^3*b*x*tan(1/2*d*x) + 16*a^3*b*x*tan(1/2
*c) - 4*a^4*tan(1/2*d*x) - 4*a^4*tan(1/2*c))/(a^5*b^2*x^4*tan(1/2*d*x)^2*ta
n(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^6*b*x^3*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1
/2*a*d/b)^2 + a^5*b^2*x^4*tan(1/2*d*x)^2*tan(1/2*c)^2 + a^5*b^2*x^4*tan(1/2
*d*x)^2*tan(1/2*a*d/b)^2 + a^5*b^2*x^4*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^7*
x^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^6*b*x^3*tan(1/2*d*x)
^2*tan(1/2*c)^2 + 2*a^6*b*x^3*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*a^6*b*x^3
*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^5*b^2*x^4*tan(1/2*d*x)^2 + a^5*b^2*x^4*t
an(1/2*c)^2 + a^7*x^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + a^5*b^2*x^4*tan(1/2*a*d
/b)^2 + a^7*x^2*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a^7*x^2*tan(1/2*c)^2*tan(
1/2*a*d/b)^2 + 2*a^6*b*x^3*tan(1/2*d*x)^2 + 2*a^6*b*x^3*tan(1/2*c)^2 + 2*a^
6*b*x^3*tan(1/2*a*d/b)^2 + a^5*b^2*x^4 + a^7*x^2*tan(1/2*d*x)^2 + a^7*x^2*t
an(1/2*c)^2 + a^7*x^2*tan(1/2*a*d/b)^2 + 2*a^6*b*x^3 + a^7*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x^3(a+bx)^3}\,dx = \int \frac{\sin\left(c+d\,x\right)}{x^3\,{\left(a+b\,x\right)}^3}\,dx$$

```
[In] int(sin(c + d*x)/(x^3*(a + b*x)^3),x)
[Out] int(sin(c + d*x)/(x^3*(a + b*x)^3), x)
```

3.40 $\int x^3(a+bx^2)\sin(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 141

$$\begin{aligned}\int x^3(a+bx^2)\sin(c+dx)\,dx = &-\frac{120bx\cos(c+dx)}{d^5}+\frac{6ax\cos(c+dx)}{d^3}\\ &+\frac{20bx^3\cos(c+dx)}{d^3}-\frac{ax^3\cos(c+dx)}{d}\\ &-\frac{bx^5\cos(c+dx)}{d}+\frac{120b\sin(c+dx)}{d^6}-\frac{6a\sin(c+dx)}{d^4}\\ &-\frac{60bx^2\sin(c+dx)}{d^4}+\frac{3ax^2\sin(c+dx)}{d^2}+\frac{5bx^4\sin(c+dx)}{d^2}\end{aligned}$$

```
[Out] -120*b*x*cos(d*x+c)/d^5+6*a*x*cos(d*x+c)/d^3+20*b*x^3*cos(d*x+c)/d^3-a*x^3*
cos(d*x+c)/d-b*x^5*cos(d*x+c)/d+120*b*sin(d*x+c)/d^6-6*a*sin(d*x+c)/d^4-60*
b*x^2*sin(d*x+c)/d^4+3*a*x^2*sin(d*x+c)/d^2+5*b*x^4*sin(d*x+c)/d^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3420, 3377, 2717}

$$\begin{aligned}\int x^3(a+bx^2)\sin(c+dx)\,dx = &-\frac{6a\sin(c+dx)}{d^4}+\frac{6ax\cos(c+dx)}{d^3}+\frac{3ax^2\sin(c+dx)}{d^2}\\ &-\frac{ax^3\cos(c+dx)}{d}+\frac{120b\sin(c+dx)}{d^6}\\ &-\frac{120bx\cos(c+dx)}{d^5}-\frac{60bx^2\sin(c+dx)}{d^4}\\ &+\frac{20bx^3\cos(c+dx)}{d^3}+\frac{5bx^4\sin(c+dx)}{d^2}-\frac{bx^5\cos(c+dx)}{d}\end{aligned}$$

```
[In] Int[x^3*(a + b*x^2)*Sin[c + d*x],x]
```

```
[Out] (-120*b*x*Cos[c + d*x])/d^5 + (6*a*x*Cos[c + d*x])/d^3 + (20*b*x^3*Cos[c +
d*x])/d^3 - (a*x^3*Cos[c + d*x])/d - (b*x^5*Cos[c + d*x])/d + (120*b*Sin[c
+ d*x])/d^6 - (6*a*Sin[c + d*x])/d^4 - (60*b*x^2*Sin[c + d*x])/d^4 + (3*a*x
^2*Sin[c + d*x])/d^2 + (5*b*x^4*Sin[c + d*x])/d^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(ax^3\sin(c+dx)+bx^5\sin(c+dx)\right)\,dx \\
&= a\int x^3\sin(c+dx)\,dx+b\int x^5\sin(c+dx)\,dx \\
&= -\frac{ax^3\cos(c+dx)}{d}-\frac{bx^5\cos(c+dx)}{d}+\frac{(3a)\int x^2\cos(c+dx)\,dx}{d}+\frac{(5b)\int x^4\cos(c+dx)\,dx}{d} \\
&= -\frac{ax^3\cos(c+dx)}{d}-\frac{bx^5\cos(c+dx)}{d}+\frac{3ax^2\sin(c+dx)}{d^2} \\
&\quad+\frac{5bx^4\sin(c+dx)}{d^2}-\frac{(6a)\int x\sin(c+dx)\,dx}{d^2}-\frac{(20b)\int x^3\sin(c+dx)\,dx}{d^2} \\
&= \frac{6ax\cos(c+dx)}{d^3}+\frac{20bx^3\cos(c+dx)}{d^3}-\frac{ax^3\cos(c+dx)}{d} \\
&\quad-\frac{bx^5\cos(c+dx)}{d}+\frac{3ax^2\sin(c+dx)}{d^2}+\frac{5bx^4\sin(c+dx)}{d^2} \\
&\quad-\frac{(6a)\int\cos(c+dx)\,dx}{d^3}-\frac{(60b)\int x^2\cos(c+dx)\,dx}{d^3} \\
&= \frac{6ax\cos(c+dx)}{d^3}+\frac{20bx^3\cos(c+dx)}{d^3}-\frac{ax^3\cos(c+dx)}{d}-\frac{bx^5\cos(c+dx)}{d}-\frac{6a\sin(c+dx)}{d^4} \\
&\quad-\frac{60bx^2\sin(c+dx)}{d^4}+\frac{3ax^2\sin(c+dx)}{d^2}+\frac{5bx^4\sin(c+dx)}{d^2}+\frac{(120b)\int x\sin(c+dx)\,dx}{d^4}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{120bx\cos(c+dx)}{d^5} + \frac{6ax\cos(c+dx)}{d^3} + \frac{20bx^3\cos(c+dx)}{d^3} \\
&\quad - \frac{ax^3\cos(c+dx)}{d} - \frac{bx^5\cos(c+dx)}{d} - \frac{6a\sin(c+dx)}{d^4} - \frac{60bx^2\sin(c+dx)}{d^4} \\
&\quad + \frac{3ax^2\sin(c+dx)}{d^2} + \frac{5bx^4\sin(c+dx)}{d^2} + \frac{(120b)\int\cos(c+dx)\,dx}{d^5} \\
&= -\frac{120bx\cos(c+dx)}{d^5} + \frac{6ax\cos(c+dx)}{d^3} + \frac{20bx^3\cos(c+dx)}{d^3} \\
&\quad - \frac{ax^3\cos(c+dx)}{d} - \frac{bx^5\cos(c+dx)}{d} + \frac{120b\sin(c+dx)}{d^6} - \frac{6a\sin(c+dx)}{d^4} \\
&\quad - \frac{60bx^2\sin(c+dx)}{d^4} + \frac{3ax^2\sin(c+dx)}{d^2} + \frac{5bx^4\sin(c+dx)}{d^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.65

$$
\int x^3(a+bx^2)\sin(c+dx)\,dx
$$

$$
= \frac{-dx(ad^2(-6+d^2x^2)+b(120-20d^2x^2+d^4x^4))\cos(c+dx)+(3ad^2(-2+d^2x^2)+5b(24-12d^2x^2+d^4x}{d^6}
$$

```
[In] Integrate[x^3*(a + b*x^2)*Sin[c + d*x],x]
```

```
[Out] (-(d*x*(a*d^2*(-6 + d^2*x^2) + b*(120 - 20*d^2*x^2 + d^4*x^4))*Cos[c + d*x]
) + (3*a*d^2*(-2 + d^2*x^2) + 5*b*(24 - 12*d^2*x^2 + d^4*x^4))*Sin[c + d*x]
)/d^6
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{x\left(b\,x^{4} d^{4}+a\,d^{4} x^{2}-20 d^{2} x^{2} b-6 a\,d^{2}+120 b\right) \cos \left(d x+c\right)}{d^{5}}+\frac{\left(5 b\,x^{4} d^{4}+3 a\,d^{4} x^{2}-60 d^{2} x^{2} b-6 a\,d^{2}+120 b\right) \sin \left(d x+c\right)}{d^{6}}$
parallelrisch	$\frac{\left(x^{2}\left(b\,x^{2}+a\right) d^{4}+\left(-20 b\,x^{2}-6 a\right) d^{2}+120 b\right) x d\left(\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)+\left(\left(10 b\,x^{4}+6 a\,x^{2}\right) d^{4}+\left(-120 b\,x^{2}-12 a\right) d^{2}+240 b\right) \tan\left(\frac{d x}{2}+\frac{c}{2}\right.}{d^{6}\left(1+\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}$
norman	$\frac{\frac{b\,x^{5}\left(\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\left(a\,d^{2}-20 b\right) x^{3}\left(\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{d^{3}}-\frac{b\,x^{5}}{d}+\frac{6\left(a\,d^{2}-20 b\right) x}{d^{5}}-\frac{\left(a\,d^{2}-20 b\right) x^{3}}{d^{3}}-\frac{12\left(a\,d^{2}-20 b\right) \tan\left(\frac{d x}{2}+\frac{c}{2}\right)}{d^{6}}+\frac{10 b\,x^{4}}{}}{1+\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)}$
meijerg	$\frac{32 b\sqrt{\pi}\,\sin\left(c\right)\left(-\frac{15}{4\sqrt{\pi}}+\frac{\left(\frac{15}{8} d^{4} x^{4}-\frac{45}{2} d^{2} x^{2}+45\right) \cos\left(d x\right)}{12\sqrt{\pi}}+\frac{x d\left(\frac{3}{8} d^{4} x^{4}-\frac{15}{2} d^{2} x^{2}+45\right) \sin\left(d x\right)}{12\sqrt{\pi}}\right)}{d^{6}}+\frac{32 b\sqrt{\pi}\,\cos\left(c\right)\left(-\frac{x d\left(\frac{7}{8} d^{4} x^{4}-\right.}{}\right.}{}$
parts	$-\frac{b\,x^{5} \cos\left(d x+c\right)}{d}-\frac{a\,x^{3} \cos\left(d x+c\right)}{d}+\frac{\frac{3 a\,c^{2} \sin\left(d x+c\right)}{d^{2}}-\frac{6 a c\left(\cos\left(d x+c\right)+\left(d x+c\right) \sin\left(d x+c\right)\right)}{d^{2}}+\frac{3 a\left(\left(d x+c\right)^{2} \sin\left(d x+c\right)-2 \sin\left(d x+\right.\right.}{d^{2}}}{}$
derivativedivides	$\frac{a\,c^{3} \cos\left(d x+c\right)+3 a\,c^{2}\left(\sin\left(d x+c\right)-\cos\left(d x+c\right)\left(d x+c\right)\right)-3 a c\left(-\left(d x+c\right)^{2} \cos\left(d x+c\right)+2 \cos\left(d x+c\right)+2\left(d x+c\right) \sin\left(d x+c\right)\right)+a\left(\right.}{}$
default	$\frac{a\,c^{3} \cos\left(d x+c\right)+3 a\,c^{2}\left(\sin\left(d x+c\right)-\cos\left(d x+c\right)\left(d x+c\right)\right)-3 a c\left(-\left(d x+c\right)^{2} \cos\left(d x+c\right)+2 \cos\left(d x+c\right)+2\left(d x+c\right) \sin\left(d x+c\right)\right)+a\left(\right.}{}$

```
[In] int(x^3*(b*x^2+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^5*x*(b*d^4*x^4+a*d^4*x^2-20*b*d^2*x^2-6*a*d^2+120*b)*cos(d*x+c)+(5*b*d
^4*x^4+3*a*d^4*x^2-60*b*d^2*x^2-6*a*d^2+120*b)/d^6*sin(d*x+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.67

$$\int x^3\left(a+bx^2\right)\sin(c+dx)\,dx = -\frac{(bd^5x^5+(ad^5-20\,bd^3)x^3-6\,(ad^3-20\,bd)x)\cos\left(dx+c\right)-(5\,bd^4x^4-6\,ad^2+3\,(ad^4-20\,bd^2)x^2+120}{d^6}$$

```
[In] integrate(x^3*(b*x^2+a)*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b*d^5*x^5 + (a*d^5 - 20*b*d^3)*x^3 - 6*(a*d^3 - 20*b*d)*x)*cos(d*x + c)
- (5*b*d^4*x^4 - 6*a*d^2 + 3*(a*d^4 - 20*b*d^2)*x^2 + 120*b)*sin(d*x + c))/
d^6
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.19

$$\int x^3(a+bx^2)\sin(c+dx)\,dx$$
$$= \begin{cases} -\frac{ax^3\cos(c+dx)}{d} + \frac{3ax^2\sin(c+dx)}{d^2} + \frac{6ax\cos(c+dx)}{d^3} - \frac{6a\sin(c+dx)}{d^4} - \frac{bx^5\cos(c+dx)}{d} + \frac{5bx^4\sin(c+dx)}{d^2} + \frac{20bx^3\cos(c+dx)}{d^3} - \\ \left(\frac{ax^4}{4} + \frac{bx^6}{6}\right)\sin(c) \end{cases}$$

```
[In] integrate(x**3*(b*x**2+a)*sin(d*x+c),x)
```

```
[Out] Piecewise((-a*x**3*cos(c + d*x)/d + 3*a*x**2*sin(c + d*x)/d**2 + 6*a*x*cos(
c + d*x)/d**3 - 6*a*sin(c + d*x)/d**4 - b*x**5*cos(c + d*x)/d + 5*b*x**4*si
n(c + d*x)/d**2 + 20*b*x**3*cos(c + d*x)/d**3 - 60*b*x**2*sin(c + d*x)/d**4
 - 120*b*x*cos(c + d*x)/d**5 + 120*b*sin(c + d*x)/d**6, Ne(d, 0)), ((a*x**4
/4 + b*x**6/6)*sin(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(141) = 282.

Time = 0.20 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.64

$$\int x^3(a+bx^2)\sin(c+dx)\,dx$$
$$= \frac{ac^3\cos(dx+c) + \frac{bc^5\cos(dx+c)}{d^2} - 3((dx+c)\cos(dx+c) - \sin(dx+c))ac^2 - \frac{5((dx+c)\cos(dx+c)-\sin(dx+c))bc^4}{d^2} -}{}$$

```
[In] integrate(x^3*(b*x^2+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] (a*c^3*cos(d*x + c) + b*c^5*cos(d*x + c)/d^2 - 3*((d*x + c)*cos(d*x + c) -
sin(d*x + c))*a*c^2 - 5*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c^4/d^2 +
 3*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a*c + 10*(((
d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b*c^3/d^2 - (((d*x
 + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*a -
 10*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x
 + c))*b*c^2/d^2 + 5*(((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d*x + c) - 4*
((d*x + c)^3 - 6*d*x - 6*c)*sin(d*x + c))*b*c/d^2 - (((d*x + c)^5 - 20*(d*x
 + c)^3 + 120*d*x + 120*c)*cos(d*x + c) - 5*((d*x + c)^4 - 12*(d*x + c)^2 +
 24)*sin(d*x + c))*b/d^2)/d^4
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.69

$$\int x^3(a+bx^2)\sin(c+dx)\,dx = -\frac{(bd^5x^5+ad^5x^3-20\,bd^3x^3-6\,ad^3x+120\,bdx)\cos(dx+c)}{d^6} + \frac{(5\,bd^4x^4+3\,ad^4x^2-60\,bd^2x^2-6\,ad^2+120\,b)\sin(dx+c)}{d^6}$$

```
[In] integrate(x^3*(b*x^2+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b*d^5*x^5 + a*d^5*x^3 - 20*b*d^3*x^3 - 6*a*d^3*x + 120*b*d*x)*cos(d*x + c
)/d^6 + (5*b*d^4*x^4 + 3*a*d^4*x^2 - 60*b*d^2*x^2 - 6*a*d^2 + 120*b)*sin(d*
x + c)/d^6
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.86

$$\int x^3(a+bx^2)\sin(c+dx)\,dx = \frac{6\,\sin(c+d\,x)\,\left(20\,b-a\,d^2\right)}{d^6} + \frac{x^3\,\cos(c+d\,x)\,\left(20\,b-a\,d^2\right)}{d^3} - \frac{3\,x^2\,\sin(c+d\,x)\,\left(20\,b-a\,d^2\right)}{d^4} - \frac{6\,x\,\cos(c+d\,x)\,\left(20\,b-a\,d^2\right)}{d^5} - \frac{b\,x^5\,\cos(c+d\,x)}{d} + \frac{5\,b\,x^4\,\sin(c+d\,x)}{d^2}$$

```
[In] int(x^3*sin(c + d*x)*(a + b*x^2),x)
```

```
[Out] (6*sin(c + d*x)*(20*b - a*d^2))/d^6 + (x^3*cos(c + d*x)*(20*b - a*d^2))/d^3
 - (3*x^2*sin(c + d*x)*(20*b - a*d^2))/d^4 - (6*x*cos(c + d*x)*(20*b - a*d^
2))/d^5 - (b*x^5*cos(c + d*x))/d + (5*b*x^4*sin(c + d*x))/d^2
```

3.41 $\int x^2(a+bx^2)\sin(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 111

$$\int x^2(a+bx^2)\sin(c+dx)\,dx = -\frac{24b\cos(c+dx)}{d^5}+\frac{2a\cos(c+dx)}{d^3}+\frac{12bx^2\cos(c+dx)}{d^3}-\frac{ax^2\cos(c+dx)}{d}-\frac{bx^4\cos(c+dx)}{d}-\frac{24bx\sin(c+dx)}{d^4}+\frac{2ax\sin(c+dx)}{d^2}+\frac{4bx^3\sin(c+dx)}{d^2}$$

```
[Out] -24*b*cos(d*x+c)/d^5+2*a*cos(d*x+c)/d^3+12*b*x^2*cos(d*x+c)/d^3-a*x^2*cos(d
*x+c)/d-b*x^4*cos(d*x+c)/d-24*b*x*sin(d*x+c)/d^4+2*a*x*sin(d*x+c)/d^2+4*b*x
^3*sin(d*x+c)/d^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3420, 3377, 2718}

$$\int x^2(a+bx^2)\sin(c+dx)\,dx = \frac{2a\cos(c+dx)}{d^3}+\frac{2ax\sin(c+dx)}{d^2}-\frac{ax^2\cos(c+dx)}{d}-\frac{24b\cos(c+dx)}{d^5}-\frac{24bx\sin(c+dx)}{d^4}+\frac{12bx^2\cos(c+dx)}{d^3}+\frac{4bx^3\sin(c+dx)}{d^2}-\frac{bx^4\cos(c+dx)}{d}$$

```
[In] Int[x^2*(a + b*x^2)*Sin[c + d*x],x]
```

```
[Out] (-24*b*Cos[c + d*x])/d^5 + (2*a*Cos[c + d*x])/d^3 + (12*b*x^2*Cos[c + d*x])
/d^3 - (a*x^2*Cos[c + d*x])/d - (b*x^4*Cos[c + d*x])/d - (24*b*x*Sin[c + d*
x])/d^4 + (2*a*x*Sin[c + d*x])/d^2 + (4*b*x^3*Sin[c + d*x])/d^2
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(ax^2\sin(c+dx)+bx^4\sin(c+dx)\right)\,dx \\
&= a\int x^2\sin(c+dx)\,dx+b\int x^4\sin(c+dx)\,dx \\
&= -\frac{ax^2\cos(c+dx)}{d}-\frac{bx^4\cos(c+dx)}{d}+\frac{(2a)\int x\cos(c+dx)\,dx}{d}+\frac{(4b)\int x^3\cos(c+dx)\,dx}{d} \\
&= -\frac{ax^2\cos(c+dx)}{d}-\frac{bx^4\cos(c+dx)}{d}+\frac{2ax\sin(c+dx)}{d^2} \\
&\quad+\frac{4bx^3\sin(c+dx)}{d^2}-\frac{(2a)\int\sin(c+dx)\,dx}{d^2}-\frac{(12b)\int x^2\sin(c+dx)\,dx}{d^2} \\
&= \frac{2a\cos(c+dx)}{d^3}+\frac{12bx^2\cos(c+dx)}{d^3}-\frac{ax^2\cos(c+dx)}{d}-\frac{bx^4\cos(c+dx)}{d} \\
&\quad+\frac{2ax\sin(c+dx)}{d^2}+\frac{4bx^3\sin(c+dx)}{d^2}-\frac{(24b)\int x\cos(c+dx)\,dx}{d^3} \\
&= \frac{2a\cos(c+dx)}{d^3}+\frac{12bx^2\cos(c+dx)}{d^3}-\frac{ax^2\cos(c+dx)}{d}-\frac{bx^4\cos(c+dx)}{d} \\
&\quad-\frac{24bx\sin(c+dx)}{d^4}+\frac{2ax\sin(c+dx)}{d^2}+\frac{4bx^3\sin(c+dx)}{d^2}+\frac{(24b)\int\sin(c+dx)\,dx}{d^4} \\
&= -\frac{24b\cos(c+dx)}{d^5}+\frac{2a\cos(c+dx)}{d^3}+\frac{12bx^2\cos(c+dx)}{d^3}-\frac{ax^2\cos(c+dx)}{d} \\
&\quad-\frac{bx^4\cos(c+dx)}{d}-\frac{24bx\sin(c+dx)}{d^4}+\frac{2ax\sin(c+dx)}{d^2}+\frac{4bx^3\sin(c+dx)}{d^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.68

$$\int x^2(a+bx^2)\sin(c+dx)\,dx$$
$$=\frac{-((ad^2(-2+d^2x^2)+b(24-12d^2x^2+d^4x^4))\cos(c+dx))+2dx(ad^2+2b(-6+d^2x^2))\sin(c+dx)}{d^5}$$

```
[In] Integrate[x^2*(a + b*x^2)*Sin[c + d*x],x]
[Out] (-((a*d^2*(-2 + d^2*x^2) + b*(24 - 12*d^2*x^2 + d^4*x^4))*Cos[c + d*x]) + 2
*d*x*(a*d^2 + 2*b*(-6 + d^2*x^2))*Sin[c + d*x])/d^5
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{(b\,x^4d^4+a\,d^4x^2-12d^2x^2b-2a\,d^2+24b)\cos(dx+c)}{d^5}+\frac{2x(2d^2x^2b+a\,d^2-12b)\sin(dx+c)}{d^4}$
parallelrisch	$\frac{((b\,x^2+a)d^2-12b)x^2d^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4((2b\,x^2+a)d^2-12b)xd\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+(-b\,x^4-a\,x^2)d^4+4(3b\,x^2+a)d^2-48b}{d^5\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$
norman	$\frac{\frac{4a\,d^2-48b}{d^5}+\frac{b\,x^4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\left(a\,d^2-12b\right)x^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d^3}-\frac{b\,x^4}{d}-\frac{\left(a\,d^2-12b\right)x^2}{d^3}+\frac{8b\,x^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^2}+\frac{4\left(a\,d^2-12b\right)x\tan}{d^4}}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$
parts	$-\frac{b\,x^4\cos(dx+c)}{d}-\frac{a\,x^2\cos(dx+c)}{d}+\frac{-\frac{2ac\sin(dx+c)}{d}+\frac{2a(\cos(dx+c)+(dx+c)\sin(dx+c))}{d}-\frac{4b\,c^3\sin(dx+c)}{d^3}+\frac{12b\,c^2(\cos(dx}{}}{}$
meijerg	$\frac{16b\sqrt{\pi}\,\sin(c)\left(-\frac{x\left(d^2\right)^{\frac{5}{2}}\left(-\frac{5d^2x^2}{2}+15\right)\cos(dx)}{10\sqrt{\pi}\,d^4}+\frac{\left(d^2\right)^{\frac{5}{2}}\left(\frac{5}{8}d^4x^4-\frac{15}{2}d^2x^2+15\right)\sin(dx)}{10\sqrt{\pi}\,d^5}\right)}{d^4\sqrt{d^2}}+\frac{16b\sqrt{\pi}\,\cos(c)\left(\frac{3}{2\sqrt{\pi}}-\frac{\left(\frac{3}{8}d^4x^4\right.}{}\right.}{}$
derivativedivides	$\frac{-a\,c^2\cos(dx+c)-2ac(\sin(dx+c)-\cos(dx+c)(dx+c))+a\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)-\frac{b\,c}{}}{}$
default	$\frac{-a\,c^2\cos(dx+c)-2ac(\sin(dx+c)-\cos(dx+c)(dx+c))+a\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)-\frac{b\,c}{}}{}$

```
[In] int(x^2*(b*x^2+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
[Out] -(b*d^4*x^4+a*d^4*x^2-12*b*d^2*x^2-2*a*d^2+24*b)/d^5*cos(d*x+c)+2/d^4*x*(2*
b*d^2*x^2+a*d^2-12*b)*sin(d*x+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.69

$$\int x^2(a+bx^2)\sin(c+dx)\,dx = -\frac{(bd^4x^4 - 2\,ad^2 + (ad^4 - 12\,bd^2)x^2 + 24\,b)\cos{(dx+c)} - 2\,(2\,bd^3x^3 + (ad^3 - 12\,bd)x)\sin{(dx+c)}}{d^5}$$

```
[In] integrate(x^2*(b*x^2+a)*sin(d*x+c),x, algorithm="fricas")
[Out] -((b*d^4*x^4 - 2*a*d^2 + (a*d^4 - 12*b*d^2)*x^2 + 24*b)*cos(d*x + c) - 2*(2
*b*d^3*x^3 + (a*d^3 - 12*b*d)*x)*sin(d*x + c))/d^5
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.21

$$\int x^2(a+bx^2)\sin(c+dx)\,dx = \begin{cases} -\frac{ax^2\cos{(c+dx)}}{d} + \frac{2ax\sin{(c+dx)}}{d^2} + \frac{2a\cos{(c+dx)}}{d^3} - \frac{bx^4\cos{(c+dx)}}{d} + \frac{4bx^3\sin{(c+dx)}}{d^2} + \frac{12bx^2\cos{(c+dx)}}{d^3} - \frac{24bx\sin{(c+dx)}}{d^4} - 24 \\ \left(\frac{ax^3}{3} + \frac{bx^5}{5}\right)\sin{(c)} \end{cases}$$

```
[In] integrate(x**2*(b*x**2+a)*sin(d*x+c),x)
[Out] Piecewise((-a*x**2*cos(c + d*x)/d + 2*a*x*sin(c + d*x)/d**2 + 2*a*cos(c + d
*x)/d**3 - b*x**4*cos(c + d*x)/d + 4*b*x**3*sin(c + d*x)/d**2 + 12*b*x**2*c
os(c + d*x)/d**3 - 24*b*x*sin(c + d*x)/d**4 - 24*b*cos(c + d*x)/d**5, Ne(d,
 0)), ((a*x**3/3 + b*x**5/5)*sin(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(111) = 222.

Time = 0.19 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.32

$$\int x^2(a+bx^2)\sin(c+dx)\,dx = -\frac{ac^2\cos{(dx+c)} + \frac{bc^4\cos(dx+c)}{d^2} - 2\,((dx+c)\cos{(dx+c)} - \sin{(dx+c)})ac - \frac{4\,((dx+c)\cos(dx+c) - \sin(dx+c))bc^3}{d^2} -}{}$$

```
[In] integrate(x^2*(b*x^2+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a*c^2*cos(d*x + c) + b*c^4*cos(d*x + c)/d^2 - 2*((d*x + c)*cos(d*x + c) -
 sin(d*x + c))*a*c - 4*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c^3/d^2 +
(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a + 6*(((d*x +
c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b*c^2/d^2 - 4*(((d*x + c
)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*b*c/d^2
 + (((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d*x + c) - 4*((d*x + c)^3 - 6*d
*x - 6*c)*sin(d*x + c))*b/d^2)/d^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.71

$$\int x^2(a+bx^2)\sin(c+dx)\,dx = -\frac{(bd^4x^4 + ad^4x^2 - 12\,bd^2x^2 - 2\,ad^2 + 24\,b)\cos{(dx+c)}}{d^5} + \frac{2\,(2\,bd^3x^3 + ad^3x - 12\,bdx)\sin{(dx+c)}}{d^5}$$

```
[In] integrate(x^2*(b*x^2+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b*d^4*x^4 + a*d^4*x^2 - 12*b*d^2*x^2 - 2*a*d^2 + 24*b)*cos(d*x + c)/d^5 +
 2*(2*b*d^3*x^3 + a*d^3*x - 12*b*d*x)*sin(d*x + c)/d^5
```

Mupad [B] (verification not implemented)

Time = 6.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.87

$$\int x^2(a+bx^2)\sin(c+dx)\,dx = \frac{x^2\,\cos{(c+d\,x)}\,\left(12\,b-a\,d^2\right)}{d^3} - \frac{2\,\cos{(c+d\,x)}\,\left(12\,b-a\,d^2\right)}{d^5} - \frac{2\,x\,\sin{(c+d\,x)}\,\left(12\,b-a\,d^2\right)}{d^4} - \frac{b\,x^4\,\cos{(c+d\,x)}}{d} + \frac{4\,b\,x^3\,\sin{(c+d\,x)}}{d^2}$$

```
[In] int(x^2*sin(c + d*x)*(a + b*x^2),x)
```

```
[Out] (x^2*cos(c + d*x)*(12*b - a*d^2))/d^3 - (2*cos(c + d*x)*(12*b - a*d^2))/d^5
 - (2*x*sin(c + d*x)*(12*b - a*d^2))/d^4 - (b*x^4*cos(c + d*x))/d + (4*b*x^
3*sin(c + d*x))/d^2
```

3.42 $\int x(a+bx^2)\sin(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int x(a+bx^2)\sin(c+dx)\,dx = \frac{6bx\cos(c+dx)}{d^3} - \frac{ax\cos(c+dx)}{d} - \frac{bx^3\cos(c+dx)}{d} - \frac{6b\sin(c+dx)}{d^4} + \frac{a\sin(c+dx)}{d^2} + \frac{3bx^2\sin(c+dx)}{d^2}$$

```
[Out] 6*b*x*cos(d*x+c)/d^3-a*x*cos(d*x+c)/d-b*x^3*cos(d*x+c)/d-6*b*sin(d*x+c)/d^4
+a*sin(d*x+c)/d^2+3*b*x^2*sin(d*x+c)/d^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3420, 3377, 2717}

$$\int x(a+bx^2)\sin(c+dx)\,dx = \frac{a\sin(c+dx)}{d^2} - \frac{ax\cos(c+dx)}{d} - \frac{6b\sin(c+dx)}{d^4} + \frac{6bx\cos(c+dx)}{d^3} + \frac{3bx^2\sin(c+dx)}{d^2} - \frac{bx^3\cos(c+dx)}{d}$$

```
[In] Int[x*(a + b*x^2)*Sin[c + d*x],x]
```

```
[Out] (6*b*x*Cos[c + d*x])/d^3 - (a*x*Cos[c + d*x])/d - (b*x^3*Cos[c + d*x])/d -
(6*b*Sin[c + d*x])/d^4 + (a*Sin[c + d*x])/d^2 + (3*b*x^2*Sin[c + d*x])/d^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(ax\sin(c+dx)+bx^3\sin(c+dx)\right)\,dx\\
&= a\int x\sin(c+dx)\,dx+b\int x^3\sin(c+dx)\,dx\\
&= -\frac{ax\cos(c+dx)}{d}-\frac{bx^3\cos(c+dx)}{d}+\frac{a\int\cos(c+dx)\,dx}{d}+\frac{(3b)\int x^2\cos(c+dx)\,dx}{d}\\
&= -\frac{ax\cos(c+dx)}{d}-\frac{bx^3\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d^2}\\
&\quad+\frac{3bx^2\sin(c+dx)}{d^2}-\frac{(6b)\int x\sin(c+dx)\,dx}{d^2}\\
&= \frac{6bx\cos(c+dx)}{d^3}-\frac{ax\cos(c+dx)}{d}-\frac{bx^3\cos(c+dx)}{d}\\
&\quad+\frac{a\sin(c+dx)}{d^2}+\frac{3bx^2\sin(c+dx)}{d^2}-\frac{(6b)\int\cos(c+dx)\,dx}{d^3}\\
&= \frac{6bx\cos(c+dx)}{d^3}-\frac{ax\cos(c+dx)}{d}-\frac{bx^3\cos(c+dx)}{d}\\
&\quad-\frac{6b\sin(c+dx)}{d^4}+\frac{a\sin(c+dx)}{d^2}+\frac{3bx^2\sin(c+dx)}{d^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int x(a+bx^2)\sin(c+dx)\,dx = \frac{-dx(ad^2+b(-6+d^2x^2))\cos(c+dx)+(ad^2+3b(-2+d^2x^2))\sin(c+dx)}{d^4}$$

```
[In] Integrate[x*(a + b*x^2)*Sin[c + d*x],x]
[Out] (-(d*x*(a*d^2 + b*(-6 + d^2*x^2))*Cos[c + d*x]) + (a*d^2 + 3*b*(-2 + d^2*x^
2))*Sin[c + d*x])/d^4
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{x(d^2x^2b+a\,d^2-6b)\cos(dx+c)}{d^3}+\frac{(3d^2x^2b+a\,d^2-6b)\sin(dx+c)}{d^4}$
parallelrisch	$\frac{((b\,x^2+a)d^2-6b)xd\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2((3b\,x^2+a)d^2-6b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-((b\,x^2+a)d^2-6b)xd}{d^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$
parts	$-\frac{b\,x^3\cos(dx+c)}{d}-\frac{ax\cos(dx+c)}{d}+\frac{a\sin(dx+c)+\frac{3b\,c^2\sin(dx+c)}{d^2}-\frac{6bc(\cos(dx+c)+(dx+c)\sin(dx+c))}{d^2}+\frac{3b\left((dx+c)^2\sin(dx+ \cdots\right.}{}}{d^2}$
norman	$\frac{\frac{b\,x^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\left(a\,d^2-6b\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d^3}-\frac{b\,x^3}{d}-\frac{\left(a\,d^2-6b\right)x}{d^3}+\frac{2\left(a\,d^2-6b\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^4}+\frac{6b\,x^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^2}}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$
meijerg	$\frac{8b\sqrt{\pi}\,\sin(c)\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(-\frac{3d^2x^2}{2}+3\right)\cos(dx)}{4\sqrt{\pi}}-\frac{dx\left(-\frac{d^2x^2}{2}+3\right)\sin(dx)}{4\sqrt{\pi}}\right)}{d^4}+\frac{8b\sqrt{\pi}\,\cos(c)\left(\frac{xd\left(-\frac{5d^2x^2}{2}+15\right)\cos(dx)}{20\sqrt{\pi}}-\frac{\left(-\frac{15d^2x \cdots\right.}{}\right.}{d^4}$
derivativedivides	$\frac{ac\cos(dx+c)+a(\sin(dx+c)-\cos(dx+c)(dx+c))+\frac{b\,c^3\cos(dx+c)}{d^2}+\frac{3b\,c^2(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^2}-\frac{3bc\left(-(dx+c)^2\cos(dx+c \cdots\right.}{}}{d^2}$
default	$\frac{ac\cos(dx+c)+a(\sin(dx+c)-\cos(dx+c)(dx+c))+\frac{b\,c^3\cos(dx+c)}{d^2}+\frac{3b\,c^2(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^2}-\frac{3bc\left(-(dx+c)^2\cos(dx+c \cdots\right.}{}}{d^2}$

```
[In] int(x*(b*x^2+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^3*x*(b*d^2*x^2+a*d^2-6*b)*cos(d*x+c)+(3*b*d^2*x^2+a*d^2-6*b)/d^4*sin(d
*x+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.75

$$\int x(a+bx^2)\sin(c+dx)\,dx = -\frac{(bd^3x^3+(ad^3-6\,bd)x)\cos(dx+c)-(3\,bd^2x^2+ad^2-6\,b)\sin(dx+c)}{d^4}$$

```
[In] integrate(x*(b*x^2+a)*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b*d^3*x^3 + (a*d^3 - 6*b*d)*x)*cos(d*x + c) - (3*b*d^2*x^2 + a*d^2 - 6*b
)*sin(d*x + c))/d^4
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.24

$$\int x(a+bx^2)\sin(c+dx)\,dx$$
$$=\begin{cases} -\frac{ax\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d^2}-\frac{bx^3\cos(c+dx)}{d}+\frac{3bx^2\sin(c+dx)}{d^2}+\frac{6bx\cos(c+dx)}{d^3}-\frac{6b\sin(c+dx)}{d^4} & \text{for } d\neq 0 \\ \left(\frac{ax^2}{2}+\frac{bx^4}{4}\right)\sin(c) & \text{otherwise} \end{cases}$$

```
[In] integrate(x*(b*x**2+a)*sin(d*x+c),x)
```

```
[Out] Piecewise((-a*x*cos(c + d*x)/d + a*sin(c + d*x)/d**2 - b*x**3*cos(c + d*x)/
d + 3*b*x**2*sin(c + d*x)/d**2 + 6*b*x*cos(c + d*x)/d**3 - 6*b*sin(c + d*x)
/d**4, Ne(d, 0)), ((a*x**2/2 + b*x**4/4)*sin(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(80) = 160.

Time = 0.19 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.06

$$\int x(a+bx^2)\sin(c+dx)\,dx$$
$$=\frac{ac\cos(dx+c)+\frac{bc^3\cos(dx+c)}{d^2}-((dx+c)\cos(dx+c)-\sin(dx+c))a-\frac{3((dx+c)\cos(dx+c)-\sin(dx+c))bc^2}{d^2}+\frac{3((}{}}{d^2}$$

```
[In] integrate(x*(b*x^2+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] (a*c*cos(d*x + c) + b*c^3*cos(d*x + c)/d^2 - ((d*x + c)*cos(d*x + c) - sin(
d*x + c))*a - 3*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c^2/d^2 + 3*(((d*
x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b*c/d^2 - (((d*x + c
)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*b/d^2)/
d^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.75

$$\int x(a+bx^2)\sin(c+dx)\,dx=-\frac{(bd^3x^3+ad^3x-6\,bdx)\cos(dx+c)}{d^4}+\frac{(3\,bd^2x^2+ad^2-6\,b)\sin(dx+c)}{d^4}$$

```
[In] integrate(x*(b*x^2+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b*d^3*x^3 + a*d^3*x - 6*b*d*x)*cos(d*x + c)/d^4 + (3*b*d^2*x^2 + a*d^2 -
6*b)*sin(d*x + c)/d^4
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

$$\int x(a+bx^2)\sin(c+dx)\,dx = \frac{x\,\cos(c+d\,x)\,(6\,b-a\,d^2)}{d^3} - \frac{\sin(c+d\,x)\,(6\,b-a\,d^2)}{d^4} - \frac{b\,x^3\,\cos(c+d\,x)}{d} + \frac{3\,b\,x^2\,\sin(c+d\,x)}{d^2}$$

```
[In] int(x*sin(c + d*x)*(a + b*x^2),x)
```

```
[Out] (x*cos(c + d*x)*(6*b - a*d^2))/d^3 - (sin(c + d*x)*(6*b - a*d^2))/d^4 - (b*
x^3*cos(c + d*x))/d + (3*b*x^2*sin(c + d*x))/d^2
```

3.43 $\int (a+bx^2)\sin(c+dx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 53

$$\int (a+bx^2)\sin(c+dx)\,dx=\frac{2b\cos(c+dx)}{d^3}-\frac{a\cos(c+dx)}{d}-\frac{bx^2\cos(c+dx)}{d}+\frac{2bx\sin(c+dx)}{d^2}$$

```
[Out] 2*b*cos(d*x+c)/d^3-a*cos(d*x+c)/d-b*x^2*cos(d*x+c)/d+2*b*x*sin(d*x+c)/d^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3410, 2718, 3377}

$$\int (a+bx^2)\sin(c+dx)\,dx=-\frac{a\cos(c+dx)}{d}+\frac{2b\cos(c+dx)}{d^3}+\frac{2bx\sin(c+dx)}{d^2}-\frac{bx^2\cos(c+dx)}{d}$$

```
[In] Int[(a + b*x^2)*Sin[c + d*x],x]
```

```
[Out] (2*b*Cos[c + d*x])/d^3 - (a*Cos[c + d*x])/d - (b*x^2*Cos[c + d*x])/d + (2*b
*x*Sin[c + d*x])/d^2
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3410

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d,
 n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a\sin(c+dx)+bx^2\sin(c+dx)\right)\,dx\\
&= a\int \sin(c+dx)\,dx+b\int x^2\sin(c+dx)\,dx\\
&= -\frac{a\cos(c+dx)}{d}-\frac{bx^2\cos(c+dx)}{d}+\frac{(2b)\int x\cos(c+dx)\,dx}{d}\\
&= -\frac{a\cos(c+dx)}{d}-\frac{bx^2\cos(c+dx)}{d}+\frac{2bx\sin(c+dx)}{d^2}-\frac{(2b)\int \sin(c+dx)\,dx}{d^2}\\
&= \frac{2b\cos(c+dx)}{d^3}-\frac{a\cos(c+dx)}{d}-\frac{bx^2\cos(c+dx)}{d}+\frac{2bx\sin(c+dx)}{d^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \left(a+bx^2\right)\sin(c+dx)\,dx=\frac{-\left(\left(ad^2+b\left(-2+d^2x^2\right)\right)\cos(c+dx)\right)+2bdx\sin(c+dx)}{d^3}$$

```
[In] Integrate[(a + b*x^2)*Sin[c + d*x],x]
[Out] (-((a*d^2 + b*(-2 + d^2*x^2))*Cos[c + d*x]) + 2*b*d*x*Sin[c + d*x])/d^3
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

method	result
risch	$-\frac{(d^2x^2b+a\,d^2-2b)\cos(dx+c)}{d^3}+\frac{2bx\sin(dx+c)}{d^2}$
parallelrisch	$\frac{((-b\,x^2-a)\,d^2+2b)\cos(dx+c)+2\sin(dx+c)bdx+a\,d^2-2b}{d^3}$
parts	$-\frac{b\,x^2\cos(dx+c)}{d}-\frac{a\cos(dx+c)}{d}+\frac{2b(\cos(dx+c)+(dx+c)\sin(dx+c)-c\sin(dx+c))}{d^3}$
norman	$\frac{\frac{b\,x^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2a\,d^2-4b}{d^3}-\frac{b\,x^2}{d}+\frac{4bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^2}}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$
derivativedivides	$\frac{-a\cos(dx+c)-\frac{b\,c^2\cos(dx+c)}{d^2}-\frac{2bc(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^2}+\frac{b\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)}{d^2}}{d}$
default	$\frac{-a\cos(dx+c)-\frac{b\,c^2\cos(dx+c)}{d^2}-\frac{2bc(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^2}+\frac{b\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)}{d^2}}{d}$
meijerg	$\frac{4b\sqrt{\pi}\,\sin(c)\left(\frac{x\left(d^2\right)^{\frac{3}{2}}\cos(dx)}{2\sqrt{\pi}\,d^2}-\frac{\left(d^2\right)^{\frac{3}{2}}\left(-\frac{3d^2x^2}{2}+3\right)\sin(dx)}{6\sqrt{\pi}\,d^3}\right)}{d^2\sqrt{d^2}}+\frac{4b\sqrt{\pi}\,\cos(c)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(-\frac{d^2x^2}{2}+1\right)\cos(dx)}{2\sqrt{\pi}}+\frac{dx\sin(dx)}{2\sqrt{\pi}}\right.}{d^3}$

```
[In] int((b*x^2+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
[Out] -(b*d^2*x^2+a*d^2-2*b)/d^3*cos(d*x+c)+2*b*x*sin(d*x+c)/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int (a+bx^2)\sin(c+dx)\,dx=\frac{2\,bdx\sin(dx+c)-(bd^2x^2+ad^2-2\,b)\cos(dx+c)}{d^3}$$

```
[In] integrate((b*x^2+a)*sin(d*x+c),x, algorithm="fricas")
[Out] (2*b*d*x*sin(d*x + c) - (b*d^2*x^2 + a*d^2 - 2*b)*cos(d*x + c))/d^3
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.23

$$\int (a+bx^2)\sin(c+dx)\,dx=\begin{cases}-\frac{a\cos(c+dx)}{d}-\frac{bx^2\cos(c+dx)}{d}+\frac{2bx\sin(c+dx)}{d^2}+\frac{2b\cos(c+dx)}{d^3} & \text{for } d\neq 0\\ \left(ax+\frac{bx^3}{3}\right)\sin(c) & \text{otherwise}\end{cases}$$

```
[In] integrate((b*x**2+a)*sin(d*x+c),x)
```

```
[Out] Piecewise((-a*cos(c + d*x)/d - b*x**2*cos(c + d*x)/d + 2*b*x*sin(c + d*x)/d
**2 + 2*b*cos(c + d*x)/d**3, Ne(d, 0)), ((a*x + b*x**3/3)*sin(c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.72

$$\int (a+bx^2)\sin(c+dx)\,dx = -\frac{a\cos(dx+c) + \frac{bc^2\cos(dx+c)}{d^2} - \frac{2\,((dx+c)\cos(dx+c)-\sin(dx+c))bc}{d^2} + \frac{\left(\left((dx+c)^2-2\right)\cos(dx+c)-2\,(dx+c)\sin(dx+c)\right)b}{d^2}}{d}$$

```
[In] integrate((b*x^2+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a*cos(d*x + c) + b*c^2*cos(d*x + c)/d^2 - 2*((d*x + c)*cos(d*x + c) - sin
(d*x + c))*b*c/d^2 + (((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x
+ c))*b/d^2)/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int (a+bx^2)\sin(c+dx)\,dx = \frac{2\,bx\sin(dx+c)}{d^2} - \frac{(bd^2x^2+ad^2-2\,b)\cos(dx+c)}{d^3}$$

```
[In] integrate((b*x^2+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] 2*b*x*sin(d*x + c)/d^2 - (b*d^2*x^2 + a*d^2 - 2*b)*cos(d*x + c)/d^3
```

Mupad [B] (verification not implemented)

Time = 5.95 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int (a+bx^2)\sin(c+dx)\,dx = \frac{\cos(c+d\,x)\,(2\,b-a\,d^2)}{d^3} + \frac{2\,b\,x\,\sin(c+d\,x)}{d^2} - \frac{b\,x^2\,\cos(c+d\,x)}{d}$$

```
[In] int(sin(c + d*x)*(a + b*x^2),x)
```

```
[Out] (cos(c + d*x)*(2*b - a*d^2))/d^3 + (2*b*x*sin(c + d*x))/d^2 - (b*x^2*cos(c
+ d*x))/d
```

3.44 $\int \frac{(a+bx^2)\sin(c+dx)}{x} dx$

Optimal result

Integrand size = 17, antiderivative size = 41

$$\int \frac{(a+bx^2)\sin(c+dx)}{x} dx = -\frac{bx\cos(c+dx)}{d} + a\,\text{CosIntegral}(dx)\sin(c) + \frac{b\sin(c+dx)}{d^2} + a\cos(c)\text{Si}(dx)$$

```
[Out] -b*x*cos(d*x+c)/d+a*cos(c)*Si(d*x)+a*Ci(d*x)*sin(c)+b*sin(d*x+c)/d^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3420, 3384, 3380, 3383, 3377, 2717}

$$\int \frac{(a+bx^2)\sin(c+dx)}{x} dx = a\sin(c)\,\text{CosIntegral}(dx) + a\cos(c)\text{Si}(dx) + \frac{b\sin(c+dx)}{d^2} - \frac{bx\cos(c+dx)}{d}$$

```
[In] Int[((a + b*x^2)*Sin[c + d*x])/x,x]
```

```
[Out] -((b*x*Cos[c + d*x])/d) + a*CosIntegral[d*x]*Sin[c] + (b*Sin[c + d*x])/d^2
+ a*Cos[c]*SinIntegral[d*x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a\sin(c+dx)}{x}+bx\sin(c+dx)\right)dx \\
&= a\int \frac{\sin(c+dx)}{x}\,dx+b\int x\sin(c+dx)\,dx \\
&= -\frac{bx\cos(c+dx)}{d}+\frac{b\int\cos(c+dx)\,dx}{d}+(a\cos(c))\int\frac{\sin(dx)}{x}\,dx+(a\sin(c))\int\frac{\cos(dx)}{x}\,dx \\
&= -\frac{bx\cos(c+dx)}{d}+a\,\text{CosIntegral}(dx)\sin(c)+\frac{b\sin(c+dx)}{d^2}+a\cos(c)\text{Si}(dx)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx^2)\sin(c+dx)}{x}\,dx = -\frac{b\cos(dx)(dx\cos(c)-\sin(c))}{d^2} + a\,\text{CosIntegral}(dx)\sin(c) + \frac{b(\cos(c)+dx\sin(c))\sin(dx)}{d^2} + a\cos(c)\text{Si}(dx)$$

```
[In] Integrate[((a + b*x^2)*Sin[c + d*x])/x,x]
```

```
[Out] -((b*Cos[d*x]*(d*x*Cos[c] - Sin[c]))/d^2) + a*CosIntegral[d*x]*Sin[c] + (b*
(Cos[c] + d*x*Sin[c])*Sin[d*x])/d^2 + a*Cos[c]*SinIntegral[d*x]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.46

method	result
derivativedivides	$a(\text{Si}(dx)\cos(c)+\text{Ci}(dx)\sin(c))+\frac{2bc\cos(dx+c)}{d^2}+\frac{(c+1)b(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^2}$
default	$a(\text{Si}(dx)\cos(c)+\text{Ci}(dx)\sin(c))+\frac{2bc\cos(dx+c)}{d^2}+\frac{(c+1)b(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^2}$
risch	$\frac{ia\,e^{ic}\,\text{Ei}_1(-idx)}{2}-\frac{i\,\text{Ei}_1(-idx)e^{-ic}a}{2}-\frac{\pi\ \text{csgn}(dx)e^{-ic}a}{2}+\text{Si}(dx)\,e^{-ic}a-\frac{bx\cos(dx+c)}{d}+\frac{b\sin(dx+c)}{d^2}$
meijerg	$\frac{2b\sqrt{\pi}\ \sin(c)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(dx)}{2\sqrt{\pi}}+\frac{dx\sin(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{2b\sqrt{\pi}\ \cos(c)\left(-\frac{dx\cos(dx)}{2\sqrt{\pi}}+\frac{\sin(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{a\sqrt{\pi}\ \sin(c)\left(\frac{2\gamma+2\ln(x)+\ln\left(d^2\right)}{\sqrt{\pi}}\right.}{}$

```
[In] int((b*x^2+a)*sin(d*x+c)/x,x,method=_RETURNVERBOSE)
```

```
[Out] a*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))+2/d^2*b*c*cos(d*x+c)+(c+1)/d^2*b*(sin(d*x
+c)-cos(d*x+c)*(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)\sin(c+dx)}{x}\,dx = \frac{ad^2\,\text{Ci}(dx)\sin(c)+ad^2\cos(c)\,\text{Si}(dx)-bdx\cos(dx+c)+b\sin(dx+c)}{d^2}$$

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x,x, algorithm="fricas")
```

```
[Out] (a*d^2*cos_integral(d*x)*sin(c) + a*d^2*cos(c)*sin_integral(d*x) - b*d*x*co
s(d*x + c) + b*sin(d*x + c))/d^2
```

Sympy [A] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.59

$$\int \frac{(a+bx^2)\sin(c+dx)}{x}\,dx = a\sin(c)\,\mathrm{Ci}(dx) + a\cos(c)\,\mathrm{Si}(dx) + bx\left(\begin{cases} x\sin(c) & \text{for } d=0 \\ -\frac{\cos(c+dx)}{d} & \text{otherwise} \end{cases}\right) - b\left(\begin{cases} \frac{x^2\sin(c)}{2} & \text{for } d=0 \\ -\frac{\begin{cases} \frac{\sin(c+dx)}{d} & \text{for } d\neq 0 \\ x\cos(c) & \text{otherwise} \end{cases}}{d} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((b*x**2+a)*sin(d*x+c)/x,x)
```

```
[Out] a*sin(c)*Ci(d*x) + a*cos(c)*Si(d*x) + b*x*Piecewise((x*sin(c), Eq(d, 0)), (
-cos(c + d*x)/d, True)) - b*Piecewise((x**2*sin(c)/2, Eq(d, 0)), (-Piecewis
e((sin(c + d*x)/d, Ne(d, 0)), (x*cos(c), True))/d, True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.61

$$\int \frac{(a+bx^2)\sin(c+dx)}{x}\,dx = -\frac{2\,bdx\cos(dx+c) - (a(-i\,\mathrm{Ei}(i\,dx) + i\,\mathrm{Ei}(-i\,dx))\cos(c) + a(\mathrm{Ei}(i\,dx) + \mathrm{Ei}(-i\,dx))\sin(c))d^2 - 2\,b\sin(d}{2\,d^2}$$

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*d*x*cos(d*x + c) - (a*(-I*Ei(I*d*x) + I*Ei(-I*d*x))*cos(c) + a*(E
i(I*d*x) + Ei(-I*d*x))*sin(c))*d^2 - 2*b*sin(d*x + c))/d^2
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.32 (sec) , antiderivative size = 432, normalized size of antiderivative = 10.54

$$\int \frac{(a+bx^2)\sin(c+dx)}{x}\,dx = -\frac{ad^2\Im(\text{Ci}\,(dx))\tan\left(\frac{1}{2}\,dx\right)^2\tan\left(\frac{1}{2}\,c\right)^2 - ad^2\Im(\text{Ci}\,(-dx))\tan\left(\frac{1}{2}\,dx\right)^2\tan\left(\frac{1}{2}\,c\right)^2 + 2\,ad^2\,\text{Si}\,(dx)\tan\left(\frac{1}{2}\,dx\right)}{}$$

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x,x, algorithm="giac")
```

```
[Out] -1/2*(a*d^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d^
2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d^2*sin_i
ntegral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*d^2*real_part(cos_integral(d
*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^2*real_part(cos_integral(-d*x))*tan(
1/2*d*x)^2*tan(1/2*c) + 2*b*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d^2*imag_pa
rt(cos_integral(d*x))*tan(1/2*d*x)^2 + a*d^2*imag_part(cos_integral(-d*x))*
tan(1/2*d*x)^2 - 2*a*d^2*sin_integral(d*x)*tan(1/2*d*x)^2 + a*d^2*imag_part
(cos_integral(d*x))*tan(1/2*c)^2 - a*d^2*imag_part(cos_integral(-d*x))*tan(
1/2*c)^2 + 2*a*d^2*sin_integral(d*x)*tan(1/2*c)^2 - 2*b*d*x*tan(1/2*d*x)^2
- 2*a*d^2*real_part(cos_integral(d*x))*tan(1/2*c) - 2*a*d^2*real_part(cos_i
ntegral(-d*x))*tan(1/2*c) - 8*b*d*x*tan(1/2*d*x)*tan(1/2*c) - 2*b*d*x*tan(1
/2*c)^2 - a*d^2*imag_part(cos_integral(d*x)) + a*d^2*imag_part(cos_integral
(-d*x)) - 2*a*d^2*sin_integral(d*x) + 4*b*tan(1/2*d*x)^2*tan(1/2*c) + 4*b*t
an(1/2*d*x)*tan(1/2*c)^2 + 2*b*d*x - 4*b*tan(1/2*d*x) - 4*b*tan(1/2*c))/(d^
2*tan(1/2*d*x)^2*tan(1/2*c)^2 + d^2*tan(1/2*d*x)^2 + d^2*tan(1/2*c)^2 + d^2
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\sin(c+dx)}{x}\,dx = a\,\text{cosint}(d\,x)\,\sin(c) + a\,\text{sinint}(d\,x)\,\cos(c) + \frac{b\,(\sin(c+d\,x) - d\,x\,\cos(c+d\,x))}{d^2}$$

```
[In] int((sin(c + d*x)*(a + b*x^2))/x,x)
```

```
[Out] a*cosint(d*x)*sin(c) + a*sinint(d*x)*cos(c) + (b*(sin(c + d*x) - d*x*cos(c
+ d*x)))/d^2
```

3.45 $\int \frac{(a+bx^2)\sin(c+dx)}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 44

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^2}\,dx = -\frac{b\cos(c+dx)}{d} + ad\cos(c)\,\text{CosIntegral}(dx) - \frac{a\sin(c+dx)}{x} - ad\sin(c)\text{Si}(dx)$$

```
[Out] a*d*Ci(d*x)*cos(c)-b*cos(d*x+c)/d-a*d*Si(d*x)*sin(c)-a*sin(d*x+c)/x
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3420, 2718, 3378, 3384, 3380, 3383}

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^2}\,dx = ad\cos(c)\,\text{CosIntegral}(dx) - ad\sin(c)\text{Si}(dx) - \frac{a\sin(c+dx)}{x} - \frac{b\cos(c+dx)}{d}$$

```
[In] Int[((a + b*x^2)*Sin[c + d*x])/x^2,x]
```

```
[Out] -((b*Cos[c + d*x])/d) + a*d*Cos[c]*CosIntegral[d*x] - (a*Sin[c + d*x])/x -
a*d*Sin[c]*SinIntegral[d*x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(b\sin(c+dx)+\frac{a\sin(c+dx)}{x^2}\right)dx\\
&= a\int \frac{\sin(c+dx)}{x^2}\,dx+b\int \sin(c+dx)\,dx\\
&= -\frac{b\cos(c+dx)}{d}-\frac{a\sin(c+dx)}{x}+(ad)\int \frac{\cos(c+dx)}{x}\,dx\\
&= -\frac{b\cos(c+dx)}{d}-\frac{a\sin(c+dx)}{x}+(ad\cos(c))\int \frac{\cos(dx)}{x}\,dx-(ad\sin(c))\int \frac{\sin(dx)}{x}\,dx\\
&= -\frac{b\cos(c+dx)}{d}+ad\cos(c)\,\text{CosIntegral}(dx)-\frac{a\sin(c+dx)}{x}-ad\sin(c)\text{Si}(dx)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^2}\,dx = -\frac{b\cos(c+dx)}{d} + ad\cos(c)\,\text{CosIntegral}(dx) - \frac{a\sin(c+dx)}{x} - ad\sin(c)\text{Si}(dx)$$

[In] Integrate[((a + b*x^2)*Sin[c + d*x])/x^2,x]

[Out] -((b*Cos[c + d*x])/d) + a*d*Cos[c]*CosIntegral[d*x] - (a*Sin[c + d*x])/x - a*d*Sin[c]*SinIntegral[d*x]

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

method	result
derivativedivides	$d\left(a\left(-\frac{\sin(dx+c)}{dx} - \text{Si}(dx)\sin(c) + \text{Ci}(dx)\cos(c)\right) - \frac{b\cos(dx+c)}{d^2}\right)$
default	$d\left(a\left(-\frac{\sin(dx+c)}{dx} - \text{Si}(dx)\sin(c) + \text{Ci}(dx)\cos(c)\right) - \frac{b\cos(dx+c)}{d^2}\right)$
risch	$-\frac{d\cos(c)a\ \text{Ei}_1(idx)}{2} - \frac{d\cos(c)a\ \text{Ei}_1(-idx)}{2} + \frac{id\sin(c)a\ \text{Ei}_1(idx)}{2} - \frac{id\sin(c)a\ \text{Ei}_1(-idx)}{2} - \frac{b\cos(dx+c)}{d} -$ a sin(
meijerg	$\frac{b\sin(c)\sin(dx)}{d} + \frac{b\sqrt{\pi}\cos(c)\left(\frac{1}{\sqrt{\pi}} - \frac{\cos(dx)}{\sqrt{\pi}}\right)}{d} + \frac{a\sqrt{\pi}\sin(c)d^2\left(-\frac{4d^2\cos\left(x\sqrt{d^2}\right)}{x(d^2)^{\frac{3}{2}}\sqrt{\pi}} - \frac{4\ \text{Si}\left(x\sqrt{d^2}\right)}{\sqrt{\pi}}\right)}{4\sqrt{d^2}} +$ a√π cos(c)d(4γ−

[In] int((b*x^2+a)*sin(d*x+c)/x^2,x,method=_RETURNVERBOSE)

[Out] d*(a*(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c))-1/d^2*b*cos(d*x+c))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^2}\,dx = \frac{ad^2x\cos(c)\,\text{Ci}(dx) - ad^2x\sin(c)\,\text{Si}(dx) - bx\cos(dx+c) - ad\sin(dx+c)}{dx}$$

[In] integrate((b*x^2+a)*sin(d*x+c)/x^2,x, algorithm="fricas")

[Out] (a*d^2*x*cos(c)*cos_integral(d*x) - a*d^2*x*sin(c)*sin_integral(d*x) - b*x*cos(d*x + c) - a*d*sin(d*x + c))/(d*x)

Sympy [F]

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^2}\,dx = \int \frac{(a+bx^2)\sin{(c+dx)}}{x^2}\,dx$$

```
[In] integrate((b*x**2+a)*sin(d*x+c)/x**2,x)
```

```
[Out] Integral((a + b*x**2)*sin(c + d*x)/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 937, normalized size of antiderivative = 21.30

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^2}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*(((I*exp_integral_e(2, I*d*x) - I*exp_integral_e(2, -I*d*x))*cos(c)^3
+ (I*exp_integral_e(2, I*d*x) - I*exp_integral_e(2, -I*d*x))*cos(c)*sin(c)^
2 + (exp_integral_e(2, I*d*x) + exp_integral_e(2, -I*d*x))*sin(c)^3 + (I*ex
p_integral_e(2, I*d*x) - I*exp_integral_e(2, -I*d*x))*cos(c) + ((exp_integr
al_e(2, I*d*x) + exp_integral_e(2, -I*d*x))*cos(c)^2 + exp_integral_e(2, I*
d*x) + exp_integral_e(2, -I*d*x))*sin(c))*b*c^2/((d*x + c)*(cos(c)^2 + sin(
c)^2)*d^2 - (c*cos(c)^2 + c*sin(c)^2)*d^2) - ((I*exp_integral_e(2, I*d*x) -
 I*exp_integral_e(2, -I*d*x))*cos(c)^3 + (I*exp_integral_e(2, I*d*x) - I*ex
p_integral_e(2, -I*d*x))*cos(c)*sin(c)^2 + (exp_integral_e(2, I*d*x) + exp_
integral_e(2, -I*d*x))*sin(c)^3 + (I*exp_integral_e(2, I*d*x) - I*exp_integ
ral_e(2, -I*d*x))*cos(c) + ((exp_integral_e(2, I*d*x) + exp_integral_e(2, -
I*d*x))*cos(c)^2 + exp_integral_e(2, I*d*x) + exp_integral_e(2, -I*d*x))*si
n(c))*a/(c*cos(c)^2 + c*sin(c)^2 - (d*x + c)*(cos(c)^2 + sin(c)^2)) + 2*(((
b*cos(c)^2 + b*sin(c)^2)*(d*x + c)^2 - 2*(b*c*cos(c)^2 + b*c*sin(c)^2)*(d*x
 + c))*cos(d*x + c)^3 + (b*c^2*(exp_integral_e(3, I*d*x) + exp_integral_e(3
, -I*d*x))*cos(c)^3 + b*c^2*(exp_integral_e(3, I*d*x) + exp_integral_e(3, -
I*d*x))*cos(c)*sin(c)^2 + b*c^2*(-I*exp_integral_e(3, I*d*x) + I*exp_integr
al_e(3, -I*d*x))*sin(c)^3 + b*c^2*(exp_integral_e(3, I*d*x) + exp_integral_
e(3, -I*d*x))*cos(c) + (b*c^2*(-I*exp_integral_e(3, I*d*x) + I*exp_integral
_e(3, -I*d*x))*cos(c)^2 + b*c^2*(-I*exp_integral_e(3, I*d*x) + I*exp_integr
al_e(3, -I*d*x)))*sin(c))*cos(d*x + c)^2 + (b*c^2*(exp_integral_e(3, I*d*x)
 + exp_integral_e(3, -I*d*x))*cos(c)^3 + b*c^2*(exp_integral_e(3, I*d*x) +
exp_integral_e(3, -I*d*x))*cos(c)*sin(c)^2 + b*c^2*(-I*exp_integral_e(3, I*
d*x) + I*exp_integral_e(3, -I*d*x))*sin(c)^3 + b*c^2*(exp_integral_e(3, I*d
*x) + exp_integral_e(3, -I*d*x))*cos(c) + ((b*cos(c)^2 + b*sin(c)^2)*(d*x +
```

```
c)^2 - 2*(b*c*cos(c)^2 + b*c*sin(c)^2)*(d*x + c))*cos(d*x + c) + (b*c^2*(-
I*exp_integral_e(3, I*d*x) + I*exp_integral_e(3, -I*d*x))*cos(c)^2 + b*c^2*
(-I*exp_integral_e(3, I*d*x) + I*exp_integral_e(3, -I*d*x)))*sin(c))*sin(d*
x + c)^2 + ((b*cos(c)^2 + b*sin(c)^2)*(d*x + c)^2 - 2*(b*c*cos(c)^2 + b*c*s
in(c)^2)*(d*x + c))*cos(d*x + c))/(((d*x + c)^2*(cos(c)^2 + sin(c)^2)*d^2 -
 2*(c*cos(c)^2 + c*sin(c)^2)*(d*x + c)*d^2 + (c^2*cos(c)^2 + c^2*sin(c)^2)*
d^2)*cos(d*x + c)^2 + ((d*x + c)^2*(cos(c)^2 + sin(c)^2)*d^2 - 2*(c*cos(c)^
2 + c*sin(c)^2)*(d*x + c)*d^2 + (c^2*cos(c)^2 + c^2*sin(c)^2)*d^2)*sin(d*x
+ c)^2))*d
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.29 (sec) , antiderivative size = 411, normalized size of antiderivative = 9.34

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^2}\,dx = -\frac{ad^2x\Re(\mathrm{Ci}\,(dx))\tan\left(\frac{1}{2}\,dx\right)^2\tan\left(\frac{1}{2}\,c\right)^2 + ad^2x\Re(\mathrm{Ci}\,(-dx))\tan\left(\frac{1}{2}\,dx\right)^2\tan\left(\frac{1}{2}\,c\right)^2 + 2\,ad^2x\Im(\mathrm{Ci}\,(dx))\tan}{}$$

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(a*d^2*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + a*
d^2*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d^2*x
*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^2*x*imag_pa
rt(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a*d^2*x*sin_integral(d
*x)*tan(1/2*d*x)^2*tan(1/2*c) - a*d^2*x*real_part(cos_integral(d*x))*tan(1/
2*d*x)^2 - a*d^2*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2 + a*d^2*x*r
eal_part(cos_integral(d*x))*tan(1/2*c)^2 + a*d^2*x*real_part(cos_integral(-
d*x))*tan(1/2*c)^2 + 2*a*d^2*x*imag_part(cos_integral(d*x))*tan(1/2*c) - 2*
a*d^2*x*imag_part(cos_integral(-d*x))*tan(1/2*c) + 4*a*d^2*x*sin_integral(d
*x)*tan(1/2*c) + 2*b*x*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d^2*x*real_part(cos_
integral(d*x)) - a*d^2*x*real_part(cos_integral(-d*x)) - 4*a*d*tan(1/2*d*x)
^2*tan(1/2*c) - 4*a*d*tan(1/2*d*x)*tan(1/2*c)^2 - 2*b*x*tan(1/2*d*x)^2 - 8*
b*x*tan(1/2*d*x)*tan(1/2*c) - 2*b*x*tan(1/2*c)^2 + 4*a*d*tan(1/2*d*x) + 4*a
*d*tan(1/2*c) + 2*b*x)/(d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + d*x*tan(1/2*d*x)^
2 + d*x*tan(1/2*c)^2 + d*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^2}\,dx = \int \frac{\sin\left(c+d\,x\right)\,\left(b\,x^2+a\right)}{x^2}\,dx$$

```
[In] int((sin(c + d*x)*(a + b*x^2))/x^2,x)
[Out] int((sin(c + d*x)*(a + b*x^2))/x^2, x)
```

3.46 $\int \frac{(a+bx^2)\sin(c+dx)}{x^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^3}\,dx = -\frac{ad\cos(c+dx)}{2x} + b\,\text{CosIntegral}(dx)\sin(c) - \frac{1}{2}ad^2\,\text{CosIntegral}(dx)\sin(c) - \frac{a\sin(c+dx)}{2x^2} + b\cos(c)\text{Si}(dx) - \frac{1}{2}ad^2\cos(c)\text{Si}(dx)$$

```
[Out] -1/2*a*d*cos(d*x+c)/x+b*cos(c)*Si(d*x)-1/2*a*d^2*cos(c)*Si(d*x)+b*Ci(d*x)*s
in(c)-1/2*a*d^2*Ci(d*x)*sin(c)-1/2*a*sin(d*x+c)/x^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3420, 3378, 3384, 3380, 3383}

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^3}\,dx = -\frac{1}{2}ad^2\sin(c)\,\text{CosIntegral}(dx) - \frac{1}{2}ad^2\cos(c)\text{Si}(dx) - \frac{a\sin(c+dx)}{2x^2} - \frac{ad\cos(c+dx)}{2x} + b\sin(c)\,\text{CosIntegral}(dx) + b\cos(c)\text{Si}(dx)$$

```
[In] Int[((a + b*x^2)*Sin[c + d*x])/x^3,x]
```

```
[Out] -1/2*(a*d*Cos[c + d*x])/x + b*CosIntegral[d*x]*Sin[c] - (a*d^2*CosIntegral[
d*x]*Sin[c])/2 - (a*Sin[c + d*x])/(2*x^2) + b*Cos[c]*SinIntegral[d*x] - (a*
d^2*Cos[c]*SinIntegral[d*x])/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a\sin(c+dx)}{x^3}+\frac{b\sin(c+dx)}{x}\right)dx \\
&= a\int \frac{\sin(c+dx)}{x^3}\,dx+b\int \frac{\sin(c+dx)}{x}\,dx \\
&= -\frac{a\sin(c+dx)}{2x^2}+\frac{1}{2}(ad)\int \frac{\cos(c+dx)}{x^2}\,dx+(b\cos(c))\int \frac{\sin(dx)}{x}\,dx+(b\sin(c))\int \frac{\cos(dx)}{x}\,dx \\
&= -\frac{ad\cos(c+dx)}{2x}+b\,\text{CosIntegral}(dx)\sin(c)-\frac{a\sin(c+dx)}{2x^2} \\
&\quad +b\cos(c)\text{Si}(dx)-\frac{1}{2}\left(ad^2\right)\int \frac{\sin(c+dx)}{x}\,dx
\end{aligned}$$

$$= -\frac{ad\cos(c+dx)}{2x} + b\,\text{CosIntegral}(dx)\sin(c) - \frac{a\sin(c+dx)}{2x^2} + b\cos(c)\text{Si}(dx) - \frac{1}{2}\left(ad^2\cos(c)\right)\int \frac{\sin(dx)}{x}\,dx - \frac{1}{2}\left(ad^2\sin(c)\right)\int \frac{\cos(dx)}{x}\,dx$$

$$= -\frac{ad\cos(c+dx)}{2x} + b\,\text{CosIntegral}(dx)\sin(c) - \frac{1}{2}ad^2\,\text{CosIntegral}(dx)\sin(c) - \frac{a\sin(c+dx)}{2x^2} + b\cos(c)\text{Si}(dx) - \frac{1}{2}ad^2\cos(c)\text{Si}(dx)$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^3}\,dx = b\,\text{CosIntegral}(dx)\sin(c) - \frac{a\cos(dx)(dx\cos(c)+\sin(c))}{2x^2} + \frac{a(-\cos(c)+dx\sin(c))\sin(dx)}{2x^2} + b\cos(c)\text{Si}(dx) - \frac{1}{2}ad^2(\text{CosIntegral}(dx)\sin(c)+\cos(c)\text{Si}(dx))$$

```
[In] Integrate[((a + b*x^2)*Sin[c + d*x])/x^3,x]
```

```
[Out] b*CosIntegral[d*x]*Sin[c] - (a*Cos[d*x]*(d*x*Cos[c] + Sin[c]))/(2*x^2) + (a
*(-Cos[c] + d*x*Sin[c])*Sin[d*x])/(2*x^2) + b*Cos[c]*SinIntegral[d*x] - (a*
d^2*(CosIntegral[d*x]*Sin[c] + Cos[c]*SinIntegral[d*x]))/2
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

method	result
derivativedivides	$d^2\left(a\left(-\frac{\sin(dx+c)}{2d^2x^2}-\frac{\cos(dx+c)}{2dx}-\frac{\text{Si}(dx)\cos(c)}{2}-\frac{\text{Ci}(dx)\sin(c)}{2}\right)+\frac{b(\text{Si}(dx)\cos(c)+\text{Ci}(dx)\sin(c))}{d^2}\right)$
default	$d^2\left(a\left(-\frac{\sin(dx+c)}{2d^2x^2}-\frac{\cos(dx+c)}{2dx}-\frac{\text{Si}(dx)\cos(c)}{2}-\frac{\text{Ci}(dx)\sin(c)}{2}\right)+\frac{b(\text{Si}(dx)\cos(c)+\text{Ci}(dx)\sin(c))}{d^2}\right)$
risch	$\frac{i\cos(c)\,\text{Ei}_1(idx)a\,d^2}{4}-\frac{i\cos(c)\,\text{Ei}_1(-idx)a\,d^2}{4}-\frac{i\cos(c)\,\text{Ei}_1(idx)b}{2}+\frac{i\cos(c)\,\text{Ei}_1(-idx)b}{2}+\frac{\sin(c)\,\text{Ei}_1(idx)a\,d^2}{4}+$
meijerg	$\frac{b\sqrt{\pi}\,\sin(c)\left(\frac{2\gamma+2\ln(x)+\ln\left(d^2\right)}{\sqrt{\pi}}-\frac{2\gamma}{\sqrt{\pi}}-\frac{2\ln(2)}{\sqrt{\pi}}-\frac{2\ln\left(\frac{dx}{2}\right)}{\sqrt{\pi}}+\frac{2\,\text{Ci}(dx)}{\sqrt{\pi}}\right)}{2}+b\cos(c)\,\text{Si}(dx)+$

```
[In] int((b*x^2+a)*sin(d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^2*(a*(-1/2*sin(d*x+c)/d^2/x^2-1/2*cos(d*x+c)/d/x-1/2*Si(d*x)*cos(c)-1/2*C
i(d*x)*sin(c))+1/d^2*b*(Si(d*x)*cos(c)+Ci(d*x)*sin(c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^3}\,dx = -\frac{(ad^2-2\,b)x^2\operatorname{Ci}(dx)\sin(c)+(ad^2-2\,b)x^2\cos(c)\operatorname{Si}(dx)+adx\cos(dx+c)+a\sin(dx+c)}{2\,x^2}$$

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*((a*d^2 - 2*b)*x^2*cos_integral(d*x)*sin(c) + (a*d^2 - 2*b)*x^2*cos(c)
*sin_integral(d*x) + a*d*x*cos(d*x + c) + a*sin(d*x + c))/x^2
```

Sympy [F]

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^3}\,dx = \int \frac{(a+bx^2)\sin(c+dx)}{x^3}\,dx$$

```
[In] integrate((b*x**2+a)*sin(d*x+c)/x**3,x)
```

```
[Out] Integral((a + b*x**2)*sin(c + d*x)/x**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.66

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^3}\,dx = -\frac{2\,bdx\cos(dx+c)+((a(-i\,\Gamma(-2,i\,dx)+i\,\Gamma(-2,-i\,dx))\cos(c)-a(\Gamma(-2,i\,dx)+\Gamma(-2,-i\,dx))\sin(c}{}$$

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*d*x*cos(d*x + c) + ((a*(-I*gamma(-2, I*d*x) + I*gamma(-2, -I*d*x)
)*cos(c) - a*(gamma(-2, I*d*x) + gamma(-2, -I*d*x))*sin(c))*d^4 - 2*(b*(-I*
gamma(-2, I*d*x) + I*gamma(-2, -I*d*x))*cos(c) - b*(gamma(-2, I*d*x) + gamm
a(-2, -I*d*x))*sin(c))*d^2)*x^2 + 2*b*sin(d*x + c))/(d^2*x^2)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.29 (sec) , antiderivative size = 766, normalized size of antiderivative = 10.35

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^3}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(a*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a
*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d^
2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*d^2*x^2*real_part
(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^2*x^2*real_part(cos_i
ntegral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - a*d^2*x^2*imag_part(cos_integral
(d*x))*tan(1/2*d*x)^2 + a*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x
)^2 - 2*a*d^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2 + a*d^2*x^2*imag_part(co
s_integral(d*x))*tan(1/2*c)^2 - a*d^2*x^2*imag_part(cos_integral(-d*x))*tan
(1/2*c)^2 + 2*a*d^2*x^2*sin_integral(d*x)*tan(1/2*c)^2 - 2*b*x^2*imag_part(
cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*b*x^2*imag_part(cos_inte
gral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*b*x^2*sin_integral(d*x)*tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*a*d^2*x^2*real_part(cos_integral(d*x))*tan(1/2*c)
- 2*a*d^2*x^2*real_part(cos_integral(-d*x))*tan(1/2*c) + 4*b*x^2*real_part(
cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*b*x^2*real_part(cos_integr
al(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 -
 a*d^2*x^2*imag_part(cos_integral(d*x)) + a*d^2*x^2*imag_part(cos_integral(
-d*x)) - 2*a*d^2*x^2*sin_integral(d*x) + 2*b*x^2*imag_part(cos_integral(d*x
))*tan(1/2*d*x)^2 - 2*b*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2 +
4*b*x^2*sin_integral(d*x)*tan(1/2*d*x)^2 - 2*b*x^2*imag_part(cos_integral(d
*x))*tan(1/2*c)^2 + 2*b*x^2*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 - 4*
b*x^2*sin_integral(d*x)*tan(1/2*c)^2 + 2*a*d*x*tan(1/2*d*x)^2 + 4*b*x^2*rea
l_part(cos_integral(d*x))*tan(1/2*c) + 4*b*x^2*real_part(cos_integral(-d*x)
)*tan(1/2*c) + 8*a*d*x*tan(1/2*d*x)*tan(1/2*c) + 2*a*d*x*tan(1/2*c)^2 + 2*b
*x^2*imag_part(cos_integral(d*x)) - 2*b*x^2*imag_part(cos_integral(-d*x)) +
 4*b*x^2*sin_integral(d*x) + 4*a*tan(1/2*d*x)^2*tan(1/2*c) + 4*a*tan(1/2*d*
x)*tan(1/2*c)^2 - 2*a*d*x - 4*a*tan(1/2*d*x) - 4*a*tan(1/2*c))/(x^2*tan(1/2
*d*x)^2*tan(1/2*c)^2 + x^2*tan(1/2*d*x)^2 + x^2*tan(1/2*c)^2 + x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^3}\,dx = \int \frac{\sin(c+d\,x)\,\left(b\,x^2+a\right)}{x^3}\,dx$$

```
[In] int((sin(c + d*x)*(a + b*x^2))/x^3,x)
[Out] int((sin(c + d*x)*(a + b*x^2))/x^3, x)
```

3.47 $\int \frac{(a+bx^2)\sin(c+dx)}{x^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 106

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^4}\,dx = -\frac{ad\cos(c+dx)}{6x^2} + bd\cos(c)\,\text{CosIntegral}(dx) - \frac{1}{6}ad^3\cos(c)\,\text{CosIntegral}(dx) - \frac{a\sin(c+dx)}{3x^3} - \frac{b\sin(c+dx)}{x} + \frac{ad^2\sin(c+dx)}{6x} - bd\sin(c)\text{Si}(dx) + \frac{1}{6}ad^3\sin(c)\text{Si}(dx)$$

```
[Out] b*d*Ci(d*x)*cos(c)-1/6*a*d^3*Ci(d*x)*cos(c)-1/6*a*d*cos(d*x+c)/x^2-b*d*Si(d
*x)*sin(c)+1/6*a*d^3*Si(d*x)*sin(c)-1/3*a*sin(d*x+c)/x^3-b*sin(d*x+c)/x+1/6
*a*d^2*sin(d*x+c)/x
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3420, 3378, 3384, 3380, 3383}

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^4}\,dx = -\frac{1}{6}ad^3\cos(c)\,\text{CosIntegral}(dx) + \frac{1}{6}ad^3\sin(c)\text{Si}(dx) + \frac{ad^2\sin(c+dx)}{6x} - \frac{a\sin(c+dx)}{3x^3} - \frac{ad\cos(c+dx)}{6x^2} + bd\cos(c)\,\text{CosIntegral}(dx) - bd\sin(c)\text{Si}(dx) - \frac{b\sin(c+dx)}{x}$$

```
[In] Int[((a + b*x^2)*Sin[c + d*x])/x^4,x]
```

```
[Out] -1/6*(a*d*Cos[c + d*x])/x^2 + b*d*Cos[c]*CosIntegral[d*x] - (a*d^3*Cos[c]*C
osIntegral[d*x])/6 - (a*Sin[c + d*x])/(3*x^3) - (b*Sin[c + d*x])/x + (a*d^2
```

```
*Sin[c + d*x])/(6*x) - b*d*Sin[c]*SinIntegral[d*x] + (a*d^3*Sin[c]*SinInteg
ral[d*x])/6
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a\sin(c+dx)}{x^4}+\frac{b\sin(c+dx)}{x^2}\right)\,dx \\
&= a\int \frac{\sin(c+dx)}{x^4}\,dx+b\int \frac{\sin(c+dx)}{x^2}\,dx \\
&= -\frac{a\sin(c+dx)}{3x^3}-\frac{b\sin(c+dx)}{x}+\frac{1}{3}(ad)\int \frac{\cos(c+dx)}{x^3}\,dx+(bd)\int \frac{\cos(c+dx)}{x}\,dx \\
&= -\frac{ad\cos(c+dx)}{6x^2}-\frac{a\sin(c+dx)}{3x^3}-\frac{b\sin(c+dx)}{x}-\frac{1}{6}\left(ad^2\right)\int \frac{\sin(c+dx)}{x^2}\,dx \\
&\quad +(bd\cos(c))\int \frac{\cos(dx)}{x}\,dx-(bd\sin(c))\int \frac{\sin(dx)}{x}\,dx
\end{aligned}
$$

$$= -\frac{ad\cos(c+dx)}{6x^2} + bd\cos(c)\,\text{CosIntegral}(dx) - \frac{a\sin(c+dx)}{3x^3} - \frac{b\sin(c+dx)}{x} + \frac{ad^2\sin(c+dx)}{6x} - bd\sin(c)\text{Si}(dx) - \frac{1}{6}\left(ad^3\right)\int \frac{\cos(c+dx)}{x}\,dx$$

$$= -\frac{ad\cos(c+dx)}{6x^2} + bd\cos(c)\,\text{CosIntegral}(dx) - \frac{a\sin(c+dx)}{3x^3} - \frac{b\sin(c+dx)}{x} + \frac{ad^2\sin(c+dx)}{6x} - bd\sin(c)\text{Si}(dx) - \frac{1}{6}\left(ad^3\cos(c)\right)\int \frac{\cos(dx)}{x}\,dx + \frac{1}{6}\left(ad^3\sin(c)\right)\int \frac{\sin(dx)}{x}\,dx$$

$$= -\frac{ad\cos(c+dx)}{6x^2} + bd\cos(c)\,\text{CosIntegral}(dx) - \frac{1}{6}ad^3\cos(c)\,\text{CosIntegral}(dx) - \frac{a\sin(c+dx)}{3x^3} - \frac{b\sin(c+dx)}{x} + \frac{ad^2\sin(c+dx)}{6x} - bd\sin(c)\text{Si}(dx) + \frac{1}{6}ad^3\sin(c)\text{Si}(dx)$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^4}\,dx = \frac{-adx\cos(c+dx) + d(6b-ad^2)\,x^3\cos(c)\,\text{CosIntegral}(dx) - 2a\sin(c+dx) - 6bx^2\sin(c+dx) + ad^2x^2\sin(}{6x^3}$$

```
[In] Integrate[((a + b*x^2)*Sin[c + d*x])/x^4,x]
```

```
[Out] (-(a*d*x*Cos[c + d*x]) + d*(6*b - a*d^2)*x^3*Cos[c]*CosIntegral[d*x] - 2*a*
Sin[c + d*x] - 6*b*x^2*Sin[c + d*x] + a*d^2*x^2*Sin[c + d*x] + d*(-6*b + a*
d^2)*x^3*Sin[c]*SinIntegral[d*x])/(6*x^3)
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.96

method	result
derivativedivides	$d^3\left(a\left(-\frac{\sin(dx+c)}{3d^3x^3}-\frac{\cos(dx+c)}{6d^2x^2}+\frac{\sin(dx+c)}{6dx}+\frac{\mathrm{Si}(dx)\sin(c)}{6}-\frac{\mathrm{Ci}(dx)\cos(c)}{6}\right)+\frac{b\left(-\frac{\sin(dx+c)}{dx}-\mathrm{Si}(dx)\sin\right.}{d^2}\right.$
default	$d^3\left(a\left(-\frac{\sin(dx+c)}{3d^3x^3}-\frac{\cos(dx+c)}{6d^2x^2}+\frac{\sin(dx+c)}{6dx}+\frac{\mathrm{Si}(dx)\sin(c)}{6}-\frac{\mathrm{Ci}(dx)\cos(c)}{6}\right)+\frac{b\left(-\frac{\sin(dx+c)}{dx}-\mathrm{Si}(dx)\sin\right.}{d^2}\right.$
risch	$\frac{\cos(c)\,\mathrm{Ei}_1(idx)a\,d^3}{12}+\frac{\cos(c)\,\mathrm{Ei}_1(-idx)a\,d^3}{12}-\frac{\cos(c)\,\mathrm{Ei}_1(idx)bd}{2}-\frac{\cos(c)\,\mathrm{Ei}_1(-idx)bd}{2}-\frac{i\sin(c)\,\mathrm{Ei}_1(idx)a\,d^3}{12}+$
meijerg	$\frac{d^2b\sqrt{\pi}\,\sin(c)\left(-\frac{4d^2\cos\left(x\sqrt{d^2}\right)}{x\left(d^2\right)^{\frac{3}{2}}\sqrt{\pi}}-\frac{4\,\mathrm{Si}\left(x\sqrt{d^2}\right)}{\sqrt{\pi}}\right)}{4\sqrt{d^2}}+\frac{db\sqrt{\pi}\,\cos(c)\left(\frac{4\gamma-4+4\ln(x)+4\ln(d)}{\sqrt{\pi}}+\frac{4}{\sqrt{\pi}}-\frac{4\gamma}{\sqrt{\pi}}-\frac{4\ln(2)}{\sqrt{\pi}}-\frac{4\ln\left(\frac{dx}{2}\right)}{\sqrt{\pi}}-\right.}{4}$

```
[In] int((b*x^2+a)*sin(d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] d^3*(a*(-1/3*sin(d*x+c)/d^3/x^3-1/6*cos(d*x+c)/d^2/x^2+1/6*sin(d*x+c)/d/x+1
/6*Si(d*x)*sin(c)-1/6*Ci(d*x)*cos(c))+1/d^2*b*(-sin(d*x+c)/d/x-Si(d*x)*sin(
c)+Ci(d*x)*cos(c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^4}\,dx = -\frac{(ad^3-6\,bd)x^3\cos(c)\,\mathrm{Ci}(dx)-(ad^3-6\,bd)x^3\sin(c)\,\mathrm{Si}(dx)+adx\cos(dx+c)-((ad^2-6\,b)x^2-2\,a)}{6\,x^3}$$

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*((a*d^3 - 6*b*d)*x^3*cos(c)*cos_integral(d*x) - (a*d^3 - 6*b*d)*x^3*si
n(c)*sin_integral(d*x) + a*d*x*cos(d*x + c) - ((a*d^2 - 6*b)*x^2 - 2*a)*sin
(d*x + c))/x^3
```

Sympy [F]

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^4}\,dx = \int \frac{(a+bx^2)\sin(c+dx)}{x^4}\,dx$$

```
[In] integrate((b*x**2+a)*sin(d*x+c)/x**4,x)
```

```
[Out] Integral((a + b*x**2)*sin(c + dx)/x**4, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^4}\,dx =$$
$$-\frac{((a(\Gamma(-3,i\,dx)+\Gamma(-3,-i\,dx))\cos{(c)}+a(-i\,\Gamma(-3,i\,dx)+i\,\Gamma(-3,-i\,dx))\sin{(c)})d^5-6\,(b(\Gamma(-3,i\,dx)}{}$$

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*(((a*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*cos(c) + a*(-I*gamma(-3, I
*d*x) + I*gamma(-3, -I*d*x))*sin(c))*d^5 - 6*(b*(gamma(-3, I*d*x) + gamma(-
3, -I*d*x))*cos(c) + b*(-I*gamma(-3, I*d*x) + I*gamma(-3, -I*d*x))*sin(c))*
d^3)*x^3 + 2*b*d*x*cos(d*x + c) + 4*b*sin(d*x + c))/(d^2*x^3)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.31 (sec) , antiderivative size = 834, normalized size of antiderivative = 7.87

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^4}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*(a*d^3*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
a*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d
^3*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^3*x^3
*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a*d^3*x^3*sin_
integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) - a*d^3*x^3*real_part(cos_integral(
d*x))*tan(1/2*d*x)^2 - a*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)
^2 + a*d^3*x^3*real_part(cos_integral(d*x))*tan(1/2*c)^2 + a*d^3*x^3*real_p
art(cos_integral(-d*x))*tan(1/2*c)^2 - 6*b*d*x^3*real_part(cos_integral(d*x
))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 6*b*d*x^3*real_part(cos_integral(-d*x))*ta
n(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d^3*x^3*imag_part(cos_integral(d*x))*tan(1/
2*c) - 2*a*d^3*x^3*imag_part(cos_integral(-d*x))*tan(1/2*c) + 4*a*d^3*x^3*s
in_integral(d*x)*tan(1/2*c) - 12*b*d*x^3*imag_part(cos_integral(d*x))*tan(1
/2*d*x)^2*tan(1/2*c) + 12*b*d*x^3*imag_part(cos_integral(-d*x))*tan(1/2*d*x
)^2*tan(1/2*c) - 24*b*d*x^3*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) - a
*d^3*x^3*real_part(cos_integral(d*x)) - a*d^3*x^3*real_part(cos_integral(-d
*x)) + 6*b*d*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2 + 6*b*d*x^3*re
al_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 4*a*d^2*x^2*tan(1/2*d*x)^2*tan
```

```
(1/2*c) - 6*b*d*x^3*real_part(cos_integral(d*x))*tan(1/2*c)^2 - 6*b*d*x^3*r
eal_part(cos_integral(-d*x))*tan(1/2*c)^2 - 4*a*d^2*x^2*tan(1/2*d*x)*tan(1/
2*c)^2 - 12*b*d*x^3*imag_part(cos_integral(d*x))*tan(1/2*c) + 12*b*d*x^3*im
ag_part(cos_integral(-d*x))*tan(1/2*c) - 24*b*d*x^3*sin_integral(d*x)*tan(1
/2*c) - 2*a*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + 6*b*d*x^3*real_part(cos_integ
ral(d*x)) + 6*b*d*x^3*real_part(cos_integral(-d*x)) + 4*a*d^2*x^2*tan(1/2*d
*x) + 4*a*d^2*x^2*tan(1/2*c) + 24*b*x^2*tan(1/2*d*x)^2*tan(1/2*c) + 24*b*x^
2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*a*d*x*tan(1/2*d*x)^2 + 8*a*d*x*tan(1/2*d*x)
*tan(1/2*c) + 2*a*d*x*tan(1/2*c)^2 - 24*b*x^2*tan(1/2*d*x) - 24*b*x^2*tan(1
/2*c) + 8*a*tan(1/2*d*x)^2*tan(1/2*c) + 8*a*tan(1/2*d*x)*tan(1/2*c)^2 - 2*a
*d*x - 8*a*tan(1/2*d*x) - 8*a*tan(1/2*c))/(x^3*tan(1/2*d*x)^2*tan(1/2*c)^2
+ x^3*tan(1/2*d*x)^2 + x^3*tan(1/2*c)^2 + x^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^4}\,dx=\int \frac{\sin\left(c+d\,x\right)\,\left(b\,x^2+a\right)}{x^4}\,dx$$

```
[In] int((sin(c + d*x)*(a + b*x^2))/x^4,x)
[Out] int((sin(c + d*x)*(a + b*x^2))/x^4, x)
```

3.48 $\int \frac{(a+bx^2)\sin(c+dx)}{x^5}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 149

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^5}\,dx = -\frac{ad\cos(c+dx)}{12x^3} - \frac{bd\cos(c+dx)}{2x} + \frac{ad^3\cos(c+dx)}{24x} - \frac{1}{2}bd^2\,\text{CosIntegral}(dx)\sin(c) + \frac{1}{24}ad^4\,\text{CosIntegral}(dx)\sin(c) - \frac{a\sin(c+dx)}{4x^4} - \frac{b\sin(c+dx)}{2x^2} + \frac{ad^2\sin(c+dx)}{24x^2} - \frac{1}{2}bd^2\cos(c)\text{Si}(dx) + \frac{1}{24}ad^4\cos(c)\text{Si}(dx)$$

```
[Out] -1/12*a*d*cos(d*x+c)/x^3-1/2*b*d*cos(d*x+c)/x+1/24*a*d^3*cos(d*x+c)/x-1/2*b
*d^2*cos(c)*Si(d*x)+1/24*a*d^4*cos(c)*Si(d*x)-1/2*b*d^2*Ci(d*x)*sin(c)+1/24
*a*d^4*Ci(d*x)*sin(c)-1/4*a*sin(d*x+c)/x^4-1/2*b*sin(d*x+c)/x^2+1/24*a*d^2*
sin(d*x+c)/x^2
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3420, 3378, 3384, 3380, 3383}

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^5}\,dx = \frac{1}{24}ad^4\sin(c)\,\text{CosIntegral}(dx) + \frac{1}{24}ad^4\cos(c)\text{Si}(dx) + \frac{ad^3\cos(c+dx)}{24x} + \frac{ad^2\sin(c+dx)}{24x^2} - \frac{a\sin(c+dx)}{4x^4} - \frac{ad\cos(c+dx)}{12x^3} - \frac{1}{2}bd^2\sin(c)\,\text{CosIntegral}(dx) - \frac{1}{2}bd^2\cos(c)\text{Si}(dx) - \frac{b\sin(c+dx)}{2x^2} - \frac{bd\cos(c+dx)}{2x}$$

```
[In] Int[((a + b*x^2)*Sin[c + d*x])/x^5,x]
```

```
[Out] -1/12*(a*d*Cos[c + d*x])/x^3 - (b*d*Cos[c + d*x])/(2*x) + (a*d^3*Cos[c + d*
x])/(24*x) - (b*d^2*CosIntegral[d*x]*Sin[c])/2 + (a*d^4*CosIntegral[d*x]*Si
n[c])/24 - (a*Sin[c + d*x])/(4*x^4) - (b*Sin[c + d*x])/(2*x^2) + (a*d^2*Sin
[c + d*x])/(24*x^2) - (b*d^2*Cos[c]*SinIntegral[d*x])/2 + (a*d^4*Cos[c]*Sin
Integral[d*x])/24
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{a\sin(c+dx)}{x^5}+\frac{b\sin(c+dx)}{x^3}\right)dx\\ &= a\int \frac{\sin(c+dx)}{x^5}\,dx+b\int \frac{\sin(c+dx)}{x^3}\,dx\\ &= -\frac{a\sin(c+dx)}{4x^4}-\frac{b\sin(c+dx)}{2x^2}+\frac{1}{4}(ad)\int \frac{\cos(c+dx)}{x^4}\,dx+\frac{1}{2}(bd)\int \frac{\cos(c+dx)}{x^2}\,dx\end{aligned}$$

$$
\begin{aligned}
&= -\frac{ad\cos(c+dx)}{12x^3} - \frac{bd\cos(c+dx)}{2x} - \frac{a\sin(c+dx)}{4x^4} - \frac{b\sin(c+dx)}{2x^2} \\
&\quad - \frac{1}{12}\left(ad^2\right)\int\frac{\sin(c+dx)}{x^3}\,dx - \frac{1}{2}\left(bd^2\right)\int\frac{\sin(c+dx)}{x}\,dx \\
&= -\frac{ad\cos(c+dx)}{12x^3} - \frac{bd\cos(c+dx)}{2x} - \frac{a\sin(c+dx)}{4x^4} - \frac{b\sin(c+dx)}{2x^2} + \frac{ad^2\sin(c+dx)}{24x^2} \\
&\quad - \frac{1}{24}\left(ad^3\right)\int\frac{\cos(c+dx)}{x^2}\,dx - \frac{1}{2}\left(bd^2\cos(c)\right)\int\frac{\sin(dx)}{x}\,dx - \frac{1}{2}\left(bd^2\sin(c)\right)\int\frac{\cos(dx)}{x}\,dx \\
&= -\frac{ad\cos(c+dx)}{12x^3} - \frac{bd\cos(c+dx)}{2x} + \frac{ad^3\cos(c+dx)}{24x} \\
&\quad - \frac{1}{2}bd^2\,\text{CosIntegral}(dx)\sin(c) - \frac{a\sin(c+dx)}{4x^4} - \frac{b\sin(c+dx)}{2x^2} \\
&\quad + \frac{ad^2\sin(c+dx)}{24x^2} - \frac{1}{2}bd^2\cos(c)\text{Si}(dx) + \frac{1}{24}\left(ad^4\right)\int\frac{\sin(c+dx)}{x}\,dx \\
&= -\frac{ad\cos(c+dx)}{12x^3} - \frac{bd\cos(c+dx)}{2x} + \frac{ad^3\cos(c+dx)}{24x} - \frac{1}{2}bd^2\,\text{CosIntegral}(dx)\sin(c) \\
&\quad - \frac{a\sin(c+dx)}{4x^4} - \frac{b\sin(c+dx)}{2x^2} + \frac{ad^2\sin(c+dx)}{24x^2} - \frac{1}{2}bd^2\cos(c)\text{Si}(dx) \\
&\quad + \frac{1}{24}\left(ad^4\cos(c)\right)\int\frac{\sin(dx)}{x}\,dx + \frac{1}{24}\left(ad^4\sin(c)\right)\int\frac{\cos(dx)}{x}\,dx \\
&= -\frac{ad\cos(c+dx)}{12x^3} - \frac{bd\cos(c+dx)}{2x} + \frac{ad^3\cos(c+dx)}{24x} - \frac{1}{2}bd^2\,\text{CosIntegral}(dx)\sin(c) \\
&\quad + \frac{1}{24}ad^4\,\text{CosIntegral}(dx)\sin(c) - \frac{a\sin(c+dx)}{4x^4} - \frac{b\sin(c+dx)}{2x^2} \\
&\quad + \frac{ad^2\sin(c+dx)}{24x^2} - \frac{1}{2}bd^2\cos(c)\text{Si}(dx) + \frac{1}{24}ad^4\cos(c)\text{Si}(dx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.84

$$
\int\frac{(a+bx^2)\sin(c+dx)}{x^5}\,dx
= \frac{-2adx\cos(c+dx) - 12bdx^3\cos(c+dx) + ad^3x^3\cos(c+dx) + d^2\left(-12b+ad^2\right)x^4\,\text{CosIntegral}(dx)\sin(c) -}{24x^4}
$$

```
[In] Integrate[((a + b*x^2)*Sin[c + d*x])/x^5,x]
```

```
[Out] (-2*a*d*x*Cos[c + d*x] - 12*b*d*x^3*Cos[c + d*x] + a*d^3*x^3*Cos[c + d*x] +
 d^2*(-12*b + a*d^2)*x^4*CosIntegral[d*x]*Sin[c] - 6*a*Sin[c + d*x] - 12*b*
x^2*Sin[c + d*x] + a*d^2*x^2*Sin[c + d*x] + d^2*(-12*b + a*d^2)*x^4*Cos[c]*
SinIntegral[d*x])/(24*x^4)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.88

method	result
derivativedivides	$d^4\left(a\left(-\frac{\sin(dx+c)}{4d^4x^4}-\frac{\cos(dx+c)}{12d^3x^3}+\frac{\sin(dx+c)}{24d^2x^2}+\frac{\cos(dx+c)}{24dx}+\frac{\mathrm{Si}(dx)\cos(c)}{24}+\frac{\mathrm{Ci}(dx)\sin(c)}{24}\right)+\frac{b\left(-\frac{\sin(dx\right.}{2d^2x}\right.$
default	$d^4\left(a\left(-\frac{\sin(dx+c)}{4d^4x^4}-\frac{\cos(dx+c)}{12d^3x^3}+\frac{\sin(dx+c)}{24d^2x^2}+\frac{\cos(dx+c)}{24dx}+\frac{\mathrm{Si}(dx)\cos(c)}{24}+\frac{\mathrm{Ci}(dx)\sin(c)}{24}\right)+\frac{b\left(-\frac{\sin(dx\right.}{2d^2x}\right.$
risch	$-\frac{i\,\mathrm{Ei}_1(idx)\cos(c)a\,d^4}{48}+\frac{i\,\mathrm{Ei}_1(-idx)\cos(c)a\,d^4}{48}+\frac{i\,\mathrm{Ei}_1(idx)\cos(c)b\,d^2}{4}-\frac{i\,\mathrm{Ei}_1(-idx)\cos(c)b\,d^2}{4}-\frac{\mathrm{Ei}_1(idx)\sin(c}{48}$
meijerg	$\frac{d^2b\sqrt{\pi}\,\sin(c)\left(-\frac{4}{\sqrt{\pi}\,x^2d^2}-\frac{2\left(2\gamma-3+2\ln(x)+\ln\left(d^2\right)\right)}{\sqrt{\pi}}+\frac{-6d^2x^2+4}{\sqrt{\pi}\,x^2d^2}+\frac{4\gamma}{\sqrt{\pi}}+\frac{4\ln(2)}{\sqrt{\pi}}+\frac{4\ln\left(\frac{dx}{2}\right)}{\sqrt{\pi}}-\frac{4\cos(dx)}{\sqrt{\pi}\,d^2x^2}+\frac{4\sin(dx)}{\sqrt{\pi}\,dx}-\frac{4\,\mathrm{Ci}(dx)}{\sqrt{\pi}}\right.}{8}$

```
[In] int((b*x^2+a)*sin(d*x+c)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] d^4*(a*(-1/4*sin(d*x+c)/d^4/x^4-1/12*cos(d*x+c)/d^3/x^3+1/24*sin(d*x+c)/d^2
/x^2+1/24*cos(d*x+c)/d/x+1/24*Si(d*x)*cos(c)+1/24*Ci(d*x)*sin(c))+1/d^2*b*(
-1/2*sin(d*x+c)/d^2/x^2-1/2*cos(d*x+c)/d/x-1/2*Si(d*x)*cos(c)-1/2*Ci(d*x)*s
in(c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.68

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^5}\,dx$$
$$=\frac{(ad^4-12\,bd^2)x^4\,\mathrm{Ci}\,(dx)\sin\,(c)+(ad^4-12\,bd^2)x^4\cos\,(c)\,\mathrm{Si}\,(dx)+((ad^3-12\,bd)x^3-2\,adx)\cos\,(dx+c)}{24\,x^4}$$

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*((a*d^4 - 12*b*d^2)*x^4*cos_integral(d*x)*sin(c) + (a*d^4 - 12*b*d^2)*
x^4*cos(c)*sin_integral(d*x) + ((a*d^3 - 12*b*d)*x^3 - 2*a*d*x)*cos(d*x + c
) + ((a*d^2 - 12*b)*x^2 - 6*a)*sin(d*x + c))/x^4
```

Sympy [F]

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^5}\,dx = \int \frac{(a+bx^2)\sin{(c+dx)}}{x^5}\,dx$$

```
[In] integrate((b*x**2+a)*sin(d*x+c)/x**5,x)
```

```
[Out] Integral((a + b*x**2)*sin(c + d*x)/x**5, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^5}\,dx = -\frac{((a(i\,\Gamma(-4,i\,dx)-i\,\Gamma(-4,-i\,dx))\cos{(c)}+a(\Gamma(-4,i\,dx)+\Gamma(-4,-i\,dx))\sin{(c)})d^6-12\,(b(i\,\Gamma(-4,i\,dx}{}$$

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x^5,x, algorithm="maxima")
```

```
[Out] -1/2*(((a*(I*gamma(-4, I*d*x) - I*gamma(-4, -I*d*x))*cos(c) + a*(gamma(-4,
I*d*x) + gamma(-4, -I*d*x))*sin(c))*d^6 - 12*(b*(I*gamma(-4, I*d*x) - I*gam
ma(-4, -I*d*x))*cos(c) + b*(gamma(-4, I*d*x) + gamma(-4, -I*d*x))*sin(c))*d
^4)*x^4 + 2*b*d*x*cos(d*x + c) + 6*b*sin(d*x + c))/(d^2*x^4)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.30 (sec) , antiderivative size = 1086, normalized size of antiderivative = 7.29

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^5}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x^5,x, algorithm="giac")
```

```
[Out] -1/48*(a*d^4*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 -
 a*d^4*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*
d^4*x^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*d^4*x^4*real_pa
rt(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^4*x^4*real_part(cos
_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - a*d^4*x^4*imag_part(cos_integr
al(d*x))*tan(1/2*d*x)^2 + a*d^4*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d
*x)^2 - 2*a*d^4*x^4*sin_integral(d*x)*tan(1/2*d*x)^2 + a*d^4*x^4*imag_part(
```

```
cos_integral(d*x))*tan(1/2*c)^2 - a*d^4*x^4*imag_part(cos_integral(-d*x))*t
an(1/2*c)^2 + 2*a*d^4*x^4*sin_integral(d*x)*tan(1/2*c)^2 - 12*b*d^2*x^4*ima
g_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 12*b*d^2*x^4*imag_p
art(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 24*b*d^2*x^4*sin_inte
gral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*d^4*x^4*real_part(cos_integral(
d*x))*tan(1/2*c) - 2*a*d^4*x^4*real_part(cos_integral(-d*x))*tan(1/2*c) + 2
4*b*d^2*x^4*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 24*b*d
^2*x^4*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^3*x^
3*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d^4*x^4*imag_part(cos_integral(d*x)) + a*
d^4*x^4*imag_part(cos_integral(-d*x)) - 2*a*d^4*x^4*sin_integral(d*x) + 12*
b*d^2*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 - 12*b*d^2*x^4*imag_p
art(cos_integral(-d*x))*tan(1/2*d*x)^2 + 24*b*d^2*x^4*sin_integral(d*x)*tan
(1/2*d*x)^2 - 12*b*d^2*x^4*imag_part(cos_integral(d*x))*tan(1/2*c)^2 + 12*b
*d^2*x^4*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 - 24*b*d^2*x^4*sin_inte
gral(d*x)*tan(1/2*c)^2 + 2*a*d^3*x^3*tan(1/2*d*x)^2 + 24*b*d^2*x^4*real_par
t(cos_integral(d*x))*tan(1/2*c) + 24*b*d^2*x^4*real_part(cos_integral(-d*x)
)*tan(1/2*c) + 8*a*d^3*x^3*tan(1/2*d*x)*tan(1/2*c) + 2*a*d^3*x^3*tan(1/2*c)
^2 + 24*b*d*x^3*tan(1/2*d*x)^2*tan(1/2*c)^2 + 12*b*d^2*x^4*imag_part(cos_in
tegral(d*x)) - 12*b*d^2*x^4*imag_part(cos_integral(-d*x)) + 24*b*d^2*x^4*si
n_integral(d*x) + 4*a*d^2*x^2*tan(1/2*d*x)^2*tan(1/2*c) + 4*a*d^2*x^2*tan(1
/2*d*x)*tan(1/2*c)^2 - 2*a*d^3*x^3 - 24*b*d*x^3*tan(1/2*d*x)^2 - 96*b*d*x^3
*tan(1/2*d*x)*tan(1/2*c) - 24*b*d*x^3*tan(1/2*c)^2 + 4*a*d*x*tan(1/2*d*x)^2
*tan(1/2*c)^2 - 4*a*d^2*x^2*tan(1/2*d*x) - 4*a*d^2*x^2*tan(1/2*c) - 48*b*x^
2*tan(1/2*d*x)^2*tan(1/2*c) - 48*b*x^2*tan(1/2*d*x)*tan(1/2*c)^2 + 24*b*d*x
^3 - 4*a*d*x*tan(1/2*d*x)^2 - 16*a*d*x*tan(1/2*d*x)*tan(1/2*c) - 4*a*d*x*ta
n(1/2*c)^2 + 48*b*x^2*tan(1/2*d*x) + 48*b*x^2*tan(1/2*c) - 24*a*tan(1/2*d*x
)^2*tan(1/2*c) - 24*a*tan(1/2*d*x)*tan(1/2*c)^2 + 4*a*d*x + 24*a*tan(1/2*d*
x) + 24*a*tan(1/2*c))/(x^4*tan(1/2*d*x)^2*tan(1/2*c)^2 + x^4*tan(1/2*d*x)^2
 + x^4*tan(1/2*c)^2 + x^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\sin(c+dx)}{x^5}\,dx = \int \frac{\sin\left(c+d\,x\right)\,\left(b\,x^2+a\right)}{x^5}\,dx$$

```
[In] int((sin(c + d*x)*(a + b*x^2))/x^5,x)
[Out] int((sin(c + d*x)*(a + b*x^2))/x^5, x)
```

3.49 $\int x^2(a+bx^2)^2\sin(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 236

$$\begin{aligned}\int x^2(a+bx^2)^2\sin(c+dx)\,dx = {} & \frac{720b^2\cos(c+dx)}{d^7}-\frac{48ab\cos(c+dx)}{d^5}+\frac{2a^2\cos(c+dx)}{d^3} \\ & -\frac{360b^2x^2\cos(c+dx)}{d^5}+\frac{24abx^2\cos(c+dx)}{d^3} \\ & -\frac{a^2x^2\cos(c+dx)}{d}+\frac{30b^2x^4\cos(c+dx)}{d^3} \\ & -\frac{2abx^4\cos(c+dx)}{d}-\frac{b^2x^6\cos(c+dx)}{d} \\ & +\frac{720b^2x\sin(c+dx)}{d^6}-\frac{48abx\sin(c+dx)}{d^4} \\ & +\frac{2a^2x\sin(c+dx)}{d^2}-\frac{120b^2x^3\sin(c+dx)}{d^4} \\ & +\frac{8abx^3\sin(c+dx)}{d^2}+\frac{6b^2x^5\sin(c+dx)}{d^2}\end{aligned}$$

```
[Out] 720*b^2*cos(d*x+c)/d^7-48*a*b*cos(d*x+c)/d^5+2*a^2*cos(d*x+c)/d^3-360*b^2*x
^2*cos(d*x+c)/d^5+24*a*b*x^2*cos(d*x+c)/d^3-a^2*x^2*cos(d*x+c)/d+30*b^2*x^4
*cos(d*x+c)/d^3-2*a*b*x^4*cos(d*x+c)/d-b^2*x^6*cos(d*x+c)/d+720*b^2*x*sin(d
*x+c)/d^6-48*a*b*x*sin(d*x+c)/d^4+2*a^2*x*sin(d*x+c)/d^2-120*b^2*x^3*sin(d*
x+c)/d^4+8*a*b*x^3*sin(d*x+c)/d^2+6*b^2*x^5*sin(d*x+c)/d^2
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3420, 3377, 2718}

$$\int x^2(a+bx^2)^2\sin(c+dx)\,dx = \frac{2a^2\cos(c+dx)}{d^3}+\frac{2a^2x\sin(c+dx)}{d^2}-\frac{a^2x^2\cos(c+dx)}{d} -\frac{48ab\cos(c+dx)}{d^5}-\frac{48abx\sin(c+dx)}{d^4} +\frac{24abx^2\cos(c+dx)}{d^3}+\frac{8abx^3\sin(c+dx)}{d^2} -\frac{2abx^4\cos(c+dx)}{d}+\frac{720b^2\cos(c+dx)}{d^7} +\frac{720b^2x\sin(c+dx)}{d^6}-\frac{360b^2x^2\cos(c+dx)}{d^5} -\frac{120b^2x^3\sin(c+dx)}{d^4}+\frac{30b^2x^4\cos(c+dx)}{d^3} +\frac{6b^2x^5\sin(c+dx)}{d^2}-\frac{b^2x^6\cos(c+dx)}{d}$$

```
[In] Int[x^2*(a + b*x^2)^2*Sin[c + d*x],x]
```

```
[Out] (720*b^2*Cos[c + d*x])/d^7 - (48*a*b*Cos[c + d*x])/d^5 + (2*a^2*Cos[c + d*x
])/d^3 - (360*b^2*x^2*Cos[c + d*x])/d^5 + (24*a*b*x^2*Cos[c + d*x])/d^3 - (
a^2*x^2*Cos[c + d*x])/d + (30*b^2*x^4*Cos[c + d*x])/d^3 - (2*a*b*x^4*Cos[c
+ d*x])/d - (b^2*x^6*Cos[c + d*x])/d + (720*b^2*x*Sin[c + d*x])/d^6 - (48*a
*b*x*Sin[c + d*x])/d^4 + (2*a^2*x*Sin[c + d*x])/d^2 - (120*b^2*x^3*Sin[c +
d*x])/d^4 + (8*a*b*x^3*Sin[c + d*x])/d^2 + (6*b^2*x^5*Sin[c + d*x])/d^2
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(a^2x^2\sin(c+dx) + 2abx^4\sin(c+dx) + b^2x^6\sin(c+dx)\right) dx$$

$$= a^2\int x^2\sin(c+dx)\,dx + (2ab)\int x^4\sin(c+dx)\,dx + b^2\int x^6\sin(c+dx)\,dx$$

$$= -\frac{a^2x^2\cos(c+dx)}{d} - \frac{2abx^4\cos(c+dx)}{d} - \frac{b^2x^6\cos(c+dx)}{d} + \frac{(2a^2)\int x\cos(c+dx)\,dx}{d} + \frac{(8ab)\int x^3\cos(c+dx)\,dx}{d} + \frac{(6b^2)\int x^5\cos(c+dx)\,dx}{d}$$

$$= -\frac{a^2x^2\cos(c+dx)}{d} - \frac{2abx^4\cos(c+dx)}{d} - \frac{b^2x^6\cos(c+dx)}{d} + \frac{2a^2x\sin(c+dx)}{d^2} + \frac{8abx^3\sin(c+dx)}{d^2} + \frac{6b^2x^5\sin(c+dx)}{d^2} - \frac{(2a^2)\int\sin(c+dx)\,dx}{d^2} - \frac{(24ab)\int x^2\sin(c+dx)\,dx}{d^2} - \frac{(30b^2)\int x^4\sin(c+dx)\,dx}{d^2}$$

$$= \frac{2a^2\cos(c+dx)}{d^3} + \frac{24abx^2\cos(c+dx)}{d^3} - \frac{a^2x^2\cos(c+dx)}{d} + \frac{30b^2x^4\cos(c+dx)}{d^3} - \frac{2abx^4\cos(c+dx)}{d} - \frac{b^2x^6\cos(c+dx)}{d} + \frac{2a^2x\sin(c+dx)}{d^2} + \frac{8abx^3\sin(c+dx)}{d^2} + \frac{6b^2x^5\sin(c+dx)}{d^2} - \frac{(48ab)\int x\cos(c+dx)\,dx}{d^3} - \frac{(120b^2)\int x^3\cos(c+dx)\,dx}{d^3}$$

$$= \frac{2a^2\cos(c+dx)}{d^3} + \frac{24abx^2\cos(c+dx)}{d^3} - \frac{a^2x^2\cos(c+dx)}{d} + \frac{30b^2x^4\cos(c+dx)}{d^3} - \frac{2abx^4\cos(c+dx)}{d} - \frac{b^2x^6\cos(c+dx)}{d} - \frac{48abx\sin(c+dx)}{d^4} + \frac{2a^2x\sin(c+dx)}{d^2} - \frac{120b^2x^3\sin(c+dx)}{d^4} + \frac{8abx^3\sin(c+dx)}{d^2} + \frac{6b^2x^5\sin(c+dx)}{d^2} + \frac{(48ab)\int\sin(c+dx)\,dx}{d^4} + \frac{(360b^2)\int x^2\sin(c+dx)\,dx}{d^4}$$

$$= -\frac{48ab\cos(c+dx)}{d^5} + \frac{2a^2\cos(c+dx)}{d^3} - \frac{360b^2x^2\cos(c+dx)}{d^5} + \frac{24abx^2\cos(c+dx)}{d^3} - \frac{a^2x^2\cos(c+dx)}{d} + \frac{30b^2x^4\cos(c+dx)}{d^3} - \frac{2abx^4\cos(c+dx)}{d} - \frac{b^2x^6\cos(c+dx)}{d} - \frac{48abx\sin(c+dx)}{d^4} + \frac{2a^2x\sin(c+dx)}{d^2} - \frac{120b^2x^3\sin(c+dx)}{d^4} + \frac{8abx^3\sin(c+dx)}{d^2} + \frac{6b^2x^5\sin(c+dx)}{d^2} + \frac{(720b^2)\int x\cos(c+dx)\,dx}{d^5}$$

$$= -\frac{48ab\cos(c+dx)}{d^5} + \frac{2a^2\cos(c+dx)}{d^3} - \frac{360b^2x^2\cos(c+dx)}{d^5} + \frac{24abx^2\cos(c+dx)}{d^3} - \frac{a^2x^2\cos(c+dx)}{d} + \frac{30b^2x^4\cos(c+dx)}{d^3} - \frac{2abx^4\cos(c+dx)}{d} - \frac{b^2x^6\cos(c+dx)}{d} + \frac{720b^2x\sin(c+dx)}{d^6} - \frac{48abx\sin(c+dx)}{d^4} + \frac{2a^2x\sin(c+dx)}{d^2} - \frac{120b^2x^3\sin(c+dx)}{d^4} + \frac{8abx^3\sin(c+dx)}{d^2} + \frac{6b^2x^5\sin(c+dx)}{d^2} - \frac{(720b^2)\int\sin(c+dx)\,dx}{d^6}$$

$$= \frac{720b^2\cos(c+dx)}{d^7} - \frac{48ab\cos(c+dx)}{d^5} + \frac{2a^2\cos(c+dx)}{d^3} - \frac{360b^2x^2\cos(c+dx)}{d^5} + \frac{24abx^2\cos(c+dx)}{d^3} - \frac{a^2x^2\cos(c+dx)}{d} + \frac{30b^2x^4\cos(c+dx)}{d^3} - \frac{2abx^4\cos(c+dx)}{d} - \frac{b^2x^6\cos(c+dx)}{d} + \frac{720b^2x\sin(c+dx)}{d^6} - \frac{48abx\sin(c+dx)}{d^4} + \frac{2a^2x\sin(c+dx)}{d^2} - \frac{120b^2x^3\sin(c+dx)}{d^4} + \frac{8abx^3\sin(c+dx)}{d^2} + \frac{6b^2x^5\sin(c+dx)}{d^2}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.59

$$\int x^2\left(a+bx^2\right)^2\sin(c+dx)\,dx = \frac{-((a^2d^4(-2+d^2x^2)+2abd^2(24-12d^2x^2+d^4x^4)+b^2(-720+360d^2x^2-30d^4x^4+d^6x^6))\cos(c+dx))}{d^7}$$

```
[In] Integrate[x^2*(a + b*x^2)^2*Sin[c + d*x],x]
```

```
[Out] (-((a^2*d^4*(-2 + d^2*x^2) + 2*a*b*d^2*(24 - 12*d^2*x^2 + d^4*x^4) + b^2*(-
720 + 360*d^2*x^2 - 30*d^4*x^4 + d^6*x^6))*Cos[c + d*x]) + 2*d*x*(a^2*d^4 +
 4*a*b*d^2*(-6 + d^2*x^2) + 3*b^2*(120 - 20*d^2*x^2 + d^4*x^4))*Sin[c + d*x
])/d^7
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{\left(b^{2} x^{6} d^{6}+2 a b\, d^{6} x^{4}+a^{2} d^{6} x^{2}-30 b^{2} x^{4} d^{4}-24 a b\, d^{4} x^{2}-2 a^{2} d^{4}+360 d^{2} x^{2} b^{2}+48 a b\, d^{2}-720 b^{2}\right) \cos \left(d x+c\right)}{d^{7}}+\frac{2 x \left(3 b^{2} x^{4} d^{4}+4\right.}{}$
parallelrisch	$\frac{\left(x^{2} \left(b\,x^{2}+a\right)^{2} d^{6}+\left(-30 b^{2} x^{4}-24 a b\,x^{2}-4 a^{2}\right) d^{4}+\left(360 x^{2} b^{2}+96 a b\right) d^{2}-1440 b^{2}\right) \left(\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)+4 \left(\left(3 b\,x^{2}+a\right) \left(b\,x^{2}+a\right) d^{4}+\right.}{d^{7} \left(1+\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}$
norman	$\frac{\frac{b^{2} x^{6} \left(\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\left(a^{2} d^{4}-24 a b\,d^{2}+360 b^{2}\right) x^{2} \left(\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{d^{5}}-\frac{b^{2} x^{6}}{d}-\frac{\left(a^{2} d^{4}-24 a b\,d^{2}+360 b^{2}\right) x^{2}}{d^{5}}-\frac{\left(4 a^{2} d^{4}-96 a b\,d^{2}+1440 b^{2}\right)}{d^{7}}}{}$
meijerg	$\frac{64 b^{2} \sqrt{\pi}\, \sin \left(c\right) \left(\frac{x \left(d^{2}\right)^{\frac{7}{2}} \left(\frac{21}{8} d^{4} x^{4}-\frac{105}{2} d^{2} x^{2}+315\right) \cos \left(d x\right)}{28 \sqrt{\pi}\, d^{6}}-\frac{\left(d^{2}\right)^{\frac{7}{2}} \left(-\frac{7}{16} d^{6} x^{6}+\frac{105}{8} d^{4} x^{4}-\frac{315}{2} d^{2} x^{2}+315\right) \sin \left(d x\right)}{28 \sqrt{\pi}\, d^{7}}\right)}{d^{6} \sqrt{d^{2}}}+\frac{64 b^{2} \sqrt{\pi}}{}$
parts	$-\frac{b^{2} x^{6} \cos \left(d x+c\right)}{d}-\frac{2 a b\,x^{4} \cos \left(d x+c\right)}{d}-\frac{a^{2} x^{2} \cos \left(d x+c\right)}{d}+\frac{-\frac{2 a^{2} c \sin \left(d x+c\right)}{d}+\frac{2 a^{2} \left(\cos \left(d x+c\right)+\left(d x+c\right) \sin \left(d x+c\right)\right)}{d}-\frac{8 a b\,c}{}}{}$
derivativedivides	$\frac{-a^{2} c^{2} \cos \left(d x+c\right)-2 a^{2} c \left(\sin \left(d x+c\right)-\cos \left(d x+c\right) \left(d x+c\right)\right)+a^{2} \left(-\left(d x+c\right)^{2} \cos \left(d x+c\right)+2 \cos \left(d x+c\right)+2 \left(d x+c\right) \sin \left(d x+c\right)\right)-2}{}$
default	$\frac{-a^{2} c^{2} \cos \left(d x+c\right)-2 a^{2} c \left(\sin \left(d x+c\right)-\cos \left(d x+c\right) \left(d x+c\right)\right)+a^{2} \left(-\left(d x+c\right)^{2} \cos \left(d x+c\right)+2 \cos \left(d x+c\right)+2 \left(d x+c\right) \sin \left(d x+c\right)\right)-2}{}$

```
[In] int(x^2*(b*x^2+a)^2*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -(b^2*d^6*x^6+2*a*b*d^6*x^4+a^2*d^6*x^2-30*b^2*d^4*x^4-24*a*b*d^4*x^2-2*a^2
*d^4+360*b^2*d^2*x^2+48*a*b*d^2-720*b^2)/d^7*cos(d*x+c)+2/d^6*x*(3*b^2*d^4*
x^4+4*a*b*d^4*x^2+a^2*d^4-60*b^2*d^2*x^2-24*a*b*d^2+360*b^2)*sin(d*x+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.65

$$\int x^2 (a+bx^2)^2 \sin(c+dx)\,dx = -\frac{(b^2d^6x^6 - 2\,a^2d^4 + 2\,(abd^6 - 15\,b^2d^4)x^4 + 48\,abd^2 + (a^2d^6 - 24\,abd^4 + 360\,b^2d^2)x^2 - 720\,b^2)\cos{(dx+c)}}{d^7}$$

```
[In] integrate(x^2*(b*x^2+a)^2*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b^2*d^6*x^6 - 2*a^2*d^4 + 2*(a*b*d^6 - 15*b^2*d^4)*x^4 + 48*a*b*d^2 + (a
^2*d^6 - 24*a*b*d^4 + 360*b^2*d^2)*x^2 - 720*b^2)*cos(d*x + c) - 2*(3*b^2*d
^5*x^5 + 4*(a*b*d^5 - 15*b^2*d^3)*x^3 + (a^2*d^5 - 24*a*b*d^3 + 360*b^2*d)*
x)*sin(d*x + c))/d^7
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.21

$$\int x^2(a+bx^2)^2\sin(c+dx)\,dx$$
$$=\begin{cases} -\frac{a^2x^2\cos{(c+dx)}}{d}+\frac{2a^2x\sin{(c+dx)}}{d^2}+\frac{2a^2\cos{(c+dx)}}{d^3}-\frac{2abx^4\cos{(c+dx)}}{d}+\frac{8abx^3\sin{(c+dx)}}{d^2}+\frac{24abx^2\cos{(c+dx)}}{d^3}-\frac{48abx\sin{(c-}}{d^4} \\ \left(\frac{a^2x^3}{3}+\frac{2abx^5}{5}+\frac{b^2x^7}{7}\right)\sin{(c)} \end{cases}$$

```
[In] integrate(x**2*(b*x**2+a)**2*sin(d*x+c),x)
[Out] Piecewise((-a**2*x**2*cos(c + d*x)/d + 2*a**2*x*sin(c + d*x)/d**2 + 2*a**2*
cos(c + d*x)/d**3 - 2*a*b*x**4*cos(c + d*x)/d + 8*a*b*x**3*sin(c + d*x)/d**
2 + 24*a*b*x**2*cos(c + d*x)/d**3 - 48*a*b*x*sin(c + d*x)/d**4 - 48*a*b*cos
(c + d*x)/d**5 - b**2*x**6*cos(c + d*x)/d + 6*b**2*x**5*sin(c + d*x)/d**2 +
 30*b**2*x**4*cos(c + d*x)/d**3 - 120*b**2*x**3*sin(c + d*x)/d**4 - 360*b**
2*x**2*cos(c + d*x)/d**5 + 720*b**2*x*sin(c + d*x)/d**6 + 720*b**2*cos(c +
d*x)/d**7, Ne(d, 0)), ((a**2*x**3/3 + 2*a*b*x**5/5 + b**2*x**7/7)*sin(c), T
rue))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 612 vs. 2(236) = 472.

Time = 0.22 (sec) , antiderivative size = 612, normalized size of antiderivative = 2.59

$$\int x^2(a+bx^2)^2\sin(c+dx)\,dx =$$
$$-\frac{a^2c^2\cos{(dx+c)}+\frac{b^2c^6\cos(dx+c)}{d^4}+\frac{2\,abc^4\cos(dx+c)}{d^2}-2\left((dx+c)\cos{(dx+c)}-\sin{(dx+c)}\right)a^2c-\frac{6\,((dx+c)\,c}{}}{}$$

```
[In] integrate(x^2*(b*x^2+a)^2*sin(d*x+c),x, algorithm="maxima")
[Out] -(a^2*c^2*cos(d*x + c) + b^2*c^6*cos(d*x + c)/d^4 + 2*a*b*c^4*cos(d*x + c)/
d^2 - 2*((d*x + c)*cos(d*x + c) - sin(d*x + c))*a^2*c - 6*((d*x + c)*cos(d*
x + c) - sin(d*x + c))*b^2*c^5/d^4 - 8*((d*x + c)*cos(d*x + c) - sin(d*x +
c))*a*b*c^3/d^2 + (((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c
))*a^2 + 15*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b^2
*c^4/d^4 + 12*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a
*b*c^2/d^2 - 20*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2
- 2)*sin(d*x + c))*b^2*c^3/d^4 - 8*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c
) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*a*b*c/d^2 + 15*(((d*x + c)^4 - 12*(d*
x + c)^2 + 24)*cos(d*x + c) - 4*((d*x + c)^3 - 6*d*x - 6*c)*sin(d*x + c))*b
^2*c^2/d^4 + 2*(((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d*x + c) - 4*((d*x
```

```
+ c)^3 - 6*d*x - 6*c)*sin(d*x + c))*a*b/d^2 - 6*(((d*x + c)^5 - 20*(d*x + c
)^3 + 120*d*x + 120*c)*cos(d*x + c) - 5*((d*x + c)^4 - 12*(d*x + c)^2 + 24)
*sin(d*x + c))*b^2*c/d^4 + (((d*x + c)^6 - 30*(d*x + c)^4 + 360*(d*x + c)^2
 - 720)*cos(d*x + c) - 6*((d*x + c)^5 - 20*(d*x + c)^3 + 120*d*x + 120*c)*s
in(d*x + c))*b^2/d^4)/d^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.69

$$\int x^2 (a + bx^2)^2 \sin(c + dx) \, dx = -\frac{\left(b^2 d^6 x^6 + 2\, abd^6 x^4 + a^2 d^6 x^2 - 30\, b^2 d^4 x^4 - 24\, abd^4 x^2 - 2\, a^2 d^4 + 360\, b^2 d^2 x^2 + 48\, abd^2 - 720\, b^2\right) \cos\left(dx +\right.}{d^7} + \frac{2 \left(3\, b^2 d^5 x^5 + 4\, abd^5 x^3 + a^2 d^5 x - 60\, b^2 d^3 x^3 - 24\, abd^3 x + 360\, b^2 dx\right) \sin\left(dx + c\right)}{d^7}$$

```
[In] integrate(x^2*(b*x^2+a)^2*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b^2*d^6*x^6 + 2*a*b*d^6*x^4 + a^2*d^6*x^2 - 30*b^2*d^4*x^4 - 24*a*b*d^4*x
^2 - 2*a^2*d^4 + 360*b^2*d^2*x^2 + 48*a*b*d^2 - 720*b^2)*cos(d*x + c)/d^7 +
 2*(3*b^2*d^5*x^5 + 4*a*b*d^5*x^3 + a^2*d^5*x - 60*b^2*d^3*x^3 - 24*a*b*d^3
*x + 360*b^2*d*x)*sin(d*x + c)/d^7
```

Mupad [B] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.79

$$\int x^2 (a + bx^2)^2 \sin(c + dx) \, dx = \frac{2\, \cos\left(c + d\,x\right)\, \left(a^2\,d^4 - 24\,a\,b\,d^2 + 360\,b^2\right)}{d^7} - \frac{b^2\,x^6\, \cos\left(c + d\,x\right)}{d} + \frac{6\,b^2\,x^5\, \sin\left(c + d\,x\right)}{d^2} + \frac{2\,x\, \sin\left(c + d\,x\right)\, \left(a^2\,d^4 - 24\,a\,b\,d^2 + 360\,b^2\right)}{d^6} - \frac{x^2\, \cos\left(c + d\,x\right)\, \left(a^2\,d^4 - 24\,a\,b\,d^2 + 360\,b^2\right)}{d^5} + \frac{2\,x^4\, \cos\left(c + d\,x\right)\, \left(15\,b^2 - a\,b\,d^2\right)}{d^3} - \frac{8\,x^3\, \sin\left(c + d\,x\right)\, \left(15\,b^2 - a\,b\,d^2\right)}{d^4}$$

```
[In] int(x^2*sin(c + d*x)*(a + b*x^2)^2,x)
```

```
[Out] (2*cos(c + d*x)*(360*b^2 + a^2*d^4 - 24*a*b*d^2))/d^7 - (b^2*x^6*cos(c + d*
x))/d + (6*b^2*x^5*sin(c + d*x))/d^2 + (2*x*sin(c + d*x)*(360*b^2 + a^2*d^4
 - 24*a*b*d^2))/d^6 - (x^2*cos(c + d*x)*(360*b^2 + a^2*d^4 - 24*a*b*d^2))/d
^5 + (2*x^4*cos(c + d*x)*(15*b^2 - a*b*d^2))/d^3 - (8*x^3*sin(c + d*x)*(15*
b^2 - a*b*d^2))/d^4
```

3.50 $\int x(a+bx^2)^2 \sin(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 185

$$\begin{aligned}\int x(a+bx^2)^2 \sin(c+dx)\,dx = &-\frac{120b^2x\cos(c+dx)}{d^5}+\frac{12abx\cos(c+dx)}{d^3}-\frac{a^2x\cos(c+dx)}{d} \\ &+\frac{20b^2x^3\cos(c+dx)}{d^3}-\frac{2abx^3\cos(c+dx)}{d}-\frac{b^2x^5\cos(c+dx)}{d} \\ &+\frac{120b^2\sin(c+dx)}{d^6}-\frac{12ab\sin(c+dx)}{d^4}+\frac{a^2\sin(c+dx)}{d^2} \\ &-\frac{60b^2x^2\sin(c+dx)}{d^4}+\frac{6abx^2\sin(c+dx)}{d^2}+\frac{5b^2x^4\sin(c+dx)}{d^2}\end{aligned}$$

```
[Out] -120*b^2*x*cos(d*x+c)/d^5+12*a*b*x*cos(d*x+c)/d^3-a^2*x*cos(d*x+c)/d+20*b^2
*x^3*cos(d*x+c)/d^3-2*a*b*x^3*cos(d*x+c)/d-b^2*x^5*cos(d*x+c)/d+120*b^2*sin
(d*x+c)/d^6-12*a*b*sin(d*x+c)/d^4+a^2*sin(d*x+c)/d^2-60*b^2*x^2*sin(d*x+c)/
d^4+6*a*b*x^2*sin(d*x+c)/d^2+5*b^2*x^4*sin(d*x+c)/d^2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3420, 3377, 2717}

$$\int x(a+bx^2)^2\sin(c+dx)\,dx = \frac{a^2\sin(c+dx)}{d^2}-\frac{a^2x\cos(c+dx)}{d}-\frac{12ab\sin(c+dx)}{d^4}+\frac{12abx\cos(c+dx)}{d^3}+\frac{6abx^2\sin(c+dx)}{d^2}-\frac{2abx^3\cos(c+dx)}{d}+\frac{120b^2\sin(c+dx)}{d^6}-\frac{120b^2x\cos(c+dx)}{d^5}-\frac{60b^2x^2\sin(c+dx)}{d^4}+\frac{20b^2x^3\cos(c+dx)}{d^3}+\frac{5b^2x^4\sin(c+dx)}{d^2}-\frac{b^2x^5\cos(c+dx)}{d}$$

```
[In] Int[x*(a + b*x^2)^2*Sin[c + d*x],x]
```

```
[Out] (-120*b^2*x*Cos[c + d*x])/d^5 + (12*a*b*x*Cos[c + d*x])/d^3 - (a^2*x*Cos[c
+ d*x])/d + (20*b^2*x^3*Cos[c + d*x])/d^3 - (2*a*b*x^3*Cos[c + d*x])/d - (b
^2*x^5*Cos[c + d*x])/d + (120*b^2*Sin[c + d*x])/d^6 - (12*a*b*Sin[c + d*x])
/d^4 + (a^2*Sin[c + d*x])/d^2 - (60*b^2*x^2*Sin[c + d*x])/d^4 + (6*a*b*x^2*
Sin[c + d*x])/d^2 + (5*b^2*x^4*Sin[c + d*x])/d^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(a^2 x \sin(c+dx) + 2abx^3 \sin(c+dx) + b^2 x^5 \sin(c+dx)\right) dx$$

$$= a^2 \int x \sin(c+dx)\,dx + (2ab) \int x^3 \sin(c+dx)\,dx + b^2 \int x^5 \sin(c+dx)\,dx$$

$$= -\frac{a^2 x \cos(c+dx)}{d} - \frac{2abx^3 \cos(c+dx)}{d} - \frac{b^2 x^5 \cos(c+dx)}{d} + \frac{a^2 \int \cos(c+dx)\,dx}{d} + \frac{(6ab) \int x^2 \cos(c+dx)\,dx}{d} + \frac{(5b^2) \int x^4 \cos(c+dx)\,dx}{d}$$

$$= -\frac{a^2 x \cos(c+dx)}{d} - \frac{2abx^3 \cos(c+dx)}{d} - \frac{b^2 x^5 \cos(c+dx)}{d} + \frac{a^2 \sin(c+dx)}{d^2} + \frac{6abx^2 \sin(c+dx)}{d^2} + \frac{5b^2 x^4 \sin(c+dx)}{d^2} - \frac{(12ab) \int x \sin(c+dx)\,dx}{d^2} - \frac{(20b^2) \int x^3 \sin(c+dx)\,dx}{d^2}$$

$$= \frac{12abx \cos(c+dx)}{d^3} - \frac{a^2 x \cos(c+dx)}{d} + \frac{20b^2 x^3 \cos(c+dx)}{d^3} - \frac{2abx^3 \cos(c+dx)}{d} - \frac{b^2 x^5 \cos(c+dx)}{d} + \frac{a^2 \sin(c+dx)}{d^2} + \frac{6abx^2 \sin(c+dx)}{d^2} + \frac{5b^2 x^4 \sin(c+dx)}{d^2} - \frac{(12ab) \int \cos(c+dx)\,dx}{d^3} - \frac{(60b^2) \int x^2 \cos(c+dx)\,dx}{d^3}$$

$$= \frac{12abx \cos(c+dx)}{d^3} - \frac{a^2 x \cos(c+dx)}{d} + \frac{20b^2 x^3 \cos(c+dx)}{d^3} - \frac{2abx^3 \cos(c+dx)}{d} - \frac{b^2 x^5 \cos(c+dx)}{d} - \frac{12ab \sin(c+dx)}{d^4} + \frac{a^2 \sin(c+dx)}{d^2} - \frac{60b^2 x^2 \sin(c+dx)}{d^4} + \frac{6abx^2 \sin(c+dx)}{d^2} + \frac{5b^2 x^4 \sin(c+dx)}{d^2} + \frac{(120b^2) \int x \sin(c+dx)\,dx}{d^4}$$

$$= -\frac{120b^2 x \cos(c+dx)}{d^5} + \frac{12abx \cos(c+dx)}{d^3} - \frac{a^2 x \cos(c+dx)}{d} + \frac{20b^2 x^3 \cos(c+dx)}{d^3} - \frac{2abx^3 \cos(c+dx)}{d} - \frac{b^2 x^5 \cos(c+dx)}{d} - \frac{12ab \sin(c+dx)}{d^4} + \frac{a^2 \sin(c+dx)}{d^2} - \frac{60b^2 x^2 \sin(c+dx)}{d^4} + \frac{6abx^2 \sin(c+dx)}{d^2} + \frac{5b^2 x^4 \sin(c+dx)}{d^2} + \frac{(120b^2) \int \cos(c+dx)\,dx}{d^5}$$

$$= -\frac{120b^2 x \cos(c+dx)}{d^5} + \frac{12abx \cos(c+dx)}{d^3} - \frac{a^2 x \cos(c+dx)}{d} + \frac{20b^2 x^3 \cos(c+dx)}{d^3} - \frac{2abx^3 \cos(c+dx)}{d} - \frac{b^2 x^5 \cos(c+dx)}{d} + \frac{120b^2 \sin(c+dx)}{d^6} - \frac{12ab \sin(c+dx)}{d^4} + \frac{a^2 \sin(c+dx)}{d^2} - \frac{60b^2 x^2 \sin(c+dx)}{d^4} + \frac{6abx^2 \sin(c+dx)}{d^2} + \frac{5b^2 x^4 \sin(c+dx)}{d^2}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.61

$$\int x(a+bx^2)^2\sin(c+dx)\,dx$$
$$=\frac{-dx(a^2d^4+2abd^2(-6+d^2x^2)+b^2(120-20d^2x^2+d^4x^4))\cos(c+dx)+(a^2d^4+6abd^2(-2+d^2x^2)+5b}{d^6}$$

```
[In] Integrate[x*(a + b*x^2)^2*Sin[c + d*x],x]
```

```
[Out] (-(d*x*(a^2*d^4 + 2*a*b*d^2*(-6 + d^2*x^2) + b^2*(120 - 20*d^2*x^2 + d^4*x^
4))*Cos[c + d*x]) + (a^2*d^4 + 6*a*b*d^2*(-2 + d^2*x^2) + 5*b^2*(24 - 12*d^
2*x^2 + d^4*x^4))*Sin[c + d*x])/d^6
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{x(b^2x^4d^4+2ab\,d^4x^2+a^2d^4-20d^2x^2b^2-12ab\,d^2+120b^2)\cos(dx+c)}{d^5}+\frac{(5b^2x^4d^4+6ab\,d^4x^2+a^2d^4-60d^2x^2b^2-12ab\,d}{d^6}$
parallelrisch	$\frac{\left((b\,x^2+a)^2d^4+(-20x^2b^2-12ab)d^2+120b^2\right)xd\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\left((10b^2x^4+12ab\,x^2+2a^2)d^4-24b(5b\,x^2+a)d^2+240b^2\right)\right.}{d^6\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$
norman	$\frac{\frac{b^2x^5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{(a^2d^4-12ab\,d^2+120b^2)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d^5}-\frac{b^2x^5}{d}-\frac{(a^2d^4-12ab\,d^2+120b^2)x}{d^5}+\frac{2(a^2d^4-12ab\,d^2+120b^2)\,\mathrm{ta}}{d^6}}{1+\tan^2\left(\frac{dx}{2}\right.}$
meijerg	$\frac{32b^2\sqrt{\pi}\,\sin(c)\left(-\frac{15}{4\sqrt{\pi}}+\frac{\left(\frac{15}{8}d^4x^4-\frac{45}{2}d^2x^2+45\right)\cos(dx)}{12\sqrt{\pi}}+\frac{xd\left(\frac{3}{8}d^4x^4-\frac{15}{2}d^2x^2+45\right)\sin(dx)}{12\sqrt{\pi}}\right)}{d^6}+\frac{32b^2\sqrt{\pi}\,\cos(c)\left(-\frac{xd\left(\frac{7}{8}d^4\right.}{}\right.}{}$
parts	$-\frac{b^2x^5\cos(dx+c)}{d}-\frac{2ab\,x^3\cos(dx+c)}{d}-\frac{a^2x\cos(dx+c)}{d}+\frac{a^2\sin(dx+c)+\frac{6ab\,c^2\sin(dx+c)}{d^2}-\frac{12abc(\cos(dx+c)+(dx+c)}{d^2}}{}$
derivativedivides	$\frac{a^2c\cos(dx+c)+a^2(\sin(dx+c)-\cos(dx+c)(dx+c))+\frac{2ab\,c^3\cos(dx+c)}{d^2}+\frac{6ab\,c^2(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^2}-\frac{6abc\left(-(dx+c)^2\right.}{}}{}$
default	$\frac{a^2c\cos(dx+c)+a^2(\sin(dx+c)-\cos(dx+c)(dx+c))+\frac{2ab\,c^3\cos(dx+c)}{d^2}+\frac{6ab\,c^2(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^2}-\frac{6abc\left(-(dx+c)^2\right.}{}}{}$

```
[In] int(x*(b*x^2+a)^2*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^5*x*(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4-20*b^2*d^2*x^2-12*a*b*d^2+120*b
^2)*cos(d*x+c)+(5*b^2*d^4*x^4+6*a*b*d^4*x^2+a^2*d^4-60*b^2*d^2*x^2-12*a*b*d
^2+120*b^2)/d^6*sin(d*x+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.68

$$\int x(a+bx^2)^2 \sin(c+dx)\,dx = -\frac{(b^2d^5x^5+2\,(abd^5-10\,b^2d^3)x^3+(a^2d^5-12\,abd^3+120\,b^2d)x)\cos{(dx+c)}-(5\,b^2d^4x^4+a^2d^4-12\,abd^2}{d^6}$$

```
[In] integrate(x*(b*x^2+a)^2*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b^2*d^5*x^5 + 2*(a*b*d^5 - 10*b^2*d^3)*x^3 + (a^2*d^5 - 12*a*b*d^3 + 120
*b^2*d)*x)*cos(d*x + c) - (5*b^2*d^4*x^4 + a^2*d^4 - 12*a*b*d^2 + 6*(a*b*d^
4 - 10*b^2*d^2)*x^2 + 120*b^2)*sin(d*x + c))/d^6
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.22

$$\int x(a+bx^2)^2 \sin(c+dx)\,dx = \begin{cases} -\frac{a^2x\cos{(c+dx)}}{d}+\frac{a^2\sin{(c+dx)}}{d^2}-\frac{2abx^3\cos{(c+dx)}}{d}+\frac{6abx^2\sin{(c+dx)}}{d^2}+\frac{12abx\cos{(c+dx)}}{d^3}-\frac{12ab\sin{(c+dx)}}{d^4}-\frac{b^2x^5\cos{(c+dx)}}{d}+ \\ \left(\frac{a^2x^2}{2}+\frac{abx^4}{2}+\frac{b^2x^6}{6}\right)\sin{(c)} \end{cases}$$

```
[In] integrate(x*(b*x**2+a)**2*sin(d*x+c),x)
```

```
[Out] Piecewise((-a**2*x*cos(c + d*x)/d + a**2*sin(c + d*x)/d**2 - 2*a*b*x**3*cos
(c + d*x)/d + 6*a*b*x**2*sin(c + d*x)/d**2 + 12*a*b*x*cos(c + d*x)/d**3 - 1
2*a*b*sin(c + d*x)/d**4 - b**2*x**5*cos(c + d*x)/d + 5*b**2*x**4*sin(c + d*
x)/d**2 + 20*b**2*x**3*cos(c + d*x)/d**3 - 60*b**2*x**2*sin(c + d*x)/d**4 -
 120*b**2*x*cos(c + d*x)/d**5 + 120*b**2*sin(c + d*x)/d**6, Ne(d, 0)), ((a*
*2*x**2/2 + a*b*x**4/2 + b**2*x**6/6)*sin(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. 2(185) = 370.

Time = 0.21 (sec) , antiderivative size = 438, normalized size of antiderivative = 2.37

$$\int x(a+bx^2)^2 \sin(c+dx)\,dx = \frac{a^2c\cos{(dx+c)}+\frac{b^2c^5\cos(dx+c)}{d^4}+\frac{2\,abc^3\cos(dx+c)}{d^2}-((dx+c)\cos{(dx+c)}-\sin{(dx+c)})a^2-\frac{5\,((dx+c)\cos(dx+c)-}{d^4}}{}$$

```
[In] integrate(x*(b*x^2+a)^2*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] (a^2*c*cos(d*x + c) + b^2*c^5*cos(d*x + c)/d^4 + 2*a*b*c^3*cos(d*x + c)/d^2
 - ((d*x + c)*cos(d*x + c) - sin(d*x + c))*a^2 - 5*((d*x + c)*cos(d*x + c)
- sin(d*x + c))*b^2*c^4/d^4 - 6*((d*x + c)*cos(d*x + c) - sin(d*x + c))*a*b
*c^2/d^2 + 10*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b
^2*c^3/d^4 + 6*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*
a*b*c/d^2 - 10*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 -
 2)*sin(d*x + c))*b^2*c^2/d^4 - 2*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c)
 - 3*((d*x + c)^2 - 2)*sin(d*x + c))*a*b/d^2 + 5*(((d*x + c)^4 - 12*(d*x +
c)^2 + 24)*cos(d*x + c) - 4*((d*x + c)^3 - 6*d*x - 6*c)*sin(d*x + c))*b^2*c
/d^4 - (((d*x + c)^5 - 20*(d*x + c)^3 + 120*d*x + 120*c)*cos(d*x + c) - 5*(
(d*x + c)^4 - 12*(d*x + c)^2 + 24)*sin(d*x + c))*b^2/d^4)/d^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.70

$$\int x(a+bx^2)^2\sin(c+dx)\,dx$$
$$= -\frac{(b^2d^5x^5+2\,abd^5x^3+a^2d^5x-20\,b^2d^3x^3-12\,abd^3x+120\,b^2dx)\cos{(dx+c)}}{d^6} + \frac{(5\,b^2d^4x^4+6\,abd^4x^2+a^2d^4-60\,b^2d^2x^2-12\,abd^2+120\,b^2)\sin{(dx+c)}}{d^6}$$

```
[In] integrate(x*(b*x^2+a)^2*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b^2*d^5*x^5 + 2*a*b*d^5*x^3 + a^2*d^5*x - 20*b^2*d^3*x^3 - 12*a*b*d^3*x +
 120*b^2*d*x)*cos(d*x + c)/d^6 + (5*b^2*d^4*x^4 + 6*a*b*d^4*x^2 + a^2*d^4 -
 60*b^2*d^2*x^2 - 12*a*b*d^2 + 120*b^2)*sin(d*x + c)/d^6
```

Mupad [B] (verification not implemented)

Time = 6.33 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.82

$$\int x(a+bx^2)^2\sin(c+dx)\,dx = \frac{\sin{(c+d\,x)}\,\left(a^2\,d^4-12\,a\,b\,d^2+120\,b^2\right)}{d^6} - \frac{b^2\,x^5\,\cos{(c+d\,x)}}{d} + \frac{5\,b^2\,x^4\,\sin{(c+d\,x)}}{d^2} - \frac{x\,\cos{(c+d\,x)}\,\left(a^2\,d^4-12\,a\,b\,d^2+120\,b^2\right)}{d^5} + \frac{2\,x^3\,\cos{(c+d\,x)}\,\left(10\,b^2-a\,b\,d^2\right)}{d^3} - \frac{6\,x^2\,\sin{(c+d\,x)}\,\left(10\,b^2-a\,b\,d^2\right)}{d^4}$$

```
[In] int(x*sin(c + d*x)*(a + b*x^2)^2,x)

[Out] (sin(c + d*x)*(120*b^2 + a^2*d^4 - 12*a*b*d^2))/d^6 - (b^2*x^5*cos(c + d*x)
)/d + (5*b^2*x^4*sin(c + d*x))/d^2 - (x*cos(c + d*x)*(120*b^2 + a^2*d^4 - 1
2*a*b*d^2))/d^5 + (2*x^3*cos(c + d*x)*(10*b^2 - a*b*d^2))/d^3 - (6*x^2*sin(
c + d*x)*(10*b^2 - a*b*d^2))/d^4
```

3.51 $\int (a+bx^2)^2 \sin(c+dx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 138

$$\int (a+bx^2)^2 \sin(c+dx)\,dx = -\frac{24b^2\cos(c+dx)}{d^5}+\frac{4ab\cos(c+dx)}{d^3}-\frac{a^2\cos(c+dx)}{d}+\frac{12b^2x^2\cos(c+dx)}{d^3}-\frac{2abx^2\cos(c+dx)}{d}-\frac{b^2x^4\cos(c+dx)}{d}-\frac{24b^2x\sin(c+dx)}{d^4}+\frac{4abx\sin(c+dx)}{d^2}+\frac{4b^2x^3\sin(c+dx)}{d^2}$$

```
[Out] -24*b^2*cos(d*x+c)/d^5+4*a*b*cos(d*x+c)/d^3-a^2*cos(d*x+c)/d+12*b^2*x^2*cos
(d*x+c)/d^3-2*a*b*x^2*cos(d*x+c)/d-b^2*x^4*cos(d*x+c)/d-24*b^2*x*sin(d*x+c)
/d^4+4*a*b*x*sin(d*x+c)/d^2+4*b^2*x^3*sin(d*x+c)/d^2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3410, 2718, 3377}

$$\int (a+bx^2)^2 \sin(c+dx)\,dx = -\frac{a^2\cos(c+dx)}{d}+\frac{4ab\cos(c+dx)}{d^3}+\frac{4abx\sin(c+dx)}{d^2}-\frac{2abx^2\cos(c+dx)}{d}-\frac{24b^2\cos(c+dx)}{d^5}-\frac{24b^2x\sin(c+dx)}{d^4}+\frac{12b^2x^2\cos(c+dx)}{d^3}+\frac{4b^2x^3\sin(c+dx)}{d^2}-\frac{b^2x^4\cos(c+dx)}{d}$$

```
[In] Int[(a + b*x^2)^2*Sin[c + d*x],x]
```

```
[Out] (-24*b^2*Cos[c + d*x])/d^5 + (4*a*b*Cos[c + d*x])/d^3 - (a^2*Cos[c + d*x])/
d + (12*b^2*x^2*Cos[c + d*x])/d^3 - (2*a*b*x^2*Cos[c + d*x])/d - (b^2*x^4*C
```

```
os[c + d*x])/d - (24*b^2*x*Sin[c + d*x])/d^4 + (4*a*b*x*Sin[c + d*x])/d^2 +
 (4*b^2*x^3*Sin[c + d*x])/d^2
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3410

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d,
 n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^2\sin(c+dx)+2abx^2\sin(c+dx)+b^2x^4\sin(c+dx)\right)\,dx\\
&= a^2\int\sin(c+dx)\,dx+(2ab)\int x^2\sin(c+dx)\,dx+b^2\int x^4\sin(c+dx)\,dx\\
&= -\frac{a^2\cos(c+dx)}{d}-\frac{2abx^2\cos(c+dx)}{d}-\frac{b^2x^4\cos(c+dx)}{d}\\
&\quad+\frac{(4ab)\int x\cos(c+dx)\,dx}{d}+\frac{(4b^2)\int x^3\cos(c+dx)\,dx}{d}\\
&= -\frac{a^2\cos(c+dx)}{d}-\frac{2abx^2\cos(c+dx)}{d}-\frac{b^2x^4\cos(c+dx)}{d}+\frac{4abx\sin(c+dx)}{d^2}\\
&\quad+\frac{4b^2x^3\sin(c+dx)}{d^2}-\frac{(4ab)\int\sin(c+dx)\,dx}{d^2}-\frac{(12b^2)\int x^2\sin(c+dx)\,dx}{d^2}\\
&= \frac{4ab\cos(c+dx)}{d^3}-\frac{a^2\cos(c+dx)}{d}+\frac{12b^2x^2\cos(c+dx)}{d^3}-\frac{2abx^2\cos(c+dx)}{d}\\
&\quad-\frac{b^2x^4\cos(c+dx)}{d}+\frac{4abx\sin(c+dx)}{d^2}+\frac{4b^2x^3\sin(c+dx)}{d^2}\\
&\quad-\frac{(24b^2)\int x\cos(c+dx)\,dx}{d^3}\\
&= \frac{4ab\cos(c+dx)}{d^3}-\frac{a^2\cos(c+dx)}{d}+\frac{12b^2x^2\cos(c+dx)}{d^3}\\
&\quad-\frac{2abx^2\cos(c+dx)}{d}-\frac{b^2x^4\cos(c+dx)}{d}-\frac{24b^2x\sin(c+dx)}{d^4}\\
&\quad+\frac{4abx\sin(c+dx)}{d^2}+\frac{4b^2x^3\sin(c+dx)}{d^2}+\frac{(24b^2)\int\sin(c+dx)\,dx}{d^4}
\end{aligned}$$

$$= -\frac{24b^2\cos(c+dx)}{d^5} + \frac{4ab\cos(c+dx)}{d^3} - \frac{a^2\cos(c+dx)}{d} + \frac{12b^2x^2\cos(c+dx)}{d^3} - \frac{2abx^2\cos(c+dx)}{d} - \frac{b^2x^4\cos(c+dx)}{d} - \frac{24b^2x\sin(c+dx)}{d^4} + \frac{4abx\sin(c+dx)}{d^2} + \frac{4b^2x^3\sin(c+dx)}{d^2}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.62

$$\int (a+bx^2)^2 \sin(c+dx)\,dx$$
$$= \frac{-((a^2d^4+2abd^2(-2+d^2x^2)+b^2(24-12d^2x^2+d^4x^4))\cos(c+dx))+4bdx(ad^2+b(-6+d^2x^2))\sin(c+ \cdots}{d^5}$$

```
[In] Integrate[(a + b*x^2)^2*Sin[c + d*x],x]
```

```
[Out] (-((a^2*d^4 + 2*a*b*d^2*(-2 + d^2*x^2) + b^2*(24 - 12*d^2*x^2 + d^4*x^4))*C
os[c + d*x]) + 4*b*d*x*(a*d^2 + b*(-6 + d^2*x^2))*Sin[c + d*x])/d^5
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{(b^2x^4d^4+2ab\,d^4x^2+a^2d^4-12d^2x^2b^2-4ab\,d^2+24b^2)\cos(dx+c)}{d^5}+\frac{4bx(d^2x^2b+a\,d^2-6b)\sin(dx+c)}{d^4}$
parallelrisch	$\frac{2x^2d^2\left(\left(\left(\frac{b\,x^2}{2}+a\right)d^2-6b\right)b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8((b\,x^2+a)d^2-6b)xdb\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+(-b^2x^4-2ab\,x^2-2a^2)d^4+(12x^2b^2+8 \cdots}{d^5\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$
norman	$\frac{\frac{b^2x^4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2a^2d^4-8ab\,d^2+48b^2}{d^5}-\frac{b^2x^4}{d}+\frac{8b^2x^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^2}-\frac{2b\left(a\,d^2-6b\right)x^2}{d^3}+\frac{8b\left(a\,d^2-6b\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^4}+\frac{2b(a\,d^2 \cdots}{}}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$
parts	$-\frac{b^2x^4\cos(dx+c)}{d}-\frac{2ab\,x^2\cos(dx+c)}{d}-\frac{a^2\cos(dx+c)}{d}+\frac{4b(-ac\sin(dx+c)+a(\cos(dx+c)+(dx+c)\sin(dx+c))-\frac{b \cdots}{}}{}$
meijerg	$\frac{16b^2\sqrt{\pi}\,\sin(c)\left(-\frac{x\left(d^2\right)^{\frac{5}{2}}\left(-\frac{5d^2x^2}{2}+15\right)\cos(dx)}{10\sqrt{\pi}\,d^4}+\frac{\left(d^2\right)^{\frac{5}{2}}\left(\frac{5}{8}d^4x^4-\frac{15}{2}d^2x^2+15\right)\sin(dx)}{10\sqrt{\pi}\,d^5}\right)}{d^4\sqrt{d^2}}+\frac{16b^2\sqrt{\pi}\,\cos(c)\left(\frac{3}{2\sqrt{\pi}}-\frac{\left(\frac{3}{8}d^4 \cdots}{}\right.}{}$
derivativedivides	$\frac{-a^2\cos(dx+c)-\frac{2ab\,c^2\cos(dx+c)}{d^2}-\frac{4abc(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^2}+\frac{2ab\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+ \cdots\right.}{d^2}}{}$
default	$\frac{-a^2\cos(dx+c)-\frac{2ab\,c^2\cos(dx+c)}{d^2}-\frac{4abc(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^2}+\frac{2ab\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+ \cdots\right.}{d^2}}{}$

```
[In] int((b*x^2+a)^2*sin(d*x+c),x,method=_RETURNVERBOSE)
```

[Out] -(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4-12*b^2*d^2*x^2-4*a*b*d^2+24*b^2)/d^5*co
s(d*x+c)+4*b*x/d^4*(b*d^2*x^2+a*d^2-6*b)*sin(d*x+c)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.70

$$\int (a+bx^2)^2 \sin(c+dx)\,dx = -\frac{(b^2d^4x^4 + a^2d^4 - 4\,abd^2 + 2\,(abd^4 - 6\,b^2d^2)x^2 + 24\,b^2)\cos{(dx+c)} - 4\,(b^2d^3x^3 + (abd^3 - 6\,b^2d)x)\sin{(dx}}{d^5}$$

[In] integrate((b*x^2+a)^2*sin(d*x+c),x, algorithm="fricas")

[Out] -((b^2*d^4*x^4 + a^2*d^4 - 4*a*b*d^2 + 2*(a*b*d^4 - 6*b^2*d^2)*x^2 + 24*b^2
)*cos(d*x + c) - 4*(b^2*d^3*x^3 + (a*b*d^3 - 6*b^2*d)*x)*sin(d*x + c))/d^5

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.25

$$\int (a+bx^2)^2 \sin(c+dx)\,dx = \begin{cases} -\frac{a^2\cos{(c+dx)}}{d} - \frac{2abx^2\cos{(c+dx)}}{d} + \frac{4abx\sin{(c+dx)}}{d^2} + \frac{4ab\cos{(c+dx)}}{d^3} - \frac{b^2x^4\cos{(c+dx)}}{d} + \frac{4b^2x^3\sin{(c+dx)}}{d^2} + \frac{12b^2x^2\cos{(c+dx)}}{d^3} \\ \left(a^2x + \frac{2abx^3}{3} + \frac{b^2x^5}{5}\right)\sin{(c)} \end{cases}$$

[In] integrate((b*x**2+a)**2*sin(d*x+c),x)

[Out] Piecewise((-a**2*cos(c + d*x)/d - 2*a*b*x**2*cos(c + d*x)/d + 4*a*b*x*sin(c
 + d*x)/d**2 + 4*a*b*cos(c + d*x)/d**3 - b**2*x**4*cos(c + d*x)/d + 4*b**2*
x**3*sin(c + d*x)/d**2 + 12*b**2*x**2*cos(c + d*x)/d**3 - 24*b**2*x*sin(c +
 d*x)/d**4 - 24*b**2*cos(c + d*x)/d**5, Ne(d, 0)), ((a**2*x + 2*a*b*x**3/3
+ b**2*x**5/5)*sin(c), True))

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(138) = 276$.

Time = 0.19 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.12

$$\int (a+bx^2)^2 \sin(c+dx)\,dx =$$

$$-\frac{a^2\cos(dx+c) + \frac{b^2c^4\cos(dx+c)}{d^4} + \frac{2\,abc^2\cos(dx+c)}{d^2} - \frac{4\,((dx+c)\cos(dx+c)-\sin(dx+c))b^2c^3}{d^4} - \frac{4\,((dx+c)\cos(dx+c)-\sin(dx+c))}{d^2}}{}$$

```
[In] integrate((b*x^2+a)^2*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a^2*cos(d*x + c) + b^2*c^4*cos(d*x + c)/d^4 + 2*a*b*c^2*cos(d*x + c)/d^2
- 4*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b^2*c^3/d^4 - 4*((d*x + c)*cos(
d*x + c) - sin(d*x + c))*a*b*c/d^2 + 6*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*
(d*x + c)*sin(d*x + c))*b^2*c^2/d^4 + 2*(((d*x + c)^2 - 2)*cos(d*x + c) - 2
*(d*x + c)*sin(d*x + c))*a*b/d^2 - 4*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x +
 c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*b^2*c/d^4 + (((d*x + c)^4 - 12*(d*x
 + c)^2 + 24)*cos(d*x + c) - 4*((d*x + c)^3 - 6*d*x - 6*c)*sin(d*x + c))*b^
2/d^4)/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.72

$$\int (a+bx^2)^2 \sin(c+dx)\,dx = -\frac{(b^2d^4x^4 + 2\,abd^4x^2 + a^2d^4 - 12\,b^2d^2x^2 - 4\,abd^2 + 24\,b^2)\cos(dx+c)}{d^5} + \frac{4\,(b^2d^3x^3 + abd^3x - 6\,b^2dx)\sin(dx+c)}{d^5}$$

```
[In] integrate((b*x^2+a)^2*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b^2*d^4*x^4 + 2*a*b*d^4*x^2 + a^2*d^4 - 12*b^2*d^2*x^2 - 4*a*b*d^2 + 24*b
^2)*cos(d*x + c)/d^5 + 4*(b^2*d^3*x^3 + a*b*d^3*x - 6*b^2*d*x)*sin(d*x + c)
/d^5
```

Mupad [B] (verification not implemented)

Time = 6.26 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.86

$$\int (a+bx^2)^2 \sin(c+dx)\,dx = \frac{4\,b^2\,x^3\,\sin(c+d\,x)}{d^2} - \frac{b^2\,x^4\,\cos(c+d\,x)}{d} - \frac{\cos(c+d\,x)\,\left(a^2\,d^4-4\,a\,b\,d^2+24\,b^2\right)}{d^5} - \frac{4\,x\,\sin(c+d\,x)\,\left(6\,b^2-a\,b\,d^2\right)}{d^4} + \frac{2\,x^2\,\cos(c+d\,x)\,\left(6\,b^2-a\,b\,d^2\right)}{d^3}$$

```
[In] int(sin(c + d*x)*(a + b*x^2)^2,x)
```

```
[Out] (4*b^2*x^3*sin(c + d*x))/d^2 - (b^2*x^4*cos(c + d*x))/d - (cos(c + d*x)*(24
*b^2 + a^2*d^4 - 4*a*b*d^2))/d^5 - (4*x*sin(c + d*x)*(6*b^2 - a*b*d^2))/d^4
 + (2*x^2*cos(c + d*x)*(6*b^2 - a*b*d^2))/d^3
```

3.52 $\int \frac{(a+bx^2)^2 \sin(c+dx)}{x} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 111

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x} \, dx = \frac{6b^2x\cos(c+dx)}{d^3} - \frac{2abx\cos(c+dx)}{d} - \frac{b^2x^3\cos(c+dx)}{d} + a^2 \operatorname{CosIntegral}(dx)\sin(c) - \frac{6b^2\sin(c+dx)}{d^4} + \frac{2ab\sin(c+dx)}{d^2} + \frac{3b^2x^2\sin(c+dx)}{d^2} + a^2\cos(c)\operatorname{Si}(dx)$$

```
[Out] 6*b^2*x*cos(d*x+c)/d^3-2*a*b*x*cos(d*x+c)/d-b^2*x^3*cos(d*x+c)/d+a^2*cos(c)
*Si(d*x)+a^2*Ci(d*x)*sin(c)-6*b^2*sin(d*x+c)/d^4+2*a*b*sin(d*x+c)/d^2+3*b^2
*x^2*sin(d*x+c)/d^2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3420, 3384, 3380, 3383, 3377, 2717}

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x} \, dx = a^2\sin(c)\operatorname{CosIntegral}(dx) + a^2\cos(c)\operatorname{Si}(dx) + \frac{2ab\sin(c+dx)}{d^2} - \frac{2abx\cos(c+dx)}{d} - \frac{6b^2\sin(c+dx)}{d^4} + \frac{6b^2x\cos(c+dx)}{d^3} + \frac{3b^2x^2\sin(c+dx)}{d^2} - \frac{b^2x^3\cos(c+dx)}{d}$$

```
[In] Int[((a + b*x^2)^2*Sin[c + d*x])/x,x]
```

```
[Out] (6*b^2*x*Cos[c + d*x])/d^3 - (2*a*b*x*Cos[c + d*x])/d - (b^2*x^3*Cos[c + d*
x])/d + a^2*CosIntegral[d*x]*Sin[c] - (6*b^2*Sin[c + d*x])/d^4 + (2*a*b*Sin
[c + d*x])/d^2 + (3*b^2*x^2*Sin[c + d*x])/d^2 + a^2*Cos[c]*SinIntegral[d*x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a^2\sin(c+dx)}{x} + 2abx\sin(c+dx) + b^2x^3\sin(c+dx)\right)dx$$

$$= a^2\int \frac{\sin(c+dx)}{x}\,dx + (2ab)\int x\sin(c+dx)\,dx + b^2\int x^3\sin(c+dx)\,dx$$

$$= -\frac{2abx\cos(c+dx)}{d} - \frac{b^2x^3\cos(c+dx)}{d} + \frac{(2ab)\int\cos(c+dx)\,dx}{d} + \frac{(3b^2)\int x^2\cos(c+dx)\,dx}{d} + (a^2\cos(c))\int\frac{\sin(dx)}{x}\,dx + (a^2\sin(c))\int\frac{\cos(dx)}{x}\,dx$$

$$= -\frac{2abx\cos(c+dx)}{d} - \frac{b^2x^3\cos(c+dx)}{d} + a^2\,\mathrm{CosIntegral}(dx)\sin(c) + \frac{2ab\sin(c+dx)}{d^2} + \frac{3b^2x^2\sin(c+dx)}{d^2} + a^2\cos(c)\mathrm{Si}(dx) - \frac{(6b^2)\int x\sin(c+dx)\,dx}{d^2}$$

$$= \frac{6b^2x\cos(c+dx)}{d^3} - \frac{2abx\cos(c+dx)}{d} - \frac{b^2x^3\cos(c+dx)}{d} + a^2\,\mathrm{CosIntegral}(dx)\sin(c) + \frac{2ab\sin(c+dx)}{d^2} + \frac{3b^2x^2\sin(c+dx)}{d^2} + a^2\cos(c)\mathrm{Si}(dx) - \frac{(6b^2)\int\cos(c+dx)\,dx}{d^3}$$

$$= \frac{6b^2x\cos(c+dx)}{d^3} - \frac{2abx\cos(c+dx)}{d} - \frac{b^2x^3\cos(c+dx)}{d} + a^2\,\mathrm{CosIntegral}(dx)\sin(c) - \frac{6b^2\sin(c+dx)}{d^4} + \frac{2ab\sin(c+dx)}{d^2} + \frac{3b^2x^2\sin(c+dx)}{d^2} + a^2\cos(c)\mathrm{Si}(dx)$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.74

$$\int\frac{(a+bx^2)^2\sin(c+dx)}{x}\,dx = -\frac{bx(2ad^2+b(-6+d^2x^2))\cos(c+dx)}{d^3} + a^2\,\mathrm{CosIntegral}(dx)\sin(c) + \frac{b(2ad^2+3b(-2+d^2x^2))\sin(c+dx)}{d^4} + a^2\cos(c)\mathrm{Si}(dx)$$

```
[In] Integrate[((a + b*x^2)^2*Sin[c + d*x])/x,x]
[Out] -((b*x*(2*a*d^2 + b*(-6 + d^2*x^2))*Cos[c + d*x])/d^3) + a^2*CosIntegral[d*
x]*Sin[c] + (b*(2*a*d^2 + 3*b*(-2 + d^2*x^2))*Sin[c + d*x])/d^4 + a^2*Cos[c
]*SinIntegral[d*x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.15

method	result
risch	$-\frac{b^2x^3\cos(dx+c)}{d}+\frac{ia^2\mathrm{e}^{ic}\operatorname{Ei}_1(-idx)}{2}-\frac{ia^2\mathrm{e}^{-ic}\operatorname{Ei}_1(idx)}{2}+\frac{3b^2x^2\sin(dx+c)}{d^2}-\frac{2abx\cos(dx+c)}{d}+\frac{2ab\sin(dx+c)}{d^2}$
derivativedivides	$a^2(\operatorname{Si}(dx)\cos(c)+\operatorname{Ci}(dx)\sin(c))+\frac{4abc\cos(dx+c)}{d^2}+\frac{2(c+1)ab(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^2}+\frac{4b^2c}{}$
default	$a^2(\operatorname{Si}(dx)\cos(c)+\operatorname{Ci}(dx)\sin(c))+\frac{4abc\cos(dx+c)}{d^2}+\frac{2(c+1)ab(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^2}+\frac{4b^2c}{}$
meijerg	$\frac{8b^2\sqrt{\pi}\sin(c)\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(-\frac{3d^2x^2}{2}+3\right)\cos(dx)}{4\sqrt{\pi}}-\frac{dx\left(-\frac{d^2x^2}{2}+3\right)\sin(dx)}{4\sqrt{\pi}}\right)}{d^4}+\frac{8b^2\sqrt{\pi}\cos(c)\left(\frac{xd\left(-\frac{5d^2x^2}{2}+15\right)\cos(dx)}{20\sqrt{\pi}}-\frac{\left(-\frac{15d}{}\right.}{}\right.}{d^4}$

```
[In] int((b*x^2+a)^2*sin(d*x+c)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -b^2*x^3*cos(d*x+c)/d+1/2*I*a^2*exp(I*c)*Ei(1,-I*d*x)-1/2*I*a^2*exp(-I*c)*E
i(1,I*d*x)+3*b^2*x^2*sin(d*x+c)/d^2-2*a*b*x*cos(d*x+c)/d+2*a*b*sin(d*x+c)/d
^2+6*b^2*x*cos(d*x+c)/d^3-6*b^2*sin(d*x+c)/d^4
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2\sin(c+dx)}{x}\,dx$$
$$=\frac{a^2d^4\operatorname{Ci}(dx)\sin(c)+a^2d^4\cos(c)\operatorname{Si}(dx)-(b^2d^3x^3+2(abd^3-3b^2d)x)\cos(dx+c)+(3b^2d^2x^2+2abd^2-}{d^4}$$

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x,x, algorithm="fricas")
```

```
[Out] (a^2*d^4*cos_integral(d*x)*sin(c) + a^2*d^4*cos(c)*sin_integral(d*x) - (b^2
*d^3*x^3 + 2*(a*b*d^3 - 3*b^2*d)*x)*cos(d*x + c) + (3*b^2*d^2*x^2 + 2*a*b*d
^2 - 6*b^2)*sin(d*x + c))/d^4
```

Sympy [A] (verification not implemented)

Time = 2.49 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.44

$$\int \frac{(a+bx^2)^2\sin(c+dx)}{x}\,dx$$
$$= a^2\sin(c)\operatorname{Ci}(dx) + a^2\cos(c)\operatorname{Si}(dx) + 2abx\left(\begin{cases} x\sin(c) & \text{for } d=0 \\ -\frac{\cos(c+dx)}{d} & \text{otherwise}\end{cases}\right) - 2ab\left(\begin{cases}\frac{x^2\sin(c)}{2} & \text{for } d=0 \\ -\frac{\begin{cases}\frac{\sin(c+dx)}{d} & \text{for } d\neq 0 \\ x\cos(c) & \text{otherwise}\end{cases}}{d} & \text{otherwise}\end{cases}\right) + b^2x^3\left(\begin{cases} x\sin(c) & \text{for } d=0 \\ -\frac{\cos(c+dx)}{d} & \text{otherwise}\end{cases}\right) - 3b^2\left(\begin{cases}\frac{x^4\sin(c)}{4} & \text{for } d=0 \\ -\frac{\begin{cases}\frac{x^2\sin(c+dx)}{d} + \frac{2x\cos(c+dx)}{d^2} - \frac{2\sin(c+dx)}{d^3} & \text{for } d\neq 0 \\ \frac{x^3\cos(c)}{3} & \text{otherwise}\end{cases}}{d} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((b*x**2+a)**2*sin(d*x+c)/x,x)
```

```
[Out] a**2*sin(c)*Ci(d*x) + a**2*cos(c)*Si(d*x) + 2*a*b*x*Piecewise((x*sin(c), Eq
(d, 0)), (-cos(c + d*x)/d, True)) - 2*a*b*Piecewise((x**2*sin(c)/2, Eq(d, 0
)), (-Piecewise((sin(c + d*x)/d, Ne(d, 0)), (x*cos(c), True))/d, True)) + b
**2*x**3*Piecewise((x*sin(c), Eq(d, 0)), (-cos(c + d*x)/d, True)) - 3*b**2*
Piecewise((x**4*sin(c)/4, Eq(d, 0)), (-Piecewise((x**2*sin(c + d*x)/d + 2*x
*cos(c + d*x)/d**2 - 2*sin(c + d*x)/d**3, Ne(d, 0)), (x**3*cos(c)/3, True))
/d, True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.38 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^2)^2\sin(c+dx)}{x}\,dx$$
$$= \frac{(a^2(-i\operatorname{Ei}(i\,dx) + i\operatorname{Ei}(-i\,dx))\cos(c) + a^2(\operatorname{Ei}(i\,dx) + \operatorname{Ei}(-i\,dx))\sin(c))d^4 - 2\,(b^2d^3x^3 + 2\,(abd^3 - 3\,b^2d)x}{2\,d^4}$$

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2*(-I*Ei(I*d*x) + I*Ei(-I*d*x))*cos(c) + a^2*(Ei(I*d*x) + Ei(-I*d*x
))*sin(c))*d^4 - 2*(b^2*d^3*x^3 + 2*(a*b*d^3 - 3*b^2*d)*x)*cos(d*x + c) + 2
*(3*b^2*d^2*x^2 + 2*a*b*d^2 - 6*b^2)*sin(d*x + c))/d^4
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.31 (sec) , antiderivative size = 725, normalized size of antiderivative = 6.53

$$\int \frac{(a+bx^2)^2\sin(c+dx)}{x}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^2*d^3*x^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - a^2*d^4*imag_part(
cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + a^2*d^4*imag_part(
cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - 2*a^2*d^4*sin_int
egral(d*x)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + 2*b^2*d^3*x^3*tan(1/2*d*x
+ 1/2*c)^2 + 2*a^2*d^4*real_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*
tan(1/2*c) + 2*a^2*d^4*real_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2
*tan(1/2*c) - 2*b^2*d^3*x^3*tan(1/2*c)^2 + 4*a*b*d^3*x*tan(1/2*d*x + 1/2*c)
^2*tan(1/2*c)^2 + a^2*d^4*imag_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)
^2 - a^2*d^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2 + 2*a^2*d
^4*sin_integral(d*x)*tan(1/2*d*x + 1/2*c)^2 - a^2*d^4*imag_part(cos_integra
l(d*x))*tan(1/2*c)^2 + a^2*d^4*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 -
 2*a^2*d^4*sin_integral(d*x)*tan(1/2*c)^2 + 12*b^2*d^2*x^2*tan(1/2*d*x + 1/
2*c)*tan(1/2*c)^2 - 2*b^2*d^3*x^3 + 4*a*b*d^3*x*tan(1/2*d*x + 1/2*c)^2 + 2*
a^2*d^4*real_part(cos_integral(d*x))*tan(1/2*c) + 2*a^2*d^4*real_part(cos_i
ntegral(-d*x))*tan(1/2*c) - 4*a*b*d^3*x*tan(1/2*c)^2 - 12*b^2*d*x*tan(1/2*d
*x + 1/2*c)^2*tan(1/2*c)^2 + a^2*d^4*imag_part(cos_integral(d*x)) - a^2*d^4
*imag_part(cos_integral(-d*x)) + 2*a^2*d^4*sin_integral(d*x) + 12*b^2*d^2*x
^2*tan(1/2*d*x + 1/2*c) + 8*a*b*d^2*tan(1/2*d*x + 1/2*c)*tan(1/2*c)^2 - 4*a
*b*d^3*x - 12*b^2*d*x*tan(1/2*d*x + 1/2*c)^2 + 12*b^2*d*x*tan(1/2*c)^2 + 8*
a*b*d^2*tan(1/2*d*x + 1/2*c) - 24*b^2*tan(1/2*d*x + 1/2*c)*tan(1/2*c)^2 + 1
2*b^2*d*x - 24*b^2*tan(1/2*d*x + 1/2*c))/(d^4*tan(1/2*d*x + 1/2*c)^2*tan(1/
2*c)^2 + d^4*tan(1/2*d*x + 1/2*c)^2 + d^4*tan(1/2*c)^2 + d^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2\sin(c+dx)}{x}\,dx = \int \frac{\sin(c+d\,x)\,\left(b\,x^2+a\right)^2}{x}\,dx$$

```
[In] int((sin(c + d*x)*(a + b*x^2)^2)/x,x)
```

```
[Out] int((sin(c + d*x)*(a + b*x^2)^2)/x, x)
```

3.53 $\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 97

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^2} \, dx = \frac{2b^2 \cos(c+dx)}{d^3} - \frac{2ab \cos(c+dx)}{d} - \frac{b^2x^2 \cos(c+dx)}{d} + a^2 d \cos(c) \operatorname{CosIntegral}(dx) - \frac{a^2 \sin(c+dx)}{x} + \frac{2b^2 x \sin(c+dx)}{d^2} - a^2 d \sin(c) \operatorname{Si}(dx)$$

```
[Out] a^2*d*Ci(d*x)*cos(c)+2*b^2*cos(d*x+c)/d^3-2*a*b*cos(d*x+c)/d-b^2*x^2*cos(d*
x+c)/d-a^2*d*Si(d*x)*sin(c)-a^2*sin(d*x+c)/x+2*b^2*x*sin(d*x+c)/d^2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3420, 2718, 3378, 3384, 3380, 3383, 3377}

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^2} \, dx = a^2 d \cos(c) \operatorname{CosIntegral}(dx) - a^2 d \sin(c) \operatorname{Si}(dx) - \frac{a^2 \sin(c+dx)}{x} - \frac{2ab \cos(c+dx)}{d} + \frac{2b^2 \cos(c+dx)}{d^3} + \frac{2b^2 x \sin(c+dx)}{d^2} - \frac{b^2x^2 \cos(c+dx)}{d}$$

```
[In] Int[((a + b*x^2)^2*Sin[c + d*x])/x^2,x]
```

```
[Out] (2*b^2*Cos[c + d*x])/d^3 - (2*a*b*Cos[c + d*x])/d - (b^2*x^2*Cos[c + d*x])/
d + a^2*d*Cos[c]*CosIntegral[d*x] - (a^2*Sin[c + d*x])/x + (2*b^2*x*Sin[c +
 d*x])/d^2 - a^2*d*Sin[c]*SinIntegral[d*x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(2ab\sin(c+dx) + \frac{a^2\sin(c+dx)}{x^2} + b^2x^2\sin(c+dx)\right) dx$$

$$
\begin{aligned}
&= a^2 \int \frac{\sin(c+dx)}{x^2}\,dx + (2ab)\int \sin(c+dx)\,dx + b^2\int x^2\sin(c+dx)\,dx \\
&= -\frac{2ab\cos(c+dx)}{d} - \frac{b^2x^2\cos(c+dx)}{d} - \frac{a^2\sin(c+dx)}{x} \\
&\quad + \frac{(2b^2)\int x\cos(c+dx)\,dx}{d} + \left(a^2d\right)\int \frac{\cos(c+dx)}{x}\,dx \\
&= -\frac{2ab\cos(c+dx)}{d} - \frac{b^2x^2\cos(c+dx)}{d} - \frac{a^2\sin(c+dx)}{x} + \frac{2b^2x\sin(c+dx)}{d^2} \\
&\quad - \frac{(2b^2)\int \sin(c+dx)\,dx}{d^2} + \left(a^2d\cos(c)\right)\int \frac{\cos(dx)}{x}\,dx - \left(a^2d\sin(c)\right)\int \frac{\sin(dx)}{x}\,dx \\
&= \frac{2b^2\cos(c+dx)}{d^3} - \frac{2ab\cos(c+dx)}{d} - \frac{b^2x^2\cos(c+dx)}{d} + a^2d\cos(c)\,\text{CosIntegral}(dx) \\
&\quad - \frac{a^2\sin(c+dx)}{x} + \frac{2b^2x\sin(c+dx)}{d^2} - a^2d\sin(c)\text{Si}(dx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$
\begin{aligned}
\int \frac{(a+bx^2)^2\sin(c+dx)}{x^2}\,dx &= \frac{2b^2\cos(c+dx)}{d^3} - \frac{2ab\cos(c+dx)}{d} - \frac{b^2x^2\cos(c+dx)}{d} \\
&\quad + a^2d\cos(c)\,\text{CosIntegral}(dx) - \frac{a^2\sin(c+dx)}{x} \\
&\quad + \frac{2b^2x\sin(c+dx)}{d^2} - a^2d\sin(c)\text{Si}(dx)
\end{aligned}
$$

```
[In] Integrate[((a + b*x^2)^2*Sin[c + d*x])/x^2,x]
```

```
[Out] (2*b^2*Cos[c + d*x])/d^3 - (2*a*b*Cos[c + d*x])/d - (b^2*x^2*Cos[c + d*x])/
d + a^2*d*Cos[c]*CosIntegral[d*x] - (a^2*Sin[c + d*x])/x + (2*b^2*x*Sin[c +
 d*x])/d^2 - a^2*d*Sin[c]*SinIntegral[d*x]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.61

method	result
derivativedivides	$d\left(a^{2}\left(-\frac{\sin(dx+c)}{dx}-\mathrm{Si}\left(dx\right)\sin\left(c\right)+\mathrm{Ci}\left(dx\right)\cos\left(c\right)\right)-\frac{2ab\cos(dx+c)}{d^{2}}-\frac{6b^{2}c^{2}\cos(dx+c)}{d^{4}}-\frac{4c\,b^{2}(2\right.$
default	$d\left(a^{2}\left(-\frac{\sin(dx+c)}{dx}-\mathrm{Si}\left(dx\right)\sin\left(c\right)+\mathrm{Ci}\left(dx\right)\cos\left(c\right)\right)-\frac{2ab\cos(dx+c)}{d^{2}}-\frac{6b^{2}c^{2}\cos(dx+c)}{d^{4}}-\frac{4c\,b^{2}(2\right.$
risch	$-\frac{-\pi\ \mathrm{csgn}(dx)\sin(c)a^{2}d^{4}x+2\ \mathrm{Si}(dx)\sin(c)a^{2}d^{4}x-i\pi\ \mathrm{csgn}(dx)\cos(c)a^{2}d^{4}x+2i\,\mathrm{Si}(dx)\cos(c)a^{2}d^{4}x+2\cos(c)\,\mathrm{Ei}_{1}(-idx)a}{2d^{3}x}$
meijerg	$\frac{4b^{2}\sqrt{\pi}\,\sin(c)\left(\frac{x\left(d^{2}\right)^{\frac{3}{2}}\cos(dx)}{2\sqrt{\pi}\,d^{2}}-\frac{\left(d^{2}\right)^{\frac{3}{2}}\left(-\frac{3d^{2}x^{2}}{2}+3\right)\sin(dx)}{6\sqrt{\pi}\,d^{3}}\right)}{d^{2}\sqrt{d^{2}}}+\frac{4b^{2}\sqrt{\pi}\,\cos(c)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(-\frac{d^{2}x^{2}}{2}+1\right)\cos(dx)}{2\sqrt{\pi}}+\frac{dx\sin(dx)}{2\sqrt{\pi}}\right.}{d^{3}}$

```
[In] int((b*x^2+a)^2*sin(d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] d*(a^2*(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c))-2/d^2*a*b*cos(d*x+c)
-6/d^4*b^2*c^2*cos(d*x+c)-4*c*b^2*(2*c+1)/d^4*(sin(d*x+c)-cos(d*x+c)*(d*x+c
))+(3*c^2+2*c+1)/d^4*b^2*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(
d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.99

$$\int\frac{\left(a+bx^{2}\right)^{2}\sin(c+dx)}{x^{2}}\,dx$$
$$=\frac{a^{2}d^{4}x\cos\left(c\right)\mathrm{Ci}\left(dx\right)-a^{2}d^{4}x\sin\left(c\right)\mathrm{Si}\left(dx\right)-\left(b^{2}d^{2}x^{3}+2\left(abd^{2}-b^{2}\right)x\right)\cos\left(dx+c\right)-\left(a^{2}d^{3}-2\,b^{2}dx^{2}\right)\sin\left(\right.}{d^{3}x}$$

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] (a^2*d^4*x*cos(c)*cos_integral(d*x) - a^2*d^4*x*sin(c)*sin_integral(d*x) -
(b^2*d^2*x^3 + 2*(a*b*d^2 - b^2)*x)*cos(d*x + c) - (a^2*d^3 - 2*b^2*d*x^2)*
sin(d*x + c))/(d^3*x)
```

Sympy [F]

$$\int\frac{\left(a+bx^{2}\right)^{2}\sin(c+dx)}{x^{2}}\,dx=\int\frac{\left(a+bx^{2}\right)^{2}\sin\left(c+dx\right)}{x^{2}}\,dx$$

```
[In] integrate((b*x**2+a)**2*sin(d*x+c)/x**2,x)
```

```
[Out] Integral((a + b*x**2)**2*sin(c + d*x)/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^2}\,dx = \frac{(a^2(\Gamma(-1,i\,dx)+\Gamma(-1,-i\,dx))\cos{(c)}+a^2(-i\,\Gamma(-1,i\,dx)+i\,\Gamma(-1,-i\,dx))\sin{(c)})d^4+4\,b^2dx\sin{(dx+}}{2\,d^3}$$

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2*(gamma(-1, I*d*x) + gamma(-1, -I*d*x))*cos(c) + a^2*(-I*gamma(-1,
 I*d*x) + I*gamma(-1, -I*d*x))*sin(c))*d^4 + 4*b^2*d*x*sin(d*x + c) - 2*(b^
2*d^2*x^2 + 2*a*b*d^2 - 2*b^2)*cos(d*x + c))/d^3
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.32 (sec) , antiderivative size = 1638, normalized size of antiderivative = 16.89

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^2}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(a^2*d^4*x*real_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2
*d*x)^2*tan(1/2*c)^2 + a^2*d^4*x*real_part(cos_integral(-d*x))*tan(1/2*d*x
+ 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^2*d^4*x*imag_part(cos_integral
(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^4*x*imag_
part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c) +
 4*a^2*d^4*x*sin_integral(d*x)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/
2*c) - 2*b^2*d^2*x^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 - a
^2*d^4*x*real_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2
 - a^2*d^4*x*real_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d
*x)^2 + a^2*d^4*x*real_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1
/2*c)^2 + a^2*d^4*x*real_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*ta
n(1/2*c)^2 + a^2*d^4*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*
c)^2 + a^2*d^4*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2
- 2*b^2*d^2*x^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2 + 2*a^2*d^4*x*imag_pa
rt(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c) - 2*a^2*d^4*x*imag_
part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c) + 4*a^2*d^4*x*si
n_integral(d*x)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c) + 2*a^2*d^4*x*imag_part(c
os_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^4*x*imag_part(cos_int
```

```
egral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^2*d^4*x*sin_integral(d*x)*tan(
1/2*d*x)^2*tan(1/2*c) - 2*b^2*d^2*x^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 +
 2*b^2*d^2*x^3*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a*b*d^2*x*tan(1/2*d*x + 1/2*
c)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d^4*x*real_part(cos_integral(d*x))*t
an(1/2*d*x + 1/2*c)^2 - a^2*d^4*x*real_part(cos_integral(-d*x))*tan(1/2*d*x
 + 1/2*c)^2 - a^2*d^4*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2 - a^2*d
^4*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 4*a^2*d^3*tan(1/2*d*x +
 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c) + a^2*d^4*x*real_part(cos_integral(d*x)
)*tan(1/2*c)^2 + a^2*d^4*x*real_part(cos_integral(-d*x))*tan(1/2*c)^2 - 4*a
^2*d^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)*tan(1/2*c)^2 - 8*b^2*d*x^2*tan(1
/2*d*x + 1/2*c)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*b^2*d^2*x^3*tan(1/2*d*x + 1
/2*c)^2 + 2*b^2*d^2*x^3*tan(1/2*d*x)^2 - 4*a*b*d^2*x*tan(1/2*d*x + 1/2*c)^2
*tan(1/2*d*x)^2 + 2*a^2*d^4*x*imag_part(cos_integral(d*x))*tan(1/2*c) - 2*a
^2*d^4*x*imag_part(cos_integral(-d*x))*tan(1/2*c) + 4*a^2*d^4*x*sin_integra
l(d*x)*tan(1/2*c) + 2*b^2*d^2*x^3*tan(1/2*c)^2 - 4*a*b*d^2*x*tan(1/2*d*x +
1/2*c)^2*tan(1/2*c)^2 + 4*a*b*d^2*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*b^2*x*t
an(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d^4*x*real_part(cos
_integral(d*x)) - a^2*d^4*x*real_part(cos_integral(-d*x)) + 4*a^2*d^3*tan(1
/2*d*x + 1/2*c)^2*tan(1/2*d*x) - 8*b^2*d*x^2*tan(1/2*d*x + 1/2*c)*tan(1/2*d
*x)^2 + 4*a^2*d^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c) - 4*a^2*d^3*tan(1/2*d*x
)^2*tan(1/2*c) - 8*b^2*d*x^2*tan(1/2*d*x + 1/2*c)*tan(1/2*c)^2 - 4*a^2*d^3*
tan(1/2*d*x)*tan(1/2*c)^2 + 2*b^2*d^2*x^3 - 4*a*b*d^2*x*tan(1/2*d*x + 1/2*c
)^2 + 4*a*b*d^2*x*tan(1/2*d*x)^2 + 4*b^2*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d
*x)^2 + 4*a*b*d^2*x*tan(1/2*c)^2 + 4*b^2*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c
)^2 - 4*b^2*x*tan(1/2*d*x)^2*tan(1/2*c)^2 - 8*b^2*d*x^2*tan(1/2*d*x + 1/2*c
) + 4*a^2*d^3*tan(1/2*d*x) + 4*a^2*d^3*tan(1/2*c) + 4*a*b*d^2*x + 4*b^2*x*t
an(1/2*d*x + 1/2*c)^2 - 4*b^2*x*tan(1/2*d*x)^2 - 4*b^2*x*tan(1/2*c)^2 - 4*b
^2*x)/(d^3*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + d^3*x*tan
(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2 + d^3*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c
)^2 + d^3*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + d^3*x*tan(1/2*d*x + 1/2*c)^2 + d^
3*x*tan(1/2*d*x)^2 + d^3*x*tan(1/2*c)^2 + d^3*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2\sin(c+dx)}{x^2}\,dx = \int \frac{\sin\left(c+d\,x\right)\,\left(b\,x^2+a\right)^2}{x^2}\,dx$$

```
[In] int((sin(c + d*x)*(a + b*x^2)^2)/x^2,x)
```

```
[Out] int((sin(c + d*x)*(a + b*x^2)^2)/x^2, x)
```

3.54 $\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 114

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^3} dx = -\frac{a^2 d\cos(c+dx)}{2x} - \frac{b^2 x\cos(c+dx)}{d} + 2ab\,\text{CosIntegral}(dx)\sin(c) - \frac{1}{2}a^2d^2\,\text{CosIntegral}(dx)\sin(c) + \frac{b^2\sin(c+dx)}{d^2} - \frac{a^2\sin(c+dx)}{2x^2} + 2ab\cos(c)\text{Si}(dx) - \frac{1}{2}a^2d^2\cos(c)\text{Si}(dx)$$

```
[Out] -1/2*a^2*d*cos(d*x+c)/x-b^2*x*cos(d*x+c)/d+2*a*b*cos(c)*Si(d*x)-1/2*a^2*d^2
*cos(c)*Si(d*x)+2*a*b*Ci(d*x)*sin(c)-1/2*a^2*d^2*Ci(d*x)*sin(c)+b^2*sin(d*x
+c)/d^2-1/2*a^2*sin(d*x+c)/x^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3420, 3378, 3384, 3380, 3383, 3377, 2717}

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^3} dx = -\frac{1}{2}a^2d^2\sin(c)\,\text{CosIntegral}(dx) - \frac{1}{2}a^2d^2\cos(c)\text{Si}(dx) - \frac{a^2\sin(c+dx)}{2x^2} - \frac{a^2d\cos(c+dx)}{2x} + 2ab\sin(c)\,\text{CosIntegral}(dx) + 2ab\cos(c)\text{Si}(dx) + \frac{b^2\sin(c+dx)}{d^2} - \frac{b^2x\cos(c+dx)}{d}$$

```
[In] Int[((a + b*x^2)^2*Sin[c + d*x])/x^3,x]
```

```
[Out] -1/2*(a^2*d*Cos[c + d*x])/x - (b^2*x*Cos[c + d*x])/d + 2*a*b*CosIntegral[d*
x]*Sin[c] - (a^2*d^2*CosIntegral[d*x]*Sin[c])/2 + (b^2*Sin[c + d*x])/d^2 -
(a^2*Sin[c + d*x])/(2*x^2) + 2*a*b*Cos[c]*SinIntegral[d*x] - (a^2*d^2*Cos[c
]*SinIntegral[d*x])/2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a^2\sin(c+dx)}{x^3}+\frac{2ab\sin(c+dx)}{x}+b^2x\sin(c+dx)\right)dx\\
&= a^2\int \frac{\sin(c+dx)}{x^3}\,dx+(2ab)\int \frac{\sin(c+dx)}{x}\,dx+b^2\int x\sin(c+dx)\,dx\\
&= -\frac{b^2x\cos(c+dx)}{d}-\frac{a^2\sin(c+dx)}{2x^2}+\frac{b^2\int\cos(c+dx)\,dx}{d}+\frac{1}{2}\left(a^2d\right)\int \frac{\cos(c+dx)}{x^2}\,dx\\
&\quad+(2ab\cos(c))\int \frac{\sin(dx)}{x}\,dx+(2ab\sin(c))\int \frac{\cos(dx)}{x}\,dx\\
&= -\frac{a^2d\cos(c+dx)}{2x}-\frac{b^2x\cos(c+dx)}{d}+2ab\,\text{CosIntegral}(dx)\sin(c)+\frac{b^2\sin(c+dx)}{d^2}\\
&\quad-\frac{a^2\sin(c+dx)}{2x^2}+2ab\cos(c)\text{Si}(dx)-\frac{1}{2}\left(a^2d^2\right)\int \frac{\sin(c+dx)}{x}\,dx\\
&= -\frac{a^2d\cos(c+dx)}{2x}-\frac{b^2x\cos(c+dx)}{d}+2ab\,\text{CosIntegral}(dx)\sin(c)\\
&\quad+\frac{b^2\sin(c+dx)}{d^2}-\frac{a^2\sin(c+dx)}{2x^2}+2ab\cos(c)\text{Si}(dx)\\
&\quad-\frac{1}{2}\left(a^2d^2\cos(c)\right)\int \frac{\sin(dx)}{x}\,dx-\frac{1}{2}\left(a^2d^2\sin(c)\right)\int \frac{\cos(dx)}{x}\,dx\\
&= -\frac{a^2d\cos(c+dx)}{2x}-\frac{b^2x\cos(c+dx)}{d}+2ab\,\text{CosIntegral}(dx)\sin(c)\\
&\quad-\frac{1}{2}a^2d^2\,\text{CosIntegral}(dx)\sin(c)+\frac{b^2\sin(c+dx)}{d^2}\\
&\quad-\frac{a^2\sin(c+dx)}{2x^2}+2ab\cos(c)\text{Si}(dx)-\frac{1}{2}a^2d^2\cos(c)\text{Si}(dx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.87

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^2\sin(c+dx)}{x^3}\,dx = \frac{1}{2}\Bigg(&-\frac{a^2d\cos(c+dx)}{x}-\frac{2b^2x\cos(c+dx)}{d}\\
&+a\left(4b-ad^2\right)\text{CosIntegral}(dx)\sin(c)+\frac{2b^2\sin(c+dx)}{d^2}\\
&-\frac{a^2\sin(c+dx)}{x^2}+a\left(4b-ad^2\right)\cos(c)\text{Si}(dx)\Bigg)
\end{aligned}
$$

```
[In] Integrate[((a + b*x^2)^2*Sin[c + d*x])/x^3,x]
```

[Out] (-((a^2*d*Cos[c + d*x])/x) - (2*b^2*x*Cos[c + d*x])/d + a*(4*b - a*d^2)*Cos Integral[d*x]*Sin[c] + (2*b^2*Sin[c + d*x])/d^2 - (a^2*Sin[c + d*x])/x^2 + a*(4*b - a*d^2)*Cos[c]*SinIntegral[d*x])/2

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.09

method	result
derivativedivides	$d^2\left(a^2\left(-\frac{\sin(dx+c)}{2d^2x^2}-\frac{\cos(dx+c)}{2dx}-\frac{\mathrm{Si}(dx)\cos(c)}{2}-\frac{\mathrm{Ci}(dx)\sin(c)}{2}\right)+\frac{2ba(\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin(c))}{d^2}+\frac{4b^2c}{}$
default	$d^2\left(a^2\left(-\frac{\sin(dx+c)}{2d^2x^2}-\frac{\cos(dx+c)}{2dx}-\frac{\mathrm{Si}(dx)\cos(c)}{2}-\frac{\mathrm{Ci}(dx)\sin(c)}{2}\right)+\frac{2ba(\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin(c))}{d^2}+\frac{4b^2c}{}$
risch	$-\frac{-\cos(c)\pi\ \mathrm{csgn}(dx)a^2d^4x^2+i\sin(c)\pi\ \mathrm{csgn}(dx)a^2d^4x^2+2\cos(c)\,\mathrm{Si}(dx)a^2d^4x^2-2i\sin(c)\,\mathrm{Si}(dx)a^2d^4x^2+4\cos(c)\pi\ \mathrm{csgn}(}{}$
meijerg	$\frac{2b^2\sqrt{\pi}\ \sin(c)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(dx)}{2\sqrt{\pi}}+\frac{dx\sin(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{2b^2\sqrt{\pi}\ \cos(c)\left(-\frac{dx\cos(dx)}{2\sqrt{\pi}}+\frac{\sin(dx)}{2\sqrt{\pi}}\right)}{d^2}+ab\sqrt{\pi}\ \sin(c)\left(\frac{2\gamma+2\ln(x)}{\sqrt{\pi}}\right.$

[In] int((b*x^2+a)^2*sin(d*x+c)/x^3,x,method=_RETURNVERBOSE)

[Out] d^2*(a^2*(-1/2*sin(d*x+c)/d^2/x^2-1/2*cos(d*x+c)/d/x-1/2*Si(d*x)*cos(c)-1/2 *Ci(d*x)*sin(c))+2/d^2*b*a*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))+4/d^4*b^2*c*cos(d*x+c)+(3*c+1)/d^4*b^2*(sin(d*x+c)-cos(d*x+c)*(d*x+c)))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^2\sin(c+dx)}{x^3}\,dx = -\frac{(a^2d^4-4\,abd^2)x^2\,\mathrm{Ci}\,(dx)\sin\,(c)+(a^2d^4-4\,abd^2)x^2\cos\,(c)\,\mathrm{Si}\,(dx)+(a^2d^3x+2\,b^2dx^3)\cos\,(dx+c)+(a}{2\,d^2x^2}$$

[In] integrate((b*x^2+a)^2*sin(d*x+c)/x^3,x, algorithm="fricas")

[Out] -1/2*((a^2*d^4 - 4*a*b*d^2)*x^2*cos_integral(d*x)*sin(c) + (a^2*d^4 - 4*a*b *d^2)*x^2*cos(c)*sin_integral(d*x) + (a^2*d^3*x + 2*b^2*d*x^3)*cos(d*x + c) + (a^2*d^2 - 2*b^2*x^2)*sin(d*x + c))/(d^2*x^2)

Sympy [F]

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^3}\,dx = \int \frac{(a+bx^2)^2 \sin{(c+dx)}}{x^3}\,dx$$

```
[In] integrate((b*x**2+a)**2*sin(d*x+c)/x**3,x)
```

```
[Out] Integral((a + b*x**2)**2*sin(c + d*x)/x**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^3}\,dx = \frac{((a^2(i\,\Gamma(-2,i\,dx)-i\,\Gamma(-2,-i\,dx))\cos{(c)}+a^2(\Gamma(-2,i\,dx)+\Gamma(-2,-i\,dx))\sin{(c)})d^4-4\,(ab(i\,\Gamma(-2,i\,d}{}$$

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(((a^2*(I*gamma(-2, I*d*x) - I*gamma(-2, -I*d*x))*cos(c) + a^2*(gamma(-
2, I*d*x) + gamma(-2, -I*d*x))*sin(c))*d^4 - 4*(a*b*(I*gamma(-2, I*d*x) - I
*gamma(-2, -I*d*x))*cos(c) + a*b*(gamma(-2, I*d*x) + gamma(-2, -I*d*x))*sin
(c))*d^2)*x^2 - 2*(b^2*d*x^3 + 2*a*b*d*x)*cos(d*x + c) + 2*(b^2*x^2 - 2*a*b
)*sin(d*x + c))/(d^2*x^2)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.30 (sec) , antiderivative size = 1058, normalized size of antiderivative = 9.28

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^3}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(a^2*d^4*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 -
 a^2*d^4*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*
a^2*d^4*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*d^4*x^2*r
eal_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^4*x^2*real_
part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - a^2*d^4*x^2*imag_part(
cos_integral(d*x))*tan(1/2*d*x)^2 + a^2*d^4*x^2*imag_part(cos_integral(-d*x
```

```
))*tan(1/2*d*x)^2 - 2*a^2*d^4*x^2*sin_integral(d*x)*tan(1/2*d*x)^2 + a^2*d^
4*x^2*imag_part(cos_integral(d*x))*tan(1/2*c)^2 - a^2*d^4*x^2*imag_part(cos
_integral(-d*x))*tan(1/2*c)^2 + 2*a^2*d^4*x^2*sin_integral(d*x)*tan(1/2*c)^
2 - 4*a*b*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2
+ 4*a*b*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 -
 8*a*b*d^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*d^4*x^
2*real_part(cos_integral(d*x))*tan(1/2*c) - 2*a^2*d^4*x^2*real_part(cos_int
egral(-d*x))*tan(1/2*c) + 8*a*b*d^2*x^2*real_part(cos_integral(d*x))*tan(1/
2*d*x)^2*tan(1/2*c) + 8*a*b*d^2*x^2*real_part(cos_integral(-d*x))*tan(1/2*d
*x)^2*tan(1/2*c) - 2*a^2*d^3*x*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*b^2*d*x^3*ta
n(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d^4*x^2*imag_part(cos_integral(d*x)) + a^2*
d^4*x^2*imag_part(cos_integral(-d*x)) - 2*a^2*d^4*x^2*sin_integral(d*x) + 4
*a*b*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 - 4*a*b*d^2*x^2*im
ag_part(cos_integral(-d*x))*tan(1/2*d*x)^2 + 8*a*b*d^2*x^2*sin_integral(d*x
)*tan(1/2*d*x)^2 - 4*a*b*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*c)^2
+ 4*a*b*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 - 8*a*b*d^2*x^2*
sin_integral(d*x)*tan(1/2*c)^2 + 2*a^2*d^3*x*tan(1/2*d*x)^2 + 4*b^2*d*x^3*t
an(1/2*d*x)^2 + 8*a*b*d^2*x^2*real_part(cos_integral(d*x))*tan(1/2*c) + 8*a
*b*d^2*x^2*real_part(cos_integral(-d*x))*tan(1/2*c) + 8*a^2*d^3*x*tan(1/2*d
*x)*tan(1/2*c) + 16*b^2*d*x^3*tan(1/2*d*x)*tan(1/2*c) + 2*a^2*d^3*x*tan(1/2
*c)^2 + 4*b^2*d*x^3*tan(1/2*c)^2 + 4*a*b*d^2*x^2*imag_part(cos_integral(d*x
)) - 4*a*b*d^2*x^2*imag_part(cos_integral(-d*x)) + 8*a*b*d^2*x^2*sin_integr
al(d*x) + 4*a^2*d^2*tan(1/2*d*x)^2*tan(1/2*c) - 8*b^2*x^2*tan(1/2*d*x)^2*ta
n(1/2*c) + 4*a^2*d^2*tan(1/2*d*x)*tan(1/2*c)^2 - 8*b^2*x^2*tan(1/2*d*x)*tan
(1/2*c)^2 - 2*a^2*d^3*x - 4*b^2*d*x^3 - 4*a^2*d^2*tan(1/2*d*x) + 8*b^2*x^2*
tan(1/2*d*x) - 4*a^2*d^2*tan(1/2*c) + 8*b^2*x^2*tan(1/2*c))/(d^2*x^2*tan(1/
2*d*x)^2*tan(1/2*c)^2 + d^2*x^2*tan(1/2*d*x)^2 + d^2*x^2*tan(1/2*c)^2 + d^2
*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2\sin(c+dx)}{x^3}\,dx=\int \frac{\sin\left(c+d\,x\right)\,{\left(b\,x^2+a\right)}^2}{x^3}\,dx$$

```
[In] int((sin(c + d*x)*(a + b*x^2)^2)/x^3,x)
[Out] int((sin(c + d*x)*(a + b*x^2)^2)/x^3, x)
```

3.55 $\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^4} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 134

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^4} \, dx = -\frac{b^2\cos(c+dx)}{d} - \frac{a^2 d\cos(c+dx)}{6x^2} + 2abd\cos(c)\,\text{CosIntegral}(dx) - \frac{1}{6}a^2d^3\cos(c)\,\text{CosIntegral}(dx) - \frac{a^2\sin(c+dx)}{3x^3} - \frac{2ab\sin(c+dx)}{x} + \frac{a^2d^2\sin(c+dx)}{6x} - 2abd\sin(c)\text{Si}(dx) + \frac{1}{6}a^2d^3\sin(c)\text{Si}(dx)$$

```
[Out] 2*a*b*d*Ci(d*x)*cos(c)-1/6*a^2*d^3*Ci(d*x)*cos(c)-b^2*cos(d*x+c)/d-1/6*a^2*
d*cos(d*x+c)/x^2-2*a*b*d*Si(d*x)*sin(c)+1/6*a^2*d^3*Si(d*x)*sin(c)-1/3*a^2*
sin(d*x+c)/x^3-2*a*b*sin(d*x+c)/x+1/6*a^2*d^2*sin(d*x+c)/x
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3420, 2718, 3378, 3384, 3380, 3383}

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^4} \, dx = -\frac{1}{6}a^2d^3\cos(c)\,\text{CosIntegral}(dx) + \frac{1}{6}a^2d^3\sin(c)\text{Si}(dx) + \frac{a^2d^2\sin(c+dx)}{6x} - \frac{a^2\sin(c+dx)}{3x^3} - \frac{a^2d\cos(c+dx)}{6x^2} + 2abd\cos(c)\,\text{CosIntegral}(dx) - 2abd\sin(c)\text{Si}(dx) - \frac{2ab\sin(c+dx)}{x} - \frac{b^2\cos(c+dx)}{d}$$

```
[In] Int[((a + b*x^2)^2*Sin[c + d*x])/x^4,x]
```

```
[Out] -((b^2*Cos[c + d*x])/d) - (a^2*d*Cos[c + d*x])/(6*x^2) + 2*a*b*d*Cos[c]*Cos
Integral[d*x] - (a^2*d^3*Cos[c]*CosIntegral[d*x])/6 - (a^2*Sin[c + d*x])/(3
*x^3) - (2*a*b*Sin[c + d*x])/x + (a^2*d^2*Sin[c + d*x])/(6*x) - 2*a*b*d*Sin
[c]*SinIntegral[d*x] + (a^2*d^3*Sin[c]*SinIntegral[d*x])/6
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(b^2 \sin(c+dx) + \frac{a^2 \sin(c+dx)}{x^4} + \frac{2ab \sin(c+dx)}{x^2}\right) dx$$

$$
\begin{aligned}
&= a^2 \int \frac{\sin(c+dx)}{x^4} \, dx + (2ab) \int \frac{\sin(c+dx)}{x^2} \, dx + b^2 \int \sin(c+dx) \, dx \\
&= -\frac{b^2 \cos(c+dx)}{d} - \frac{a^2 \sin(c+dx)}{3x^3} - \frac{2ab \sin(c+dx)}{x} \\
&\quad + \frac{1}{3}(a^2 d) \int \frac{\cos(c+dx)}{x^3} \, dx + (2abd) \int \frac{\cos(c+dx)}{x} \, dx \\
&= -\frac{b^2 \cos(c+dx)}{d} - \frac{a^2 d \cos(c+dx)}{6x^2} - \frac{a^2 \sin(c+dx)}{3x^3} \\
&\quad - \frac{2ab \sin(c+dx)}{x} - \frac{1}{6}(a^2 d^2) \int \frac{\sin(c+dx)}{x^2} \, dx \\
&\quad + (2abd \cos(c)) \int \frac{\cos(dx)}{x} \, dx - (2abd \sin(c)) \int \frac{\sin(dx)}{x} \, dx \\
&= -\frac{b^2 \cos(c+dx)}{d} - \frac{a^2 d \cos(c+dx)}{6x^2} + 2abd \cos(c) \operatorname{CosIntegral}(dx) - \frac{a^2 \sin(c+dx)}{3x^3} \\
&\quad - \frac{2ab \sin(c+dx)}{x} + \frac{a^2 d^2 \sin(c+dx)}{6x} - 2abd \sin(c) \operatorname{Si}(dx) - \frac{1}{6}(a^2 d^3) \int \frac{\cos(c+dx)}{x} \, dx \\
&= -\frac{b^2 \cos(c+dx)}{d} - \frac{a^2 d \cos(c+dx)}{6x^2} + 2abd \cos(c) \operatorname{CosIntegral}(dx) \\
&\quad - \frac{a^2 \sin(c+dx)}{3x^3} - \frac{2ab \sin(c+dx)}{x} + \frac{a^2 d^2 \sin(c+dx)}{6x} - 2abd \sin(c) \operatorname{Si}(dx) \\
&\quad - \frac{1}{6}(a^2 d^3 \cos(c)) \int \frac{\cos(dx)}{x} \, dx + \frac{1}{6}(a^2 d^3 \sin(c)) \int \frac{\sin(dx)}{x} \, dx \\
&= -\frac{b^2 \cos(c+dx)}{d} - \frac{a^2 d \cos(c+dx)}{6x^2} + 2abd \cos(c) \operatorname{CosIntegral}(dx) \\
&\quad - \frac{1}{6} a^2 d^3 \cos(c) \operatorname{CosIntegral}(dx) - \frac{a^2 \sin(c+dx)}{3x^3} - \frac{2ab \sin(c+dx)}{x} \\
&\quad + \frac{a^2 d^2 \sin(c+dx)}{6x} - 2abd \sin(c) \operatorname{Si}(dx) + \frac{1}{6} a^2 d^3 \sin(c) \operatorname{Si}(dx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.85

$$
\begin{aligned}
\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^4} \, dx = \frac{1}{6} \Bigg(& -\frac{6b^2 \cos(c+dx)}{d} - \frac{a^2 d \cos(c+dx)}{x^2} \\
& - ad(-12b + ad^2) \cos(c) \operatorname{CosIntegral}(dx) - \frac{2a^2 \sin(c+dx)}{x^3} \\
& - \frac{12ab \sin(c+dx)}{x} + \frac{a^2 d^2 \sin(c+dx)}{x} \\
& + ad(-12b + ad^2) \sin(c) \operatorname{Si}(dx) \Bigg)
\end{aligned}
$$

```
[In] Integrate[((a + b*x^2)^2*Sin[c + d*x])/x^4,x]
```

```
[Out] ((-6*b^2*Cos[c + d*x])/d - (a^2*d*Cos[c + d*x])/x^2 - a*d*(-12*b + a*d^2)*C
os[c]*CosIntegral[d*x] - (2*a^2*Sin[c + d*x])/x^3 - (12*a*b*Sin[c + d*x])/x
 + (a^2*d^2*Sin[c + d*x])/x + a*d*(-12*b + a*d^2)*Sin[c]*SinIntegral[d*x])/
6
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.90

method	result
derivativedivides	$d^3\left(a^2\left(-\frac{\sin(dx+c)}{3d^3x^3}-\frac{\cos(dx+c)}{6d^2x^2}+\frac{\sin(dx+c)}{6dx}+\frac{\mathrm{Si}(dx)\sin(c)}{6}-\frac{\mathrm{Ci}(dx)\cos(c)}{6}\right)+\frac{2ab\left(-\frac{\sin(dx+c)}{dx}-\mathrm{Si}(dx)\,\mathrm{si}\right.}{d^2}\right.$
default	$d^3\left(a^2\left(-\frac{\sin(dx+c)}{3d^3x^3}-\frac{\cos(dx+c)}{6d^2x^2}+\frac{\sin(dx+c)}{6dx}+\frac{\mathrm{Si}(dx)\sin(c)}{6}-\frac{\mathrm{Ci}(dx)\cos(c)}{6}\right)+\frac{2ab\left(-\frac{\sin(dx+c)}{dx}-\mathrm{Si}(dx)\,\mathrm{si}\right.}{d^2}\right.$
risch	$\frac{\cos(c)\,\mathrm{Ei}_1(idx)a^2d^3}{12}+\frac{\cos(c)\,\mathrm{Ei}_1(-idx)a^2d^3}{12}-d\cos(c)\,\mathrm{Ei}_1(idx)\,ab-d\cos(c)\,\mathrm{Ei}_1(-idx)\,ab-\frac{i\sin(}{}$
meijerg	$\frac{b^2\sin(c)\sin(dx)}{d}+\frac{b^2\sqrt{\pi}\cos(c)\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(dx)}{\sqrt{\pi}}\right)}{d}+\frac{d^2ab\sqrt{\pi}\sin(c)\left(-\frac{4d^2\cos\left(x\sqrt{d^2}\right)}{x\left(d^2\right)^{\frac{3}{2}}\sqrt{\pi}}-\frac{4\,\mathrm{Si}\left(x\sqrt{d^2}\right)}{\sqrt{\pi}}\right)}{2\sqrt{d^2}}+\frac{dab\sqrt{\pi}\cos(c)\left(\right.}{}$

```
[In] int((b*x^2+a)^2*sin(d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] d^3*(a^2*(-1/3*sin(d*x+c)/d^3/x^3-1/6*cos(d*x+c)/d^2/x^2+1/6*sin(d*x+c)/d/x
+1/6*Si(d*x)*sin(c)-1/6*Ci(d*x)*cos(c))+2/d^2*a*b*(-sin(d*x+c)/d/x-Si(d*x)*
sin(c)+Ci(d*x)*cos(c))-1/d^4*b^2*cos(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.87

$$\int\frac{\left(a+bx^2\right)^2\sin(c+dx)}{x^4}\,dx = -\frac{(a^2d^4-12\,abd^2)x^3\cos(c)\,\mathrm{Ci}(dx)-(a^2d^4-12\,abd^2)x^3\sin(c)\,\mathrm{Si}(dx)+(a^2d^2x+6\,b^2x^3)\cos(dx+c)+(2}{6\,dx^3}$$

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*((a^2*d^4 - 12*a*b*d^2)*x^3*cos(c)*cos_integral(d*x) - (a^2*d^4 - 12*a
*b*d^2)*x^3*sin(c)*sin_integral(d*x) + (a^2*d^2*x + 6*b^2*x^3)*cos(d*x + c)
 + (2*a^2*d - (a^2*d^3 - 12*a*b*d)*x^2)*sin(d*x + c))/(d*x^3)
```

Sympy [F]

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^4}\,dx = \int \frac{(a+bx^2)^2 \sin{(c+dx)}}{x^4}\,dx$$

```
[In] integrate((b*x**2+a)**2*sin(d*x+c)/x**4,x)
[Out] Integral((a + b*x**2)**2*sin(c + d*x)/x**4, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.49 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^4}\,dx = -\frac{((a^2(\Gamma(-3,i\,dx)+\Gamma(-3,-i\,dx))\cos{(c)}+a^2(-i\,\Gamma(-3,i\,dx)+i\,\Gamma(-3,-i\,dx))\sin{(c)})d^5-12\,(ab(\Gamma(-3}{}$$

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x^4,x, algorithm="maxima")
[Out] -1/2*(((a^2*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*cos(c) + a^2*(-I*gamma(-
3, I*d*x) + I*gamma(-3, -I*d*x))*sin(c))*d^5 - 12*(a*b*(gamma(-3, I*d*x) +
gamma(-3, -I*d*x))*cos(c) + a*b*(-I*gamma(-3, I*d*x) + I*gamma(-3, -I*d*x))
*sin(c))*d^3)*x^3 + 8*a*b*sin(d*x + c) + 2*(b^2*d*x^3 + 2*a*b*d*x)*cos(d*x
+ c))/(d^2*x^3)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.30 (sec) , antiderivative size = 1032, normalized size of antiderivative = 7.70

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^4}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x^4,x, algorithm="giac")
[Out] 1/12*(a^2*d^4*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2
+ a^2*d^4*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2
*a^2*d^4*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2
*d^4*x^3*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^2*d^
4*x^3*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) - a^2*d^4*x^3*real_part(c
os_integral(d*x))*tan(1/2*d*x)^2 - a^2*d^4*x^3*real_part(cos_integral(-d*x)
```

```
)*tan(1/2*d*x)^2 + a^2*d^4*x^3*real_part(cos_integral(d*x))*tan(1/2*c)^2 +
a^2*d^4*x^3*real_part(cos_integral(-d*x))*tan(1/2*c)^2 - 12*a*b*d^2*x^3*rea
l_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 12*a*b*d^2*x^3*real
_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^2*d^4*x^3*imag_
part(cos_integral(d*x))*tan(1/2*c) - 2*a^2*d^4*x^3*imag_part(cos_integral(-
d*x))*tan(1/2*c) + 4*a^2*d^4*x^3*sin_integral(d*x)*tan(1/2*c) - 24*a*b*d^2*
x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 24*a*b*d^2*x^3
*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 48*a*b*d^2*x^3*s
in_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) - a^2*d^4*x^3*real_part(cos_inte
gral(d*x)) - a^2*d^4*x^3*real_part(cos_integral(-d*x)) + 12*a*b*d^2*x^3*rea
l_part(cos_integral(d*x))*tan(1/2*d*x)^2 + 12*a*b*d^2*x^3*real_part(cos_int
egral(-d*x))*tan(1/2*d*x)^2 - 4*a^2*d^3*x^2*tan(1/2*d*x)^2*tan(1/2*c) - 12*
a*b*d^2*x^3*real_part(cos_integral(d*x))*tan(1/2*c)^2 - 12*a*b*d^2*x^3*real
_part(cos_integral(-d*x))*tan(1/2*c)^2 - 4*a^2*d^3*x^2*tan(1/2*d*x)*tan(1/2
*c)^2 - 24*a*b*d^2*x^3*imag_part(cos_integral(d*x))*tan(1/2*c) + 24*a*b*d^2
*x^3*imag_part(cos_integral(-d*x))*tan(1/2*c) - 48*a*b*d^2*x^3*sin_integral
(d*x)*tan(1/2*c) - 2*a^2*d^2*x*tan(1/2*d*x)^2*tan(1/2*c)^2 - 12*b^2*x^3*tan
(1/2*d*x)^2*tan(1/2*c)^2 + 12*a*b*d^2*x^3*real_part(cos_integral(d*x)) + 12
*a*b*d^2*x^3*real_part(cos_integral(-d*x)) + 4*a^2*d^3*x^2*tan(1/2*d*x) + 4
*a^2*d^3*x^2*tan(1/2*c) + 48*a*b*d*x^2*tan(1/2*d*x)^2*tan(1/2*c) + 48*a*b*d
*x^2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*a^2*d^2*x*tan(1/2*d*x)^2 + 12*b^2*x^3*ta
n(1/2*d*x)^2 + 8*a^2*d^2*x*tan(1/2*d*x)*tan(1/2*c) + 48*b^2*x^3*tan(1/2*d*x
)*tan(1/2*c) + 2*a^2*d^2*x*tan(1/2*c)^2 + 12*b^2*x^3*tan(1/2*c)^2 - 48*a*b*
d*x^2*tan(1/2*d*x) - 48*a*b*d*x^2*tan(1/2*c) + 8*a^2*d*tan(1/2*d*x)^2*tan(1
/2*c) + 8*a^2*d*tan(1/2*d*x)*tan(1/2*c)^2 - 2*a^2*d^2*x - 12*b^2*x^3 - 8*a^
2*d*tan(1/2*d*x) - 8*a^2*d*tan(1/2*c))/(d*x^3*tan(1/2*d*x)^2*tan(1/2*c)^2 +
 d*x^3*tan(1/2*d*x)^2 + d*x^3*tan(1/2*c)^2 + d*x^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^4} \, dx = \int \frac{\sin(c+d\,x)\,\left(b\,x^2+a\right)^2}{x^4} \, dx$$

```
[In] int((sin(c + d*x)*(a + b*x^2)^2)/x^4,x)
```

```
[Out] int((sin(c + d*x)*(a + b*x^2)^2)/x^4, x)
```

3.56 $\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^5} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 177

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^5} \, dx = -\frac{a^2 d \cos(c+dx)}{12x^3} - \frac{abd \cos(c+dx)}{x} + \frac{a^2 d^3 \cos(c+dx)}{24x} + b^2 \operatorname{CosIntegral}(dx) \sin(c) - abd^2 \operatorname{CosIntegral}(dx) \sin(c) + \frac{1}{24} a^2 d^4 \operatorname{CosIntegral}(dx) \sin(c) - \frac{a^2 \sin(c+dx)}{4x^4} - \frac{ab \sin(c+dx)}{x^2} + \frac{a^2 d^2 \sin(c+dx)}{24x^2} + b^2 \cos(c) \operatorname{Si}(dx) - abd^2 \cos(c) \operatorname{Si}(dx) + \frac{1}{24} a^2 d^4 \cos(c) \operatorname{Si}(dx)$$

```
[Out] -1/12*a^2*d*cos(d*x+c)/x^3-a*b*d*cos(d*x+c)/x+1/24*a^2*d^3*cos(d*x+c)/x+b^2
*cos(c)*Si(d*x)-a*b*d^2*cos(c)*Si(d*x)+1/24*a^2*d^4*cos(c)*Si(d*x)+b^2*Ci(d
*x)*sin(c)-a*b*d^2*Ci(d*x)*sin(c)+1/24*a^2*d^4*Ci(d*x)*sin(c)-1/4*a^2*sin(d
*x+c)/x^4-a*b*sin(d*x+c)/x^2+1/24*a^2*d^2*sin(d*x+c)/x^2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used

= {3420, 3378, 3384, 3380, 3383}

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^5} \, dx = \frac{1}{24} a^2 d^4 \sin(c) \operatorname{CosIntegral}(dx) + \frac{1}{24} a^2 d^4 \cos(c) \operatorname{Si}(dx) + \frac{a^2 d^3 \cos(c+dx)}{24x} + \frac{a^2 d^2 \sin(c+dx)}{24x^2} - \frac{a^2 \sin(c+dx)}{4x^4} - \frac{a^2 d \cos(c+dx)}{12x^3} - abd^2 \sin(c) \operatorname{CosIntegral}(dx) - abd^2 \cos(c) \operatorname{Si}(dx) - \frac{ab \sin(c+dx)}{x^2} - \frac{abd \cos(c+dx)}{x} + b^2 \sin(c) \operatorname{CosIntegral}(dx) + b^2 \cos(c) \operatorname{Si}(dx)$$

```
[In] Int[((a + b*x^2)^2*Sin[c + d*x])/x^5,x]
```

```
[Out] -1/12*(a^2*d*Cos[c + d*x])/x^3 - (a*b*d*Cos[c + d*x])/x + (a^2*d^3*Cos[c +
d*x])/(24*x) + b^2*CosIntegral[d*x]*Sin[c] - a*b*d^2*CosIntegral[d*x]*Sin[c
] + (a^2*d^4*CosIntegral[d*x]*Sin[c])/24 - (a^2*Sin[c + d*x])/(4*x^4) - (a*
b*Sin[c + d*x])/x^2 + (a^2*d^2*Sin[c + d*x])/(24*x^2) + b^2*Cos[c]*SinInteg
ral[d*x] - a*b*d^2*Cos[c]*SinIntegral[d*x] + (a^2*d^4*Cos[c]*SinIntegral[d*
x])/24
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a^2\sin(c+dx)}{x^5}+\frac{2ab\sin(c+dx)}{x^3}+\frac{b^2\sin(c+dx)}{x}\right)dx\\
&= a^2\int \frac{\sin(c+dx)}{x^5}\,dx+(2ab)\int \frac{\sin(c+dx)}{x^3}\,dx+b^2\int \frac{\sin(c+dx)}{x}\,dx\\
&= -\frac{a^2\sin(c+dx)}{4x^4}-\frac{ab\sin(c+dx)}{x^2}+\frac{1}{4}\left(a^2d\right)\int \frac{\cos(c+dx)}{x^4}\,dx\\
&\quad+(abd)\int \frac{\cos(c+dx)}{x^2}\,dx+\left(b^2\cos(c)\right)\int \frac{\sin(dx)}{x}\,dx+\left(b^2\sin(c)\right)\int \frac{\cos(dx)}{x}\,dx\\
&= -\frac{a^2d\cos(c+dx)}{12x^3}-\frac{abd\cos(c+dx)}{x}+b^2\,\text{CosIntegral}(dx)\sin(c)-\frac{a^2\sin(c+dx)}{4x^4}\\
&\quad-\frac{ab\sin(c+dx)}{x^2}+b^2\cos(c)\text{Si}(dx)-\frac{1}{12}\left(a^2d^2\right)\int \frac{\sin(c+dx)}{x^3}\,dx-\left(abd^2\right)\int \frac{\sin(c+dx)}{x}\,dx\\
&= -\frac{a^2d\cos(c+dx)}{12x^3}-\frac{abd\cos(c+dx)}{x}+b^2\,\text{CosIntegral}(dx)\sin(c)-\frac{a^2\sin(c+dx)}{4x^4}\\
&\quad-\frac{ab\sin(c+dx)}{x^2}+\frac{a^2d^2\sin(c+dx)}{24x^2}+b^2\cos(c)\text{Si}(dx)-\frac{1}{24}\left(a^2d^3\right)\int \frac{\cos(c+dx)}{x^2}\,dx\\
&\quad-\left(abd^2\cos(c)\right)\int \frac{\sin(dx)}{x}\,dx-\left(abd^2\sin(c)\right)\int \frac{\cos(dx)}{x}\,dx\\
&= -\frac{a^2d\cos(c+dx)}{12x^3}-\frac{abd\cos(c+dx)}{x}+\frac{a^2d^3\cos(c+dx)}{24x}+b^2\,\text{CosIntegral}(dx)\sin(c)\\
&\quad-abd^2\,\text{CosIntegral}(dx)\sin(c)-\frac{a^2\sin(c+dx)}{4x^4}-\frac{ab\sin(c+dx)}{x^2}+\frac{a^2d^2\sin(c+dx)}{24x^2}\\
&\quad+b^2\cos(c)\text{Si}(dx)-abd^2\cos(c)\text{Si}(dx)+\frac{1}{24}\left(a^2d^4\right)\int \frac{\sin(c+dx)}{x}\,dx\\
&= -\frac{a^2d\cos(c+dx)}{12x^3}-\frac{abd\cos(c+dx)}{x}+\frac{a^2d^3\cos(c+dx)}{24x}\\
&\quad+b^2\,\text{CosIntegral}(dx)\sin(c)-abd^2\,\text{CosIntegral}(dx)\sin(c)-\frac{a^2\sin(c+dx)}{4x^4}\\
&\quad-\frac{ab\sin(c+dx)}{x^2}+\frac{a^2d^2\sin(c+dx)}{24x^2}+b^2\cos(c)\text{Si}(dx)-abd^2\cos(c)\text{Si}(dx)\\
&\quad+\frac{1}{24}\left(a^2d^4\cos(c)\right)\int \frac{\sin(dx)}{x}\,dx+\frac{1}{24}\left(a^2d^4\sin(c)\right)\int \frac{\cos(dx)}{x}\,dx
\end{aligned}$$

$$= -\frac{a^2 d\cos(c+dx)}{12x^3} - \frac{abd\cos(c+dx)}{x} + \frac{a^2 d^3\cos(c+dx)}{24x} + b^2\,\text{CosIntegral}(dx)\sin(c) - abd^2\,\text{CosIntegral}(dx)\sin(c) + \frac{1}{24}a^2 d^4\,\text{CosIntegral}(dx)\sin(c) - \frac{a^2\sin(c+dx)}{4x^4} - \frac{ab\sin(c+dx)}{x^2} + \frac{a^2 d^2\sin(c+dx)}{24x^2} + b^2\cos(c)\text{Si}(dx) - abd^2\cos(c)\text{Si}(dx) + \frac{1}{24}a^2 d^4\cos(c)\text{Si}(dx)$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.69

$$\int \frac{(a+bx^2)^2\sin(c+dx)}{x^5}\,dx = \frac{adx(-24bx^2 + a(-2+d^2x^2))\cos(c+dx) + (24b^2 - 24abd^2 + a^2d^4)\,x^4\,\text{CosIntegral}(dx)\sin(c) + a(-24bx^2 +}{24x^4}$$

```
[In] Integrate[((a + b*x^2)^2*Sin[c + d*x])/x^5,x]
```

```
[Out] (a*d*x*(-24*b*x^2 + a*(-2 + d^2*x^2))*Cos[c + d*x] + (24*b^2 - 24*a*b*d^2 +
 a^2*d^4)*x^4*CosIntegral[d*x]*Sin[c] + a*(-24*b*x^2 + a*(-6 + d^2*x^2))*Si
n[c + d*x] + (24*b^2 - 24*a*b*d^2 + a^2*d^4)*x^4*Cos[c]*SinIntegral[d*x])/(
24*x^4)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.89

method	result
derivativedivides	$d^4\left(a^2\left(-\frac{\sin(dx+c)}{4d^4x^4} - \frac{\cos(dx+c)}{12d^3x^3} + \frac{\sin(dx+c)}{24d^2x^2} + \frac{\cos(dx+c)}{24dx} + \frac{\text{Si}(dx)\cos(c)}{24} + \frac{\text{Ci}(dx)\sin(c)}{24}\right) + \frac{2ab\left(-\frac{\sin(d}{2d^2}\right.}{}\right.$
default	$d^4\left(a^2\left(-\frac{\sin(dx+c)}{4d^4x^4} - \frac{\cos(dx+c)}{12d^3x^3} + \frac{\sin(dx+c)}{24d^2x^2} + \frac{\cos(dx+c)}{24dx} + \frac{\text{Si}(dx)\cos(c)}{24} + \frac{\text{Ci}(dx)\sin(c)}{24}\right) + \frac{2ab\left(-\frac{\sin(d}{2d^2}\right.}{}\right.$
risch	$\frac{i\,\text{Ei}_1(-idx)\cos(c)a^2d^4}{48} - \frac{i\cos(c)\,\text{Ei}_1(idx)a^2d^4}{48} - \frac{i\cos(c)\,\text{Ei}_1(-idx)ab\,d^2}{2} + \frac{i\cos(c)\,\text{Ei}_1(idx)ab\,d^2}{2} + \frac{i\,\text{Ei}_1(-idx)\,\text{co}}{2}$
meijerg	$\frac{b^2\sqrt{\pi}\sin(c)\left(\frac{2\gamma+2\ln(x)+\ln(d^2)}{\sqrt{\pi}} - \frac{2\gamma}{\sqrt{\pi}} - \frac{2\ln(2)}{\sqrt{\pi}} - \frac{2\ln\left(\frac{dx}{2}\right)}{\sqrt{\pi}} + \frac{2\,\text{Ci}(dx)}{\sqrt{\pi}}\right)}{2} + b^2\cos(c)\,\text{Si}(dx) + \frac{d^2ab\sqrt{\pi}\sin(c)\left(-\frac{}{\sqrt{\pi}}\right.}{}$

```
[In] int((b*x^2+a)^2*sin(d*x+c)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] d^4*(a^2*(-1/4*sin(d*x+c)/d^4/x^4-1/12*cos(d*x+c)/d^3/x^3+1/24*sin(d*x+c)/d
^2/x^2+1/24*cos(d*x+c)/d/x+1/24*Si(d*x)*cos(c)+1/24*Ci(d*x)*sin(c))+2/d^2*a
```

```
*b*(-1/2*sin(d*x+c)/d^2/x^2-1/2*cos(d*x+c)/d/x-1/2*Si(d*x)*cos(c)-1/2*Ci(d*
x)*sin(c))+1/d^4*b^2*(Si(d*x)*cos(c)+Ci(d*x)*sin(c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^5} \, dx = \frac{(a^2d^4 - 24\,abd^2 + 24\,b^2)x^4 \operatorname{Ci}(dx) \sin(c) + (a^2d^4 - 24\,abd^2 + 24\,b^2)x^4 \cos(c) \operatorname{Si}(dx) - (2\,a^2dx - (a^2d^3 -}{24\,x^4}$$

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*((a^2*d^4 - 24*a*b*d^2 + 24*b^2)*x^4*cos_integral(d*x)*sin(c) + (a^2*d
^4 - 24*a*b*d^2 + 24*b^2)*x^4*cos(c)*sin_integral(d*x) - (2*a^2*d*x - (a^2*
d^3 - 24*a*b*d)*x^3)*cos(d*x + c) + ((a^2*d^2 - 24*a*b)*x^2 - 6*a^2)*sin(d*
x + c))/x^4
```

Sympy [F]

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^5} \, dx = \int \frac{(a+bx^2)^2 \sin(c+dx)}{x^5} \, dx$$

```
[In] integrate((b*x**2+a)**2*sin(d*x+c)/x**5,x)
```

```
[Out] Integral((a + b*x**2)**2*sin(c + d*x)/x**5, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.17 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^5} \, dx = -\frac{((a^2(i\,\Gamma(-4,i\,dx) - i\,\Gamma(-4,-i\,dx))\cos(c) + a^2(\Gamma(-4,i\,dx) + \Gamma(-4,-i\,dx))\sin(c))d^8 - 24\,(ab(i\,\Gamma(-4}{}$$

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x^5,x, algorithm="maxima")
```

```
[Out] -1/2*(((a^2*(I*gamma(-4, I*d*x) - I*gamma(-4, -I*d*x))*cos(c) + a^2*(gamma(
-4, I*d*x) + gamma(-4, -I*d*x))*sin(c))*d^8 - 24*(a*b*(I*gamma(-4, I*d*x) -
```

```
 I*gamma(-4, -I*d*x))*cos(c) + a*b*(gamma(-4, I*d*x) + gamma(-4, -I*d*x))*s
in(c))*d^6 - 24*(b^2*(-I*gamma(-4, I*d*x) + I*gamma(-4, -I*d*x))*cos(c) - b
^2*(gamma(-4, I*d*x) + gamma(-4, -I*d*x))*sin(c))*d^4)*x^4 + 2*(b^2*d^3*x^3
 + 2*(a*b*d^3 - b^2*d)*x)*cos(d*x + c) + 2*(b^2*d^2*x^2 + 6*a*b*d^2 - 6*b^2
)*sin(d*x + c))/(d^4*x^4)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.30 (sec) , antiderivative size = 1497, normalized size of antiderivative = 8.46

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^5}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x^5,x, algorithm="giac")
```

```
[Out] -1/48*(a^2*d^4*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2
 - a^2*d^4*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
2*a^2*d^4*x^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*d^4*x^4
*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^4*x^4*rea
l_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - a^2*d^4*x^4*imag_par
t(cos_integral(d*x))*tan(1/2*d*x)^2 + a^2*d^4*x^4*imag_part(cos_integral(-d
*x))*tan(1/2*d*x)^2 - 2*a^2*d^4*x^4*sin_integral(d*x)*tan(1/2*d*x)^2 + a^2*
d^4*x^4*imag_part(cos_integral(d*x))*tan(1/2*c)^2 - a^2*d^4*x^4*imag_part(c
os_integral(-d*x))*tan(1/2*c)^2 + 2*a^2*d^4*x^4*sin_integral(d*x)*tan(1/2*c
)^2 - 24*a*b*d^2*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)
^2 + 24*a*b*d^2*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)
^2 - 48*a*b*d^2*x^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*d
^4*x^4*real_part(cos_integral(d*x))*tan(1/2*c) - 2*a^2*d^4*x^4*real_part(co
s_integral(-d*x))*tan(1/2*c) + 48*a*b*d^2*x^4*real_part(cos_integral(d*x))*
tan(1/2*d*x)^2*tan(1/2*c) + 48*a*b*d^2*x^4*real_part(cos_integral(-d*x))*ta
n(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^3*x^3*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d
^4*x^4*imag_part(cos_integral(d*x)) + a^2*d^4*x^4*imag_part(cos_integral(-d
*x)) - 2*a^2*d^4*x^4*sin_integral(d*x) + 24*a*b*d^2*x^4*imag_part(cos_integ
ral(d*x))*tan(1/2*d*x)^2 - 24*a*b*d^2*x^4*imag_part(cos_integral(-d*x))*tan
(1/2*d*x)^2 + 48*a*b*d^2*x^4*sin_integral(d*x)*tan(1/2*d*x)^2 - 24*a*b*d^2*
x^4*imag_part(cos_integral(d*x))*tan(1/2*c)^2 + 24*a*b*d^2*x^4*imag_part(co
s_integral(-d*x))*tan(1/2*c)^2 - 48*a*b*d^2*x^4*sin_integral(d*x)*tan(1/2*c
)^2 + 24*b^2*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 -
 24*b^2*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 48*
b^2*x^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^2*d^3*x^3*tan(1
/2*d*x)^2 + 48*a*b*d^2*x^4*real_part(cos_integral(d*x))*tan(1/2*c) + 48*a*b
*d^2*x^4*real_part(cos_integral(-d*x))*tan(1/2*c) + 8*a^2*d^3*x^3*tan(1/2*d
*x)*tan(1/2*c) - 48*b^2*x^4*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan
```

```
(1/2*c) - 48*b^2*x^4*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c
) + 2*a^2*d^3*x^3*tan(1/2*c)^2 + 48*a*b*d*x^3*tan(1/2*d*x)^2*tan(1/2*c)^2 +
 24*a*b*d^2*x^4*imag_part(cos_integral(d*x)) - 24*a*b*d^2*x^4*imag_part(cos
_integral(-d*x)) + 48*a*b*d^2*x^4*sin_integral(d*x) - 24*b^2*x^4*imag_part(
cos_integral(d*x))*tan(1/2*d*x)^2 + 24*b^2*x^4*imag_part(cos_integral(-d*x)
)*tan(1/2*d*x)^2 - 48*b^2*x^4*sin_integral(d*x)*tan(1/2*d*x)^2 + 4*a^2*d^2*
x^2*tan(1/2*d*x)^2*tan(1/2*c) + 24*b^2*x^4*imag_part(cos_integral(d*x))*tan
(1/2*c)^2 - 24*b^2*x^4*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 + 48*b^2*
x^4*sin_integral(d*x)*tan(1/2*c)^2 + 4*a^2*d^2*x^2*tan(1/2*d*x)*tan(1/2*c)^
2 - 2*a^2*d^3*x^3 - 48*a*b*d*x^3*tan(1/2*d*x)^2 - 48*b^2*x^4*real_part(cos_
integral(d*x))*tan(1/2*c) - 48*b^2*x^4*real_part(cos_integral(-d*x))*tan(1/
2*c) - 192*a*b*d*x^3*tan(1/2*d*x)*tan(1/2*c) - 48*a*b*d*x^3*tan(1/2*c)^2 +
4*a^2*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 - 24*b^2*x^4*imag_part(cos_integral(d
*x)) + 24*b^2*x^4*imag_part(cos_integral(-d*x)) - 48*b^2*x^4*sin_integral(d
*x) - 4*a^2*d^2*x^2*tan(1/2*d*x) - 4*a^2*d^2*x^2*tan(1/2*c) - 96*a*b*x^2*ta
n(1/2*d*x)^2*tan(1/2*c) - 96*a*b*x^2*tan(1/2*d*x)*tan(1/2*c)^2 + 48*a*b*d*x
^3 - 4*a^2*d*x*tan(1/2*d*x)^2 - 16*a^2*d*x*tan(1/2*d*x)*tan(1/2*c) - 4*a^2*
d*x*tan(1/2*c)^2 + 96*a*b*x^2*tan(1/2*d*x) + 96*a*b*x^2*tan(1/2*c) - 24*a^2
*tan(1/2*d*x)^2*tan(1/2*c) - 24*a^2*tan(1/2*d*x)*tan(1/2*c)^2 + 4*a^2*d*x +
 24*a^2*tan(1/2*d*x) + 24*a^2*tan(1/2*c))/(x^4*tan(1/2*d*x)^2*tan(1/2*c)^2
+ x^4*tan(1/2*d*x)^2 + x^4*tan(1/2*c)^2 + x^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^5}\,dx = \int \frac{\sin(c+d\,x)\,\left(b\,x^2+a\right)^2}{x^5}\,dx$$

```
[In] int((sin(c + d*x)*(a + b*x^2)^2)/x^5,x)
[Out] int((sin(c + d*x)*(a + b*x^2)^2)/x^5, x)
```

3.57 $\int \frac{x^4 \sin(c+dx)}{a+bx^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 273

$$\begin{aligned}\int \frac{x^4 \sin(c+dx)}{a+bx^2} dx = {} & \frac{2\cos(c+dx)}{bd^3} + \frac{a\cos(c+dx)}{b^2d} - \frac{x^2\cos(c+dx)}{bd} \\ & - \frac{(-a)^{3/2}\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{5/2}} \\ & + \frac{(-a)^{3/2}\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{5/2}} \\ & + \frac{2x\sin(c+dx)}{bd^2} - \frac{(-a)^{3/2}\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{5/2}} \\ & - \frac{(-a)^{3/2}\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^{5/2}}\end{aligned}$$

```
[Out] 2*cos(d*x+c)/b/d^3+a*cos(d*x+c)/b^2/d-x^2*cos(d*x+c)/b/d+1/2*(-a)^(3/2)*cos
(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b^(1/2))/b^(5/2)-1/2*(-a)^(3/2
)*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))/b^(5/2)+2*x*sin(
d*x+c)/b/d^2-1/2*(-a)^(3/2)*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)
/b^(1/2))/b^(5/2)+1/2*(-a)^(3/2)*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)
^(1/2)/b^(1/2))/b^(5/2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3426, 2718, 3377, 3414, 3384, 3380, 3383}

$$\int \frac{x^4 \sin(c+dx)}{a+bx^2} \, dx = -\frac{(-a)^{3/2} \sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{5/2}} + \frac{(-a)^{3/2} \sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right) \text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{5/2}} - \frac{(-a)^{3/2} \cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right) \text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{5/2}} - \frac{(-a)^{3/2} \cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{5/2}} + \frac{a\cos(c+dx)}{b^2 d} + \frac{2\cos(c+dx)}{bd^3} + \frac{2x\sin(c+dx)}{bd^2} - \frac{x^2\cos(c+dx)}{bd}$$

```
[In] Int[(x^4*Sin[c + d*x])/(a + b*x^2),x]
```

```
[Out] (2*Cos[c + d*x])/(b*d^3) + (a*Cos[c + d*x])/(b^2*d) - (x^2*Cos[c + d*x])/(b
*d) - ((-a)^(3/2)*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]
*d)/Sqrt[b]])/(2*b^(5/2)) + ((-a)^(3/2)*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] -
d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*b^(5/2)) + (2*x*Sin[c + d*x])/(b*d^2
) - ((-a)^(3/2)*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt
[b] - d*x])/(2*b^(5/2)) - ((-a)^(3/2)*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinInte
gral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b^(5/2))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{a\sin(c+dx)}{b^2}+\frac{x^2\sin(c+dx)}{b}+\frac{a^2\sin(c+dx)}{b^2\left(a+bx^2\right)}\right) dx \\
&= -\frac{a\int \sin(c+dx)\,dx}{b^2}+\frac{a^2\int \frac{\sin(c+dx)}{a+bx^2}\,dx}{b^2}+\frac{\int x^2\sin(c+dx)\,dx}{b} \\
&= \frac{a\cos(c+dx)}{b^2d}-\frac{x^2\cos(c+dx)}{bd} \\
&\quad +\frac{a^2\int \left(\frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right) dx}{b^2}+\frac{2\int x\cos(c+dx)\,dx}{bd} \\
&= \frac{a\cos(c+dx)}{b^2d}-\frac{x^2\cos(c+dx)}{bd}+\frac{2x\sin(c+dx)}{bd^2} \\
&\quad -\frac{(-a)^{3/2}\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2b^2}-\frac{(-a)^{3/2}\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2b^2}-\frac{2\int \sin(c+dx)\,dx}{bd^2}
\end{aligned}$$

$$= \frac{2\cos(c+dx)}{bd^3} + \frac{a\cos(c+dx)}{b^2d} - \frac{x^2\cos(c+dx)}{bd} + \frac{2x\sin(c+dx)}{bd^2} - \frac{\left((-a)^{3/2}\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}dx}{2b^2} + \frac{\left((-a)^{3/2}\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}dx}{2b^2} - \frac{\left((-a)^{3/2}\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}dx}{2b^2} - \frac{\left((-a)^{3/2}\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}dx}{2b^2}$$

$$= \frac{2\cos(c+dx)}{bd^3} + \frac{a\cos(c+dx)}{b^2d} - \frac{x^2\cos(c+dx)}{bd} - \frac{(-a)^{3/2}\operatorname{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{5/2}} + \frac{(-a)^{3/2}\operatorname{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{5/2}} + \frac{2x\sin(c+dx)}{bd^2} - \frac{(-a)^{3/2}\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{5/2}} - \frac{(-a)^{3/2}\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^{5/2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.46 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.99

$$\int \frac{x^4\sin(c+dx)}{a+bx^2}\,dx = \frac{e^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(a^{3/2}d^3e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\operatorname{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)-a^{3/2}d^3\operatorname{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)+e^{ic}\left(a^{3/2}d^3e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\right.\right.}{}$$

```
[In] Integrate[(x^4*Sin[c + d*x])/(a + b*x^2),x]
```

```
[Out] (E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*(a^(3/2)*d^3*E^((2*Sqrt[a]*d)/Sqrt[b])*Ex
pIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x] - a^(3/2)*d^3*ExpIntegralEi[(Sq
rt[a]*d)/Sqrt[b] - I*d*x] + E^(I*c)*(a^(3/2)*d^3*E^((2*Sqrt[a]*d)/Sqrt[b])*
ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]*(Cos[c] + I*Sin[c]) - a^(3/2)
*d^3*ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*(Cos[c] + I*Sin[c]) - 4*Sqr
t[b]*E^((Sqrt[a]*d)/Sqrt[b])*((-2*b - a*d^2 + b*d^2*x^2)*Cos[c + d*x] - 2*b
*d*x*Sin[c + d*x]))))/(4*b^(5/2)*d^3)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.18

method	result
risch	$-\frac{\sqrt{ab}\,\mathrm{e}^{\frac{icb+d\sqrt{ab}}{b}}\,\mathrm{Ei}_1\left(\frac{icb+d\sqrt{ab}-b(idx+ic)}{b}\right)a}{4b^3}+\frac{\sqrt{ab}\,\mathrm{e}^{\frac{icb-d\sqrt{ab}}{b}}\,\mathrm{Ei}_1\left(\frac{icb-d\sqrt{ab}-b(idx+ic)}{b}\right)a}{4b^3}+\frac{\mathrm{e}^{-\frac{icb+d\sqrt{ab}}{b}}\,\mathrm{Ei}_1\left(-\frac{icb}{}\right.}{4}$
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int(x^4*sin(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b^3*(a*b)^(1/2)*exp((I*c*b+d*(a*b)^(1/2))/b)*Ei(1,(I*c*b+d*(a*b)^(1/2)
-b*(I*d*x+I*c))/b)*a+1/4/b^3*(a*b)^(1/2)*exp((I*c*b-d*(a*b)^(1/2))/b)*Ei(1,
(I*c*b-d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*a+1/4/b^3*exp(-(I*c*b+d*(a*b)^(1/2))
/b)*Ei(1,-(I*c*b+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*(a*b)^(1/2)*a-1/4/b^3*exp(
-(I*c*b-d*(a*b)^(1/2))/b)*Ei(1,-(I*c*b-d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*(a*b
)^(1/2)*a-(b*d^2*x^2-a*d^2-2*b)/b^2/d^3*cos(d*x+c)-2/d^3/b*(d^2*x^2+3*c*d*x
)/(-d*x-3*c)*sin(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.88

$$\int \frac{x^4\sin(c+dx)}{a+bx^2}\,dx = \frac{\sqrt{\frac{ad^2}{b}}ad^2\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}-\sqrt{\frac{ad^2}{b}}ad^2\mathrm{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)}+\sqrt{\frac{ad^2}{b}}ad^2\mathrm{Ei}\left(-i\,dx-\sqrt{}\right.}{}$$

```
[In] integrate(x^4*sin(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(a*d^2/b)*a*d^2*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b))
- sqrt(a*d^2/b)*a*d^2*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + s
qrt(a*d^2/b)*a*d^2*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) - sq
rt(a*d^2/b)*a*d^2*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) + 8*b
*d*x*sin(d*x + c) - 4*(b*d^2*x^2 - a*d^2 - 2*b)*cos(d*x + c))/(b^2*d^3)
```

Sympy [F]

$$\int \frac{x^4 \sin(c+dx)}{a+bx^2}\,dx = \int \frac{x^4 \sin(c+dx)}{a+bx^2}\,dx$$

```
[In] integrate(x**4*sin(d*x+c)/(b*x**2+a),x)
```

```
[Out] Integral(x**4*sin(c + d*x)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{x^4 \sin(c+dx)}{a+bx^2}\,dx = \int \frac{x^4 \sin(dx+c)}{bx^2+a}\,dx$$

```
[In] integrate(x^4*sin(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*(((b*d^2*x^4*cos(c) + 2*b*d*x^3*sin(c) - 2*b*x^2*cos(c) + 2*a*d*x*sin(
c))*cos(d*x + c)^2 + (b*d^2*x^4*cos(c) + 2*b*d*x^3*sin(c) - 2*b*x^2*cos(c)
+ 2*a*d*x*sin(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + ((b*cos(c)^2 + b*sin(c)^
2)*d^2*x^4 - 2*(b*cos(c)^2 + b*sin(c)^2)*x^2)*cos(d*x + c) + 2*(((b^2*cos(c
)^2 + b^2*sin(c)^2)*d^3*x^2 + (a*b*cos(c)^2 + a*b*sin(c)^2)*d^3)*cos(d*x +
c)^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^2 + (a*b*cos(c)^2 + a*b*sin(c)^
2)*d^3)*sin(d*x + c)^2)*integrate(((a^2*d^2 + 2*a*b)*x*cos(d*x + c) + (a*b*
d*x^2 + a^2*d)*sin(d*x + c))/(b^3*d^3*x^4 + 2*a*b^2*d^3*x^2 + a^2*b*d^3), x
) + 2*(((b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^2 + (a*b*cos(c)^2 + a*b*sin(c)^
2)*d^3)*cos(d*x + c)^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^2 + (a*b*cos(
c)^2 + a*b*sin(c)^2)*d^3)*sin(d*x + c)^2)*integrate(((a^2*d^2 + 2*a*b)*x*co
s(d*x + c) + (a*b*d*x^2 + a^2*d)*sin(d*x + c))/((b^3*d^3*x^4 + 2*a*b^2*d^3*
x^2 + a^2*b*d^3)*cos(d*x + c)^2 + (b^3*d^3*x^4 + 2*a*b^2*d^3*x^2 + a^2*b*d^
3)*sin(d*x + c)^2), x) + ((b*d^2*x^4*sin(c) - 2*b*d*x^3*cos(c) - 2*a*d*x*co
s(c) - 2*b*x^2*sin(c))*cos(d*x + c)^2 + (b*d^2*x^4*sin(c) - 2*b*d*x^3*cos(c
) - 2*a*d*x*cos(c) - 2*b*x^2*sin(c))*sin(d*x + c)^2)*sin(d*x + 2*c) - 2*((b
*cos(c)^2 + b*sin(c)^2)*d*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d*x)*sin(d*x + c)
)/(((b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^2 + (a*b*cos(c)^2 + a*b*sin(c)^2)*d
^3)*cos(d*x + c)^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^2 + (a*b*cos(c)^2
 + a*b*sin(c)^2)*d^3)*sin(d*x + c)^2)
```

Giac [F]

$$\int \frac{x^4 \sin(c+dx)}{a+bx^2} \, dx = \int \frac{x^4 \sin(dx+c)}{bx^2+a} \, dx$$

```
[In] integrate(x^4*sin(d*x+c)/(b*x^2+a),x, algorithm="giac")
[Out] integrate(x^4*sin(d*x + c)/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \sin(c+dx)}{a+bx^2} \, dx = \int \frac{x^4 \sin(c+d\,x)}{b\,x^2+a} \, dx$$

```
[In] int((x^4*sin(c + d*x))/(a + b*x^2),x)
[Out] int((x^4*sin(c + d*x))/(a + b*x^2), x)
```

3.58 $\int \frac{x^3 \sin(c+dx)}{a+bx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 209

$$\int \frac{x^3 \sin(c+dx)}{a+bx^2}\,dx = -\frac{x\cos(c+dx)}{bd} - \frac{a\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2} - \frac{a\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2} + \frac{\sin(c+dx)}{bd^2} + \frac{a\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2} - \frac{a\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^2}$$

```
[Out] -x*cos(d*x+c)/b/d-1/2*a*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b^(
1/2))/b^2-1/2*a*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))/b^
2+sin(d*x+c)/b/d^2-1/2*a*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^
(1/2))/b^2-1/2*a*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/
b^2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used

$= \{3426, 3377, 2717, 3384, 3380, 3383\}$

$$\int \frac{x^3 \sin(c+dx)}{a+bx^2}\,dx = -\frac{a \sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2} - \frac{a \sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2} + \frac{a \cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2} - \frac{a \cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2} + \frac{\sin(c+dx)}{bd^2} - \frac{x\cos(c+dx)}{bd}$$

```
[In] Int[(x^3*Sin[c + d*x])/(a + b*x^2),x]
```

```
[Out] -((x*Cos[c + d*x])/(b*d)) - (a*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[
c - (Sqrt[-a]*d)/Sqrt[b]])/(2*b^2) - (a*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] -
d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*b^2) + Sin[c + d*x]/(b*d^2) + (a*Cos
[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b^2)
 - (a*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]
)/(2*b^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{x\sin(c+dx)}{b} - \frac{ax\sin(c+dx)}{b\left(a+bx^2\right)}\right) dx \\
&= \frac{\int x\sin(c+dx)\,dx}{b} - \frac{a\int \frac{x\sin(c+dx)}{a+bx^2}\,dx}{b} \\
&= -\frac{x\cos(c+dx)}{bd} - \frac{a\int \left(-\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right) dx}{b} + \frac{\int \cos(c+dx)\,dx}{bd} \\
&= -\frac{x\cos(c+dx)}{bd} + \frac{\sin(c+dx)}{bd^2} + \frac{a\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2b^{3/2}} - \frac{a\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2b^{3/2}} \\
&= -\frac{x\cos(c+dx)}{bd} + \frac{\sin(c+dx)}{bd^2} - \frac{\left(a\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2b^{3/2}} \\
&\quad - \frac{\left(a\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2b^{3/2}} - \frac{\left(a\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2b^{3/2}} \\
&\quad + \frac{\left(a\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2b^{3/2}} \\
&= -\frac{x\cos(c+dx)}{bd} - \frac{a\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2} \\
&\quad - \frac{a\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2} + \frac{\sin(c+dx)}{bd^2} \\
&\quad + \frac{a\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2} - \frac{a\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^2}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.11 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.02

$$\int \frac{x^3 \sin(c+dx)}{a+bx^2} \, dx$$
$$= \frac{-iae^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)+\text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)\right)+iae^{ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIn}\right.}{4b^2}$$

[In] Integrate[(x^3*Sin[c + d*x])/(a + b*x^2),x]

[Out] ((-I)*a*E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*(E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x] + ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] - I*d*x]) + I*a*E^(I*c - (Sqrt[a]*d)/Sqrt[b])*(E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] + ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] + I*d*x]) - (4*b*Cos[d*x]*(d*x*Cos[c] - Sin[c]))/d^2 + (4*b*(Cos[c] + d*x*Sin[c])*Sin[d*x])/d^2)/(4*b^2)

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.27

method	result
risch	$-\frac{i\,\mathrm{Ei}_1\left(\frac{icb-d\sqrt{ab}-b(idx+ic)}{b}\right)\mathrm{e}^{\frac{icb-d\sqrt{ab}}{b}}a}{4b^2}-\frac{i\,\mathrm{Ei}_1\left(\frac{icb+d\sqrt{ab}-b(idx+ic)}{b}\right)\mathrm{e}^{\frac{icb+d\sqrt{ab}}{b}}a}{4b^2}+\frac{i\,\mathrm{Ei}_1\left(-\frac{icb-d\sqrt{ab}-b(idx+ic)}{b}\right)\mathrm{e}}{4b^2}$
derivativedivides	Expression too large to display
default	Expression too large to display

[In] int(x^3*sin(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)

[Out] -1/4*I/b^2*Ei(1,(I*c*b-d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*exp((I*c*b-d*(a*b)^(1/2))/b)*a-1/4*I/b^2*Ei(1,(I*c*b+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*exp((I*c*b+d*(a*b)^(1/2))/b)*a+1/4*I/b^2*Ei(1,-(I*c*b-d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*exp(-(I*c*b-d*(a*b)^(1/2))/b)*a+1/4*I/b^2*Ei(1,-(I*c*b+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*exp(-(I*c*b+d*(a*b)^(1/2))/b)*a-x*cos(d*x+c)/b/d+sin(d*x+c)/b/d^2

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.89

$$\int \frac{x^3 \sin(c+dx)}{a+bx^2}\,dx$$
$$= \frac{i\,ad^2 \mathrm{Ei}\left(i\,dx - \sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)} + i\,ad^2 \mathrm{Ei}\left(i\,dx + \sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)} - i\,ad^2 \mathrm{Ei}\left(-i\,dx - \sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c\right.}}{4\,b^2d^2}$$

```
[In] integrate(x^3*sin(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(I*a*d^2*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + I*a*d^2*Ei
(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) - I*a*d^2*Ei(-I*d*x - sqrt(
a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) - I*a*d^2*Ei(-I*d*x + sqrt(a*d^2/b))*e^(
-I*c - sqrt(a*d^2/b)) - 4*b*d*x*cos(d*x + c) + 4*b*sin(d*x + c))/(b^2*d^2)
```

Sympy [F]

$$\int \frac{x^3 \sin(c+dx)}{a+bx^2}\,dx = \int \frac{x^3 \sin(c+dx)}{a+bx^2}\,dx$$

```
[In] integrate(x**3*sin(d*x+c)/(b*x**2+a),x)
```

```
[Out] Integral(x**3*sin(c + d*x)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{x^3 \sin(c+dx)}{a+bx^2}\,dx = \int \frac{x^3 \sin(dx+c)}{bx^2+a}\,dx$$

```
[In] integrate(x^3*sin(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*((cos(c)^2 + sin(c)^2)*d*x^3*cos(d*x + c) - (cos(c)^2 + sin(c)^2)*x^2*
sin(d*x + c) + ((d*x^3*cos(c) + x^2*sin(c))*cos(d*x + c)^2 + (d*x^3*cos(c)
+ x^2*sin(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b*cos(c)^2 + b*sin(c)^2
)*d^2*x^2 + (a*cos(c)^2 + a*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b*cos(c)^2 +
b*sin(c)^2)*d^2*x^2 + (a*cos(c)^2 + a*sin(c)^2)*d^2)*sin(d*x + c)^2)*integr
ate(-(a*d*x^2*cos(d*x + c) - a*x*sin(d*x + c))/(b^2*d^2*x^4 + 2*a*b*d^2*x^2
 + a^2*d^2), x) + 2*(((b*cos(c)^2 + b*sin(c)^2)*d^2*x^2 + (a*cos(c)^2 + a*s
in(c)^2)*d^2)*cos(d*x + c)^2 + ((b*cos(c)^2 + b*sin(c)^2)*d^2*x^2 + (a*cos(
```

```
c)^2 + a*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(-(a*d*x^2*cos(d*x + c) -
a*x*sin(d*x + c))/((b^2*d^2*x^4 + 2*a*b*d^2*x^2 + a^2*d^2)*cos(d*x + c)^2 +
 (b^2*d^2*x^4 + 2*a*b*d^2*x^2 + a^2*d^2)*sin(d*x + c)^2), x) + ((d*x^3*sin(
c) - x^2*cos(c))*cos(d*x + c)^2 + (d*x^3*sin(c) - x^2*cos(c))*sin(d*x + c)^
2)*sin(d*x + 2*c))/(((b*cos(c)^2 + b*sin(c)^2)*d^2*x^2 + (a*cos(c)^2 + a*si
n(c)^2)*d^2)*cos(d*x + c)^2 + ((b*cos(c)^2 + b*sin(c)^2)*d^2*x^2 + (a*cos(c
)^2 + a*sin(c)^2)*d^2)*sin(d*x + c)^2)
```

Giac [F]

$$\int \frac{x^3 \sin(c+dx)}{a+bx^2}\,dx = \int \frac{x^3 \sin\left(dx+c\right)}{bx^2+a}\,dx$$

```
[In] integrate(x^3*sin(d*x+c)/(b*x^2+a),x, algorithm="giac")
[Out] integrate(x^3*sin(d*x + c)/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \sin(c+dx)}{a+bx^2}\,dx = \int \frac{x^3\,\sin\left(c+d\,x\right)}{b\,x^2+a}\,dx$$

```
[In] int((x^3*sin(c + d*x))/(a + b*x^2),x)
[Out] int((x^3*sin(c + d*x))/(a + b*x^2), x)
```

3.59 $\int \frac{x^2 \sin(c+dx)}{a+bx^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 227

$$
\begin{aligned}
\int \frac{x^2 \sin(c+dx)}{a+bx^2} \, dx = & -\frac{\cos(c+dx)}{bd} - \frac{\sqrt{-a}\, \text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right) \sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{3/2}} \\
& + \frac{\sqrt{-a}\, \text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right) \sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{3/2}} \\
& - \frac{\sqrt{-a} \cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}} \\
& - \frac{\sqrt{-a} \cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^{3/2}}
\end{aligned}
$$

```
[Out] -cos(d*x+c)/b/d+1/2*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b^(1/2)
)*(-a)^(1/2)/b^(3/2)-1/2*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^
(1/2))*(-a)^(1/2)/b^(3/2)-1/2*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/
2)/b^(1/2))*(-a)^(1/2)/b^(3/2)+1/2*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-
a)^(1/2)/b^(1/2))*(-a)^(1/2)/b^(3/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used

$= \{3426, 2718, 3414, 3384, 3380, 3383\}$

$$\int \frac{x^2 \sin(c+dx)}{a+bx^2}\,dx = -\frac{\sqrt{-a}\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{3/2}} + \frac{\sqrt{-a}\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}} - \frac{\sqrt{-a}\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}} - \frac{\sqrt{-a}\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{3/2}} - \frac{\cos(c+dx)}{bd}$$

```
[In] Int[(x^2*Sin[c + d*x])/(a + b*x^2),x]
```

```
[Out] -(Cos[c + d*x]/(b*d)) - (Sqrt[-a]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*S
in[c - (Sqrt[-a]*d)/Sqrt[b]])/(2*b^(3/2)) + (Sqrt[-a]*CosIntegral[(Sqrt[-a]
*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*b^(3/2)) - (Sqrt[-a]*C
os[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b^
(3/2)) - (Sqrt[-a]*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/S
qrt[b] + d*x])/(2*b^(3/2))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\sin(c+dx)}{b} - \frac{a\sin(c+dx)}{b\,(a+bx^2)}\right) dx$$

$$= \frac{\int \sin(c+dx)\,dx}{b} - \frac{a\int \frac{\sin(c+dx)}{a+bx^2}\,dx}{b}$$

$$= -\frac{\cos(c+dx)}{bd} - \frac{a\int \left(\frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right) dx}{b}$$

$$= -\frac{\cos(c+dx)}{bd} - \frac{\sqrt{-a}\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2b} - \frac{\sqrt{-a}\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2b}$$

$$= -\frac{\cos(c+dx)}{bd} - \frac{\left(\sqrt{-a}\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2b} + \frac{\left(\sqrt{-a}\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2b} - \frac{\left(\sqrt{-a}\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2b} - \frac{\left(\sqrt{-a}\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2b}$$

$$= -\frac{\cos(c+dx)}{bd} - \frac{\sqrt{-a}\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{3/2}} + \frac{\sqrt{-a}\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{3/2}} - \frac{\sqrt{-a}\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}} - \frac{\sqrt{-a}\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^{3/2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.95

$$\begin{aligned}\int \frac{x^2 \sin(c+dx)}{a+bx^2}\,dx &= -\frac{\cos(c)\cos(dx)}{bd} \\ &+ \frac{\sqrt{a}e^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(-e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\,\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)+\text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)\right)}{4b^{3/2}} \\ &+ \frac{\sqrt{a}e^{ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(-e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\,\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}+idx\right)+\text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}+idx\right)\right)}{4b^{3/2}} \\ &+ \frac{\sin(c)\sin(dx)}{bd}\end{aligned}$$

```
[In] Integrate[(x^2*Sin[c + d*x])/(a + b*x^2),x]
```

```
[Out] -((Cos[c]*Cos[d*x])/(b*d)) + (Sqrt[a]*E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*(-(E
^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x]) + E
xpIntegralEi[(Sqrt[a]*d)/Sqrt[b] - I*d*x]))/(4*b^(3/2)) + (Sqrt[a]*E^(I*c -
 (Sqrt[a]*d)/Sqrt[b])*(-(E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]
*d)/Sqrt[b]) + I*d*x]) + ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] + I*d*x]))/(4*b^
(3/2)) + (Sin[c]*Sin[d*x])/(b*d)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.16

method	result
risch	$\frac{\sqrt{ab}\,\mathrm{e}^{\frac{icb+d\sqrt{ab}}{b}}\,\mathrm{Ei}_1\left(\frac{icb+d\sqrt{ab}-b(idx+ic)}{b}\right)}{4b^2}-\frac{\sqrt{ab}\,\mathrm{e}^{\frac{icb-d\sqrt{ab}}{b}}\,\mathrm{Ei}_1\left(\frac{icb-d\sqrt{ab}-b(idx+ic)}{b}\right)}{4b^2}-\frac{\mathrm{e}^{-\frac{icb+d\sqrt{ab}}{b}}\,\mathrm{Ei}_1\left(-\frac{icb+d\sqrt{a}}{}\right.}{4b^2}\ldots$
derivativedivides	$d^2c^2\left(-\frac{\mathrm{Si}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\cos\left(\frac{d\sqrt{-ab}+cb}{b}\right)+\mathrm{Ci}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\sin\left(\frac{d\sqrt{-ab}+cb}{b}\right)}{2b\left(-\frac{d\sqrt{-ab}+cb}{b}+c\right)}-\frac{\mathrm{Si}\left(dx+c+\frac{d\sqrt{-ab}-cb}{b}\right)\cos\left(\frac{d\sqrt{-ab}}{b}\right)}{2b}\right.$
default	$d^2c^2\left(-\frac{\mathrm{Si}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\cos\left(\frac{d\sqrt{-ab}+cb}{b}\right)+\mathrm{Ci}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\sin\left(\frac{d\sqrt{-ab}+cb}{b}\right)}{2b\left(-\frac{d\sqrt{-ab}+cb}{b}+c\right)}-\frac{\mathrm{Si}\left(dx+c+\frac{d\sqrt{-ab}-cb}{b}\right)\cos\left(\frac{d\sqrt{-ab}}{b}\right)}{2b}\right.$

```
[In] int(x^2*sin(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b^2*(a*b)^(1/2)*exp((I*c*b+d*(a*b)^(1/2))/b)*Ei(1,(I*c*b+d*(a*b)^(1/2)-
b*(I*d*x+I*c))/b)-1/4/b^2*(a*b)^(1/2)*exp((I*c*b-d*(a*b)^(1/2))/b)*Ei(1,(I*
c*b-d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)-1/4/b^2*exp(-(I*c*b+d*(a*b)^(1/2))/b)*E
i(1,-(I*c*b+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*(a*b)^(1/2)+1/4/b^2*exp(-(I*c*b
-d*(a*b)^(1/2))/b)*Ei(1,-(I*c*b-d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*(a*b)^(1/2)
-cos(d*x+c)/b/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.86

$$\int \frac{x^2 \sin(c+dx)}{a+bx^2}\,dx = -\frac{\sqrt{\frac{ad^2}{b}}\,\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}-\sqrt{\frac{ad^2}{b}}\,\mathrm{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)}+\sqrt{\frac{ad^2}{b}}\,\mathrm{Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right)e}{4\,bd}$$

```
[In] integrate(x^2*sin(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(a*d^2/b)*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) - sqr
t(a*d^2/b)*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + sqrt(a*d^2/b
)*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) - sqrt(a*d^2/b)*Ei(-I
*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) + 4*cos(d*x + c))/(b*d)
```

Sympy [F]

$$\int \frac{x^2 \sin(c+dx)}{a+bx^2}\,dx = \int \frac{x^2 \sin{(c+dx)}}{a+bx^2}\,dx$$

```
[In] integrate(x**2*sin(d*x+c)/(b*x**2+a),x)
```

```
[Out] Integral(x**2*sin(c + d*x)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{x^2 \sin(c+dx)}{a+bx^2}\,dx = \int \frac{x^2 \sin{(dx+c)}}{bx^2+a}\,dx$$

```
[In] integrate(x^2*sin(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*((cos(c)^2 + sin(c)^2)*x^2*cos(d*x + c) + (x^2*cos(d*x + c)^2*cos(c) +
 x^2*cos(c)*sin(d*x + c)^2)*cos(d*x + 2*c) - 2*(((a*b*cos(c)^2 + a*b*sin(c)
```

```
^2)*d*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*cos(d*x + c)^2 + ((a*b*cos(c)^
2 + a*b*sin(c)^2)*d*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*sin(d*x + c)^2)*
integrate(x*cos(d*x + c)/(b^2*d*x^4 + 2*a*b*d*x^2 + a^2*d), x) - 2*(((a*b*c
os(c)^2 + a*b*sin(c)^2)*d*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*cos(d*x +
c)^2 + ((a*b*cos(c)^2 + a*b*sin(c)^2)*d*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)
*d)*sin(d*x + c)^2)*integrate(x*cos(d*x + c)/((b^2*d*x^4 + 2*a*b*d*x^2 + a^
2*d)*cos(d*x + c)^2 + (b^2*d*x^4 + 2*a*b*d*x^2 + a^2*d)*sin(d*x + c)^2), x)
 + (x^2*cos(d*x + c)^2*sin(c) + x^2*sin(d*x + c)^2*sin(c))*sin(d*x + 2*c))/
(((b*cos(c)^2 + b*sin(c)^2)*d*x^2 + (a*cos(c)^2 + a*sin(c)^2)*d)*cos(d*x +
c)^2 + ((b*cos(c)^2 + b*sin(c)^2)*d*x^2 + (a*cos(c)^2 + a*sin(c)^2)*d)*sin(
d*x + c)^2)
```

Giac [F]

$$\int \frac{x^2 \sin(c+dx)}{a+bx^2} \, dx = \int \frac{x^2 \sin{(dx+c)}}{bx^2+a} \, dx$$

```
[In] integrate(x^2*sin(d*x+c)/(b*x^2+a),x, algorithm="giac")
[Out] integrate(x^2*sin(d*x + c)/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \sin(c+dx)}{a+bx^2} \, dx = \int \frac{x^2 \sin{(c+d\,x)}}{b\,x^2+a} \, dx$$

```
[In] int((x^2*sin(c + d*x))/(a + b*x^2),x)
[Out] int((x^2*sin(c + d*x))/(a + b*x^2), x)
```

3.60 $\int \frac{x\sin(c+dx)}{a+bx^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 177

$$\int \frac{x\sin(c+dx)}{a+bx^2}\,dx = \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b} + \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b} - \frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b} + \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b}$$

```
[Out] 1/2*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b^(1/2))/b+1/2*cos(c-d*
(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))/b+1/2*Ci(d*x+d*(-a)^(1/2)/
b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))/b+1/2*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*si
n(c+d*(-a)^(1/2)/b^(1/2))/b
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3426, 3384, 3380, 3383}

$$\int \frac{x\sin(c+dx)}{a+bx^2}\,dx = \frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b} + \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b} - \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b} + \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b}$$

```
[In] Int[(x*Sin[c + d*x])/(a + b*x^2),x]
```

```
[Out] (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(2*
b) + (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]
)/(2*b) - (Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] -
 d*x])/(2*b) + (Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt
[b] + d*x])/(2*b)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)\,dx \\
&= -\frac{\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2\sqrt{b}}+\frac{\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2\sqrt{b}} \\
&= \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2\sqrt{b}}+\frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2\sqrt{b}} \\
&\quad +\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2\sqrt{b}}-\frac{\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2\sqrt{b}}
\end{aligned}
$$

$$= \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b} + \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b} - \frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b} + \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.88

$$\int \frac{x\sin(c+dx)}{a+bx^2}\,dx = \frac{ie^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)+\text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)-e^{2ic}\left(e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\right.\right.}{4b}$$

```
[In] Integrate[(x*Sin[c + d*x])/(a + b*x^2),x]
```

```
[Out] ((I/4)*E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*(E^((2*Sqrt[a]*d)/Sqrt[b])*ExpInteg
ralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x] + ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] -
 I*d*x] - E^((2*I)*c)*(E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d
)/Sqrt[b]) + I*d*x] + ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] + I*d*x])))/b
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.32

method	result
risch	$\frac{i\mathrm{e}^{\frac{icb+d\sqrt{ab}}{b}}\operatorname{Ei}_1\left(\frac{icb+d\sqrt{ab}-b(idx+ic)}{b}\right)}{4b}+\frac{i\mathrm{e}^{\frac{icb-d\sqrt{ab}}{b}}\operatorname{Ei}_1\left(\frac{icb-d\sqrt{ab}-b(idx+ic)}{b}\right)}{4b}-\frac{i\operatorname{Ei}_1\left(-\frac{icb+d\sqrt{ab}-b(idx+ic)}{b}\right)\mathrm{e}^{-\frac{ic}{}}}{4b}$
derivativedivides	$-\frac{d^2(d\sqrt{-ab}+cb)\left(\operatorname{Si}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\cos\left(\frac{d\sqrt{-ab}+cb}{b}\right)+\operatorname{Ci}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\sin\left(\frac{d\sqrt{-ab}+cb}{b}\right)\right)}{2b^2\left(-\frac{d\sqrt{-ab}+cb}{b}+c\right)}+\frac{d^2(d\sqrt{-ab}-cb)\left(\operatorname{Si}\left(dx\right.\right.}{}$
default	$-\frac{d^2(d\sqrt{-ab}+cb)\left(\operatorname{Si}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\cos\left(\frac{d\sqrt{-ab}+cb}{b}\right)+\operatorname{Ci}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\sin\left(\frac{d\sqrt{-ab}+cb}{b}\right)\right)}{2b^2\left(-\frac{d\sqrt{-ab}+cb}{b}+c\right)}+\frac{d^2(d\sqrt{-ab}-cb)\left(\operatorname{Si}\left(dx\right.\right.}{}$

```
[In] int(x*sin(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I/b*exp((I*c*b+d*(a*b)^(1/2))/b)*Ei(1,(I*c*b+d*(a*b)^(1/2)-b*(I*d*x+I*c
))/b)+1/4*I/b*exp((I*c*b-d*(a*b)^(1/2))/b)*Ei(1,(I*c*b-d*(a*b)^(1/2)-b*(I*d
*x+I*c))/b)-1/4*I/b*Ei(1,-(I*c*b+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*exp(-(I*c*
b+d*(a*b)^(1/2))/b)-1/4*I/b*Ei(1,-(I*c*b-d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*ex
p(-(I*c*b-d*(a*b)^(1/2))/b)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.82

$$\int \frac{x\sin(c+dx)}{a+bx^2}\,dx = \frac{-i\,\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}-i\,\mathrm{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)}+i\,\mathrm{Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c+\sqrt{\frac{ad^2}{b}}\right)}+}{4\,b}$$

```
[In] integrate(x*sin(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(-I*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) - I*Ei(I*d*x + sq
rt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + I*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c
 + sqrt(a*d^2/b)) + I*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)))/
b
```

Sympy [F]

$$\int \frac{x\sin(c+dx)}{a+bx^2}\,dx = \int \frac{x\sin{(c+dx)}}{a+bx^2}\,dx$$

```
[In] integrate(x*sin(d*x+c)/(b*x**2+a),x)
```

```
[Out] Integral(x*sin(c + d*x)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{x\sin(c+dx)}{a+bx^2}\,dx = \int \frac{x\sin{(dx+c)}}{bx^2+a}\,dx$$

```
[In] integrate(x*sin(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*((cos(c)^2 + sin(c)^2)*x*cos(d*x + c) + (x*cos(d*x + c)^2*cos(c) + x*c
os(c)*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b*cos(c)^2 + b*sin(c)^2)*d*x^2
```

```
+ (a*cos(c)^2 + a*sin(c)^2)*d)*cos(d*x + c)^2 + ((b*cos(c)^2 + b*sin(c)^2)*
d*x^2 + (a*cos(c)^2 + a*sin(c)^2)*d)*sin(d*x + c)^2)*integrate(1/2*(b*x^2 -
 a)*cos(d*x + c)/(b^2*d*x^4 + 2*a*b*d*x^2 + a^2*d), x) + 2*(((b*cos(c)^2 +
b*sin(c)^2)*d*x^2 + (a*cos(c)^2 + a*sin(c)^2)*d)*cos(d*x + c)^2 + ((b*cos(c
)^2 + b*sin(c)^2)*d*x^2 + (a*cos(c)^2 + a*sin(c)^2)*d)*sin(d*x + c)^2)*inte
grate(1/2*(b*x^2 - a)*cos(d*x + c)/((b^2*d*x^4 + 2*a*b*d*x^2 + a^2*d)*cos(d
*x + c)^2 + (b^2*d*x^4 + 2*a*b*d*x^2 + a^2*d)*sin(d*x + c)^2), x) + (x*cos(
d*x + c)^2*sin(c) + x*sin(d*x + c)^2*sin(c))*sin(d*x + 2*c))/(((b*cos(c)^2
+ b*sin(c)^2)*d*x^2 + (a*cos(c)^2 + a*sin(c)^2)*d)*cos(d*x + c)^2 + ((b*cos
(c)^2 + b*sin(c)^2)*d*x^2 + (a*cos(c)^2 + a*sin(c)^2)*d)*sin(d*x + c)^2)
```

Giac [F]

$$\int \frac{x\sin(c+dx)}{a+bx^2}\,dx = \int \frac{x\sin\left(dx+c\right)}{bx^2+a}\,dx$$

```
[In] integrate(x*sin(d*x+c)/(b*x^2+a),x, algorithm="giac")
[Out] integrate(x*sin(d*x + c)/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sin(c+dx)}{a+bx^2}\,dx = \int \frac{x\,\sin\left(c+d\,x\right)}{b\,x^2+a}\,dx$$

```
[In] int((x*sin(c + d*x))/(a + b*x^2),x)
[Out] int((x*sin(c + d*x))/(a + b*x^2), x)
```

3.61 $\int \frac{\sin(c+dx)}{a+bx^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 213

$$\int \frac{\sin(c+dx)}{a+bx^2}\,dx = -\frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}} + \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}} - \frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}} - \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2\sqrt{-a}\sqrt{b}}$$

```
[Out] 1/2*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b^(1/2))/(-a)^(1/2)/b^(
1/2)-1/2*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))/(-a)^(1/2
)/b^(1/2)-1/2*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))/(-a)
^(1/2)/b^(1/2)+1/2*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2)
)/(-a)^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

= {3414, 3384, 3380, 3383}

$$\int \frac{\sin(c+dx)}{a+bx^2}\,dx = -\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}} + \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}} - \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}} - \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}$$

```
[In] Int[Sin[c + d*x]/(a + b*x^2),x]
```

```
[Out] -1/2*(CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]
)/(Sqrt[-a]*Sqrt[b]) + (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sq
rt[-a]*d)/Sqrt[b]])/(2*Sqrt[-a]*Sqrt[b]) - (Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*S
inIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*Sqrt[-a]*Sqrt[b]) - (Cos[c - (Sq
rt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*Sqrt[-a]*Sqr
t[b])
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\text{integral}=\int\left(\frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx$$

$$=-\frac{\int\frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2\sqrt{-a}}-\frac{\int\frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2\sqrt{-a}}$$

$$=-\frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2\sqrt{-a}}+\frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2\sqrt{-a}}$$
$$-\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2\sqrt{-a}}-\frac{\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2\sqrt{-a}}$$

$$=-\frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}$$
$$+\frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}$$
$$-\frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2\sqrt{-a}\sqrt{b}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.77

$$\int\frac{\sin(c+dx)}{a+bx^2}\,dx$$
$$=\frac{e^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)-\text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)+e^{2ic}\left(e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\right.\right.\right.}{4\sqrt{a}\sqrt{b}}$$

```
[In] Integrate[Sin[c + d*x]/(a + b*x^2),x]
```

```
[Out] (E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*(E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[
-((Sqrt[a]*d)/Sqrt[b]) - I*d*x] - ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] - I*d*x
] + E^((2*I)*c)*(E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt
[b]) + I*d*x] - ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] + I*d*x])))/(4*Sqrt[a]*Sq
rt[b])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.06

method	result
derivativedivides	$d\left(-\frac{\mathrm{Si}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\cos\left(\frac{d\sqrt{-ab}+cb}{b}\right)+\mathrm{Ci}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\sin\left(\frac{d\sqrt{-ab}+cb}{b}\right)}{2b\left(-\frac{d\sqrt{-ab}+cb}{b}+c\right)}-\frac{\mathrm{Si}\left(dx+c+\frac{d\sqrt{-ab}-cb}{b}\right)\cos(}{}\right.$
default	$d\left(-\frac{\mathrm{Si}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\cos\left(\frac{d\sqrt{-ab}+cb}{b}\right)+\mathrm{Ci}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\sin\left(\frac{d\sqrt{-ab}+cb}{b}\right)}{2b\left(-\frac{d\sqrt{-ab}+cb}{b}+c\right)}-\frac{\mathrm{Si}\left(dx+c+\frac{d\sqrt{-ab}-cb}{b}\right)\cos(}{}\right.$
risch	$\frac{\sqrt{ab}\,\mathrm{e}^{-\frac{icb+d\sqrt{ab}}{b}}\,\mathrm{Ei}_1\left(-\frac{icb+d\sqrt{ab}-b(idx+ic)}{b}\right)}{4ab}-\frac{\sqrt{ab}\,\mathrm{e}^{-\frac{icb-d\sqrt{ab}}{b}}\,\mathrm{Ei}_1\left(-\frac{icb-d\sqrt{ab}-b(idx+ic)}{b}\right)}{4ab}-\frac{\sqrt{ab}\,\mathrm{e}^{\frac{icb+d\sqrt{ab}}{b}}\,\mathrm{Ei}_1}{}$

```
[In] int(sin(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] d*(-1/2/b/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos
((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/
2)+c*b)/b))-1/2/b/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)
/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a
*b)^(1/2)-c*b)/b)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.88

$$\int \frac{\sin(c+dx)}{a+bx^2}\,dx = \frac{\sqrt{\frac{ad^2}{b}}\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}-\sqrt{\frac{ad^2}{b}}\mathrm{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)}+\sqrt{\frac{ad^2}{b}}\mathrm{Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{(-}}{4\,ad}$$

```
[In] integrate(sin(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(a*d^2/b)*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) - sqrt
(a*d^2/b)*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + sqrt(a*d^2/b)
*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) - sqrt(a*d^2/b)*Ei(-I*
d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)))/(a*d)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{a+bx^2}\,dx = \int \frac{\sin\left(c+dx\right)}{a+bx^2}\,dx$$

```
[In] integrate(sin(d*x+c)/(b*x**2+a),x)
[Out] Integral(sin(c + d*x)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{a+bx^2}\,dx = \int \frac{\sin\left(dx+c\right)}{bx^2+a}\,dx$$

```
[In] integrate(sin(d*x+c)/(b*x^2+a),x, algorithm="maxima")
[Out] integrate(sin(d*x + c)/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{a+bx^2}\,dx = \int \frac{\sin\left(dx+c\right)}{bx^2+a}\,dx$$

```
[In] integrate(sin(d*x+c)/(b*x^2+a),x, algorithm="giac")
[Out] integrate(sin(d*x + c)/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{a+bx^2}\,dx = \int \frac{\sin\left(c+d\,x\right)}{b\,x^2+a}\,dx$$

```
[In] int(sin(c + d*x)/(a + b*x^2),x)
[Out] int(sin(c + d*x)/(a + b*x^2), x)
```

3.62 $\int \frac{\sin(c+dx)}{x(a+bx^2)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 197

$$\int \frac{\sin(c+dx)}{x(a+bx^2)}\,dx = \frac{\text{CosIntegral}(dx)\sin(c)}{a} - \frac{\text{CosIntegral}\left(\frac{\sqrt{-ad}}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2a} - \frac{\text{CosIntegral}\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2a} + \frac{\cos(c)\text{Si}(dx)}{a} + \frac{\cos\left(c+\frac{\sqrt{-ad}}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)}{2a} - \frac{\cos\left(c-\frac{\sqrt{-ad}}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-ad}}{\sqrt{b}}+dx\right)}{2a}$$

```
[Out] cos(c)*Si(d*x)/a-1/2*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b^(1/2
))/a-1/2*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))/a+Ci(d*x)
*sin(c)/a-1/2*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))/a-1/
2*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/a
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3426, 3384, 3380, 3383}

$$\int \frac{\sin(c+dx)}{x(a+bx^2)}\,dx = -\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a} - \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a} + \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a} - \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a} + \frac{\sin(c)\,\text{CosIntegral}(dx)}{a} + \frac{\cos(c)\text{Si}(dx)}{a}$$

```
[In] Int[Sin[c + d*x]/(x*(a + b*x^2)),x]
```

```
[Out] (CosIntegral[d*x]*Sin[c])/a - (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[
c - (Sqrt[-a]*d)/Sqrt[b]])/(2*a) - (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]
*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*a) + (Cos[c]*SinIntegral[d*x])/a + (Cos[
c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a) -
(Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*
a)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
```

```
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\sin(c+dx)}{ax} - \frac{bx\sin(c+dx)}{a(a+bx^2)} \right) dx \\
&= \frac{\int \frac{\sin(c+dx)}{x} dx}{a} - \frac{b\int \frac{x\sin(c+dx)}{a+bx^2} dx}{a} \\
&= -\frac{b\int \left(-\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} \right) dx}{a} + \frac{\cos(c)\int \frac{\sin(dx)}{x} dx}{a} + \frac{\sin(c)\int \frac{\cos(dx)}{x} dx}{a} \\
&= \frac{\text{CosIntegral}(dx)\sin(c)}{a} + \frac{\cos(c)\text{Si}(dx)}{a} + \frac{\sqrt{b}\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x} dx}{2a} - \frac{\sqrt{b}\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x} dx}{2a} \\
&= \frac{\text{CosIntegral}(dx)\sin(c)}{a} + \frac{\cos(c)\text{Si}(dx)}{a} - \frac{\left(\sqrt{b}\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x} dx}{2a} \\
&\quad - \frac{\left(\sqrt{b}\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x} dx}{2a} \\
&\quad - \frac{\left(\sqrt{b}\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x} dx}{2a} \\
&\quad + \frac{\left(\sqrt{b}\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x} dx}{2a} \\
&= \frac{\text{CosIntegral}(dx)\sin(c)}{a} - \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a} \\
&\quad - \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a} + \frac{\cos(c)\text{Si}(dx)}{a} \\
&\quad + \frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a} - \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.88

$$\int \frac{\sin(c+dx)}{x(a+bx^2)}\,dx = \frac{-ie^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)+\text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)-e^{2ic}\left(e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\right.\right.}{4a}$$

[In] Integrate[Sin[c + d*x]/(x*(a + b*x^2)),x]

[Out] ((-I)*E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*(E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x] + ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - E^((2*I)*c)*(E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] + ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] + I*d*x])) + 4*CosIntegral[d*x]*Sin[c] + 4*Cos[c]*SinIntegral[d*x])/(4*a)

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.02

method	result
derivativedivides	$-\frac{\mathrm{Si}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\cos\left(\frac{d\sqrt{-ab}+cb}{b}\right)+\mathrm{Ci}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\sin\left(\frac{d\sqrt{-ab}+cb}{b}\right)}{2a}-\frac{\mathrm{Si}\left(dx+c+\frac{d\sqrt{-ab}-cb}{b}\right)\cos\left(\frac{d\sqrt{-a}}{b}\right.}{}$
default	$-\frac{\mathrm{Si}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\cos\left(\frac{d\sqrt{-ab}+cb}{b}\right)+\mathrm{Ci}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\sin\left(\frac{d\sqrt{-ab}+cb}{b}\right)}{2a}-\frac{\mathrm{Si}\left(dx+c+\frac{d\sqrt{-ab}-cb}{b}\right)\cos\left(\frac{d\sqrt{-a}}{b}\right.}{}$
risch	$-\frac{ie^{\frac{icb+d\sqrt{ab}}{b}}\mathrm{Ei}_1\left(\frac{icb+d\sqrt{ab}-b(idx+ic)}{b}\right)}{4a}-\frac{ie^{\frac{icb-d\sqrt{ab}}{b}}\mathrm{Ei}_1\left(\frac{icb-d\sqrt{ab}-b(idx+ic)}{b}\right)}{4a}+\frac{ie^{ic}\mathrm{Ei}_1(-idx)}{2a}+\frac{ie^{-\frac{icb+d\sqrt{ab}}{b}}}{}$

[In] int(sin(d*x+c)/x/(b*x^2+a),x,method=_RETURNVERBOSE)

[Out] -1/2/a*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))-1/2/a*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))+1/a*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.82

$$\int \frac{\sin(c+dx)}{x\,(a+bx^2)}\,dx$$
$$= \frac{i\,\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}+i\,\mathrm{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)}-i\,\mathrm{Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c+\sqrt{\frac{ad^2}{b}}\right)}-}{4\,a}$$

```
[In] integrate(sin(d*x+c)/x/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(I*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + I*Ei(I*d*x + sqr
t(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) - I*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c
+ sqrt(a*d^2/b)) - I*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) +
4*cos_integral(d*x)*sin(c) + 4*cos(c)*sin_integral(d*x))/a
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{x\,(a+bx^2)}\,dx = \int \frac{\sin\left(c+dx\right)}{x\,(a+bx^2)}\,dx$$

```
[In] integrate(sin(d*x+c)/x/(b*x**2+a),x)
```

```
[Out] Integral(sin(c + d*x)/(x*(a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x\,(a+bx^2)}\,dx = \int \frac{\sin\left(dx+c\right)}{(bx^2+a)x}\,dx$$

```
[In] integrate(sin(d*x+c)/x/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x^2 + a)*x), x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{x(a+bx^2)}\,dx = \int \frac{\sin(dx+c)}{(bx^2+a)x}\,dx$$

```
[In] integrate(sin(d*x+c)/x/(b*x^2+a),x, algorithm="giac")
[Out] integrate(sin(d*x + c)/((b*x^2 + a)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x(a+bx^2)}\,dx = \int \frac{\sin(c+d\,x)}{x\,(b\,x^2+a)}\,dx$$

```
[In] int(sin(c + d*x)/(x*(a + b*x^2)),x)
[Out] int(sin(c + d*x)/(x*(a + b*x^2)), x)
```

3.63 $\int \frac{\sin(c+dx)}{x^2(a+bx^2)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 250

$$\begin{aligned}\int \frac{\sin(c+dx)}{x^2(a+bx^2)}\,dx = & \frac{d\cos(c)\,\text{CosIntegral}(dx)}{a}-\frac{\sqrt{b}\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2(-a)^{3/2}} \\ & +\frac{\sqrt{b}\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2(-a)^{3/2}}-\frac{\sin(c+dx)}{ax} \\ & -\frac{d\sin(c)\text{Si}(dx)}{a}-\frac{\sqrt{b}\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2(-a)^{3/2}} \\ & -\frac{\sqrt{b}\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2(-a)^{3/2}}\end{aligned}$$

```
[Out] d*Ci(d*x)*cos(c)/a-d*Si(d*x)*sin(c)/a-sin(d*x+c)/a/x+1/2*cos(c+d*(-a)^(1/2)
/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(3/2)-1/2*cos(c-d*(-a)^
(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(3/2)-1/2*Ci(d*x+d
*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(3/2)+1/2*Ci(
-d*x+d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(3/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3426, 3378, 3384, 3380, 3383, 3414}

$$\int \frac{\sin(c+dx)}{x^2(a+bx^2)}\,dx = -\frac{\sqrt{b}\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2(-a)^{3/2}} + \frac{\sqrt{b}\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2(-a)^{3/2}} - \frac{\sqrt{b}\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2(-a)^{3/2}} - \frac{\sqrt{b}\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2(-a)^{3/2}} + \frac{d\cos(c)\text{CosIntegral}(dx)}{a} - \frac{d\sin(c)\text{Si}(dx)}{a} - \frac{\sin(c+dx)}{ax}$$

```
[In] Int[Sin[c + d*x]/(x^2*(a + b*x^2)),x]
```

```
[Out] (d*Cos[c]*CosIntegral[d*x])/a - (Sqrt[b]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] +
 d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(2*(-a)^(3/2)) + (Sqrt[b]*CosIntegral[
(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*(-a)^(3/2)) -
 Sin[c + d*x]/(a*x) - (d*Sin[c]*SinIntegral[d*x])/a - (Sqrt[b]*Cos[c + (Sqr
t[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*(-a)^(3/2)) -
 (Sqrt[b]*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] +
d*x])/(2*(-a)^(3/2))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{\sin(c+dx)}{ax^2} - \frac{b\sin(c+dx)}{a\left(a+bx^2\right)}\right) dx \\
&= \frac{\int \frac{\sin(c+dx)}{x^2}\,dx}{a} - \frac{b\int \frac{\sin(c+dx)}{a+bx^2}\,dx}{a} \\
&= -\frac{\sin(c+dx)}{ax} - \frac{b\int \left(\frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right) dx}{a} + \frac{d\int \frac{\cos(c+dx)}{x}\,dx}{a} \\
&= -\frac{\sin(c+dx)}{ax} - \frac{b\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2(-a)^{3/2}} - \frac{b\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2(-a)^{3/2}} \\
&\quad + \frac{(d\cos(c))\int \frac{\cos(dx)}{x}\,dx}{a} - \frac{(d\sin(c))\int \frac{\sin(dx)}{x}\,dx}{a} \\
&= \frac{d\cos(c)\,\text{CosIntegral}(dx)}{a} - \frac{\sin(c+dx)}{ax} - \frac{d\sin(c)\text{Si}(dx)}{a} \\
&\quad - \frac{\left(b\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2(-a)^{3/2}} + \frac{\left(b\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2(-a)^{3/2}} \\
&\quad - \frac{\left(b\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2(-a)^{3/2}} - \frac{\left(b\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2(-a)^{3/2}}
\end{aligned}
$$

$$= \frac{d\cos(c)\,\text{CosIntegral}(dx)}{a} - \frac{\sqrt{b}\,\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}} + dx\right)\sin\left(c - \frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2(-a)^{3/2}} + \frac{\sqrt{b}\,\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}} - dx\right)\sin\left(c + \frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2(-a)^{3/2}} - \frac{\sin(c+dx)}{ax} - \frac{d\sin(c)\text{Si}(dx)}{a} - \frac{\sqrt{b}\cos\left(c + \frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}} - dx\right)}{2(-a)^{3/2}} - \frac{\sqrt{b}\cos\left(c - \frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}} + dx\right)}{2(-a)^{3/2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.95

$$\int \frac{\sin(c+dx)}{x^2(a+bx^2)}\,dx = \frac{\sqrt{b}e^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(-e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\,\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}} - idx\right) + \text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}} - idx\right)\right)}{4a^{3/2}} + \frac{\sqrt{b}e^{ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(-e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\,\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}} + idx\right) + \text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}} + idx\right)\right)}{4a^{3/2}} - \frac{\cos(dx)\sin(c)}{ax} - \frac{\cos(c)\sin(dx)}{ax} + \frac{d(\cos(c)\,\text{CosIntegral}(dx) - \sin(c)\text{Si}(dx))}{a}$$

```
[In] Integrate[Sin[c + d*x]/(x^2*(a + b*x^2)),x]
```

```
[Out] (Sqrt[b]*E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*(-(E^((2*Sqrt[a]*d)/Sqrt[b])*ExpI
ntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x]) + ExpIntegralEi[(Sqrt[a]*d)/Sqrt
[b] - I*d*x]))/(4*a^(3/2)) + (Sqrt[b]*E^(I*c - (Sqrt[a]*d)/Sqrt[b])*(-(E^((
2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]) + ExpI
ntegralEi[(Sqrt[a]*d)/Sqrt[b] + I*d*x]))/(4*a^(3/2)) - (Cos[d*x]*Sin[c])/(a
*x) - (Cos[c]*Sin[d*x])/(a*x) + (d*(Cos[c]*CosIntegral[d*x] - Sin[c]*SinInt
egral[d*x]))/a
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.06

method	result
derivativedivides	$d\left(-\frac{b\left(-\frac{\mathrm{Si}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\cos\left(\frac{d\sqrt{-ab}+cb}{b}\right)+\mathrm{Ci}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\sin\left(\frac{d\sqrt{-ab}+cb}{b}\right)}{2b\left(-\frac{d\sqrt{-ab}+cb}{b}+c\right)}-\frac{\mathrm{Si}\left(dx+c+\frac{d\sqrt{-ab}-cb}{b}\right)\cos\left(\frac{d}{}\right.}{}\right.}{a}\right.$
default	$d\left(-\frac{b\left(-\frac{\mathrm{Si}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\cos\left(\frac{d\sqrt{-ab}+cb}{b}\right)+\mathrm{Ci}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\sin\left(\frac{d\sqrt{-ab}+cb}{b}\right)}{2b\left(-\frac{d\sqrt{-ab}+cb}{b}+c\right)}-\frac{\mathrm{Si}\left(dx+c+\frac{d\sqrt{-ab}-cb}{b}\right)\cos\left(\right.}{}\right.}{a}\right.$
risch	$-\frac{d\,\mathrm{Ei}_1(-idx)\mathrm{e}^{ic}}{2a}+\frac{\sqrt{ab}\,\mathrm{e}^{\frac{icb+d\sqrt{ab}}{b}}\,\mathrm{Ei}_1\left(\frac{icb+d\sqrt{ab}-b(idx+ic)}{b}\right)}{4a^2}-\frac{\sqrt{ab}\,\mathrm{e}^{\frac{icb-d\sqrt{ab}}{b}}\,\mathrm{Ei}_1\left(-\frac{-icb+d\sqrt{ab}+b(idx+ic)}{b}\right)}{4a^2}-$

```
[In] int(sin(d*x+c)/x^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] d*(-b/a*(-1/2/b/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/
b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*
b)^(1/2)+c*b)/b))-1/2/b/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2
)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin(
(d*(-a*b)^(1/2)-c*b)/b)))+1/a*(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c
)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.94

$$\int \frac{\sin(c+dx)}{x^2(a+bx^2)}\,dx$$
$$=\frac{4\,ad^2x\cos(c)\,\mathrm{Ci}\,(dx)-4\,ad^2x\sin(c)\,\mathrm{Si}\,(dx)-\sqrt{\frac{ad^2}{b}}bx\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}+\sqrt{\frac{ad^2}{b}}bx\mathrm{Ei}\left(i\,dx+\right.}{}$$

```
[In] integrate(sin(d*x+c)/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(4*a*d^2*x*cos(c)*cos_integral(d*x) - 4*a*d^2*x*sin(c)*sin_integral(d*x
) - sqrt(a*d^2/b)*b*x*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + s
qrt(a*d^2/b)*b*x*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) - sqrt(a
*d^2/b)*b*x*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) + sqrt(a*d^
2/b)*b*x*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) - 4*a*d*sin(d*
x + c))/(a^2*d*x)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{x^2(a+bx^2)}\,dx = \int \frac{\sin(c+dx)}{x^2(a+bx^2)}\,dx$$

```
[In] integrate(sin(d*x+c)/x**2/(b*x**2+a),x)
[Out] Integral(sin(c + d*x)/(x**2*(a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x^2(a+bx^2)}\,dx = \int \frac{\sin(dx+c)}{(bx^2+a)x^2}\,dx$$

```
[In] integrate(sin(d*x+c)/x^2/(b*x^2+a),x, algorithm="maxima")
[Out] integrate(sin(d*x + c)/((b*x^2 + a)*x^2), x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{x^2(a+bx^2)}\,dx = \int \frac{\sin(dx+c)}{(bx^2+a)x^2}\,dx$$

```
[In] integrate(sin(d*x+c)/x^2/(b*x^2+a),x, algorithm="giac")
[Out] integrate(sin(d*x + c)/((b*x^2 + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x^2(a+bx^2)}\,dx = \int \frac{\sin(c+d\,x)}{x^2\,(b\,x^2+a)}\,dx$$

```
[In] int(sin(c + d*x)/(x^2*(a + b*x^2)),x)
[Out] int(sin(c + d*x)/(x^2*(a + b*x^2)), x)
```

3.64 $\int \frac{\sin(c+dx)}{x^3(a+bx^2)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 270

$$\begin{aligned}\int \frac{\sin(c+dx)}{x^3(a+bx^2)}\,dx =& -\frac{d\cos(c+dx)}{2ax}-\frac{b\,\text{CosIntegral}(dx)\sin(c)}{a^2}-\frac{d^2\,\text{CosIntegral}(dx)\sin(c)}{2a} \\ &+\frac{b\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2} \\ &+\frac{b\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2} \\ &-\frac{\sin(c+dx)}{2ax^2}-\frac{b\cos(c)\text{Si}(dx)}{a^2}-\frac{d^2\cos(c)\text{Si}(dx)}{2a} \\ &-\frac{b\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2}+\frac{b\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^2}\end{aligned}$$

```
[Out] -1/2*d*cos(d*x+c)/a/x-b*cos(c)*Si(d*x)/a^2-1/2*d^2*cos(c)*Si(d*x)/a+1/2*b*c
os(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b^(1/2))/a^2+1/2*b*cos(c-d*(
-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))/a^2-b*Ci(d*x)*sin(c)/a^2-1/
2*d^2*Ci(d*x)*sin(c)/a-1/2*sin(d*x+c)/a/x^2+1/2*b*Ci(d*x+d*(-a)^(1/2)/b^(1/
2))*sin(c-d*(-a)^(1/2)/b^(1/2))/a^2+1/2*b*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*sin
(c+d*(-a)^(1/2)/b^(1/2))/a^2
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3426, 3378, 3384, 3380, 3383}

$$\int \frac{\sin(c+dx)}{x^3(a+bx^2)}\,dx = -\frac{b\sin(c)\,\text{CosIntegral}(dx)}{a^2}+\frac{b\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2}+\frac{b\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2}-\frac{b\cos(c)\text{Si}(dx)}{a^2}-\frac{b\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2}+\frac{b\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2}-\frac{d^2\sin(c)\,\text{CosIntegral}(dx)}{2a}-\frac{d^2\cos(c)\text{Si}(dx)}{2a}-\frac{\sin(c+dx)}{2ax^2}-\frac{d\cos(c+dx)}{2ax}$$

```
[In] Int[Sin[c + d*x]/(x^3*(a + b*x^2)),x]
```

```
[Out] -1/2*(d*Cos[c + d*x])/(a*x) - (b*CosIntegral[d*x]*Sin[c])/a^2 - (d^2*CosInt
egral[d*x]*Sin[c])/(2*a) + (b*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c
 - (Sqrt[-a]*d)/Sqrt[b]])/(2*a^2) + (b*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d
*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*a^2) - Sin[c + d*x]/(2*a*x^2) - (b*Co
s[c]*SinIntegral[d*x])/a^2 - (d^2*Cos[c]*SinIntegral[d*x])/(2*a) - (b*Cos[c
 + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^2) +
 (b*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/
(2*a^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\sin(c+dx)}{ax^3} - \frac{b\sin(c+dx)}{a^2x} + \frac{b^2x\sin(c+dx)}{a^2(a+bx^2)}\right) dx$$

$$= \frac{\int \frac{\sin(c+dx)}{x^3}\,dx}{a} - \frac{b\int \frac{\sin(c+dx)}{x}\,dx}{a^2} + \frac{b^2\int \frac{x\sin(c+dx)}{a+bx^2}\,dx}{a^2}$$

$$= -\frac{\sin(c+dx)}{2ax^2} + \frac{b^2\int \left(-\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right) dx}{a^2} + \frac{d\int \frac{\cos(c+dx)}{x^2}\,dx}{2a} - \frac{(b\cos(c))\int \frac{\sin(dx)}{x}\,dx}{a^2} - \frac{(b\sin(c))\int \frac{\cos(dx)}{x}\,dx}{a^2}$$

$$= -\frac{d\cos(c+dx)}{2ax} - \frac{b\,\text{CosIntegral}(dx)\sin(c)}{a^2} - \frac{\sin(c+dx)}{2ax^2} - \frac{b\cos(c)\text{Si}(dx)}{a^2} - \frac{b^{3/2}\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2a^2} + \frac{b^{3/2}\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2a^2} - \frac{d^2\int \frac{\sin(c+dx)}{x}\,dx}{2a}$$

$$= -\frac{d\cos(c+dx)}{2ax} - \frac{b\,\mathrm{CosIntegral}(dx)\sin(c)}{a^2} - \frac{\sin(c+dx)}{2ax^2} - \frac{b\cos(c)\mathrm{Si}(dx)}{a^2} - \frac{(d^2\cos(c))\int\frac{\sin(dx)}{x}\,dx}{2a} + \frac{\left(b^{3/2}\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2a^2} + \frac{\left(b^{3/2}\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2a^2} - \frac{(d^2\sin(c))\int\frac{\cos(dx)}{x}\,dx}{2a} + \frac{\left(b^{3/2}\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2a^2} - \frac{\left(b^{3/2}\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2a^2}$$

$$= -\frac{d\cos(c+dx)}{2ax} - \frac{b\,\mathrm{CosIntegral}(dx)\sin(c)}{a^2} - \frac{d^2\,\mathrm{CosIntegral}(dx)\sin(c)}{2a} + \frac{b\,\mathrm{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2} + \frac{b\,\mathrm{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2} - \frac{\sin(c+dx)}{2ax^2} - \frac{b\cos(c)\mathrm{Si}(dx)}{a^2} - \frac{d^2\cos(c)\mathrm{Si}(dx)}{2a} - \frac{b\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2} + \frac{b\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^2}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.89

$$\int\frac{\sin(c+dx)}{x^3(a+bx^2)}\,dx$$

$$= \frac{ibe^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\,\mathrm{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)+\mathrm{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)\right)-ibe^{ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\,\mathrm{ExpInte}\right.}{}$$

```
[In] Integrate[Sin[c + d*x]/(x^3*(a + b*x^2)),x]
```

```
[Out] (I*b*E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*(E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegra
lEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x] + ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] - I
*d*x]) - I*b*E^(I*c - (Sqrt[a]*d)/Sqrt[b])*(E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIn
tegralEi[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] + ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b
] + I*d*x]) - (2*a*Cos[d*x]*(d*x*Cos[c] + Sin[c]))/x^2 + (2*a*(-Cos[c] + d*
x*Sin[c])*Sin[d*x])/x^2 - 2*(2*b + a*d^2)*(CosIntegral[d*x]*Sin[c] + Cos[c]
*SinIntegral[d*x]))/(4*a^2)
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.96

method	result
derivativedivides	$d^2\left(-\frac{\sin(dx+c)}{2a\,d^2x^2}-\frac{\cos(dx+c)}{2adx}+\frac{b\left(\mathrm{Si}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\cos\left(\frac{d\sqrt{-ab}+cb}{b}\right)+\mathrm{Ci}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\sin\left(\frac{d\sqrt{-ab}+cb}{b}\right.\right.}{2a^2d^2}\right.$
default	$d^2\left(-\frac{\sin(dx+c)}{2a\,d^2x^2}-\frac{\cos(dx+c)}{2adx}+\frac{b\left(\mathrm{Si}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\cos\left(\frac{d\sqrt{-ab}+cb}{b}\right)+\mathrm{Ci}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\sin\left(\frac{d\sqrt{-ab}+cb}{b}\right.\right.}{2a^2d^2}\right.$
risch	$\frac{ib\,\mathrm{e}^{\frac{icb+d\sqrt{ab}}{b}}\,\mathrm{Ei}_1\left(\frac{icb+d\sqrt{ab}-b(idx+ic)}{b}\right)}{4a^2}+\frac{ib\,\mathrm{e}^{\frac{icb-d\sqrt{ab}}{b}}\,\mathrm{Ei}_1\left(\frac{icb-d\sqrt{ab}-b(idx+ic)}{b}\right)}{4a^2}-\frac{id^2\mathrm{e}^{ic}\,\mathrm{Ei}_1(-idx)}{4a}-\frac{i\mathrm{e}^{ic}\,\mathrm{Ei}_1(-}{2a^2}$

```
[In] int(sin(d*x+c)/x^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] d^2*(-1/2*sin(d*x+c)/a/d^2/x^2-1/2*cos(d*x+c)/a/d/x+1/2*b/a^2/d^2*(Si(d*x+c
-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/
2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))+1/2*b/a^2/d^2*(Si(d*x+c+(d*(-a*b)^(
1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*s
in((d*(-a*b)^(1/2)-c*b)/b))-1/2/a^2*(a*d^2+2*b)/d^2*(Si(d*x)*cos(c)+Ci(d*x)
*sin(c)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.83

$$\int \frac{\sin(c+dx)}{x^3\,(a+bx^2)}\,dx$$
$$=\frac{-i\,bx^2\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}-i\,bx^2\mathrm{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)}+i\,bx^2\mathrm{Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\right.}}{}$$

```
[In] integrate(sin(d*x+c)/x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(-I*b*x^2*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) - I*b*x^2*E
i(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + I*b*x^2*Ei(-I*d*x - sqrt
(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) + I*b*x^2*Ei(-I*d*x + sqrt(a*d^2/b))*e^
(-I*c - sqrt(a*d^2/b)) - 2*(a*d^2 + 2*b)*x^2*cos_integral(d*x)*sin(c) - 2*(
a*d^2 + 2*b)*x^2*cos(c)*sin_integral(d*x) - 2*a*d*x*cos(d*x + c) - 2*a*sin(
d*x + c))/(a^2*x^2)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{x^3(a+bx^2)}\,dx = \int \frac{\sin(c+dx)}{x^3(a+bx^2)}\,dx$$

```
[In] integrate(sin(d*x+c)/x**3/(b*x**2+a),x)
[Out] Integral(sin(c + d*x)/(x**3*(a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x^3(a+bx^2)}\,dx = \int \frac{\sin(dx+c)}{(bx^2+a)x^3}\,dx$$

```
[In] integrate(sin(d*x+c)/x^3/(b*x^2+a),x, algorithm="maxima")
[Out] integrate(sin(d*x + c)/((b*x^2 + a)*x^3), x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{x^3(a+bx^2)}\,dx = \int \frac{\sin(dx+c)}{(bx^2+a)x^3}\,dx$$

```
[In] integrate(sin(d*x+c)/x^3/(b*x^2+a),x, algorithm="giac")
[Out] integrate(sin(d*x + c)/((b*x^2 + a)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x^3(a+bx^2)}\,dx = \int \frac{\sin(c+d\,x)}{x^3\,(b\,x^2+a)}\,dx$$

```
[In] int(sin(c + d*x)/(x^3*(a + b*x^2)),x)
[Out] int(sin(c + d*x)/(x^3*(a + b*x^2)), x)
```

3.65 $\int \frac{x^4 \sin(c+dx)}{(a+bx^2)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 450

$$\int \frac{x^4 \sin(c+dx)}{(a+bx^2)^2} dx = -\frac{\cos(c+dx)}{b^2 d} - \frac{ad\cos\left(c+\frac{\sqrt{-ad}}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)}{4b^3} - \frac{ad\cos\left(c-\frac{\sqrt{-ad}}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-ad}}{\sqrt{b}}+dx\right)}{4b^3} - \frac{3\sqrt{-a}\,\text{CosIntegral}\left(\frac{\sqrt{-ad}}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-ad}}{\sqrt{b}}\right)}{4b^{5/2}} + \frac{3\sqrt{-a}\,\text{CosIntegral}\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-ad}}{\sqrt{b}}\right)}{4b^{5/2}} + \frac{x\sin(c+dx)}{2b^2} - \frac{x^3\sin(c+dx)}{2b(a+bx^2)} - \frac{3\sqrt{-a}\cos\left(c+\frac{\sqrt{-ad}}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)}{4b^{5/2}} - \frac{ad\sin\left(c+\frac{\sqrt{-ad}}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)}{4b^3} - \frac{3\sqrt{-a}\cos\left(c-\frac{\sqrt{-ad}}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-ad}}{\sqrt{b}}+dx\right)}{4b^{5/2}} + \frac{ad\sin\left(c-\frac{\sqrt{-ad}}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-ad}}{\sqrt{b}}+dx\right)}{4b^3}$$

```
[Out] -cos(d*x+c)/b^2/d-1/4*a*d*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*cos(c-d*(-a)^(1/2)/b
^(1/2))/b^3-1/4*a*d*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*cos(c+d*(-a)^(1/2)/b^(1/2
))/b^3+1/2*x*sin(d*x+c)/b^2-1/2*x^3*sin(d*x+c)/b/(b*x^2+a)+1/4*a*d*Si(d*x+d
*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))/b^3+1/4*a*d*Si(d*x-d*(-a)^
```

```
(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/b^3+3/4*cos(c+d*(-a)^(1/2)/b^(1/
2))*Si(d*x-d*(-a)^(1/2)/b^(1/2))*(-a)^(1/2)/b^(5/2)-3/4*cos(c-d*(-a)^(1/2)/
b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))*(-a)^(1/2)/b^(5/2)-3/4*Ci(d*x+d*(-a)^
(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))*(-a)^(1/2)/b^(5/2)+3/4*Ci(-d*x+d
*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))*(-a)^(1/2)/b^(5/2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3424, 3426, 2718, 3414, 3384, 3380, 3383, 3427, 3377}

$$\int \frac{x^4 \sin(c+dx)}{(a+bx^2)^2}\,dx = -\frac{3\sqrt{-a}\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^{5/2}} + \frac{3\sqrt{-a}\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^{5/2}} - \frac{3\sqrt{-a}\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^{5/2}} - \frac{3\sqrt{-a}\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^{5/2}} - \frac{ad\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^3} - \frac{ad\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^3} - \frac{ad\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^3} + \frac{ad\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^3} - \frac{x^3\sin(c+dx)}{2b(a+bx^2)} + \frac{x\sin(c+dx)}{2b^2} - \frac{\cos(c+dx)}{b^2 d}$$

```
[In] Int[(x^4*Sin[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] -(Cos[c + d*x]/(b^2*d)) - (a*d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(S
qrt[-a]*d)/Sqrt[b] - d*x])/(4*b^3) - (a*d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*Cos
Integral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*b^3) - (3*Sqrt[-a]*CosIntegral[(Sq
rt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(4*b^(5/2)) + (3*Sq
rt[-a]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]
])/(4*b^(5/2)) + (x*Sin[c + d*x])/(2*b^2) - (x^3*Sin[c + d*x])/(2*b*(a + b*
x^2)) - (3*Sqrt[-a]*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/
```

```
Sqrt[b] - d*x])/(4*b^(5/2)) - (a*d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegra
l[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*b^3) - (3*Sqrt[-a]*Cos[c - (Sqrt[-a]*d)/S
qrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*b^(5/2)) + (a*d*Sin[c -
 (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*b^3)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3424

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
```

```
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3427

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^3\sin(c+dx)}{2b\,(a+bx^2)}+\frac{3\int\frac{x^2\sin(c+dx)}{a+bx^2}\,dx}{2b}+\frac{d\int\frac{x^3\cos(c+dx)}{a+bx^2}\,dx}{2b}\\
&= -\frac{x^3\sin(c+dx)}{2b\,(a+bx^2)}+\frac{3\int\left(\frac{\sin(c+dx)}{b}-\frac{a\sin(c+dx)}{b(a+bx^2)}\right)dx}{2b}+\frac{d\int\left(\frac{x\cos(c+dx)}{b}-\frac{ax\cos(c+dx)}{b(a+bx^2)}\right)dx}{2b}\\
&= -\frac{x^3\sin(c+dx)}{2b\,(a+bx^2)}+\frac{3\int\sin(c+dx)\,dx}{2b^2}-\frac{(3a)\int\frac{\sin(c+dx)}{a+bx^2}\,dx}{2b^2}\\
&\quad+\frac{d\int x\cos(c+dx)\,dx}{2b^2}-\frac{(ad)\int\frac{x\cos(c+dx)}{a+bx^2}\,dx}{2b^2}\\
&= -\frac{3\cos(c+dx)}{2b^2d}+\frac{x\sin(c+dx)}{2b^2}-\frac{x^3\sin(c+dx)}{2b\,(a+bx^2)}\\
&\quad-\frac{\int\sin(c+dx)\,dx}{2b^2}-\frac{(3a)\int\left(\frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{2b^2}\\
&\quad-\frac{(ad)\int\left(-\frac{\cos(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\cos(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{2b^2}\\
&= -\frac{\cos(c+dx)}{b^2d}+\frac{x\sin(c+dx)}{2b^2}-\frac{x^3\sin(c+dx)}{2b\,(a+bx^2)}-\frac{\left(3\sqrt{-a}\right)\int\frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4b^2}\\
&\quad-\frac{\left(3\sqrt{-a}\right)\int\frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4b^2}+\frac{(ad)\int\frac{\cos(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4b^{5/2}}-\frac{(ad)\int\frac{\cos(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4b^{5/2}}
\end{aligned}$$

$$= -\frac{\cos(c+dx)}{b^2d} + \frac{x\sin(c+dx)}{2b^2} - \frac{x^3\sin(c+dx)}{2b(a+bx^2)}$$
$$- \frac{\left(3\sqrt{-a}\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4b^2}$$
$$- \frac{\left(ad\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4b^{5/2}}$$
$$+ \frac{\left(3\sqrt{-a}\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4b^2}$$
$$+ \frac{\left(ad\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4b^{5/2}}$$
$$- \frac{\left(3\sqrt{-a}\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4b^2}$$
$$+ \frac{\left(ad\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4b^{5/2}}$$
$$- \frac{\left(3\sqrt{-a}\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4b^2}$$
$$+ \frac{\left(ad\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4b^{5/2}}$$

$$= -\frac{\cos(c+dx)}{b^2d} - \frac{ad\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^3}$$
$$- \frac{ad\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4b^3}$$
$$- \frac{3\sqrt{-a}\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^{5/2}}$$
$$+ \frac{3\sqrt{-a}\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^{5/2}} + \frac{x\sin(c+dx)}{2b^2} - \frac{x^3\sin(c+dx)}{2b(a+bx^2)}$$
$$- \frac{3\sqrt{-a}\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^{5/2}} - \frac{ad\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^3}$$
$$- \frac{3\sqrt{-a}\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4b^{5/2}} + \frac{ad\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4b^3}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.15 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.66

$$\int \frac{x^4 \sin(c+dx)}{(a+bx^2)^2}\,dx = -\frac{\sqrt{a}e^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(\left(3\sqrt{b}+\sqrt{a}d\right)e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)+\left(-3\sqrt{b}+\sqrt{a}d\right)\text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}-\right.\right.}{}$$

```
[In] Integrate[(x^4*Sin[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] -1/8*(Sqrt[a]*E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*((3*Sqrt[b] + Sqrt[a]*d)*E^(
(2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x] + (-3*
Sqrt[b] + Sqrt[a]*d)*ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] - I*d*x]) + Sqrt[a]*
E^(I*c - (Sqrt[a]*d)/Sqrt[b])*((3*Sqrt[b] + Sqrt[a]*d)*E^((2*Sqrt[a]*d)/Sqr
t[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] + (-3*Sqrt[b] + Sqrt[a]
*d)*ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] + I*d*x]) - 4*b*Cos[d*x]*((-2*Cos[c])
/d + (a*x*Sin[c])/(a + b*x^2)) - 4*b*((a*x*Cos[c])/(a + b*x^2) + (2*Sin[c])
/d)*Sin[d*x])/b^3
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.16

method	result
risch	$\frac{\mathrm{e}^{\frac{icb+d\sqrt{ab}}{b}}\,\mathrm{Ei}_1\left(\frac{icb+d\sqrt{ab}-b(idx+ic)}{b}\right)ad}{8b^3}+\frac{\mathrm{e}^{\frac{icb-d\sqrt{ab}}{b}}\,\mathrm{Ei}_1\left(-\frac{-icb+d\sqrt{ab}+b(idx+ic)}{b}\right)ad}{8b^3}+\frac{3\sqrt{ab}\,\mathrm{e}^{\frac{icb+d\sqrt{ab}}{b}}\,\mathrm{Ei}_1\left(\frac{icb+d\sqrt{}}{}\right.}{8b^3}$
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int(x^4*sin(d*x+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/b^3*exp((I*c*b+d*(a*b)^(1/2))/b)*Ei(1,(I*c*b+d*(a*b)^(1/2)-b*(I*d*x+I*c
))/b)*a*d+1/8/b^3*exp((I*c*b-d*(a*b)^(1/2))/b)*Ei(1,-(-I*c*b+d*(a*b)^(1/2)+
b*(I*d*x+I*c))/b)*a*d+3/8/b^3*(a*b)^(1/2)*exp((I*c*b+d*(a*b)^(1/2))/b)*Ei(1
,(I*c*b+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)-3/8/b^3*(a*b)^(1/2)*exp((I*c*b-d*(a
*b)^(1/2))/b)*Ei(1,-(-I*c*b+d*(a*b)^(1/2)+b*(I*d*x+I*c))/b)+1/8/b^3*exp(-(I
*c*b+d*(a*b)^(1/2))/b)*Ei(1,-(I*c*b+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*a*d+1/8
/b^3*exp(-(I*c*b-d*(a*b)^(1/2))/b)*Ei(1,(-I*c*b+d*(a*b)^(1/2)+b*(I*d*x+I*c)
)/b)*a*d-3/8/b^3*(a*b)^(1/2)*exp(-(I*c*b+d*(a*b)^(1/2))/b)*Ei(1,-(I*c*b+d*(
a*b)^(1/2)-b*(I*d*x+I*c))/b)+3/8/b^3*(a*b)^(1/2)*exp(-(I*c*b-d*(a*b)^(1/2))
```

```
/b)*Ei(1,(-I*c*b+d*(a*b)^(1/2)+b*(I*d*x+I*c))/b)-cos(d*x+c)/b^2/d+1/2*d^2*a
*x/b^2/(b*d^2*x^2+a*d^2)*sin(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.78

$$\int \frac{x^4 \sin(c+dx)}{(a+bx^2)^2} \, dx = \frac{4\,abdx\sin\left(dx+c\right) - \left(abd^2x^2 + a^2d^2 + 3\left(b^2x^2+ab\right)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(i\,dx - \sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)} - \left(abd^2x^2 + a\right.}{}$$

```
[In] integrate(x^4*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*a*b*d*x*sin(d*x + c) - (a*b*d^2*x^2 + a^2*d^2 + 3*(b^2*x^2 + a*b)*sq
rt(a*d^2/b))*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) - (a*b*d^2*x
^2 + a^2*d^2 - 3*(b^2*x^2 + a*b)*sqrt(a*d^2/b))*Ei(I*d*x + sqrt(a*d^2/b))*e
^(I*c - sqrt(a*d^2/b)) - (a*b*d^2*x^2 + a^2*d^2 + 3*(b^2*x^2 + a*b)*sqrt(a*
d^2/b))*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) - (a*b*d^2*x^2
+ a^2*d^2 - 3*(b^2*x^2 + a*b)*sqrt(a*d^2/b))*Ei(-I*d*x + sqrt(a*d^2/b))*e^(
-I*c - sqrt(a*d^2/b)) - 8*(b^2*x^2 + a*b)*cos(d*x + c))/(b^4*d*x^2 + a*b^3*
d)
```

Sympy [F]

$$\int \frac{x^4 \sin(c+dx)}{(a+bx^2)^2} \, dx = \int \frac{x^4 \sin\left(c+dx\right)}{\left(a+bx^2\right)^2} \, dx$$

```
[In] integrate(x**4*sin(d*x+c)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**4*sin(c + d*x)/(a + b*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x^4 \sin(c+dx)}{(a+bx^2)^2} \, dx = \int \frac{x^4 \sin(dx+c)}{(bx^2+a)^2} \, dx$$

```
[In] integrate(x^4*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((b*cos(c)^2 + b*sin(c)^2)*d*x^4*cos(d*x + c) - 4*(a*cos(c)^2 + a*sin(
c)^2)*x*sin(d*x + c) + ((b*d*x^4*cos(c) + 4*a*x*sin(c))*cos(d*x + c)^2 + (b
*d*x^4*cos(c) + 4*a*x*sin(c))*sin(d*x + c)^2)*cos(d*x + 2*c) - 2*(((b^3*cos
(c)^2 + b^3*sin(c)^2)*d^2*x^4 + 2*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^2*x^2
 + (a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b^3*cos(c)^2 +
 b^3*sin(c)^2)*d^2*x^4 + 2*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^2*x^2 + (a^2
*b*cos(c)^2 + a^2*b*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(-2*(a^2*d*x*co
s(d*x + c) - (3*a*b*x^2 - a^2)*sin(d*x + c))/(b^4*d^2*x^6 + 3*a*b^3*d^2*x^4
 + 3*a^2*b^2*d^2*x^2 + a^3*b*d^2), x) - 2*(((b^3*cos(c)^2 + b^3*sin(c)^2)*d
^2*x^4 + 2*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^2*x^2 + (a^2*b*cos(c)^2 + a^
2*b*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d^2*x^4
+ 2*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^2*x^2 + (a^2*b*cos(c)^2 + a^2*b*sin
(c)^2)*d^2)*sin(d*x + c)^2)*integrate(-2*(a^2*d*x*cos(d*x + c) - (3*a*b*x^2
 - a^2)*sin(d*x + c))/((b^4*d^2*x^6 + 3*a*b^3*d^2*x^4 + 3*a^2*b^2*d^2*x^2 +
 a^3*b*d^2)*cos(d*x + c)^2 + (b^4*d^2*x^6 + 3*a*b^3*d^2*x^4 + 3*a^2*b^2*d^2
*x^2 + a^3*b*d^2)*sin(d*x + c)^2), x) + ((b*d*x^4*sin(c) - 4*a*x*cos(c))*co
s(d*x + c)^2 + (b*d*x^4*sin(c) - 4*a*x*cos(c))*sin(d*x + c)^2)*sin(d*x + 2*
c))/(((b^3*cos(c)^2 + b^3*sin(c)^2)*d^2*x^4 + 2*(a*b^2*cos(c)^2 + a*b^2*sin
(c)^2)*d^2*x^2 + (a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((
b^3*cos(c)^2 + b^3*sin(c)^2)*d^2*x^4 + 2*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*
d^2*x^2 + (a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^2)*sin(d*x + c)^2)
```

Giac [F]

$$\int \frac{x^4 \sin(c+dx)}{(a+bx^2)^2} \, dx = \int \frac{x^4 \sin(dx+c)}{(bx^2+a)^2} \, dx$$

```
[In] integrate(x^4*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^4*sin(d*x + c)/(b*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \sin(c + dx)}{(a + bx^2)^2} \, dx = \int \frac{x^4 \sin(c + d\,x)}{(b\,x^2 + a)^2} \, dx$$

```
[In] int((x^4*sin(c + d*x))/(a + b*x^2)^2,x)
[Out] int((x^4*sin(c + d*x))/(a + b*x^2)^2, x)
```

3.66 $\int \frac{x^3 \sin(c+dx)}{(a+bx^2)^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 431

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^2)^2} \, dx = \frac{\sqrt{-a}d \cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^{5/2}} - \frac{\sqrt{-a}d \cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4b^{5/2}} + \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right) \sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2} + \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right) \sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2} + \frac{\sin(c+dx)}{2b^2} - \frac{x^2 \sin(c+dx)}{2b(a+bx^2)} - \frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2} + \frac{\sqrt{-a}d \sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^{5/2}} + \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^2} + \frac{\sqrt{-a}d \sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4b^{5/2}}$$

```
[Out] 1/2*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b^(1/2))/b^2+1/2*cos(c-
d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))/b^2+1/2*sin(d*x+c)/b^2-1
/2*x^2*sin(d*x+c)/b/(b*x^2+a)+1/2*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)
^(1/2)/b^(1/2))/b^2+1/2*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^
```

```
(1/2))/b^2-1/4*d*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*cos(c-d*(-a)^(1/2)/b^(1/2))*(
-a)^(1/2)/b^(5/2)+1/4*d*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*cos(c+d*(-a)^(1/2)/b^
(1/2))*(-a)^(1/2)/b^(5/2)+1/4*d*Si(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(
1/2)/b^(1/2))*(-a)^(1/2)/b^(5/2)-1/4*d*Si(d*x-d*(-a)^(1/2)/b^(1/2))*sin(c+d
*(-a)^(1/2)/b^(1/2))*(-a)^(1/2)/b^(5/2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3424, 3426, 3384, 3380, 3383, 3427, 2717, 3415}

$$\begin{aligned}\int \frac{x^3\sin(c+dx)}{(a+bx^2)^2}\,dx = & \frac{\sqrt{-a}d\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^{5/2}} \\ & -\frac{\sqrt{-a}d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^{5/2}} \\ & +\frac{\sqrt{-a}d\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^{5/2}} \\ & +\frac{\sqrt{-a}d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^{5/2}} \\ & +\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2} \\ & +\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2} \\ & -\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2} \\ & +\frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2}-\frac{x^2\sin(c+dx)}{2b(a+bx^2)}+\frac{\sin(c+dx)}{2b^2}\end{aligned}$$

```
[In] Int[(x^3*Sin[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] (Sqrt[-a]*d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b]
- d*x])/(4*b^(5/2)) - (Sqrt[-a]*d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral
[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*b^(5/2)) + (CosIntegral[(Sqrt[-a]*d)/Sqrt[
b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(2*b^2) + (CosIntegral[(Sqrt[-a]*d
)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*b^2) + Sin[c + d*x]/(2*b
^2) - (x^2*Sin[c + d*x])/(2*b*(a + b*x^2)) - (Cos[c + (Sqrt[-a]*d)/Sqrt[b]]
*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b^2) + (Sqrt[-a]*d*Sin[c + (Sq
rt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*b^(5/2)) + (
Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b
```

```
^2) + (Sqrt[-a]*d*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sq
rt[b] + d*x])/(4*b^(5/2))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3415

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3424

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3427

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\frac{x^2\sin(c+dx)}{2b(a+bx^2)} + \frac{\int \frac{x\sin(c+dx)}{a+bx^2}\,dx}{b} + \frac{d\int \frac{x^2\cos(c+dx)}{a+bx^2}\,dx}{2b}$$

$$= -\frac{x^2\sin(c+dx)}{2b(a+bx^2)} + \frac{\int \left(-\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{b} + \frac{d\int \left(\frac{\cos(c+dx)}{b} - \frac{a\cos(c+dx)}{b(a+bx^2)}\right)dx}{2b}$$

$$= -\frac{x^2\sin(c+dx)}{2b(a+bx^2)} - \frac{\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2b^{3/2}} + \frac{\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2b^{3/2}} + \frac{d\int \cos(c+dx)\,dx}{2b^2} - \frac{(ad)\int \frac{\cos(c+dx)}{a+bx^2}\,dx}{2b^2}$$

$$= \frac{\sin(c+dx)}{2b^2} - \frac{x^2\sin(c+dx)}{2b(a+bx^2)} - \frac{(ad)\int \left(\frac{\sqrt{-a}\cos(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{-a}\cos(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{2b^2}$$
$$+ \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2b^{3/2}} + \frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2b^{3/2}}$$
$$+ \frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2b^{3/2}} - \frac{\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2b^{3/2}}$$

$$= \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2}$$
$$+ \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2} + \frac{\sin(c+dx)}{2b^2} - \frac{x^2\sin(c+dx)}{2b(a+bx^2)}$$
$$- \frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2} + \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^2}$$
$$- \frac{\left(\sqrt{-a}d\right)\int \frac{\cos(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4b^2} - \frac{\left(\sqrt{-a}d\right)\int \frac{\cos(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4b^2}$$

$$= \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2} + \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2} + \frac{\sin(c+dx)}{2b^2} - \frac{x^2\sin(c+dx)}{2b(a+bx^2)} - \frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2} + \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^2} - \frac{\left(\sqrt{-a}d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4b^2} - \frac{\left(\sqrt{-a}d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4b^2} + \frac{\left(\sqrt{-a}d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4b^2} - \frac{\left(\sqrt{-a}d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4b^2}$$

$$= \frac{\sqrt{-a}d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^{5/2}} - \frac{\sqrt{-a}d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4b^{5/2}} + \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2} + \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2} + \frac{\sin(c+dx)}{2b^2} - \frac{x^2\sin(c+dx)}{2b(a+bx^2)} - \frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2} + \frac{\sqrt{-a}d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^{5/2}} + \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^2} + \frac{\sqrt{-a}d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4b^{5/2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.82 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.66

$$\int \frac{x^3\sin(c+dx)}{(a+bx^2)^2}\,dx = \frac{ie^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(\left(2\sqrt{b}+\sqrt{a}d\right)e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\,\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)+\left(2\sqrt{b}-\sqrt{a}d\right)\text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right.\right.}{}$$

```
[In] Integrate[(x^3*Sin[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] (I*E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*((2*Sqrt[b] + Sqrt[a]*d)*E^((2*Sqrt[a]*
d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x] + (2*Sqrt[b] - Sq
rt[a]*d)*ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] - I*d*x]) - I*E^(I*c - (Sqrt[a]*
d)/Sqrt[b])*((2*Sqrt[b] + Sqrt[a]*d)*E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralE
i[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] + (2*Sqrt[b] - Sqrt[a]*d)*ExpIntegralEi[(
Sqrt[a]*d)/Sqrt[b] + I*d*x]) + (4*a*Sqrt[b]*Cos[d*x]*Sin[c])/(a + b*x^2) +
(4*a*Sqrt[b]*Cos[c]*Sin[d*x])/(a + b*x^2))/(8*b^(5/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 852, normalized size of antiderivative = 1.98

method	result
risch	$\frac{i\sqrt{ab}\,\mathrm{e}^{\frac{icb+d\sqrt{ab}}{b}}\,\mathrm{Ei}_1\left(\frac{icb+d\sqrt{ab}-b(idx+ic)}{b}\right)d}{8b^3}-\frac{i\sqrt{ab}\,\mathrm{e}^{\frac{icb-d\sqrt{ab}}{b}}\,\mathrm{Ei}_1\left(\frac{icb-d\sqrt{ab}-b(idx+ic)}{b}\right)d}{8b^3}+\frac{i\mathrm{e}^{\frac{icb+d\sqrt{ab}}{b}}\,\mathrm{Ei}_1\left(\frac{icb+\cdots}{}\right.}{4b^2}$
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int(x^3*sin(d*x+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*I/b^3*(a*b)^(1/2)*exp((I*c*b+d*(a*b)^(1/2))/b)*Ei(1,(I*c*b+d*(a*b)^(1/2
)-b*(I*d*x+I*c))/b)*d-1/8*I/b^3*(a*b)^(1/2)*exp((I*c*b-d*(a*b)^(1/2))/b)*Ei
(1,(I*c*b-d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*d+1/4*I/b^2*exp((I*c*b+d*(a*b)^(1
/2))/b)*Ei(1,(I*c*b+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)+1/4*I/b^2*exp((I*c*b-d*
(a*b)^(1/2))/b)*Ei(1,(I*c*b-d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)+1/8*I/b^3*exp(-
(I*c*b+d*(a*b)^(1/2))/b)*Ei(1,-(I*c*b+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*(a*b)
^(1/2)*d-1/8*I/b^3*exp(-(I*c*b-d*(a*b)^(1/2))/b)*Ei(1,-(I*c*b-d*(a*b)^(1/2)
-b*(I*d*x+I*c))/b)*(a*b)^(1/2)*d-1/4*I/b^2*exp(-(I*c*b+d*(a*b)^(1/2))/b)*Ei
(1,-(I*c*b+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)-1/4*I/b^2*exp(-(I*c*b-d*(a*b)^(1
/2))/b)*Ei(1,-(I*c*b-d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)+1/d^4*(1/2/(2*I*(I*d*x
```

```
+I*c)*b*c-b*(I*d*x+I*c)^2+a*d^2+c^2*b)/b^2/a*(3*I*(I*d*x+I*c)*a*b*c*d^2-I*(
I*d*x+I*c)*b^2*c^3+a^2*d^4-b^2*c^4)*d^2+1/2*c^3*d^3*x/(-2*I*(I*d*x+I*c)*b*c
+b*(I*d*x+I*c)^2-a*d^2-c^2*b)/a-3/2*c^2*d^2*(I*(I*d*x+I*c)*b*c+a*d^2+c^2*b)
/a/b/(2*I*(I*d*x+I*c)*b*c-b*(I*d*x+I*c)^2+a*d^2+c^2*b)-3/2*I*c*d^2*(-I*a*c*
d^2-I*b*c^3-(I*d*x+I*c)*a*d^2+(I*d*x+I*c)*b*c^2)/a/b/(-2*I*(I*d*x+I*c)*b*c+
b*(I*d*x+I*c)^2-a*d^2-c^2*b))*sin(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.68

$$\int \frac{x^3\sin(c+dx)}{(a+bx^2)^2}\,dx = -\frac{\left(2i\,bx^2-(-i\,bx^2-i\,a)\sqrt{\frac{ad^2}{b}}+2i\,a\right)\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}+\left(2i\,bx^2-(i\,bx^2+i\,a)\sqrt{\frac{ad^2}{b}}+2i\,a\right.}{}$$

```
[In] integrate(x^3*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*((2*I*b*x^2 - (-I*b*x^2 - I*a)*sqrt(a*d^2/b) + 2*I*a)*Ei(I*d*x - sqrt(
a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + (2*I*b*x^2 - (I*b*x^2 + I*a)*sqrt(a*d^2
/b) + 2*I*a)*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + (-2*I*b*x^
2 - (I*b*x^2 + I*a)*sqrt(a*d^2/b) - 2*I*a)*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I
*c + sqrt(a*d^2/b)) + (-2*I*b*x^2 - (-I*b*x^2 - I*a)*sqrt(a*d^2/b) - 2*I*a)
*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) - 4*a*sin(d*x + c))/(b
^3*x^2 + a*b^2)
```

Sympy [F]

$$\int \frac{x^3\sin(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x^3\sin(c+dx)}{(a+bx^2)^2}\,dx$$

```
[In] integrate(x**3*sin(d*x+c)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**3*sin(c + d*x)/(a + b*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x^3 \sin(dx+c)}{(bx^2+a)^2}\,dx$$

[In] integrate(x^3*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")

[Out] -1/2*((cos(c)^2 + sin(c)^2)*d*x^2*sin(d*x + c) + ((d^2*x^3*cos(c) - d*x^2*sin(c) - 2*x*cos(c))*cos(d*x + c)^2 + (d^2*x^3*cos(c) - d*x^2*sin(c) - 2*x*cos(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + ((cos(c)^2 + sin(c)^2)*d^2*x^3 - 2*(cos(c)^2 + sin(c)^2)*x)*cos(d*x + c) - 2*(((b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^4 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^3*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^3)*cos(d*x + c)^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^4 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^3*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^3)*sin(d*x + c)^2)*integrate((2*a*d*x*sin(d*x + c) + ((2*a*d^2 + 3*b)*x^2 - a)*cos(d*x + c))/(b^3*d^3*x^6 + 3*a*b^2*d^3*x^4 + 3*a^2*b*d^3*x^2 + a^3*d^3), x) - 2*(((b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^4 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^3*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^3)*cos(d*x + c)^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^4 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^3*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^3)*sin(d*x + c)^2)*integrate((2*a*d*x*sin(d*x + c) + ((2*a*d^2 + 3*b)*x^2 - a)*cos(d*x + c))/((b^3*d^3*x^6 + 3*a*b^2*d^3*x^4 + 3*a^2*b*d^3*x^2 + a^3*d^3)*cos(d*x + c)^2 + (b^3*d^3*x^6 + 3*a*b^2*d^3*x^4 + 3*a^2*b*d^3*x^2 + a^3*d^3)*sin(d*x + c)^2), x) + ((d^2*x^3*sin(c) + d*x^2*cos(c) - 2*x*sin(c))*cos(d*x + c)^2 + (d^2*x^3*sin(c) + d*x^2*cos(c) - 2*x*sin(c))*sin(d*x + c)^2)*sin(d*x + 2*c))/(((b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^4 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^3*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^3)*cos(d*x + c)^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^4 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^3*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^3)*sin(d*x + c)^2)

Giac [F]

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x^3 \sin(dx+c)}{(bx^2+a)^2}\,dx$$

[In] integrate(x^3*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")

[Out] integrate(x^3*sin(d*x + c)/(b*x^2 + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x^3\,\sin(c+d\,x)}{(b\,x^2+a)^2}\,dx$$

```
[In] int((x^3*sin(c + d*x))/(a + b*x^2)^2,x)
[Out] int((x^3*sin(c + d*x))/(a + b*x^2)^2, x)
```

3.67 $\int \frac{x^2 \sin(c+dx)}{(a+bx^2)^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 416

$$\begin{aligned}\int \frac{x^2 \sin(c+dx)}{(a+bx^2)^2}\,dx &= \frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^2} \\ &+ \frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4b^2} \\ &- \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4\sqrt{-a}b^{3/2}} \\ &+ \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4\sqrt{-a}b^{3/2}} - \frac{x\sin(c+dx)}{2b(a+bx^2)} \\ &- \frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}} + \frac{d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^2} \\ &- \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4\sqrt{-a}b^{3/2}} - \frac{d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4b^2}\end{aligned}$$

```
[Out] 1/4*d*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*cos(c-d*(-a)^(1/2)/b^(1/2))/b^2+1/4*d*Ci
(-d*x+d*(-a)^(1/2)/b^(1/2))*cos(c+d*(-a)^(1/2)/b^(1/2))/b^2-1/2*x*sin(d*x+c
)/b/(b*x^2+a)-1/4*d*Si(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2)
)/b^2-1/4*d*Si(d*x-d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/b^2+1/
4*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b^(1/2))/b^(3/2)/(-a)^(1/
2)-1/4*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))/b^(3/2)/(-a
)^(1/2)-1/4*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))/b^(3/2
)/(-a)^(1/2)+1/4*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/
b^(3/2)/(-a)^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3424, 3414, 3384, 3380, 3383, 3427}

$$\int \frac{x^2\sin(c+dx)}{(a+bx^2)^2}\,dx = \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}} - \frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4\sqrt{-a}b^{3/2}} - \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}} - \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4\sqrt{-a}b^{3/2}} + \frac{d\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^2} + \frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^2} + \frac{d\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^2} - \frac{d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^2} - \frac{x\sin(c+dx)}{2b\,(a+bx^2)}$$

```
[In] Int[(x^2*Sin[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] (d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(
4*b^2) + (d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b]
+ d*x])/(4*b^2) - (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a
]*d)/Sqrt[b]])/(4*Sqrt[-a]*b^(3/2)) + (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d
*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(4*Sqrt[-a]*b^(3/2)) - (x*Sin[c + d*x])/
(2*b*(a + b*x^2)) - (Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)
/Sqrt[b] - d*x])/(4*Sqrt[-a]*b^(3/2)) + (d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*Si
nIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*b^2) - (Cos[c - (Sqrt[-a]*d)/Sqrt
[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*Sqrt[-a]*b^(3/2)) - (d*Sin
[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*b^2)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3424

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3427

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\frac{x\sin(c+dx)}{2b(a+bx^2)} + \frac{\int \frac{\sin(c+dx)}{a+bx^2}\,dx}{2b} + \frac{d\int \frac{x\cos(c+dx)}{a+bx^2}\,dx}{2b}$$

$$= -\frac{x\sin(c+dx)}{2b(a+bx^2)} + \frac{\int \left(\frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{2b}$$
$$+ \frac{d\int \left(-\frac{\cos(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cos(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{2b}$$

$$= -\frac{x\sin(c+dx)}{2b(a+bx^2)} - \frac{\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}dx}{4\sqrt{-a}b} - \frac{\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}dx}{4\sqrt{-a}b} - \frac{d\int \frac{\cos(c+dx)}{\sqrt{-a}-\sqrt{b}x}dx}{4b^{3/2}} + \frac{d\int \frac{\cos(c+dx)}{\sqrt{-a}+\sqrt{b}x}dx}{4b^{3/2}}$$

$$= -\frac{x\sin(c+dx)}{2b(a+bx^2)} - \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}dx}{4\sqrt{-a}b} + \frac{\left(d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}dx}{4b^{3/2}}$$
$$+ \frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}dx}{4\sqrt{-a}b} - \frac{\left(d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}dx}{4b^{3/2}}$$
$$- \frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}dx}{4\sqrt{-a}b} - \frac{\left(d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}dx}{4b^{3/2}}$$
$$- \frac{\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}dx}{4\sqrt{-a}b} - \frac{\left(d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}dx}{4b^{3/2}}$$

$$= \frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^2}$$
$$+ \frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4b^2}$$
$$- \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4\sqrt{-a}b^{3/2}}$$
$$+ \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4\sqrt{-a}b^{3/2}} - \frac{x\sin(c+dx)}{2b(a+bx^2)}$$
$$- \frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}} + \frac{d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^2}$$
$$- \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4\sqrt{-a}b^{3/2}} - \frac{d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4b^2}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.01 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.66

$$\int \frac{x^2\sin(c+dx)}{(a+bx^2)^2}dx$$
$$= \frac{\frac{e^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(\left(\sqrt{b}+\sqrt{a}d\right)e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)+\left(-\sqrt{b}+\sqrt{a}d\right)\text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)\right)}{\sqrt{a}} + \frac{e^{ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(\left(\sqrt{b}+\sqrt{a}d\right)e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIn}\right.}{}}{8b^2}$$

```
[In] Integrate[(x^2*Sin[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] ((E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*((Sqrt[b] + Sqrt[a]*d)*E^((2*Sqrt[a]*d)/
Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x] + (-Sqrt[b] + Sqrt[a
]*d)*ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] - I*d*x]))/Sqrt[a] + (E^(I*c - (Sqrt
[a]*d)/Sqrt[b])*((Sqrt[b] + Sqrt[a]*d)*E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegra
lEi[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] + (-Sqrt[b] + Sqrt[a]*d)*ExpIntegralEi[
(Sqrt[a]*d)/Sqrt[b] + I*d*x]))/Sqrt[a] - (4*b*x*Cos[d*x]*Sin[c])/(a + b*x^2
) - (4*b*x*Cos[c]*Sin[d*x])/(a + b*x^2))/(8*b^2)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 748, normalized size of antiderivative = 1.80

method	result
risch	$-\frac{\mathrm{Ei}_{1}\left(\frac{icb+d\sqrt{ab}-b(idx+ic)}{b}\right)\mathrm{e}^{\frac{icb+d\sqrt{ab}}{b}}d}{8b^{2}}-\frac{\mathrm{Ei}_{1}\left(\frac{icb-d\sqrt{ab}-b(idx+ic)}{b}\right)\mathrm{e}^{\frac{icb-d\sqrt{ab}}{b}}d}{8b^{2}}-\frac{\mathrm{Ei}_{1}\left(\frac{icb+d\sqrt{ab}-b(idx+ic)}{b}\right)\mathrm{e}^{\frac{icb-}{}}}{8a\,b^{2}}$
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int(x^2*sin(d*x+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/b^2*Ei(1,(I*c*b+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*exp((I*c*b+d*(a*b)^(1/
2))/b)*d-1/8/b^2*Ei(1,(I*c*b-d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*exp((I*c*b-d*(
a*b)^(1/2))/b)*d-1/8/a/b^2*Ei(1,(I*c*b+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*exp(
(I*c*b+d*(a*b)^(1/2))/b)*(a*b)^(1/2)+1/8/a/b^2*Ei(1,(I*c*b-d*(a*b)^(1/2)-b*
(I*d*x+I*c))/b)*exp((I*c*b-d*(a*b)^(1/2))/b)*(a*b)^(1/2)-1/8/b^2*Ei(1,-(I*c
*b+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*exp(-(I*c*b+d*(a*b)^(1/2))/b)*d-1/8/b^2*
Ei(1,-(I*c*b-d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*exp(-(I*c*b-d*(a*b)^(1/2))/b)*
d+1/8/a/b^2*Ei(1,-(I*c*b+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*(a*b)^(1/2)*exp(-(
I*c*b+d*(a*b)^(1/2))/b)-1/8/a/b^2*Ei(1,-(I*c*b-d*(a*b)^(1/2)-b*(I*d*x+I*c))
/b)*(a*b)^(1/2)*exp(-(I*c*b-d*(a*b)^(1/2))/b)+I/d^3*(1/2/(-2*I*(I*d*x+I*c)*
b*c+b*(I*d*x+I*c)^2-a*d^2-c^2*b)/b/a*(-I*a*c*d^2-I*b*c^3-(I*d*x+I*c)*a*d^2+
(I*d*x+I*c)*b*c^2)*d^2+1/2*I*c^2*d^3*x/(-2*I*(I*d*x+I*c)*b*c+b*(I*d*x+I*c)^
2-a*d^2-c^2*b)/a-I*c*d^2*(I*(I*d*x+I*c)*b*c+a*d^2+c^2*b)/a/b/(2*I*(I*d*x+I*
c)*b*c-b*(I*d*x+I*c)^2+a*d^2+c^2*b))*sin(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.80

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^2)^2}\,dx = -\frac{4\,abdx\sin\left(dx+c\right) - \left(abd^2x^2 + a^2d^2 + (b^2x^2+ab)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(i\,dx - \sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)} - \left(abd^2x^2 + a\right.}{}$$

```
[In] integrate(x^2*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*a*b*d*x*sin(d*x + c) - (a*b*d^2*x^2 + a^2*d^2 + (b^2*x^2 + a*b)*sqr
t(a*d^2/b))*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) - (a*b*d^2*x^
2 + a^2*d^2 - (b^2*x^2 + a*b)*sqrt(a*d^2/b))*Ei(I*d*x + sqrt(a*d^2/b))*e^(I
*c - sqrt(a*d^2/b)) - (a*b*d^2*x^2 + a^2*d^2 + (b^2*x^2 + a*b)*sqrt(a*d^2/b
))*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) - (a*b*d^2*x^2 + a^2
*d^2 - (b^2*x^2 + a*b)*sqrt(a*d^2/b))*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c -
sqrt(a*d^2/b)))/(a*b^3*d*x^2 + a^2*b^2*d)
```

Sympy [F]

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x^2 \sin\left(c+dx\right)}{\left(a+bx^2\right)^2}\,dx$$

```
[In] integrate(x**2*sin(d*x+c)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**2*sin(c + d*x)/(a + b*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x^2 \sin\left(dx+c\right)}{\left(bx^2+a\right)^2}\,dx$$

```
[In] integrate(x^2*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((cos(c)^2 + sin(c)^2)*d*x^2*cos(d*x + c) + 2*(cos(c)^2 + sin(c)^2)*x*
sin(d*x + c) + ((d*x^2*cos(c) - 2*x*sin(c))*cos(d*x + c)^2 + (d*x^2*cos(c)
- 2*x*sin(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b^2*cos(c)^2 + b^2*sin(
c)^2)*d^2*x^4 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^2*x^2 + (a^2*cos(c)^2 + a
^2*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d^2*x^4 +
```

```
2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^2*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^
2)*sin(d*x + c)^2)*integrate(-(2*a*d*x*cos(d*x + c) - (3*b*x^2 - a)*sin(d*x
 + c))/(b^3*d^2*x^6 + 3*a*b^2*d^2*x^4 + 3*a^2*b*d^2*x^2 + a^3*d^2), x) + 2*
(((b^2*cos(c)^2 + b^2*sin(c)^2)*d^2*x^4 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d
^2*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b^2*cos(c)^2
 + b^2*sin(c)^2)*d^2*x^4 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^2*x^2 + (a^2*c
os(c)^2 + a^2*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(-(2*a*d*x*cos(d*x +
c) - (3*b*x^2 - a)*sin(d*x + c))/((b^3*d^2*x^6 + 3*a*b^2*d^2*x^4 + 3*a^2*b*
d^2*x^2 + a^3*d^2)*cos(d*x + c)^2 + (b^3*d^2*x^6 + 3*a*b^2*d^2*x^4 + 3*a^2*
b*d^2*x^2 + a^3*d^2)*sin(d*x + c)^2), x) + ((d*x^2*sin(c) + 2*x*cos(c))*cos
(d*x + c)^2 + (d*x^2*sin(c) + 2*x*cos(c))*sin(d*x + c)^2)*sin(d*x + 2*c))/(
((b^2*cos(c)^2 + b^2*sin(c)^2)*d^2*x^4 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^
2*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b^2*cos(c)^2
+ b^2*sin(c)^2)*d^2*x^4 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^2*x^2 + (a^2*co
s(c)^2 + a^2*sin(c)^2)*d^2)*sin(d*x + c)^2)
```

Giac [F]

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x^2 \sin(dx+c)}{(bx^2+a)^2}\,dx$$

```
[In] integrate(x^2*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
[Out] integrate(x^2*sin(d*x + c)/(b*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x^2\,\sin(c+d\,x)}{(b\,x^2+a)^2}\,dx$$

```
[In] int((x^2*sin(c + d*x))/(a + b*x^2)^2,x)
[Out] int((x^2*sin(c + d*x))/(a + b*x^2)^2, x)
```

3.68 $\int \frac{x\sin(c+dx)}{(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 239

$$\int \frac{x\sin(c+dx)}{(a+bx^2)^2}\,dx = \frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}} - \frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4\sqrt{-a}b^{3/2}} - \frac{\sin(c+dx)}{2b(a+bx^2)} + \frac{d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}} + \frac{d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4\sqrt{-a}b^{3/2}}$$

```
[Out] -1/2*sin(d*x+c)/b/(b*x^2+a)-1/4*d*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*cos(c-d*(-a)
^(1/2)/b^(1/2))/b^(3/2)/(-a)^(1/2)+1/4*d*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*cos(
c+d*(-a)^(1/2)/b^(1/2))/b^(3/2)/(-a)^(1/2)+1/4*d*Si(d*x+d*(-a)^(1/2)/b^(1/2
))*sin(c-d*(-a)^(1/2)/b^(1/2))/b^(3/2)/(-a)^(1/2)-1/4*d*Si(d*x-d*(-a)^(1/2)
/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/b^(3/2)/(-a)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used

$= \{3422, 3415, 3384, 3380, 3383\}$

$$\int \frac{x \sin(c+dx)}{(a+bx^2)^2} \, dx = \frac{d \cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right) \text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}} - \frac{d \cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4\sqrt{-a}b^{3/2}} + \frac{d \sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right) \text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}} + \frac{d \sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4\sqrt{-a}b^{3/2}} - \frac{\sin(c+dx)}{2b\,(a+bx^2)}$$

```
[In] Int[(x*Sin[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] (d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(
4*Sqrt[-a]*b^(3/2)) - (d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a
]*d)/Sqrt[b] + d*x])/(4*Sqrt[-a]*b^(3/2)) - Sin[c + d*x]/(2*b*(a + b*x^2))
+ (d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])
/(4*Sqrt[-a]*b^(3/2)) + (d*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[
-a]*d)/Sqrt[b] + d*x])/(4*Sqrt[-a]*b^(3/2))
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3415

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3422

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))),
x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rubi steps

$$\text{integral} = -\frac{\sin(c+dx)}{2b(a+bx^2)} + \frac{d\int \frac{\cos(c+dx)}{a+bx^2}\,dx}{2b}$$

$$= -\frac{\sin(c+dx)}{2b(a+bx^2)} + \frac{d\int \left(\frac{\sqrt{-a}\cos(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{-a}\cos(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{2b}$$

$$= -\frac{\sin(c+dx)}{2b(a+bx^2)} - \frac{d\int \frac{\cos(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4\sqrt{-a}b} - \frac{d\int \frac{\cos(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4\sqrt{-a}b}$$

$$= -\frac{\sin(c+dx)}{2b(a+bx^2)} - \frac{\left(d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4\sqrt{-a}b}$$
$$- \frac{\left(d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4\sqrt{-a}b}$$
$$+ \frac{\left(d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4\sqrt{-a}b} - \frac{\left(d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4\sqrt{-a}b}$$

$$= \frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}} - \frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4\sqrt{-a}b^{3/2}}$$
$$- \frac{\sin(c+dx)}{2b(a+bx^2)} + \frac{d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}} + \frac{d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4\sqrt{-a}b^{3/2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.99

$$\int \frac{x\sin(c+dx)}{(a+bx^2)^2}\,dx$$
$$= \frac{-\frac{ide^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)-\text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)\right)}{\sqrt{a}}+\frac{ide^{ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}+idx\right)-\text{ExpIn}\right.}{\sqrt{a}}}{8b^{3/2}}$$

[In] Integrate[(x*Sin[c + d*x])/(a + b*x^2)^2,x]

[Out] (((-I)*d*E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*(E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x] - ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] - I*d*x]))/Sqrt[a] + (I*d*E^(I*c - (Sqrt[a]*d)/Sqrt[b])*(E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] - ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] + I*d*x]))/Sqrt[a] - (4*Sqrt[b]*Cos[d*x]*Sin[c])/(a + b*x^2) - (4*Sqrt[b]*Cos[c]*Sin[d*x])/(a + b*x^2))/(8*b^(3/2))

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.72

method	result
risch	$-\frac{id\sqrt{ab}\,\text{Ei}_1\left(\frac{icb+d\sqrt{ab}-b(idx+ic)}{b}\right)e^{\frac{icb+d\sqrt{ab}}{b}}}{8a\,b^2}+\frac{id\sqrt{ab}\,e^{\frac{icb-d\sqrt{ab}}{b}}\,\text{Ei}_1\left(-\frac{-icb+d\sqrt{ab}+b(idx+ic)}{b}\right)}{8a\,b^2}-\frac{id\sqrt{ab}\,e^{-\frac{icb+d\sqrt{}}{b}}}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

[In] int(x*sin(d*x+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)

[Out] -1/8*I*d*(a*b)^(1/2)/a/b^2*Ei(1,(I*c*b+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*exp((I*c*b+d*(a*b)^(1/2))/b)+1/8*I*d*(a*b)^(1/2)/a/b^2*exp((I*c*b-d*(a*b)^(1/2))/b)*Ei(1,-(-I*c*b+d*(a*b)^(1/2)+b*(I*d*x+I*c))/b)-1/8*I*d*(a*b)^(1/2)/a/b^2*exp(-(I*c*b+d*(a*b)^(1/2))/b)*Ei(1,-(I*c*b+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)+1/8*I*d*(a*b)^(1/2)/a/b^2*exp(-(I*c*b-d*(a*b)^(1/2))/b)*Ei(1,(-I*c*b+d*(a*b)^(1/2)+b*(I*d*x+I*c))/b)-1/d^2*(1/2/(2*I*(I*d*x+I*c)*b*c-b*(I*d*x+I*c)^2+a*d^2+c^2*b)/b/a*(I*(I*d*x+I*c)*b*c+a*d^2+c^2*b)*d^2-1/2*c*d^3*x/(-2*I*(I*d*x+I*c)*b*c+b*(I*d*x+I*c)^2-a*d^2-c^2*b)/a)*sin(d*x+c)

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.02

$$\int \frac{x\sin(c+dx)}{(a+bx^2)^2}\,dx$$
$$= \frac{(i\,bx^2+i\,a)\sqrt{\frac{ad^2}{b}}\,\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}+(-i\,bx^2-i\,a)\sqrt{\frac{ad^2}{b}}\,\mathrm{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)}+(-i\,b}{8}$$

```
[In] integrate(x*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((I*b*x^2 + I*a)*sqrt(a*d^2/b)*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(
a*d^2/b)) + (-I*b*x^2 - I*a)*sqrt(a*d^2/b)*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c
 - sqrt(a*d^2/b)) + (-I*b*x^2 - I*a)*sqrt(a*d^2/b)*Ei(-I*d*x - sqrt(a*d^2/b
))*e^(-I*c + sqrt(a*d^2/b)) + (I*b*x^2 + I*a)*sqrt(a*d^2/b)*Ei(-I*d*x + sqr
t(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) - 4*a*sin(d*x + c))/(a*b^2*x^2 + a^2*b
)
```

Sympy [F]

$$\int \frac{x\sin(c+dx)}{(a+bx^2)^2}\,dx=\int \frac{x\sin(c+dx)}{(a+bx^2)^2}\,dx$$

```
[In] integrate(x*sin(d*x+c)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x*sin(c + d*x)/(a + b*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x\sin(c+dx)}{(a+bx^2)^2}\,dx=\int \frac{x\sin(dx+c)}{(bx^2+a)^2}\,dx$$

```
[In] integrate(x*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((cos(c)^2 + sin(c)^2)*x*cos(d*x + c) + (x*cos(d*x + c)^2*cos(c) + x*c
os(c)*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b^2*cos(c)^2 + b^2*sin(c)^2)*d*
x^4 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)
*d)*cos(d*x + c)^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d*x^4 + 2*(a*b*cos(c)^2
 + a*b*sin(c)^2)*d*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*sin(d*x + c)^2)*i
ntegrate(1/2*(3*b*x^2 - a)*cos(d*x + c)/(b^3*d*x^6 + 3*a*b^2*d*x^4 + 3*a^2*
```

```
b*d*x^2 + a^3*d), x) + 2*(((b^2*cos(c)^2 + b^2*sin(c)^2)*d*x^4 + 2*(a*b*cos
(c)^2 + a*b*sin(c)^2)*d*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*cos(d*x + c)
^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d*x^4 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)
*d*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*sin(d*x + c)^2)*integrate(1/2*(3*
b*x^2 - a)*cos(d*x + c)/((b^3*d*x^6 + 3*a*b^2*d*x^4 + 3*a^2*b*d*x^2 + a^3*d
)*cos(d*x + c)^2 + (b^3*d*x^6 + 3*a*b^2*d*x^4 + 3*a^2*b*d*x^2 + a^3*d)*sin(
d*x + c)^2), x) + (x*cos(d*x + c)^2*sin(c) + x*sin(d*x + c)^2*sin(c))*sin(d
*x + 2*c))/(((b^2*cos(c)^2 + b^2*sin(c)^2)*d*x^4 + 2*(a*b*cos(c)^2 + a*b*si
n(c)^2)*d*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*cos(d*x + c)^2 + ((b^2*cos
(c)^2 + b^2*sin(c)^2)*d*x^4 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d*x^2 + (a^2*
cos(c)^2 + a^2*sin(c)^2)*d)*sin(d*x + c)^2)
```

Giac [F]

$$\int \frac{x\sin(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x\sin\left(dx+c\right)}{\left(bx^2+a\right)^2}\,dx$$

```
[In] integrate(x*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
[Out] integrate(x*sin(d*x + c)/(b*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sin(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x\,\sin\left(c+d\,x\right)}{\left(b\,x^2+a\right)^2}\,dx$$

```
[In] int((x*sin(c + d*x))/(a + b*x^2)^2,x)
[Out] int((x*sin(c + d*x))/(a + b*x^2)^2, x)
```

3.69 $\int \frac{\sin(c+dx)}{(a+bx^2)^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 476

$$\begin{aligned}\int \frac{\sin(c+dx)}{(a+bx^2)^2} dx = & -\frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4ab} \\ & -\frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4ab} \\ & +\frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}} \\ & -\frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}} \\ & -\frac{\sin(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sin(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} \\ & +\frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}-\frac{d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4ab} \\ & +\frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4ab}\end{aligned}$$

```
[Out] -1/4*d*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*cos(c-d*(-a)^(1/2)/b^(1/2))/a/b-1/4*d*C
i(-d*x+d*(-a)^(1/2)/b^(1/2))*cos(c+d*(-a)^(1/2)/b^(1/2))/a/b+1/4*d*Si(d*x+d
*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))/a/b+1/4*d*Si(d*x-d*(-a)^(1
/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/a/b-1/4*cos(c+d*(-a)^(1/2)/b^(1/2)
)*Si(d*x-d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(1/2)+1/4*cos(c-d*(-a)^(1/2)/b^
(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(1/2)+1/4*Ci(d*x+d*(-a)^(1
```

```
/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(1/2)-1/4*Ci(-d*x+d*(
-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(1/2)-1/4*sin(d
*x+c)/a/b^(1/2)/((-a)^(1/2)-x*b^(1/2))+1/4*sin(d*x+c)/a/b^(1/2)/((-a)^(1/2)
+x*b^(1/2))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3414, 3378, 3384, 3380, 3383}

$$\begin{aligned}\int \frac{\sin(c+dx)}{(a+bx^2)^2}\,dx = &-\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}} \\ &+\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}} \\ &-\frac{d\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4ab} \\ &-\frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4ab} \\ &-\frac{d\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4ab}+\frac{d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4ab} \\ &+\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}} \\ &-\frac{\sin(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sin(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\end{aligned}$$

```
[In] Int[Sin[c + d*x]/(a + b*x^2)^2,x]
```

```
[Out] -1/4*(d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*
x])/(a*b) - (d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[
b] + d*x])/(4*a*b) + (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt
[-a]*d)/Sqrt[b]])/(4*(-a)^(3/2)*Sqrt[b]) - (CosIntegral[(Sqrt[-a]*d)/Sqrt[b
] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(4*(-a)^(3/2)*Sqrt[b]) - Sin[c + d*
x]/(4*a*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)) + Sin[c + d*x]/(4*a*Sqrt[b]*(Sqrt[-
a] + Sqrt[b]*x)) + (Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/
Sqrt[b] - d*x])/(4*(-a)^(3/2)*Sqrt[b]) - (d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*S
inIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*a*b) + (Cos[c - (Sqrt[-a]*d)/Sqr
t[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(3/2)*Sqrt[b]) + (d*
Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*a
*b)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{b\sin(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}-bx\right)^2}-\frac{b\sin(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}+bx\right)^2}-\frac{b\sin(c+dx)}{2a\left(-ab-b^2x^2\right)}\right)dx \\
&= -\frac{b\int\frac{\sin(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}\,dx}{4a}-\frac{b\int\frac{\sin(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^2}\,dx}{4a}-\frac{b\int\frac{\sin(c+dx)}{-ab-b^2x^2}\,dx}{2a} \\
&= -\frac{\sin(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sin(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} \\
&\quad -\frac{b\int\left(-\frac{\sqrt{-a}\sin(c+dx)}{2ab\left(\sqrt{-a}-\sqrt{b}x\right)}-\frac{\sqrt{-a}\sin(c+dx)}{2ab\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{2a}+\frac{d\int\frac{\cos(c+dx)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{4a}-\frac{d\int\frac{\cos(c+dx)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{4a}
\end{aligned}$$

$$= -\frac{\sin(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sin(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} + \frac{\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4(-a)^{3/2}} + \frac{\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4(-a)^{3/2}}$$
$$- \frac{\left(d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{4a} + \frac{\left(d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{4a}$$
$$+ \frac{\left(d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{4a} + \frac{\left(d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{4a}$$

$$= -\frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4ab}$$
$$- \frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4ab}$$
$$- \frac{\sin(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sin(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}$$
$$- \frac{d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4ab} + \frac{d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4ab}$$
$$+ \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4(-a)^{3/2}} - \frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4(-a)^{3/2}}$$
$$+ \frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4(-a)^{3/2}} + \frac{\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4(-a)^{3/2}}$$

$$
\begin{aligned}
&= -\frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4ab} \\
&\quad - \frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4ab} \\
&\quad + \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}} \\
&\quad - \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}} \\
&\quad - \frac{\sin(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sin(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} \\
&\quad + \frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}} - \frac{d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4ab} \\
&\quad + \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{3/2}\sqrt{b}} + \frac{d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4ab}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.95 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.59

$$
\int \frac{\sin(c+dx)}{(a+bx^2)^2}\,dx
= \frac{\frac{e^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(\left(\sqrt{b}-\sqrt{a}d\right)e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)-\left(\sqrt{b}+\sqrt{a}d\right)\text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)\right)}{b}+\frac{e^{ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(\left(\sqrt{b}-\sqrt{a}d\right)e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpInte}\right.}{}}{8a^{3/2}}
$$

```
[In] Integrate[Sin[c + d*x]/(a + b*x^2)^2,x]
```

```
[Out] ((E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*((Sqrt[b] - Sqrt[a]*d)*E^((2*Sqrt[a]*d)/
Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x] - (Sqrt[b] + Sqrt[a]
*d)*ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] - I*d*x]))/b + (E^(I*c - (Sqrt[a]*d)/
Sqrt[b])*((Sqrt[b] - Sqrt[a]*d)*E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((
Sqrt[a]*d)/Sqrt[b]) + I*d*x] - (Sqrt[b] + Sqrt[a]*d)*ExpIntegralEi[(Sqrt[a]
*d)/Sqrt[b] + I*d*x]))/b + (4*Sqrt[a]*x*Cos[d*x]*Sin[c])/(a + b*x^2) + (4*S
qrt[a]*x*Cos[c]*Sin[d*x])/(a + b*x^2))/(8*a^(3/2))
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.03

method	result
derivativedivides	$d^{3}\left(\frac{\sin(dx+c)\left(\frac{dx+c}{2a\,d^{2}}-\frac{c}{2a\,d^{2}}\right)}{a\,d^{2}+c^{2}b-2bc(dx+c)+b(dx+c)^{2}}-\frac{\mathrm{Si}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\cos\left(\frac{d\sqrt{-ab}+cb}{b}\right)+\mathrm{Ci}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\sin\left(\frac{d\sqrt{-ab}+}{b}\right.}{4a\,d^{2}b\left(-\frac{d\sqrt{-ab}+cb}{b}+c\right)}\right.$
default	$d^{3}\left(\frac{\sin(dx+c)\left(\frac{dx+c}{2a\,d^{2}}-\frac{c}{2a\,d^{2}}\right)}{a\,d^{2}+c^{2}b-2bc(dx+c)+b(dx+c)^{2}}-\frac{\mathrm{Si}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\cos\left(\frac{d\sqrt{-ab}+cb}{b}\right)+\mathrm{Ci}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\sin\left(\frac{d\sqrt{-ab}+}{b}\right.}{4a\,d^{2}b\left(-\frac{d\sqrt{-ab}+cb}{b}+c\right)}\right.$
risch	$\frac{d\,\mathrm{e}^{\frac{icb+d\sqrt{ab}}{b}}\mathrm{Ei}_{1}\left(\frac{icb+d\sqrt{ab}-b(idx+ic)}{b}\right)}{8ab}+\frac{d\,\mathrm{e}^{\frac{icb-d\sqrt{ab}}{b}}\mathrm{Ei}_{1}\left(\frac{icb-d\sqrt{ab}-b(idx+ic)}{b}\right)}{8ab}-\frac{\sqrt{ab}\,\mathrm{e}^{\frac{icb+d\sqrt{ab}}{b}}\mathrm{Ei}_{1}\left(\frac{icb+d\sqrt{ab}-}{b}\right.}{8a^{2}b}$

```
[In] int(sin(d*x+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^3*(sin(d*x+c)*(1/2/a/d^2*(d*x+c)-1/2*c/a/d^2)/(a*d^2+c^2*b-2*b*c*(d*x+c)+
b*(d*x+c)^2)-1/4/a/d^2/b/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(Si(d*x+c-(d*(-a*b)^(1
/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*si
n((d*(-a*b)^(1/2)+c*b)/b))-1/4/a/d^2/b/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c
+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/
2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-1/4/a/b/d^2*(-Si(d*x+c-(d*(-a*b)^(1
/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*co
s((d*(-a*b)^(1/2)+c*b)/b))-1/4/a/b/d^2*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*si
n((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1
/2)-c*b)/b)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.70

$$\int \frac{\sin(c+dx)}{(a+bx^2)^2}\,dx$$

$$= \frac{4\,abdx\sin\left(dx+c\right)-\left(abd^{2}x^{2}+a^{2}d^{2}-\left(b^{2}x^{2}+ab\right)\sqrt{\frac{ad^{2}}{b}}\right)\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^{2}}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^{2}}{b}}\right)}-\left(abd^{2}x^{2}+a^{2}\right.}{}$$

```
[In] integrate(sin(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*a*b*d*x*sin(d*x + c) - (a*b*d^2*x^2 + a^2*d^2 - (b^2*x^2 + a*b)*sqrt
(a*d^2/b))*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) - (a*b*d^2*x^2
 + a^2*d^2 + (b^2*x^2 + a*b)*sqrt(a*d^2/b))*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*
c - sqrt(a*d^2/b)) - (a*b*d^2*x^2 + a^2*d^2 - (b^2*x^2 + a*b)*sqrt(a*d^2/b)
)*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) - (a*b*d^2*x^2 + a^2*
```

```
d^2 + (b^2*x^2 + a*b)*sqrt(a*d^2/b))*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - s
qrt(a*d^2/b)))/(a^2*b^2*d*x^2 + a^3*b*d)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{(a+bx^2)^2}\,dx=\int \frac{\sin{(c+dx)}}{(a+bx^2)^2}\,dx$$

```
[In] integrate(sin(d*x+c)/(b*x**2+a)**2,x)
[Out] Integral(sin(c + d*x)/(a + b*x**2)**2, x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{(a+bx^2)^2}\,dx=\int \frac{\sin{(dx+c)}}{(bx^2+a)^2}\,dx$$

```
[In] integrate(sin(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
[Out] integrate(sin(d*x + c)/(b*x^2 + a)^2, x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{(a+bx^2)^2}\,dx=\int \frac{\sin{(dx+c)}}{(bx^2+a)^2}\,dx$$

```
[In] integrate(sin(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
[Out] integrate(sin(d*x + c)/(b*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{(a+bx^2)^2}\,dx=\int \frac{\sin{(c+d\,x)}}{(b\,x^2+a)^2}\,dx$$

```
[In] int(sin(c + d*x)/(a + b*x^2)^2,x)
[Out] int(sin(c + d*x)/(a + b*x^2)^2, x)
```

3.70 $\int \frac{\sin(c+dx)}{x(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 435

$$\begin{aligned}\int \frac{\sin(c+dx)}{x(a+bx^2)^2}\,dx &= \frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}} \\ &- \frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{3/2}\sqrt{b}} \\ &+ \frac{\text{CosIntegral}(dx)\sin(c)}{a^2} - \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2} \\ &- \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2} + \frac{\sin(c+dx)}{2a(a+bx^2)} \\ &+ \frac{\cos(c)\text{Si}(dx)}{a^2} + \frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2} \\ &+ \frac{d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}} - \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^2} \\ &+ \frac{d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{3/2}\sqrt{b}}\end{aligned}$$

```
[Out] cos(c)*Si(d*x)/a^2-1/2*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b^(1
/2))/a^2-1/2*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))/a^2+C
i(d*x)*sin(c)/a^2+1/2*sin(d*x+c)/a/(b*x^2+a)-1/2*Ci(d*x+d*(-a)^(1/2)/b^(1/2
))*sin(c-d*(-a)^(1/2)/b^(1/2))/a^2-1/2*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*sin(c+
d*(-a)^(1/2)/b^(1/2))/a^2-1/4*d*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*cos(c-d*(-a)^(
1/2)/b^(1/2))/(-a)^(3/2)/b^(1/2)+1/4*d*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*cos(c+
```

```
d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(1/2)+1/4*d*Si(d*x+d*(-a)^(1/2)/b^(1/2))
*sin(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(1/2)-1/4*d*Si(d*x-d*(-a)^(1/2)/b
^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3426, 3384, 3380, 3383, 3422, 3415}

$$\int \frac{\sin(c+dx)}{x(a+bx^2)^2}\,dx = -\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2} - \frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2} + \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2} - \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2} + \frac{\sin(c)\text{CosIntegral}(dx)}{a^2} + \frac{\cos(c)\text{Si}(dx)}{a^2} + \frac{d\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}} - \frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}} + \frac{d\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}} + \frac{d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}} + \frac{\sin(c+dx)}{2a(a+bx^2)}$$

```
[In] Int[Sin[c + d*x]/(x*(a + b*x^2)^2),x]
```

```
[Out] (d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(
4*(-a)^(3/2)*Sqrt[b]) - (d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[
-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(3/2)*Sqrt[b]) + (CosIntegral[d*x]*Sin[c])/a
^2 - (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]
)/(2*a^2) - (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/S
qrt[b]])/(2*a^2) + Sin[c + d*x]/(2*a*(a + b*x^2)) + (Cos[c]*SinIntegral[d*x
])/a^2 + (Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] -
d*x])/(2*a^2) + (d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/S
qrt[b] - d*x])/(4*(-a)^(3/2)*Sqrt[b]) - (Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinI
ntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^2) + (d*Sin[c - (Sqrt[-a]*d)/Sqrt
[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(3/2)*Sqrt[b])
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3415

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3422

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))),
x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{\sin(c+dx)}{a^2x} - \frac{bx\sin(c+dx)}{a\left(a+bx^2\right)^2} - \frac{bx\sin(c+dx)}{a^2\left(a+bx^2\right)}\right)\,dx \\ &= \frac{\int \frac{\sin(c+dx)}{x}\,dx}{a^2} - \frac{b\int \frac{x\sin(c+dx)}{a+bx^2}\,dx}{a^2} - \frac{b\int \frac{x\sin(c+dx)}{\left(a+bx^2\right)^2}\,dx}{a}\end{aligned}$$

$$= \frac{\sin(c+dx)}{2a\left(a+bx^2\right)} - \frac{b\int \left(-\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{a^2} - \frac{d\int \frac{\cos(c+dx)}{a+bx^2}dx}{2a} + \frac{\cos(c)\int \frac{\sin(dx)}{x}dx}{a^2} + \frac{\sin(c)\int \frac{\cos(dx)}{x}dx}{a^2}$$

$$= \frac{\text{CosIntegral}(dx)\sin(c)}{a^2} + \frac{\sin(c+dx)}{2a\left(a+bx^2\right)} + \frac{\cos(c)\text{Si}(dx)}{a^2} + \frac{\sqrt{b}\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}dx}{2a^2} - \frac{\sqrt{b}\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}dx}{2a^2} - \frac{d\int \left(\frac{\sqrt{-a}\cos(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{-a}\cos(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{2a}$$

$$= \frac{\text{CosIntegral}(dx)\sin(c)}{a^2} + \frac{\sin(c+dx)}{2a\left(a+bx^2\right)} + \frac{\cos(c)\text{Si}(dx)}{a^2} - \frac{d\int \frac{\cos(c+dx)}{\sqrt{-a}-\sqrt{b}x}dx}{4(-a)^{3/2}} - \frac{d\int \frac{\cos(c+dx)}{\sqrt{-a}+\sqrt{b}x}dx}{4(-a)^{3/2}} - \frac{\left(\sqrt{b}\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}dx}{2a^2} - \frac{\left(\sqrt{b}\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}dx}{2a^2} - \frac{\left(\sqrt{b}\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}dx}{2a^2} + \frac{\left(\sqrt{b}\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}dx}{2a^2}$$

$$= \frac{\text{CosIntegral}(dx)\sin(c)}{a^2} - \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2} - \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2} + \frac{\sin(c+dx)}{2a\left(a+bx^2\right)} + \frac{\cos(c)\text{Si}(dx)}{a^2} + \frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2} - \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^2} - \frac{\left(d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}dx}{4(-a)^{3/2}} - \frac{\left(d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}dx}{4(-a)^{3/2}} + \frac{\left(d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}dx}{4(-a)^{3/2}} - \frac{\left(d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}dx}{4(-a)^{3/2}}$$

$$
\begin{aligned}
&= \frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}} \\
&\quad - \frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{3/2}\sqrt{b}} \\
&\quad + \frac{\text{CosIntegral}(dx)\sin(c)}{a^2} - \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2} \\
&\quad - \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2} + \frac{\sin(c+dx)}{2a(a+bx^2)} + \frac{\cos(c)\text{Si}(dx)}{a^2} \\
&\quad + \frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2} + \frac{d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}} \\
&\quad - \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^2} + \frac{d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{3/2}\sqrt{b}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 419, normalized size of antiderivative = 0.96

$$
\int \frac{\sin(c+dx)}{x\,(a+bx^2)^2}\,dx
$$

$$
= \frac{\frac{i\sqrt{a}de^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)-\text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)\right)}{\sqrt{b}} - 2ie^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}} -\right.\right.}{}
$$

```
[In] Integrate[Sin[c + d*x]/(x*(a + b*x^2)^2),x]
```

```
[Out] ((I*Sqrt[a]*d*E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*(E^((2*Sqrt[a]*d)/Sqrt[b])*E
xpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x] - ExpIntegralEi[(Sqrt[a]*d)/Sq
rt[b] - I*d*x]))/Sqrt[b] - (2*I)*E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*(E^((2*Sq
rt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x] + ExpIntegr
alEi[(Sqrt[a]*d)/Sqrt[b] - I*d*x]) - (I*Sqrt[a]*d*E^(I*c - (Sqrt[a]*d)/Sqrt
[b])*(E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) + I*d*
x] - ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] + I*d*x]))/Sqrt[b] + (2*I)*E^(I*c -
(Sqrt[a]*d)/Sqrt[b])*(E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)
/Sqrt[b]) + I*d*x] + ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] + I*d*x]) + (4*a*Cos
[d*x]*Sin[c])/(a + b*x^2) + (4*a*Cos[c]*Sin[d*x])/(a + b*x^2) + 8*(CosInteg
ral[d*x]*Sin[c] + Cos[c]*SinIntegral[d*x]))/(8*a^2)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{\sin(dx+c)d^2}{2a\left(a\,d^2+c^2b-2bc(dx+c)+b(dx+c)^2\right)}+\frac{\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin(c)}{a^2}-\frac{\mathrm{Si}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\cos\left(\frac{d\sqrt{-ab}+cb}{b}\right)+\mathrm{Ci}\left(d\right.}{2a^2}$
default	$\frac{\sin(dx+c)d^2}{2a\left(a\,d^2+c^2b-2bc(dx+c)+b(dx+c)^2\right)}+\frac{\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin(c)}{a^2}-\frac{\mathrm{Si}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\cos\left(\frac{d\sqrt{-ab}+cb}{b}\right)+\mathrm{Ci}\left(d\right.}{2a^2}$
risch	$\frac{ie^{\frac{icb+d\sqrt{ab}}{b}}\mathrm{Ei}_1\left(\frac{icb+d\sqrt{ab}-b(idx+ic)}{b}\right)d}{8a\sqrt{ab}}-\frac{ie^{\frac{icb-d\sqrt{ab}}{b}}\mathrm{Ei}_1\left(\frac{icb-d\sqrt{ab}-b(idx+ic)}{b}\right)d}{8a\sqrt{ab}}-\frac{ie^{\frac{icb+d\sqrt{ab}}{b}}\mathrm{Ei}_1\left(\frac{icb+d\sqrt{ab}-b(idx\right.}{b}}{4a^2}$

```
[In] int(sin(d*x+c)/x/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sin(d*x+c)*d^2/a/(a*d^2+c^2*b-2*b*c*(d*x+c)+b*(d*x+c)^2)+1/a^2*(Si(d*x)
*cos(c)+Ci(d*x)*sin(c))-1/2/a^2*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-
a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b
)/b))-1/2/a^2*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)
-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))+1/4*d^2/a/b/
(-(d*(-a*b)^(1/2)+c*b)/b+c)*(-Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b
)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b
))+1/4*d^2/a/b/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)
*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)
^(1/2)-c*b)/b))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.74

$$\int \frac{\sin(c+dx)}{x\,(a+bx^2)^2}\,dx =$$
$$-\frac{\left(-2i\,bx^2-(-i\,bx^2-i\,a)\sqrt{\frac{ad^2}{b}}-2i\,a\right)\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}+\left(-2i\,bx^2-(i\,bx^2+i\,a)\sqrt{\frac{ad^2}{b}}-\right.}{}$$

```
[In] integrate(sin(d*x+c)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*((-2*I*b*x^2 - (-I*b*x^2 - I*a)*sqrt(a*d^2/b) - 2*I*a)*Ei(I*d*x - sqrt
(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + (-2*I*b*x^2 - (I*b*x^2 + I*a)*sqrt(a*d
^2/b) - 2*I*a)*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + (2*I*b*x
^2 - (I*b*x^2 + I*a)*sqrt(a*d^2/b) + 2*I*a)*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-
I*c + sqrt(a*d^2/b)) + (2*I*b*x^2 - (-I*b*x^2 - I*a)*sqrt(a*d^2/b) + 2*I*a)
```

```
*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) - 8*(b*x^2 + a)*cos_in
tegral(d*x)*sin(c) - 8*(b*x^2 + a)*cos(c)*sin_integral(d*x) - 4*a*sin(d*x +
 c))/(a^2*b*x^2 + a^3)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{x\,(a+bx^2)^2}\,dx=\int \frac{\sin{(c+dx)}}{x\,(a+bx^2)^2}\,dx$$

```
[In] integrate(sin(d*x+c)/x/(b*x**2+a)**2,x)
[Out] Integral(sin(c + d*x)/(x*(a + b*x**2)**2), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x\,(a+bx^2)^2}\,dx=\int \frac{\sin{(dx+c)}}{(bx^2+a)^2x}\,dx$$

```
[In] integrate(sin(d*x+c)/x/(b*x^2+a)^2,x, algorithm="maxima")
[Out] integrate(sin(d*x + c)/((b*x^2 + a)^2*x), x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{x\,(a+bx^2)^2}\,dx=\int \frac{\sin{(dx+c)}}{(bx^2+a)^2x}\,dx$$

```
[In] integrate(sin(d*x+c)/x/(b*x^2+a)^2,x, algorithm="giac")
[Out] integrate(sin(d*x + c)/((b*x^2 + a)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x\,(a+bx^2)^2}\,dx=\int \frac{\sin{(c+d\,x)}}{x\,(b\,x^2+a)^2}\,dx$$

```
[In] int(sin(c + d*x)/(x*(a + b*x^2)^2),x)
[Out] int(sin(c + d*x)/(x*(a + b*x^2)^2), x)
```

3.71 $\int \frac{\sin(c+dx)}{x^2(a+bx^2)^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 501

$$\begin{aligned}\int \frac{\sin(c+dx)}{x^2(a+bx^2)^2} \, dx = {} & \frac{d\cos(c)\,\mathrm{CosIntegral}(dx)}{a^2} + \frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4a^2} \\ & + \frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4a^2} \\ & + \frac{3\sqrt{b}\,\mathrm{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{5/2}} \\ & - \frac{3\sqrt{b}\,\mathrm{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{5/2}} \\ & - \frac{\sin(c+dx)}{a^2x} + \frac{\sqrt{b}\sin(c+dx)}{4a^2\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{\sqrt{b}\sin(c+dx)}{4a^2\left(\sqrt{-a}+\sqrt{b}x\right)} \\ & - \frac{d\sin(c)\mathrm{Si}(dx)}{a^2} + \frac{3\sqrt{b}\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{5/2}} \\ & + \frac{d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4a^2} \\ & + \frac{3\sqrt{b}\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{5/2}} \\ & - \frac{d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4a^2}\end{aligned}$$

```
[Out] d*Ci(d*x)*cos(c)/a^2+1/4*d*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*cos(c-d*(-a)^(1/2)/
b^(1/2))/a^2+1/4*d*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*cos(c+d*(-a)^(1/2)/b^(1/2)
```

```
)/a^2-d*Si(d*x)*sin(c)/a^2-sin(d*x+c)/a^2/x-1/4*d*Si(d*x+d*(-a)^(1/2)/b^(1/
2))*sin(c-d*(-a)^(1/2)/b^(1/2))/a^2-1/4*d*Si(d*x-d*(-a)^(1/2)/b^(1/2))*sin(
c+d*(-a)^(1/2)/b^(1/2))/a^2-3/4*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(
1/2)/b^(1/2))*b^(1/2)/(-a)^(5/2)+3/4*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(
-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(5/2)+3/4*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*sin(
c-d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(5/2)-3/4*Ci(-d*x+d*(-a)^(1/2)/b^(1/2)
)*sin(c+d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(5/2)+1/4*sin(d*x+c)*b^(1/2)/a^2
/((-a)^(1/2)-x*b^(1/2))-1/4*sin(d*x+c)*b^(1/2)/a^2/((-a)^(1/2)+x*b^(1/2))
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3426, 3378, 3384, 3380, 3383, 3414}

$$\int \frac{\sin(c+dx)}{x^2\left(a+bx^2\right)^2}\,dx = \frac{d\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4a^2} + \frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4a^2} + \frac{d\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4a^2} - \frac{d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4a^2} + \frac{\sqrt{b}\sin(c+dx)}{4a^2\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{\sqrt{b}\sin(c+dx)}{4a^2\left(\sqrt{-a}+\sqrt{b}x\right)} + \frac{d\cos(c)\text{CosIntegral}(dx)}{a^2} - \frac{d\sin(c)\text{Si}(dx)}{a^2} - \frac{\sin(c+dx)}{a^2x} + \frac{3\sqrt{b}\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{5/2}} - \frac{3\sqrt{b}\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{5/2}} + \frac{3\sqrt{b}\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{5/2}} + \frac{3\sqrt{b}\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{5/2}}$$

```
[In] Int[Sin[c + d*x]/(x^2*(a + b*x^2)^2),x]
```

```
[Out] (d*Cos[c]*CosIntegral[d*x])/a^2 + (d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosInteg
ral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*a^2) + (d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]
```

```
*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*a^2) + (3*Sqrt[b]*CosIntegral[
(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(4*(-a)^(5/2)) -
 (3*Sqrt[b]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sq
rt[b]])/(4*(-a)^(5/2)) - Sin[c + d*x]/(a^2*x) + (Sqrt[b]*Sin[c + d*x])/(4*a
^2*(Sqrt[-a] - Sqrt[b]*x)) - (Sqrt[b]*Sin[c + d*x])/(4*a^2*(Sqrt[-a] + Sqrt
[b]*x)) - (d*Sin[c]*SinIntegral[d*x])/a^2 + (3*Sqrt[b]*Cos[c + (Sqrt[-a]*d)
/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*(-a)^(5/2)) + (d*Sin[
c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*a^2)
+ (3*Sqrt[b]*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b]
 + d*x])/(4*(-a)^(5/2)) - (d*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqr
t[-a]*d)/Sqrt[b] + d*x])/(4*a^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
```

```
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\sin(c+dx)}{a^2x^2} - \frac{b\sin(c+dx)}{a\,(a+bx^2)^2} - \frac{b\sin(c+dx)}{a^2\,(a+bx^2)} \right) dx$$

$$= \frac{\int \frac{\sin(c+dx)}{x^2}\,dx}{a^2} - \frac{b\int \frac{\sin(c+dx)}{a+bx^2}\,dx}{a^2} - \frac{b\int \frac{\sin(c+dx)}{(a+bx^2)^2}\,dx}{a}$$

$$= -\frac{\sin(c+dx)}{a^2x} - \frac{b\int \left(\frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)} \right) dx}{a^2}$$
$$- \frac{b\int \left(-\frac{b\sin(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}-bx\right)^2} - \frac{b\sin(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}+bx\right)^2} - \frac{b\sin(c+dx)}{2a(-ab-b^2x^2)} \right) dx}{a} + \frac{d\int \frac{\cos(c+dx)}{x}\,dx}{a^2}$$

$$= -\frac{\sin(c+dx)}{a^2x} + \frac{b\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2(-a)^{5/2}} + \frac{b\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2(-a)^{5/2}}$$
$$+ \frac{b^2\int \frac{\sin(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}\,dx}{4a^2} + \frac{b^2\int \frac{\sin(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^2}\,dx}{4a^2} + \frac{b^2\int \frac{\sin(c+dx)}{-ab-b^2x^2}\,dx}{2a^2}$$
$$+ \frac{(d\cos(c))\int \frac{\cos(dx)}{x}\,dx}{a^2} - \frac{(d\sin(c))\int \frac{\sin(dx)}{x}\,dx}{a^2}$$

$$= \frac{d\cos(c)\,\text{CosIntegral}(dx)}{a^2} - \frac{\sin(c+dx)}{a^2x} + \frac{\sqrt{b}\sin(c+dx)}{4a^2\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{\sqrt{b}\sin(c+dx)}{4a^2\left(\sqrt{-a}+\sqrt{b}x\right)}$$
$$- \frac{d\sin(c)\text{Si}(dx)}{a^2} + \frac{b^2\int \left(-\frac{\sqrt{-a}\sin(c+dx)}{2ab\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{\sqrt{-a}\sin(c+dx)}{2ab\left(\sqrt{-a}+\sqrt{b}x\right)} \right) dx}{2a^2}$$
$$- \frac{(bd)\int \frac{\cos(c+dx)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{4a^2} + \frac{(bd)\int \frac{\cos(c+dx)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{4a^2}$$
$$+ \frac{\left(b\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2(-a)^{5/2}} - \frac{\left(b\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2(-a)^{5/2}}$$
$$+ \frac{\left(b\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2(-a)^{5/2}} + \frac{\left(b\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2(-a)^{5/2}}$$

$$= \frac{d\cos(c)\,\text{CosIntegral}(dx)}{a^2} + \frac{\sqrt{b}\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}} + dx\right)\sin\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2(-a)^{5/2}} - \frac{\sqrt{b}\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)\sin\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2(-a)^{5/2}} - \frac{\sin(c+dx)}{a^2x} + \frac{\sqrt{b}\sin(c+dx)}{4a^2\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{\sqrt{b}\sin(c+dx)}{4a^2\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{d\sin(c)\text{Si}(dx)}{a^2} + \frac{\sqrt{b}\cos\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2(-a)^{5/2}} + \frac{\sqrt{b}\cos\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}} + dx\right)}{2(-a)^{5/2}} + \frac{b\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}dx}{4(-a)^{5/2}} + \frac{b\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}dx}{4(-a)^{5/2}} + \frac{\left(bd\cos\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}\sqrt{b}+bx}dx}{4a^2} - \frac{\left(bd\cos\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}\sqrt{b}-bx}dx}{4a^2} - \frac{\left(bd\sin\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}\sqrt{b}+bx}dx}{4a^2} - \frac{\left(bd\sin\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}\sqrt{b}-bx}dx}{4a^2}$$

$$= \frac{d\cos(c)\,\text{CosIntegral}(dx)}{a^2} + \frac{d\cos\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{4a^2} + \frac{d\cos\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}} + dx\right)}{4a^2} + \frac{\sqrt{b}\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}} + dx\right)\sin\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2(-a)^{5/2}} - \frac{\sqrt{b}\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)\sin\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2(-a)^{5/2}} - \frac{\sin(c+dx)}{a^2x} + \frac{\sqrt{b}\sin(c+dx)}{4a^2\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{\sqrt{b}\sin(c+dx)}{4a^2\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{d\sin(c)\text{Si}(dx)}{a^2} + \frac{\sqrt{b}\cos\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2(-a)^{5/2}} + \frac{d\sin\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{4a^2} + \frac{\sqrt{b}\cos\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}} + dx\right)}{2(-a)^{5/2}} - \frac{d\sin\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}} + dx\right)}{4a^2} + \frac{\left(b\cos\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}dx}{4(-a)^{5/2}} - \frac{\left(b\cos\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}dx}{4(-a)^{5/2}} + \frac{\left(b\sin\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}dx}{4(-a)^{5/2}} + \frac{\left(b\sin\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}dx}{4(-a)^{5/2}}$$

$$= \frac{d\cos(c)\,\text{CosIntegral}(dx)}{a^2} + \frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4a^2} + \frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4a^2} + \frac{3\sqrt{b}\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{5/2}} - \frac{3\sqrt{b}\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{5/2}} - \frac{\sin(c+dx)}{a^2x} + \frac{\sqrt{b}\sin(c+dx)}{4a^2\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{\sqrt{b}\sin(c+dx)}{4a^2\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{d\sin(c)\text{Si}(dx)}{a^2} + \frac{3\sqrt{b}\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{5/2}} + \frac{d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4a^2} + \frac{3\sqrt{b}\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{5/2}} - \frac{d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4a^2}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.30 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.66

$$\int \frac{\sin(c+dx)}{x^2\left(a+bx^2\right)^2}\,dx = \frac{e^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(-\left(\left(3\sqrt{b}-\sqrt{a}d\right)e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)\right)+\left(3\sqrt{b}+\sqrt{a}d\right)\text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}-\right.\right.}{}$$

```
[In] Integrate[Sin[c + d*x]/(x^2*(a + b*x^2)^2),x]
```

```
[Out] (E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*(-((3*Sqrt[b] - Sqrt[a]*d)*E^((2*Sqrt[a]*
d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x]) + (3*Sqrt[b] + S
qrt[a]*d)*ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] - I*d*x]) + E^(I*c - (Sqrt[a]*d
)/Sqrt[b])*(-((3*Sqrt[b] - Sqrt[a]*d)*E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegral
Ei[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]) + (3*Sqrt[b] + Sqrt[a]*d)*ExpIntegralEi
[(Sqrt[a]*d)/Sqrt[b] + I*d*x]) - (4*Sqrt[a]*(2*a + 3*b*x^2)*Cos[d*x]*Sin[c]
)/(x*(a + b*x^2)) - (4*Sqrt[a]*(2*a + 3*b*x^2)*Cos[c]*Sin[d*x])/(x*(a + b*x
^2)) + 8*Sqrt[a]*d*(Cos[c]*CosIntegral[d*x] - Sin[c]*SinIntegral[d*x]))/(8*
a^(5/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 622, normalized size of antiderivative = 1.24

method	result
risch	$-\frac{d\,\mathrm{e}^{\frac{icb+d\sqrt{ab}}{b}}\,\mathrm{Ei}_1\left(\frac{icb+d\sqrt{ab}-b(idx+ic)}{b}\right)}{8a^2}-\frac{d\,\mathrm{e}^{\frac{icb-d\sqrt{ab}}{b}}\,\mathrm{Ei}_1\left(\frac{icb-d\sqrt{ab}-b(idx+ic)}{b}\right)}{8a^2}+\frac{3\,\mathrm{e}^{\frac{icb+d\sqrt{ab}}{b}}\,\mathrm{Ei}_1\left(\frac{icb+d\sqrt{ab}-b(i}{b}\right.}{8a^2\sqrt{ab}}$
derivativedivides	$d\left(\frac{-\frac{\sin(dx+c)}{dx}-\mathrm{Si}(dx)\sin(c)+\mathrm{Ci}(dx)\cos(c)}{a^2}-\frac{b\left(-\frac{\mathrm{Si}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\cos\left(\frac{d\sqrt{-ab}+cb}{b}\right)+\mathrm{Ci}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\sin\left(\right.}{2b\left(-\frac{d\sqrt{-ab}+cb}{b}+c\right)}\right.}{}\right.$
default	$d\left(\frac{-\frac{\sin(dx+c)}{dx}-\mathrm{Si}(dx)\sin(c)+\mathrm{Ci}(dx)\cos(c)}{a^2}-\frac{b\left(-\frac{\mathrm{Si}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\cos\left(\frac{d\sqrt{-ab}+cb}{b}\right)+\mathrm{Ci}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\sin\left(\right.}{2b\left(-\frac{d\sqrt{-ab}+cb}{b}+c\right)}\right.}{}\right.$

```
[In] int(sin(d*x+c)/x^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*d/a^2*exp((I*c*b+d*(a*b)^(1/2))/b)*Ei(1,(I*c*b+d*(a*b)^(1/2)-b*(I*d*x+
I*c))/b)-1/8*d/a^2*exp((I*c*b-d*(a*b)^(1/2))/b)*Ei(1,(I*c*b-d*(a*b)^(1/2)-b
*(I*d*x+I*c))/b)+3/8/a^2/(a*b)^(1/2)*exp((I*c*b+d*(a*b)^(1/2))/b)*Ei(1,(I*c
*b+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*b-3/8/a^2/(a*b)^(1/2)*exp((I*c*b-d*(a*b)
^(1/2))/b)*Ei(1,(I*c*b-d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*b-1/2*d/a^2*Ei(1,-I*
d*x)*exp(I*c)-1/8*d/a^2*exp(-(I*c*b-d*(a*b)^(1/2))/b)*Ei(1,-(I*c*b-d*(a*b)^
(1/2)-b*(I*d*x+I*c))/b)-1/8*d/a^2*exp(-(I*c*b+d*(a*b)^(1/2))/b)*Ei(1,-(I*c*
b+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)+3/8/a^2/(a*b)^(1/2)*exp(-(I*c*b-d*(a*b)^(
1/2))/b)*Ei(1,-(I*c*b-d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*b-3/8/a^2/(a*b)^(1/2)
*exp(-(I*c*b+d*(a*b)^(1/2))/b)*Ei(1,-(I*c*b+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)
*b-1/2*d/a^2*Ei(1,I*d*x)*exp(-I*c)-1/2*(6*I*(I*d*x+I*c)*b*c-3*b*(I*d*x+I*c)
^2+2*a*d^2+3*c^2*b)/a^2/(2*I*(I*d*x+I*c)*b*c-b*(I*d*x+I*c)^2+a*d^2+c^2*b)/x
*sin(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 400, normalized size of antiderivative = 0.80

$$\int \frac{\sin(c+dx)}{x^2\,(a+bx^2)^2}\,dx$$

$$=\frac{8\,(abd^2x^3+a^2d^2x)\cos(c)\,\mathrm{Ci}(dx)+\left(abd^2x^3+a^2d^2x-3\,(b^2x^3+abx)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right.}}{}$$

```
[In] integrate(sin(d*x+c)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(8*(a*b*d^2*x^3 + a^2*d^2*x)*cos(c)*cos_integral(d*x) + (a*b*d^2*x^3 +
a^2*d^2*x - 3*(b^2*x^3 + a*b*x)*sqrt(a*d^2/b))*Ei(I*d*x - sqrt(a*d^2/b))*e^
(I*c + sqrt(a*d^2/b)) + (a*b*d^2*x^3 + a^2*d^2*x + 3*(b^2*x^3 + a*b*x)*sqrt
(a*d^2/b))*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + (a*b*d^2*x^3
 + a^2*d^2*x - 3*(b^2*x^3 + a*b*x)*sqrt(a*d^2/b))*Ei(-I*d*x - sqrt(a*d^2/b)
)*e^(-I*c + sqrt(a*d^2/b)) + (a*b*d^2*x^3 + a^2*d^2*x + 3*(b^2*x^3 + a*b*x)
*sqrt(a*d^2/b))*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) - 8*(a*
b*d^2*x^3 + a^2*d^2*x)*sin(c)*sin_integral(d*x) - 4*(3*a*b*d*x^2 + 2*a^2*d)
*sin(d*x + c))/(a^3*b*d*x^3 + a^4*d*x)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{x^2\left(a+bx^2\right)^2}\,dx = \int \frac{\sin\left(c+dx\right)}{x^2\left(a+bx^2\right)^2}\,dx$$

```
[In] integrate(sin(d*x+c)/x**2/(b*x**2+a)**2,x)
```

```
[Out] Integral(sin(c + d*x)/(x**2*(a + b*x**2)**2), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x^2\left(a+bx^2\right)^2}\,dx = \int \frac{\sin\left(dx+c\right)}{\left(bx^2+a\right)^2x^2}\,dx$$

```
[In] integrate(sin(d*x+c)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x^2 + a)^2*x^2), x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{x^2\left(a+bx^2\right)^2}\,dx = \int \frac{\sin\left(dx+c\right)}{\left(bx^2+a\right)^2x^2}\,dx$$

```
[In] integrate(sin(d*x+c)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/((b*x^2 + a)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x^2 (a+bx^2)^2} \, dx = \int \frac{\sin (c+d\,x)}{x^2\,(b\,x^2+a)^2} \, dx$$

```
[In] int(sin(c + d*x)/(x^2*(a + b*x^2)^2),x)
[Out] int(sin(c + d*x)/(x^2*(a + b*x^2)^2), x)
```

3.72 $\int \frac{x^3 \sin(c+dx)}{(a+bx^2)^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 476

$$\begin{aligned}\int \frac{x^3 \sin(c+dx)}{(a+bx^2)^3} \, dx = & -\frac{dx \cos(c+dx)}{8b^2(a+bx^2)} + \frac{3d \cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}b^{5/2}} \\ & -\frac{3d \cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16\sqrt{-a}b^{5/2}} \\ & -\frac{d^2 \text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right) \sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16b^3} \\ & -\frac{d^2 \text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right) \sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16b^3} - \frac{x^2 \sin(c+dx)}{4b(a+bx^2)^2} \\ & -\frac{\sin(c+dx)}{4b^2(a+bx^2)} + \frac{d^2 \cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16b^3} \\ & +\frac{3d \sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}b^{5/2}} \\ & -\frac{d^2 \cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16b^3} \\ & +\frac{3d \sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16\sqrt{-a}b^{5/2}}\end{aligned}$$

```
[Out] -1/8*d*x*cos(d*x+c)/b^2/(b*x^2+a)-1/16*d^2*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d
*x-d*(-a)^(1/2)/b^(1/2))/b^3-1/16*d^2*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*
(-a)^(1/2)/b^(1/2))/b^3-1/4*x^2*sin(d*x+c)/b/(b*x^2+a)^2-1/4*sin(d*x+c)/b^2
```

```
/(b*x^2+a)-1/16*d^2*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2)
)/b^3-1/16*d^2*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/b^
3-3/16*d*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*cos(c-d*(-a)^(1/2)/b^(1/2))/b^(5/2)/(
-a)^(1/2)+3/16*d*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*cos(c+d*(-a)^(1/2)/b^(1/2))/
b^(5/2)/(-a)^(1/2)+3/16*d*Si(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b
^(1/2))/b^(5/2)/(-a)^(1/2)-3/16*d*Si(d*x-d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)
^(1/2)/b^(1/2))/b^(5/2)/(-a)^(1/2)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3424, 3422, 3415, 3384, 3380, 3383, 3425, 3426}

$$\begin{aligned}\int \frac{x^3\sin(c+dx)}{(a+bx^2)^3}\,dx = {} & \frac{3d\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}b^{5/2}} \\ & -\frac{3d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16\sqrt{-a}b^{5/2}} \\ & +\frac{3d\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}b^{5/2}} \\ & +\frac{3d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16\sqrt{-a}b^{5/2}} \\ & -\frac{d^2\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16b^3} \\ & -\frac{d^2\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16b^3} \\ & +\frac{d^2\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16b^3} \\ & -\frac{d^2\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16b^3} \\ & -\frac{\sin(c+dx)}{4b^2(a+bx^2)}-\frac{dx\cos(c+dx)}{8b^2(a+bx^2)}-\frac{x^2\sin(c+dx)}{4b(a+bx^2)^2}\end{aligned}$$

```
[In] Int[(x^3*Sin[c + d*x])/(a + b*x^2)^3,x]
```

```
[Out] -1/8*(d*x*Cos[c + d*x])/(b^2*(a + b*x^2)) + (3*d*Cos[c + (Sqrt[-a]*d)/Sqrt[
b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*Sqrt[-a]*b^(5/2)) - (3*d*C
os[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*S
qrt[-a]*b^(5/2)) - (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sq
```

```
rt[-a]*d)/Sqrt[b]])/(16*b^3) - (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]
*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*b^3) - (x^2*Sin[c + d*x])/(4*b*(a + b*x
^2)^2) - Sin[c + d*x]/(4*b^2*(a + b*x^2)) + (d^2*Cos[c + (Sqrt[-a]*d)/Sqrt[
b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*b^3) + (3*d*Sin[c + (Sqrt[
-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*Sqrt[-a]*b^(5/
2)) - (d^2*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] +
 d*x])/(16*b^3) + (3*d*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*
d)/Sqrt[b] + d*x])/(16*Sqrt[-a]*b^(5/2))
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3415

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3422

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))),
x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rule 3424

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
```

```
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3425

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cos[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Cos[c + d*x], x], x] + Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Sin[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^2\sin(c+dx)}{4b\left(a+bx^2\right)^2}+\frac{\int\frac{x\sin(c+dx)}{\left(a+bx^2\right)^2}\,dx}{2b}+\frac{d\int\frac{x^2\cos(c+dx)}{\left(a+bx^2\right)^2}\,dx}{4b}\\
&= -\frac{dx\cos(c+dx)}{8b^2\left(a+bx^2\right)}-\frac{x^2\sin(c+dx)}{4b\left(a+bx^2\right)^2}-\frac{\sin(c+dx)}{4b^2\left(a+bx^2\right)}\\
&\quad+\frac{d\int\frac{\cos(c+dx)}{a+bx^2}\,dx}{8b^2}+\frac{d\int\frac{\cos(c+dx)}{a+bx^2}\,dx}{4b^2}-\frac{d^2\int\frac{x\sin(c+dx)}{a+bx^2}\,dx}{8b^2}\\
&= -\frac{dx\cos(c+dx)}{8b^2\left(a+bx^2\right)}-\frac{x^2\sin(c+dx)}{4b\left(a+bx^2\right)^2}-\frac{\sin(c+dx)}{4b^2\left(a+bx^2\right)}+\frac{d\int\left(\frac{\sqrt{-a}\cos(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\cos(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{8b^2}\\
&\quad+\frac{d\int\left(\frac{\sqrt{-a}\cos(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\cos(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{4b^2}-\frac{d^2\int\left(-\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{8b^2}\\
&= -\frac{dx\cos(c+dx)}{8b^2\left(a+bx^2\right)}-\frac{x^2\sin(c+dx)}{4b\left(a+bx^2\right)^2}-\frac{\sin(c+dx)}{4b^2\left(a+bx^2\right)}-\frac{d\int\frac{\cos(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16\sqrt{-a}b^2}\\
&\quad-\frac{d\int\frac{\cos(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16\sqrt{-a}b^2}-\frac{d\int\frac{\cos(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{8\sqrt{-a}b^2}-\frac{d\int\frac{\cos(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{8\sqrt{-a}b^2}+\frac{d^2\int\frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16b^{5/2}}\\
&\quad-\frac{d^2\int\frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16b^{5/2}}
\end{aligned}$$

$$= -\frac{dx\cos(c+dx)}{8b^2(a+bx^2)} - \frac{x^2\sin(c+dx)}{4b(a+bx^2)^2} - \frac{\sin(c+dx)}{4b^2(a+bx^2)}$$
$$- \frac{\left(d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16\sqrt{-a}b^2} - \frac{\left(d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{8\sqrt{-a}b^2}$$
$$- \frac{\left(d^2\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16b^{5/2}} - \frac{\left(d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16\sqrt{-a}b^2}$$
$$- \frac{\left(d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{8\sqrt{-a}b^2} - \frac{\left(d^2\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16b^{5/2}}$$
$$+ \frac{\left(d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16\sqrt{-a}b^2} + \frac{\left(d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{8\sqrt{-a}b^2}$$
$$- \frac{\left(d^2\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16b^{5/2}} - \frac{\left(d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16\sqrt{-a}b^2}$$
$$- \frac{\left(d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{8\sqrt{-a}b^2} + \frac{\left(d^2\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16b^{5/2}}$$

$$= -\frac{dx\cos(c+dx)}{8b^2(a+bx^2)} + \frac{3d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}b^{5/2}}$$
$$- \frac{3d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16\sqrt{-a}b^{5/2}}$$
$$- \frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16b^3}$$
$$- \frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16b^3} - \frac{x^2\sin(c+dx)}{4b(a+bx^2)^2} - \frac{\sin(c+dx)}{4b^2(a+bx^2)}$$
$$+ \frac{d^2\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16b^3} + \frac{3d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}b^{5/2}}$$
$$- \frac{d^2\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16b^3} + \frac{3d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16\sqrt{-a}b^{5/2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.48 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.69

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^2)^3} \, dx$$
$$= \frac{-\frac{ide^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(\left(3\sqrt{b}+\sqrt{a}d\right)e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)+\left(-3\sqrt{b}+\sqrt{a}d\right)\text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)\right)}{\sqrt{a}}+\frac{ide^{ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(\left(3\sqrt{b}+\sqrt{a}d\right)e^{\frac{2\sqrt{\ldots}}{\ldots}}\ldots\right.}{\ldots}}{\ldots}$$

[In] Integrate[(x^3*Sin[c + d*x])/(a + b*x^2)^3,x]

[Out] (((-I)*d*E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*((3*Sqrt[b] + Sqrt[a]*d)*E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x] + (-3*Sqrt[b] + Sqrt[a]*d)*ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] - I*d*x]))/Sqrt[a] + (I*d*E^(I*c - (Sqrt[a]*d)/Sqrt[b])*((3*Sqrt[b] + Sqrt[a]*d)*E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] + (-3*Sqrt[b] + Sqrt[a]*d)*ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] + I*d*x]))/Sqrt[a] - (4*b*Cos[d*x]*(d*x*(a + b*x^2)*Cos[c] + 2*(a + 2*b*x^2)*Sin[c]))/(a + b*x^2)^2 + (4*b*(-2*(a + 2*b*x^2)*Cos[c] + d*x*(a + b*x^2)*Sin[c])*Sin[d*x])/(a + b*x^2)^2)/(32*b^3)

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.10 (sec) , antiderivative size = 640, normalized size of antiderivative = 1.34

method	result
risch	$-\frac{id^2 e^{\frac{icb+d\sqrt{ab}}{b}}\,\mathrm{Ei}_1\left(\frac{icb+d\sqrt{ab}-b(idx+ic)}{b}\right)}{32b^3}-\frac{id^2 e^{\frac{icb-d\sqrt{ab}}{b}}\,\mathrm{Ei}_1\left(-\frac{-icb+d\sqrt{ab}+b(idx+ic)}{b}\right)}{32b^3}-\frac{3id\sqrt{ab}\,e^{\frac{icb+d\sqrt{ab}}{b}}\,\mathrm{Ei}_1\left(\ldots\right.}{32a\,b^{\ldots}}$
derivativedivides	Expression too large to display
default	Expression too large to display

[In] int(x^3*sin(d*x+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)

[Out] -1/32*I*d^2/b^3*exp((I*c*b+d*(a*b)^(1/2))/b)*Ei(1,(I*c*b+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)-1/32*I*d^2/b^3*exp((I*c*b-d*(a*b)^(1/2))/b)*Ei(1,-(-I*c*b+d*(a*b)^(1/2)+b*(I*d*x+I*c))/b)-3/32*I*d/a/b^3*(a*b)^(1/2)*exp((I*c*b+d*(a*b)^(1/2))/b)*Ei(1,(I*c*b+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)+3/32*I*d/a/b^3*(a*b)^(1/2)*exp((I*c*b-d*(a*b)^(1/2))/b)*Ei(1,-(-I*c*b+d*(a*b)^(1/2)+b*(I*d*x+I*c))/b)+1/32*I*d^2/b^3*Ei(1,-(I*c*b+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*exp(-(I*c*b+d*(a*b)^(1/2))/b)+1/32*I*d^2/b^3*exp(-(I*c*b-d*(a*b)^(1/2))/b)*Ei(1,(-I*

```
c*b+d*(a*b)^(1/2)+b*(I*d*x+I*c))/b)-3/32*I*d/a/b^3*Ei(1,-(I*c*b+d*(a*b)^(1/
2)-b*(I*d*x+I*c))/b)*(a*b)^(1/2)*exp(-(I*c*b+d*(a*b)^(1/2))/b)+3/32*I*d/a/b
^3*exp(-(I*c*b-d*(a*b)^(1/2))/b)*Ei(1,(-I*c*b+d*(a*b)^(1/2)+b*(I*d*x+I*c))/
b)*(a*b)^(1/2)+1/8/a*(a*b*d^5*x^3+a^2*d^5*x)/b^2/(-b^2*d^4*x^4-2*a*b*d^4*x^
2-a^2*d^4)*cos(d*x+c)-1/8/d^2*(-4*a^2*b*d^6*x^2-2*a^3*d^6)/a^2/b^2/(-b^2*d^
4*x^4-2*a*b*d^4*x^2-a^2*d^4)*sin(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.03

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^2)^3}\,dx = -\frac{\left(-i\,ab^2d^2x^4 - 2i\,a^2bd^2x^2 - i\,a^3d^2 + 3\left(-i\,b^3x^4 - 2i\,ab^2x^2 - i\,a^2b\right)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(i\,dx - \sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}}{}$$

```
[In] integrate(x^3*sin(d*x+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/32*((-I*a*b^2*d^2*x^4 - 2*I*a^2*b*d^2*x^2 - I*a^3*d^2 + 3*(-I*b^3*x^4 -
2*I*a*b^2*x^2 - I*a^2*b)*sqrt(a*d^2/b))*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c +
sqrt(a*d^2/b)) + (-I*a*b^2*d^2*x^4 - 2*I*a^2*b*d^2*x^2 - I*a^3*d^2 + 3*(I*b
^3*x^4 + 2*I*a*b^2*x^2 + I*a^2*b)*sqrt(a*d^2/b))*Ei(I*d*x + sqrt(a*d^2/b))*
e^(I*c - sqrt(a*d^2/b)) + (I*a*b^2*d^2*x^4 + 2*I*a^2*b*d^2*x^2 + I*a^3*d^2
+ 3*(I*b^3*x^4 + 2*I*a*b^2*x^2 + I*a^2*b)*sqrt(a*d^2/b))*Ei(-I*d*x - sqrt(a
*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) + (I*a*b^2*d^2*x^4 + 2*I*a^2*b*d^2*x^2 +
I*a^3*d^2 + 3*(-I*b^3*x^4 - 2*I*a*b^2*x^2 - I*a^2*b)*sqrt(a*d^2/b))*Ei(-I*d
*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) + 4*(a*b^2*d*x^3 + a^2*b*d*x)*
cos(d*x + c) + 8*(2*a*b^2*x^2 + a^2*b)*sin(d*x + c))/(a*b^5*x^4 + 2*a^2*b^4
*x^2 + a^3*b^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^2)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*sin(d*x+c)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3\sin(c+dx)}{(a+bx^2)^3}\,dx = \int \frac{x^3\sin\left(dx+c\right)}{\left(bx^2+a\right)^3}\,dx$$

[In] integrate(x^3*sin(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")

[Out] -1/2*(3*(cos(c)^2 + sin(c)^2)*d*x^2*sin(d*x + c) + ((d^2*x^3*cos(c) - 3*d*x^2*sin(c) - 12*x*cos(c))*cos(d*x + c)^2 + (d^2*x^3*cos(c) - 3*d*x^2*sin(c) - 12*x*cos(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + ((cos(c)^2 + sin(c)^2)*d^2*x^3 - 12*(cos(c)^2 + sin(c)^2)*x)*cos(d*x + c) - 2*(((b^3*cos(c)^2 + b^3*sin(c)^2)*d^3*x^6 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^3*x^4 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^3*x^2 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^3)*cos(d*x + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d^3*x^6 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^3*x^4 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^3*x^2 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^3)*sin(d*x + c)^2)*integrate(3*(3*a*d*x*sin(d*x + c) + ((a*d^2 + 10*b)*x^2 - 2*a)*cos(d*x + c))/(b^4*d^3*x^8 + 4*a*b^3*d^3*x^6 + 6*a^2*b^2*d^3*x^4 + 4*a^3*b*d^3*x^2 + a^4*d^3), x) - 2*(((b^3*cos(c)^2 + b^3*sin(c)^2)*d^3*x^6 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^3*x^4 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^3*x^2 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^3)*cos(d*x + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d^3*x^6 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^3*x^4 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^3*x^2 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^3)*sin(d*x + c)^2)*integrate(3*(3*a*d*x*sin(d*x + c) + ((a*d^2 + 10*b)*x^2 - 2*a)*cos(d*x + c))/((b^4*d^3*x^8 + 4*a*b^3*d^3*x^6 + 6*a^2*b^2*d^3*x^4 + 4*a^3*b*d^3*x^2 + a^4*d^3)*cos(d*x + c)^2 + (b^4*d^3*x^8 + 4*a*b^3*d^3*x^6 + 6*a^2*b^2*d^3*x^4 + 4*a^3*b*d^3*x^2 + a^4*d^3)*sin(d*x + c)^2), x) + ((d^2*x^3*sin(c) + 3*d*x^2*cos(c) - 12*x*sin(c))*cos(d*x + c)^2 + (d^2*x^3*sin(c) + 3*d*x^2*cos(c) - 12*x*sin(c))*sin(d*x + c)^2)*sin(d*x + 2*c))/(((b^3*cos(c)^2 + b^3*sin(c)^2)*d^3*x^6 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^3*x^4 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^3*x^2 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^3)*cos(d*x + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d^3*x^6 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^3*x^4 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^3*x^2 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^3)*sin(d*x + c)^2)

Giac [F]

$$\int \frac{x^3\sin(c+dx)}{(a+bx^2)^3}\,dx = \int \frac{x^3\sin\left(dx+c\right)}{\left(bx^2+a\right)^3}\,dx$$

[In] integrate(x^3*sin(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")

[Out] integrate(x^3*sin(d*x + c)/(b*x^2 + a)^3, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^2)^3}\,dx = \int \frac{x^3\,\sin\left(c+d\,x\right)}{{\left(b\,x^2+a\right)}^3}\,dx$$

```
[In] int((x^3*sin(c + d*x))/(a + b*x^2)^3,x)
[Out] int((x^3*sin(c + d*x))/(a + b*x^2)^3, x)
```

3.73 $\int \frac{x^2 \sin(c+dx)}{(a+bx^2)^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 746

$$\int \frac{x^2\sin(c+dx)}{(a+bx^2)^3}\,dx = -\frac{d\cos(c+dx)}{8b^2(a+bx^2)} - \frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2}$$
$$-\frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16ab^2}$$
$$+\frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{3/2}b^{3/2}}$$
$$+\frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16\sqrt{-a}b^{5/2}}$$
$$-\frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{3/2}b^{3/2}}$$
$$-\frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16\sqrt{-a}b^{5/2}}$$
$$-\frac{\sin(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sin(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)}$$
$$-\frac{x\sin(c+dx)}{4b(a+bx^2)^2}+\frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}}$$
$$+\frac{d^2\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}b^{5/2}}$$
$$-\frac{d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2}+\frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{3/2}b^{3/2}}$$
$$+\frac{d^2\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16\sqrt{-a}b^{5/2}}$$
$$+\frac{d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16ab^2}$$

```
[Out] -1/8*d*cos(d*x+c)/b^2/(b*x^2+a)-1/16*d*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*cos(c-d
*(-a)^(1/2)/b^(1/2))/a/b^2-1/16*d*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*cos(c+d*(-a
)^(1/2)/b^(1/2))/a/b^2-1/16*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)
/b^(1/2))/(-a)^(3/2)/b^(3/2)+1/16*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)
^(1/2)/b^(1/2))/(-a)^(3/2)/b^(3/2)-1/4*x*sin(d*x+c)/b/(b*x^2+a)^2+1/16*Ci(d
*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(3/2)+1/1
6*d*Si(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))/a/b^2-1/16*Ci(
```

```
-d*x+d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(3/2)+1
/16*d*Si(d*x-d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/a/b^2-1/16*d
^2*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b^(1/2))/b^(5/2)/(-a)^(1
/2)+1/16*d^2*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))/b^(5/
2)/(-a)^(1/2)+1/16*d^2*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1
/2))/b^(5/2)/(-a)^(1/2)-1/16*d^2*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)
^(1/2)/b^(1/2))/b^(5/2)/(-a)^(1/2)-1/16*sin(d*x+c)/a/b^(3/2)/((-a)^(1/2)-x*
b^(1/2))+1/16*sin(d*x+c)/a/b^(3/2)/((-a)^(1/2)+x*b^(1/2))
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 746, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used

$= \{3424, 3414, 3378, 3384, 3380, 3383, 3423\}$

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^2)^3}\,dx = \frac{d^2 \sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16\sqrt{-a}b^{5/2}} - \frac{d^2 \sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}b^{5/2}} + \frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{3/2}b^{3/2}} - \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}} + \frac{d^2 \cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}b^{5/2}} + \frac{d^2 \cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16\sqrt{-a}b^{5/2}} + \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}} + \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{3/2}b^{3/2}} - \frac{\sin(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sin(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{d\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2} - \frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16ab^2} - \frac{d\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2} + \frac{d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16ab^2} - \frac{d\cos(c+dx)}{8b^2(a+bx^2)} - \frac{x\sin(c+dx)}{4b(a+bx^2)^2}$$

```
[In] Int[(x^2*Sin[c + d*x])/(a + b*x^2)^3,x]
```

```
[Out] -1/8*(d*Cos[c + d*x])/(b^2*(a + b*x^2)) - (d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*
CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a*b^2) - (d*Cos[c - (Sqrt[-a]*
d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a*b^2) + (CosInteg
ral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(3/
2)*b^(3/2)) + (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a
]*d)/Sqrt[b]])/(16*Sqrt[-a]*b^(5/2)) - (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] -
d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(3/2)*b^(3/2)) - (d^2*CosInteg
```

```
ral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*Sqrt[-a]
*b^(5/2)) - Sin[c + d*x]/(16*a*b^(3/2)*(Sqrt[-a] - Sqrt[b]*x)) + Sin[c + d*
x]/(16*a*b^(3/2)*(Sqrt[-a] + Sqrt[b]*x)) - (x*Sin[c + d*x])/(4*b*(a + b*x^2
)^2) + (Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*
x])/(16*(-a)^(3/2)*b^(3/2)) + (d^2*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegra
l[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*Sqrt[-a]*b^(5/2)) - (d*Sin[c + (Sqrt[-a]
*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a*b^2) + (Cos[c -
 (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(3
/2)*b^(3/2)) + (d^2*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/
Sqrt[b] + d*x])/(16*Sqrt[-a]*b^(5/2)) + (d*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*Si
nIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a*b^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3423

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_
), x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Cos[c + d*x]/(b*n*(p + 1))),
```

```
x] + Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Sin[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rule 3424

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x\sin(c+dx)}{4b\left(a+bx^2\right)^2}+\frac{\int \frac{\sin(c+dx)}{\left(a+bx^2\right)^2}\,dx}{4b}+\frac{d\int \frac{x\cos(c+dx)}{\left(a+bx^2\right)^2}\,dx}{4b}\\
&= -\frac{d\cos(c+dx)}{8b^2\left(a+bx^2\right)}-\frac{x\sin(c+dx)}{4b\left(a+bx^2\right)^2}\\
&\quad+\frac{\int \left(-\frac{b\sin(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}-bx\right)^2}-\frac{b\sin(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}+bx\right)^2}-\frac{b\sin(c+dx)}{2a\left(-ab-b^2x^2\right)}\right)dx}{4b}-\frac{d^2\int \frac{\sin(c+dx)}{a+bx^2}\,dx}{8b^2}\\
&= -\frac{d\cos(c+dx)}{8b^2\left(a+bx^2\right)}-\frac{x\sin(c+dx)}{4b\left(a+bx^2\right)^2}-\frac{\int \frac{\sin(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}\,dx}{16a}-\frac{\int \frac{\sin(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^2}\,dx}{16a}\\
&\quad-\frac{\int \frac{\sin(c+dx)}{-ab-b^2x^2}\,dx}{8a}-\frac{d^2\int \left(\frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{8b^2}\\
&= -\frac{d\cos(c+dx)}{8b^2\left(a+bx^2\right)}-\frac{\sin(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sin(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)}\\
&\quad-\frac{x\sin(c+dx)}{4b\left(a+bx^2\right)^2}-\frac{\int \left(-\frac{\sqrt{-a}\sin(c+dx)}{2ab\left(\sqrt{-a}-\sqrt{b}x\right)}-\frac{\sqrt{-a}\sin(c+dx)}{2ab\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{8a}\\
&\quad+\frac{d\int \frac{\cos(c+dx)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16ab}-\frac{d\int \frac{\cos(c+dx)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16ab}+\frac{d^2\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16\sqrt{-a}b^2}+\frac{d^2\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16\sqrt{-a}b^2}
\end{aligned}
$$

$$= -\frac{d\cos(c+dx)}{8b^2\left(a+bx^2\right)} - \frac{\sin(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sin(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{x\sin(c+dx)}{4b\left(a+bx^2\right)^2} + \frac{\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16(-a)^{3/2}b} + \frac{\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16(-a)^{3/2}b} - \frac{\left(d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16ab} + \frac{\left(d^2\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16\sqrt{-a}b^2} + \frac{\left(d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16ab} - \frac{\left(d^2\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16\sqrt{-a}b^2} + \frac{\left(d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16ab} + \frac{\left(d^2\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16\sqrt{-a}b^2} + \frac{\left(d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16ab} + \frac{\left(d^2\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16\sqrt{-a}b^2}$$

$$= -\frac{d\cos(c+dx)}{8b^2\left(a+bx^2\right)} - \frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2} - \frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16ab^2} + \frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16\sqrt{-a}b^{5/2}} - \frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16\sqrt{-a}b^{5/2}} - \frac{\sin(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sin(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{x\sin(c+dx)}{4b\left(a+bx^2\right)^2} + \frac{d^2\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}b^{5/2}} - \frac{d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2} + \frac{d^2\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16\sqrt{-a}b^{5/2}} + \frac{d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16ab^2} + \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16(-a)^{3/2}b} - \frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16(-a)^{3/2}b} + \frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16(-a)^{3/2}b} + \frac{\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16(-a)^{3/2}b}$$

$$= -\frac{d\cos(c+dx)}{8b^2(a+bx^2)} - \frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2}$$
$$- \frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16ab^2}$$
$$+ \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{3/2}b^{3/2}}$$
$$+ \frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16\sqrt{-a}b^{5/2}}$$
$$- \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{3/2}b^{3/2}}$$
$$- \frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16\sqrt{-a}b^{5/2}}$$
$$- \frac{\sin(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sin(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{x\sin(c+dx)}{4b(a+bx^2)^2}$$
$$+ \frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}} + \frac{d^2\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}b^{5/2}}$$
$$- \frac{d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2} + \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{3/2}b^{3/2}}$$
$$+ \frac{d^2\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16\sqrt{-a}b^{5/2}} + \frac{d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16ab^2}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.36 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.49

$$\int \frac{x^2\sin(c+dx)}{(a+bx^2)^3}\,dx$$
$$= \frac{e^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(\left(b-\sqrt{a}\sqrt{b}d-ad^2\right)e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)-\left(b+\sqrt{a}\sqrt{b}d-ad^2\right)\text{ExpIntegralEi}\right.}{}$$

```
[In] Integrate[(x^2*Sin[c + d*x])/(a + b*x^2)^3,x]
```

```
[Out] (E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*((b - Sqrt[a]*Sqrt[b]*d - a*d^2)*E^((2*Sq
rt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x] - (b + Sqrt
[a]*Sqrt[b]*d - a*d^2)*ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] - I*d*x]) + E^(I*c
```

 - (Sqrt[a]*d)/Sqrt[b])*((b - Sqrt[a]*Sqrt[b]*d - a*d^2)*E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] - (b + Sqrt[a]*Sqrt[b]*d - a*d^2)*ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] + I*d*x]) - (4*Sqrt[a]*Sqrt[b]*Cos[d*x]*(a*d*(a + b*x^2)*Cos[c] + b*x*(a - b*x^2)*Sin[c]))/(a + b*x^2)^2 + (4*Sqrt[a]*Sqrt[b]*(b*x*(-a + b*x^2)*Cos[c] + a*d*(a + b*x^2)*Sin[c])*Sin[d*x])/(a + b*x^2)^2)/(32*a^(3/2)*b^(5/2))

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 912, normalized size of antiderivative = 1.22

method	result
risch	$\frac{d^2 \operatorname{Ei}_1\left(\frac{icb+d\sqrt{ab}-b(idx+ic)}{b}\right) e^{\frac{icb+d\sqrt{ab}}{b}} \sqrt{ab}}{32b^3a} - \frac{d^2 \operatorname{Ei}_1\left(\frac{icb-d\sqrt{ab}-b(idx+ic)}{b}\right) e^{\frac{icb-d\sqrt{ab}}{b}} \sqrt{ab}}{32b^3a} + \frac{d \operatorname{Ei}_1\left(\frac{icb+d\sqrt{ab}-b(idx+}{b}\right.}{32b^2a}$
derivativedivides	Expression too large to display
default	Expression too large to display

[In] int(x^2*sin(d*x+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)

[Out] 1/32*d^2/b^3/a*Ei(1,(I*c*b+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*exp((I*c*b+d*(a*b)^(1/2))/b)*(a*b)^(1/2)-1/32*d^2/b^3/a*Ei(1,(I*c*b-d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*exp((I*c*b-d*(a*b)^(1/2))/b)*(a*b)^(1/2)+1/32*d/b^2/a*Ei(1,(I*c*b+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*exp((I*c*b+d*(a*b)^(1/2))/b)+1/32*d/b^2/a*Ei(1,(I*c*b-d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*exp((I*c*b-d*(a*b)^(1/2))/b)-1/32/b^2/a^2*Ei(1,(I*c*b+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*exp((I*c*b+d*(a*b)^(1/2))/b)*(a*b)^(1/2)+1/32/b^2/a^2*Ei(1,(I*c*b-d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*exp((I*c*b-d*(a*b)^(1/2))/b)*(a*b)^(1/2)-1/32*d^2/b^3/a*exp(-(I*c*b+d*(a*b)^(1/2))/b)*Ei(1,-(I*c*b+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*(a*b)^(1/2)+1/32*d^2/b^3/a*exp(-(I*c*b-d*(a*b)^(1/2))/b)*Ei(1,-(I*c*b-d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*(a*b)^(1/2)+1/32*d/b^2/a*exp(-(I*c*b+d*(a*b)^(1/2))/b)*Ei(1,-(I*c*b+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)+1/32*d/b^2/a*exp(-(I*c*b-d*(a*b)^(1/2))/b)*Ei(1,-(I*c*b-d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)+1/32/b^2/a^2*exp(-(I*c*b+d*(a*b)^(1/2))/b)*Ei(1,-(I*c*b+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*(a*b)^(1/2)-1/32/b^2/a^2*exp(-(I*c*b-d*(a*b)^(1/2))/b)*Ei(1,-(I*c*b-d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*(a*b)^(1/2)-1/8*d/a*(-a*b*d^4*x^2-a^2*d^4)/b^2/(-b^2*d^4*x^4-2*a*b*d^4*x^2-a^2*d^4)*cos(d*x+c)+1/8/d/b*(-a*b*d^5*x^3+a^2*d^5*x)/a^2/(-b^2*d^4*x^4-2*a*b*d^4*x^2-a^2*d^4)*sin(d*x+c)

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 604, normalized size of antiderivative = 0.81

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^2)^3}\,dx =$$

$$-\frac{\left(ab^2d^2x^4 + 2\,a^2bd^2x^2 + a^3d^2 + (a^3d^2 + (ab^2d^2 - b^3)x^4 - a^2b + 2\,(a^2bd^2 - ab^2)x^2)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(i\,dx - \sqrt{\right.}{}$$

```
[In] integrate(x^2*sin(d*x+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/32*((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2 + (a^3*d^2 + (a*b^2*d^2 -
 b^3)*x^4 - a^2*b + 2*(a^2*b*d^2 - a*b^2)*x^2)*sqrt(a*d^2/b))*Ei(I*d*x - sq
rt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a
^3*d^2 - (a^3*d^2 + (a*b^2*d^2 - b^3)*x^4 - a^2*b + 2*(a^2*b*d^2 - a*b^2)*x
^2)*sqrt(a*d^2/b))*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + (a*b
^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2 + (a^3*d^2 + (a*b^2*d^2 - b^3)*x^4 -
 a^2*b + 2*(a^2*b*d^2 - a*b^2)*x^2)*sqrt(a*d^2/b))*Ei(-I*d*x - sqrt(a*d^2/b
))*e^(-I*c + sqrt(a*d^2/b)) + (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2 -
(a^3*d^2 + (a*b^2*d^2 - b^3)*x^4 - a^2*b + 2*(a^2*b*d^2 - a*b^2)*x^2)*sqrt(
a*d^2/b))*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) + 4*(a^2*b*d^
2*x^2 + a^3*d^2)*cos(d*x + c) - 4*(a*b^2*d*x^3 - a^2*b*d*x)*sin(d*x + c))/(
a^2*b^4*d*x^4 + 2*a^3*b^3*d*x^2 + a^4*b^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^2)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*sin(d*x+c)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^2)^3} \, dx = \int \frac{x^2 \sin{(dx+c)}}{(bx^2+a)^3} \, dx$$

```
[In] integrate(x^2*sin(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*((cos(c)^2 + sin(c)^2)*d*x^2*cos(d*x + c) + 4*(cos(c)^2 + sin(c)^2)*x*
sin(d*x + c) + ((d*x^2*cos(c) - 4*x*sin(c))*cos(d*x + c)^2 + (d*x^2*cos(c)
- 4*x*sin(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b^3*cos(c)^2 + b^3*sin(
c)^2)*d^2*x^6 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^2*x^4 + 3*(a^2*b*cos(
c)^2 + a^2*b*sin(c)^2)*d^2*x^2 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^2)*cos(d*x
 + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d^2*x^6 + 3*(a*b^2*cos(c)^2 + a*b^
2*sin(c)^2)*d^2*x^4 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^2*x^2 + (a^3*co
s(c)^2 + a^3*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(-(3*a*d*x*cos(d*x + c
) - 2*(5*b*x^2 - a)*sin(d*x + c))/(b^4*d^2*x^8 + 4*a*b^3*d^2*x^6 + 6*a^2*b^
2*d^2*x^4 + 4*a^3*b*d^2*x^2 + a^4*d^2), x) + 2*(((b^3*cos(c)^2 + b^3*sin(c)
^2)*d^2*x^6 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^2*x^4 + 3*(a^2*b*cos(c)
^2 + a^2*b*sin(c)^2)*d^2*x^2 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^2)*cos(d*x +
 c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d^2*x^6 + 3*(a*b^2*cos(c)^2 + a*b^2*
sin(c)^2)*d^2*x^4 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^2*x^2 + (a^3*cos(
c)^2 + a^3*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(-(3*a*d*x*cos(d*x + c)
- 2*(5*b*x^2 - a)*sin(d*x + c))/((b^4*d^2*x^8 + 4*a*b^3*d^2*x^6 + 6*a^2*b^2
*d^2*x^4 + 4*a^3*b*d^2*x^2 + a^4*d^2)*cos(d*x + c)^2 + (b^4*d^2*x^8 + 4*a*b
^3*d^2*x^6 + 6*a^2*b^2*d^2*x^4 + 4*a^3*b*d^2*x^2 + a^4*d^2)*sin(d*x + c)^2)
, x) + ((d*x^2*sin(c) + 4*x*cos(c))*cos(d*x + c)^2 + (d*x^2*sin(c) + 4*x*co
s(c))*sin(d*x + c)^2)*sin(d*x + 2*c))/(((b^3*cos(c)^2 + b^3*sin(c)^2)*d^2*x
^6 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^2*x^4 + 3*(a^2*b*cos(c)^2 + a^2*
b*sin(c)^2)*d^2*x^2 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^2)*cos(d*x + c)^2 + (
(b^3*cos(c)^2 + b^3*sin(c)^2)*d^2*x^6 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)
*d^2*x^4 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^2*x^2 + (a^3*cos(c)^2 + a^
3*sin(c)^2)*d^2)*sin(d*x + c)^2)
```

Giac [F]

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^2)^3} \, dx = \int \frac{x^2 \sin{(dx+c)}}{(bx^2+a)^3} \, dx$$

```
[In] integrate(x^2*sin(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*sin(d*x + c)/(b*x^2 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^2)^3} \, dx = \int \frac{x^2 \, \sin\left(c+d\,x\right)}{\left(b\,x^2+a\right)^3} \, dx$$

```
[In] int((x^2*sin(c + d*x))/(a + b*x^2)^3,x)
[Out] int((x^2*sin(c + d*x))/(a + b*x^2)^3, x)
```

3.74 $\int \frac{x\sin(c+dx)}{(a+bx^2)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 512

$$\begin{aligned}\int \frac{x\sin(c+dx)}{(a+bx^2)^3}\,dx = &-\frac{d\cos(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{d\cos(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)}\\ &-\frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}}\\ &+\frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{3/2}b^{3/2}}\\ &+\frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16ab^2}\\ &+\frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16ab^2}\\ &-\frac{\sin(c+dx)}{4b\left(a+bx^2\right)^2}-\frac{d^2\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2}\\ &-\frac{d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}}\\ &+\frac{d^2\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16ab^2}\\ &-\frac{d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{3/2}b^{3/2}}\end{aligned}$$

```
[Out] 1/16*d*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*cos(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/
b^(3/2)-1/16*d*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*cos(c+d*(-a)^(1/2)/b^(1/2))/(-
```

```
a)^(3/2)/b^(3/2)+1/16*d^2*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b
^(1/2))/a/b^2+1/16*d^2*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1
/2))/a/b^2-1/4*sin(d*x+c)/b/(b*x^2+a)^2+1/16*d^2*Ci(d*x+d*(-a)^(1/2)/b^(1/2
))*sin(c-d*(-a)^(1/2)/b^(1/2))/a/b^2-1/16*d*Si(d*x+d*(-a)^(1/2)/b^(1/2))*si
n(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(3/2)+1/16*d^2*Ci(-d*x+d*(-a)^(1/2)/
b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/a/b^2+1/16*d*Si(d*x-d*(-a)^(1/2)/b^(1/
2))*sin(c+d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(3/2)-1/16*d*cos(d*x+c)/a/b^(3
/2)/((-a)^(1/2)-x*b^(1/2))+1/16*d*cos(d*x+c)/a/b^(3/2)/((-a)^(1/2)+x*b^(1/2
))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3422, 3415, 3378, 3384, 3380, 3383}

$$\int \frac{x\sin(c+dx)}{(a+bx^2)^3}\,dx = -\frac{d\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}} + \frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{3/2}b^{3/2}} - \frac{d\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}} - \frac{d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{3/2}b^{3/2}} - \frac{d\cos(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{d\cos(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)} + \frac{d^2\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16ab^2} + \frac{d^2\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2} - \frac{d^2\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2} + \frac{d^2\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16ab^2} - \frac{\sin(c+dx)}{4b\left(a+bx^2\right)^2}$$

```
[In] Int[(x*Sin[c + d*x])/(a + b*x^2)^3,x]
```

```
[Out] -1/16*(d*Cos[c + d*x])/(a*b^(3/2)*(Sqrt[-a] - Sqrt[b]*x)) + (d*Cos[c + d*x]
)/(16*a*b^(3/2)*(Sqrt[-a] + Sqrt[b]*x)) - (d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*
CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(3/2)*b^(3/2)) + (d*Cos[c
 - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^
```

```
(3/2)*b^(3/2)) + (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt
[-a]*d)/Sqrt[b]])/(16*a*b^2) + (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]
*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*a*b^2) - Sin[c + d*x]/(4*b*(a + b*x^2)^
2) - (d^2*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] -
d*x])/(16*a*b^2) - (d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d
)/Sqrt[b] - d*x])/(16*(-a)^(3/2)*b^(3/2)) + (d^2*Cos[c - (Sqrt[-a]*d)/Sqrt[
b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a*b^2) - (d*Sin[c - (Sqrt[
-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(3/2)*b^(
3/2))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3415

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3422

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))),
x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
```

```
ntegerQ[n] || GtQ[e, 0])
```

Rubi steps

$$\text{integral} = -\frac{\sin(c+dx)}{4b\left(a+bx^2\right)^2} + \frac{d\int \frac{\cos(c+dx)}{\left(a+bx^2\right)^2}\,dx}{4b}$$

$$= -\frac{\sin(c+dx)}{4b\left(a+bx^2\right)^2} + \frac{d\int \left(-\frac{b\cos(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}-bx\right)^2} - \frac{b\cos(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}+bx\right)^2} - \frac{b\cos(c+dx)}{2a\left(-ab-b^2x^2\right)}\right)dx}{4b}$$

$$= -\frac{\sin(c+dx)}{4b\left(a+bx^2\right)^2} - \frac{d\int \frac{\cos(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}\,dx}{16a} - \frac{d\int \frac{\cos(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^2}\,dx}{16a} - \frac{d\int \frac{\cos(c+dx)}{-ab-b^2x^2}\,dx}{8a}$$

$$= -\frac{d\cos(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{d\cos(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{\sin(c+dx)}{4b\left(a+bx^2\right)^2} - \frac{d\int \left(-\frac{\sqrt{-a}\cos(c+dx)}{2ab\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{\sqrt{-a}\cos(c+dx)}{2ab\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{8a} - \frac{d^2\int \frac{\sin(c+dx)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16ab} + \frac{d^2\int \frac{\sin(c+dx)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16ab}$$

$$= -\frac{d\cos(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{d\cos(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{\sin(c+dx)}{4b\left(a+bx^2\right)^2} + \frac{d\int \frac{\cos(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16(-a)^{3/2}b} + \frac{d\int \frac{\cos(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16(-a)^{3/2}b} + \frac{\left(d^2\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16ab} + \frac{\left(d^2\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16ab} + \frac{\left(d^2\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16ab} - \frac{\left(d^2\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16ab}$$

$$= -\frac{d\cos(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{d\cos(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)} + \frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16ab^2} + \frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16ab^2} - \frac{\sin(c+dx)}{4b\left(a+bx^2\right)^2} - \frac{d^2\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2} + \frac{d^2\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16ab^2} + \frac{\left(d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}dx}{16(-a)^{3/2}b} + \frac{\left(d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}dx}{16(-a)^{3/2}b} - \frac{\left(d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}dx}{16(-a)^{3/2}b} + \frac{\left(d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}dx}{16(-a)^{3/2}b}$$

$$= -\frac{d\cos(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{d\cos(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}} + \frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{3/2}b^{3/2}} + \frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16ab^2} + \frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16ab^2} - \frac{\sin(c+dx)}{4b\left(a+bx^2\right)^2} - \frac{d^2\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2} - \frac{d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}} + \frac{d^2\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16ab^2} - \frac{d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{3/2}b^{3/2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.53 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.62

$$\int \frac{x\sin(c+dx)}{(a+bx^2)^3}\,dx$$
$$= \frac{ide^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(-\left(\left(\sqrt{b}-\sqrt{a}d\right)e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)\right)+\left(\sqrt{b}+\sqrt{a}d\right)\text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}-\right.\right.}{}$$

```
[In] Integrate[(x*Sin[c + d*x])/(a + b*x^2)^3,x]
```

```
[Out] (I*d*E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*(-((Sqrt[b] - Sqrt[a]*d)*E^((2*Sqrt[a
]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x]) + (Sqrt[b] + S
qrt[a]*d)*ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] - I*d*x]) - I*d*E^(I*c - (Sqrt[
a]*d)/Sqrt[b])*(-((Sqrt[b] - Sqrt[a]*d)*E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegr
alEi[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]) + (Sqrt[b] + Sqrt[a]*d)*ExpIntegralEi
[(Sqrt[a]*d)/Sqrt[b] + I*d*x]) + (4*Sqrt[a]*b*Cos[d*x]*(d*x*(a + b*x^2)*Cos
[c] - 2*a*Sin[c]))/(a + b*x^2)^2 - (4*Sqrt[a]*b*(2*a*Cos[c] + d*x*(a + b*x^
2)*Sin[c])*Sin[d*x])/(a + b*x^2)^2)/(32*a^(3/2)*b^2)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 628, normalized size of antiderivative = 1.23

method	result
risch	$\frac{id^2 e^{\frac{icb+d\sqrt{ab}}{b}}\,\text{Ei}_1\left(\frac{icb+d\sqrt{ab}-b(idx+ic)}{b}\right)}{32a\,b^2}+\frac{id^2 e^{\frac{icb-d\sqrt{ab}}{b}}\,\text{Ei}_1\left(-\frac{-icb+d\sqrt{ab}+b(idx+ic)}{b}\right)}{32a\,b^2}-\frac{id\,e^{\frac{icb+d\sqrt{ab}}{b}}\sqrt{ab}\,\text{Ei}_1\left(\right.}{32a^2b^2}$
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int(x*sin(d*x+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/32*I*d^2/a/b^2*exp((I*c*b+d*(a*b)^(1/2))/b)*Ei(1,(I*c*b+d*(a*b)^(1/2)-b*(
I*d*x+I*c))/b)+1/32*I*d^2/a/b^2*exp((I*c*b-d*(a*b)^(1/2))/b)*Ei(1,-(-I*c*b+
d*(a*b)^(1/2)+b*(I*d*x+I*c))/b)-1/32*I*d/a^2/b^2*exp((I*c*b+d*(a*b)^(1/2))/
b)*(a*b)^(1/2)*Ei(1,(I*c*b+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)+1/32*I*d/a^2/b^2
*(a*b)^(1/2)*exp((I*c*b-d*(a*b)^(1/2))/b)*Ei(1,-(-I*c*b+d*(a*b)^(1/2)+b*(I*
d*x+I*c))/b)-1/32*I*d^2/a/b^2*exp(-(I*c*b-d*(a*b)^(1/2))/b)*Ei(1,(-I*c*b+d*
(a*b)^(1/2)+b*(I*d*x+I*c))/b)-1/32*I*d^2/a/b^2*Ei(1,-(I*c*b+d*(a*b)^(1/2)-b
*(I*d*x+I*c))/b)*exp(-(I*c*b+d*(a*b)^(1/2))/b)+1/32*I*d/a^2/b^2*(a*b)^(1/2)
*exp(-(I*c*b-d*(a*b)^(1/2))/b)*Ei(1,(-I*c*b+d*(a*b)^(1/2)+b*(I*d*x+I*c))/b)
```

```
-1/32*I*d/a^2/b^2*(a*b)^(1/2)*Ei(1,-(I*c*b+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)*
exp(-(I*c*b+d*(a*b)^(1/2))/b)-1/8*d^3/a*x*(b*d^2*x^2+a*d^2)/b/(-b^2*d^4*x^4
-2*a*b*d^4*x^2-a^2*d^4)*cos(d*x+c)+1/4*d^4/b/(-b^2*d^4*x^4-2*a*b*d^4*x^2-a^
2*d^4)*sin(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 483, normalized size of antiderivative = 0.94

$$\int \frac{x\sin(c+dx)}{(a+bx^2)^3}\,dx = -\frac{8\,a^2b\sin(dx+c) + \left(i\,ab^2d^2x^4 + 2i\,a^2bd^2x^2 + i\,a^3d^2 - (i\,b^3x^4 + 2i\,ab^2x^2 + i\,a^2b)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(i\,dx - \sqrt{\frac{ad^2}{b}}\right.}{}$$

```
[In] integrate(x*sin(d*x+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/32*(8*a^2*b*sin(d*x + c) + (I*a*b^2*d^2*x^4 + 2*I*a^2*b*d^2*x^2 + I*a^3*
d^2 - (I*b^3*x^4 + 2*I*a*b^2*x^2 + I*a^2*b)*sqrt(a*d^2/b))*Ei(I*d*x - sqrt(
a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + (I*a*b^2*d^2*x^4 + 2*I*a^2*b*d^2*x^2 +
I*a^3*d^2 - (-I*b^3*x^4 - 2*I*a*b^2*x^2 - I*a^2*b)*sqrt(a*d^2/b))*Ei(I*d*x
+ sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + (-I*a*b^2*d^2*x^4 - 2*I*a^2*b*d^
2*x^2 - I*a^3*d^2 - (-I*b^3*x^4 - 2*I*a*b^2*x^2 - I*a^2*b)*sqrt(a*d^2/b))*E
i(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) + (-I*a*b^2*d^2*x^4 - 2*
I*a^2*b*d^2*x^2 - I*a^3*d^2 - (I*b^3*x^4 + 2*I*a*b^2*x^2 + I*a^2*b)*sqrt(a*
d^2/b))*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) - 4*(a*b^2*d*x^
3 + a^2*b*d*x)*cos(d*x + c))/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x\sin(c+dx)}{(a+bx^2)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x*sin(d*x+c)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x\sin(c+dx)}{(a+bx^2)^3}\,dx = \int \frac{x\sin{(dx+c)}}{(bx^2+a)^3}\,dx$$

```
[In] integrate(x*sin(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*((cos(c)^2 + sin(c)^2)*x*cos(d*x + c) + (x*cos(d*x + c)^2*cos(c) + x*c
os(c)*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b^3*cos(c)^2 + b^3*sin(c)^2)*d*
x^6 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x^4 + 3*(a^2*b*cos(c)^2 + a^2*b
*sin(c)^2)*d*x^2 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d)*cos(d*x + c)^2 + ((b^3*
cos(c)^2 + b^3*sin(c)^2)*d*x^6 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x^4
+ 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d*x^2 + (a^3*cos(c)^2 + a^3*sin(c)^2)
*d)*sin(d*x + c)^2)*integrate(1/2*(5*b*x^2 - a)*cos(d*x + c)/(b^4*d*x^8 + 4
*a*b^3*d*x^6 + 6*a^2*b^2*d*x^4 + 4*a^3*b*d*x^2 + a^4*d), x) + 2*(((b^3*cos(
c)^2 + b^3*sin(c)^2)*d*x^6 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x^4 + 3*
(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d*x^2 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d)*
cos(d*x + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d*x^6 + 3*(a*b^2*cos(c)^2 +
 a*b^2*sin(c)^2)*d*x^4 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d*x^2 + (a^3*c
os(c)^2 + a^3*sin(c)^2)*d)*sin(d*x + c)^2)*integrate(1/2*(5*b*x^2 - a)*cos(
d*x + c)/((b^4*d*x^8 + 4*a*b^3*d*x^6 + 6*a^2*b^2*d*x^4 + 4*a^3*b*d*x^2 + a^
4*d)*cos(d*x + c)^2 + (b^4*d*x^8 + 4*a*b^3*d*x^6 + 6*a^2*b^2*d*x^4 + 4*a^3*
b*d*x^2 + a^4*d)*sin(d*x + c)^2), x) + (x*cos(d*x + c)^2*sin(c) + x*sin(d*x
 + c)^2*sin(c))*sin(d*x + 2*c))/(((b^3*cos(c)^2 + b^3*sin(c)^2)*d*x^6 + 3*(
a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x^4 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2
)*d*x^2 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d)*cos(d*x + c)^2 + ((b^3*cos(c)^2
+ b^3*sin(c)^2)*d*x^6 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x^4 + 3*(a^2*
b*cos(c)^2 + a^2*b*sin(c)^2)*d*x^2 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d)*sin(d
*x + c)^2)
```

Giac [F]

$$\int \frac{x\sin(c+dx)}{(a+bx^2)^3}\,dx = \int \frac{x\sin{(dx+c)}}{(bx^2+a)^3}\,dx$$

```
[In] integrate(x*sin(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x*sin(d*x + c)/(b*x^2 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sin(c+dx)}{(a+bx^2)^3}\,dx = \int \frac{x\,\sin\left(c+d\,x\right)}{\left(b\,x^2+a\right)^3}\,dx$$

```
[In] int((x*sin(c + d*x))/(a + b*x^2)^3,x)
[Out] int((x*sin(c + d*x))/(a + b*x^2)^3, x)
```

3.75 $\int \frac{\sin(c+dx)}{(a+bx^2)^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 856

$$\int \frac{\sin(c+dx)}{(a+bx^2)^3}\,dx = \frac{d\cos(c+dx)}{16(-a)^{3/2}b\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{d\cos(c+dx)}{16(-a)^{3/2}b\left(\sqrt{-a}+\sqrt{b}x\right)}$$
$$-\frac{3d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^2b}$$
$$-\frac{3d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16a^2b}$$
$$-\frac{3\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{5/2}\sqrt{b}}$$
$$+\frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{3/2}b^{3/2}}$$
$$+\frac{3\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{5/2}\sqrt{b}}$$
$$-\frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{3/2}b^{3/2}}$$
$$-\frac{\sin(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)^2}-\frac{3\sin(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}$$
$$+\frac{\sin(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)^2}+\frac{3\sin(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}$$
$$-\frac{3\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{5/2}\sqrt{b}}+\frac{d^2\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}}$$
$$-\frac{3d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^2b}-\frac{3\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{5/2}\sqrt{b}}$$
$$+\frac{d^2\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{3/2}b^{3/2}}$$
$$+\frac{3d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16a^2b}$$

```
[Out] -3/16*d*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*cos(c-d*(-a)^(1/2)/b^(1/2))/a^2/b-3/16
*d*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*cos(c+d*(-a)^(1/2)/b^(1/2))/a^2/b-1/16*d^2
*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(3/2
)+1/16*d^2*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))/(-a)^(3
```

```
/2)/b^(3/2)+1/16*d^2*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2
))/(-a)^(3/2)/b^(3/2)+3/16*d*Si(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2
)/b^(1/2))/a^2/b-1/16*d^2*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/
b^(1/2))/(-a)^(3/2)/b^(3/2)+3/16*d*Si(d*x-d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a
)^(1/2)/b^(1/2))/a^2/b+3/16*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)
/b^(1/2))/(-a)^(5/2)/b^(1/2)-3/16*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)
^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)-3/16*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-
d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)+3/16*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))
*sin(c+d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)-1/16*sin(d*x+c)/(-a)^(3/2)/
b^(1/2)/((-a)^(1/2)-x*b^(1/2))^2+1/16*d*cos(d*x+c)/(-a)^(3/2)/b/((-a)^(1/2)
-x*b^(1/2))-3/16*sin(d*x+c)/a^2/b^(1/2)/((-a)^(1/2)-x*b^(1/2))+1/16*sin(d*x
+c)/(-a)^(3/2)/b^(1/2)/((-a)^(1/2)+x*b^(1/2))^2+1/16*d*cos(d*x+c)/(-a)^(3/2
)/b/((-a)^(1/2)+x*b^(1/2))+3/16*sin(d*x+c)/a^2/b^(1/2)/((-a)^(1/2)+x*b^(1/2
))
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 856, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used

$= \{3414, 3378, 3384, 3380, 3383\}$

$$\int \frac{\sin(c+dx)}{(a+bx^2)^3}\,dx = \frac{\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)d^2}{16(-a)^{3/2}b^{3/2}}$$
$$-\frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)d^2}{16(-a)^{3/2}b^{3/2}}$$
$$+\frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)d^2}{16(-a)^{3/2}b^{3/2}}$$
$$+\frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)d^2}{16(-a)^{3/2}b^{3/2}}$$
$$+\frac{\cos(c+dx)d}{16(-a)^{3/2}b\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\cos(c+dx)d}{16(-a)^{3/2}b\left(\sqrt{b}x+\sqrt{-a}\right)}$$
$$-\frac{3\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)d}{16a^2b}$$
$$-\frac{3\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)d}{16a^2b}$$
$$-\frac{3\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)d}{16a^2b}$$
$$+\frac{3\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)d}{16a^2b}$$
$$-\frac{3\,\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{5/2}\sqrt{b}}$$
$$+\frac{3\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{5/2}\sqrt{b}}$$
$$-\frac{3\sin(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{3\sin(c+dx)}{16a^2\sqrt{b}\left(\sqrt{b}x+\sqrt{-a}\right)}$$
$$-\frac{\sin(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)^2}+\frac{\sin(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{b}x+\sqrt{-a}\right)^2}$$
$$-\frac{3\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{5/2}\sqrt{b}}-\frac{3\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{5/2}\sqrt{b}}$$

```
[In] Int[Sin[c + d*x]/(a + b*x^2)^3,x]
```

```
[Out] (d*Cos[c + d*x])/(16*(-a)^(3/2)*b*(Sqrt[-a] - Sqrt[b]*x)) + (d*Cos[c + d*x]
)/(16*(-a)^(3/2)*b*(Sqrt[-a] + Sqrt[b]*x)) - (3*d*Cos[c + (Sqrt[-a]*d)/Sqrt
```

```
[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^2*b) - (3*d*Cos[c - (Sq
rt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^2*b) - (3
*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(16
*(-a)^(5/2)*Sqrt[b]) + (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c -
 (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(3/2)*b^(3/2)) + (3*CosIntegral[(Sqrt[-a]*
d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(5/2)*Sqrt[b]) -
(d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])
/(16*(-a)^(3/2)*b^(3/2)) - Sin[c + d*x]/(16*(-a)^(3/2)*Sqrt[b]*(Sqrt[-a] -
Sqrt[b]*x)^2) - (3*Sin[c + d*x])/(16*a^2*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)) +
Sin[c + d*x]/(16*(-a)^(3/2)*Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)^2) + (3*Sin[c +
d*x])/(16*a^2*Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)) - (3*Cos[c + (Sqrt[-a]*d)/Sqr
t[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(5/2)*Sqrt[b]) + (d
^2*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(
16*(-a)^(3/2)*b^(3/2)) - (3*d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sq
rt[-a]*d)/Sqrt[b] - d*x])/(16*a^2*b) - (3*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*Sin
Integral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(5/2)*Sqrt[b]) + (d^2*Cos[c
- (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(
3/2)*b^(3/2)) + (3*d*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)
/Sqrt[b] + d*x])/(16*a^2*b)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\text{integral} = \int \left(-\frac{b^{3/2}\sin(c+dx)}{8(-a)^{3/2}\left(\sqrt{-a}\sqrt{b}-bx\right)^3} - \frac{3b\sin(c+dx)}{16a^2\left(\sqrt{-a}\sqrt{b}-bx\right)^2} - \frac{b^{3/2}\sin(c+dx)}{8(-a)^{3/2}\left(\sqrt{-a}\sqrt{b}+bx\right)^3} - \frac{3b\sin(c+dx)}{16a^2\left(\sqrt{-a}\sqrt{b}+bx\right)^2} - \frac{3b\sin(c+dx)}{8a^2\left(-ab-b^2x^2\right)} \right) dx$$

$$= -\frac{(3b)\int \frac{\sin(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}\,dx}{16a^2} - \frac{(3b)\int \frac{\sin(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^2}\,dx}{16a^2} - \frac{(3b)\int \frac{\sin(c+dx)}{-ab-b^2x^2}\,dx}{8a^2} - \frac{b^{3/2}\int \frac{\sin(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^3}\,dx}{8(-a)^{3/2}} - \frac{b^{3/2}\int \frac{\sin(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^3}\,dx}{8(-a)^{3/2}}$$

$$= -\frac{\sin(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)^2} - \frac{3\sin(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sin(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)^2} + \frac{3\sin(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{(3b)\int \left(-\frac{\sqrt{-a}\sin(c+dx)}{2ab\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{\sqrt{-a}\sin(c+dx)}{2ab\left(\sqrt{-a}+\sqrt{b}x\right)}\right) dx}{8a^2} + \frac{(3d)\int \frac{\cos(c+dx)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16a^2} - \frac{(3d)\int \frac{\cos(c+dx)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16a^2} + \frac{\left(\sqrt{b}d\right)\int \frac{\cos(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}\,dx}{16(-a)^{3/2}} - \frac{\left(\sqrt{b}d\right)\int \frac{\cos(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^2}\,dx}{16(-a)^{3/2}}$$

$$= \frac{d\cos(c+dx)}{16(-a)^{3/2}b\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{d\cos(c+dx)}{16(-a)^{3/2}b\left(\sqrt{-a}+\sqrt{b}x\right)}-\frac{\sin(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)^2}$$
$$-\frac{3\sin(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sin(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)^2}+\frac{3\sin(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}$$
$$-\frac{3\int\frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16(-a)^{5/2}}-\frac{3\int\frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16(-a)^{5/2}}+\frac{d^2\int\frac{\sin(c+dx)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16(-a)^{3/2}\sqrt{b}}+\frac{d^2\int\frac{\sin(c+dx)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16(-a)^{3/2}\sqrt{b}}$$
$$-\frac{\left(3d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16a^2}+\frac{\left(3d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16a^2}$$
$$+\frac{\left(3d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16a^2}+\frac{\left(3d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16a^2}$$
$$= \frac{d\cos(c+dx)}{16(-a)^{3/2}b\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{d\cos(c+dx)}{16(-a)^{3/2}b\left(\sqrt{-a}+\sqrt{b}x\right)}$$
$$-\frac{3d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^2b}$$
$$-\frac{3d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16a^2b}$$
$$-\frac{\sin(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)^2}-\frac{3\sin(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}$$
$$+\frac{\sin(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)^2}+\frac{3\sin(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}$$
$$-\frac{3d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^2b}+\frac{3d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16a^2b}$$
$$-\frac{\left(3\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16(-a)^{5/2}}+\frac{\left(d^2\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16(-a)^{3/2}\sqrt{b}}$$
$$+\frac{\left(3\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16(-a)^{5/2}}-\frac{\left(d^2\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16(-a)^{3/2}\sqrt{b}}$$
$$-\frac{\left(3\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16(-a)^{5/2}}+\frac{\left(d^2\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16(-a)^{3/2}\sqrt{b}}$$
$$-\frac{\left(3\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16(-a)^{5/2}}+\frac{\left(d^2\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16(-a)^{3/2}\sqrt{b}}$$

$$= \frac{d\cos(c+dx)}{16(-a)^{3/2}b\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{d\cos(c+dx)}{16(-a)^{3/2}b\left(\sqrt{-a}+\sqrt{b}x\right)}$$
$$- \frac{3d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^2b}$$
$$- \frac{3d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16a^2b}$$
$$- \frac{3\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{5/2}\sqrt{b}}$$
$$+ \frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{3/2}b^{3/2}}$$
$$+ \frac{3\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{5/2}\sqrt{b}}$$
$$- \frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{3/2}b^{3/2}}$$
$$- \frac{\sin(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)^2} - \frac{3\sin(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}$$
$$+ \frac{\sin(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)^2} + \frac{3\sin(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}$$
$$- \frac{3\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{5/2}\sqrt{b}} + \frac{d^2\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}}$$
$$- \frac{3d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^2b} - \frac{3\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{5/2}\sqrt{b}}$$
$$+ \frac{d^2\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{3/2}b^{3/2}} + \frac{3d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16a^2b}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.99 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.44

$$\int \frac{\sin(c+dx)}{(a+bx^2)^3}\,dx$$
$$= \frac{e^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(\left(3b-3\sqrt{a}\sqrt{b}d+ad^2\right)e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)-\left(3b+3\sqrt{a}\sqrt{b}d+ad^2\right)\text{ExpIntegra}\right.}{}$$

[In] Integrate[Sin[c + d*x]/(a + b*x^2)^3,x]

[Out] (E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*((3*b - 3*Sqrt[a]*Sqrt[b]*d + a*d^2)*E^((
2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x] - (3*b
+ 3*Sqrt[a]*Sqrt[b]*d + a*d^2)*ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] - I*d*x])
+ E^(I*c - (Sqrt[a]*d)/Sqrt[b])*((3*b - 3*Sqrt[a]*Sqrt[b]*d + a*d^2)*E^((2*
Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] - (3*b +
3*Sqrt[a]*Sqrt[b]*d + a*d^2)*ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] + I*d*x]) +
(4*Sqrt[a]*Sqrt[b]*Cos[d*x]*(a*d*(a + b*x^2)*Cos[c] + b*x*(5*a + 3*b*x^2)*S
in[c]))/(a + b*x^2)^2 - (4*Sqrt[a]*Sqrt[b]*(-(b*x*(5*a + 3*b*x^2)*Cos[c]) +
a*d*(a + b*x^2)*Sin[c])*Sin[d*x])/(a + b*x^2)^2)/(32*a^(5/2)*b^(3/2))

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 598, normalized size of antiderivative = 0.70

method	result
derivativedivides	$d^5\left(-\frac{\sin(dx+c)\left(5ac\,d^2-5a\,d^2(dx+c)+3c^3b-9b\,c^2(dx+c)+9bc(dx+c)^2-3b(dx+c)^3\right)}{8a^2d^4\left(a\,d^2+c^2b-2bc(dx+c)+b(dx+c)^2\right)^2}+\frac{\cos(dx+c)}{8ab\,d^2\left(a\,d^2+c^2b-2bc(dx+\right.}\right.$
default	$d^5\left(-\frac{\sin(dx+c)\left(5ac\,d^2-5a\,d^2(dx+c)+3c^3b-9b\,c^2(dx+c)+9bc(dx+c)^2-3b(dx+c)^3\right)}{8a^2d^4\left(a\,d^2+c^2b-2bc(dx+c)+b(dx+c)^2\right)^2}+\frac{\cos(dx+c)}{8ab\,d^2\left(a\,d^2+c^2b-2bc(dx+\right.}\right.$
risch	$-\frac{d^2\sqrt{ab}\,e^{\frac{icb+d\sqrt{ab}}{b}}\operatorname{Ei}_1\left(\frac{icb+d\sqrt{ab}-b(idx+ic)}{b}\right)}{32a^2b^2}+\frac{d^2\sqrt{ab}\,e^{\frac{icb-d\sqrt{ab}}{b}}\operatorname{Ei}_1\left(\frac{icb-d\sqrt{ab}-b(idx+ic)}{b}\right)}{32a^2b^2}+\frac{3d\,e^{\frac{icb+d\sqrt{ab}}{b}}\operatorname{Ei}_1\left(\right.}{3}$

[In] int(sin(d*x+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)

[Out] d^5*(-1/8*sin(d*x+c)*(5*a*c*d^2-5*a*d^2*(d*x+c)+3*c^3*b-9*b*c^2*(d*x+c)+9*b
c(d*x+c)^2-3*b*(d*x+c)^3)/a^2/d^4/(a*d^2+c^2*b-2*b*c*(d*x+c)+b*(d*x+c)^2)
^2+1/8*cos(d*x+c)/a/b/d^2/(a*d^2+c^2*b-2*b*c*(d*x+c)+b*(d*x+c)^2)-1/16*(a*d
^2+3*b)/a^2/d^4/b^2/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c
*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*
(-a*b)^(1/2)+c*b)/b))-1/16*(a*d^2+3*b)/a^2/d^4/b^2/((d*(-a*b)^(1/2)-c*b)/b+
c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(
d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-3/16/a^2/d^4/b*(-Si(d*x
+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(
1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))-3/16/a^2/d^4/b*(Si(d*x+c+(d*(-a*b
)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b
)*cos((d*(-a*b)^(1/2)-c*b)/b)))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 611, normalized size of antiderivative = 0.71

$$\int \frac{\sin(c+dx)}{(a+bx^2)^3}\,dx =$$

$$-\frac{\left(3\,ab^2d^2x^4+6\,a^2bd^2x^2+3\,a^3d^2-(a^3d^2+(ab^2d^2+3\,b^3)x^4+3\,a^2b+2\,(a^2bd^2+3\,ab^2)x^2)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(i\,dx\right.}{}$$

```
[In] integrate(sin(d*x+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/32*((3*a*b^2*d^2*x^4 + 6*a^2*b*d^2*x^2 + 3*a^3*d^2 - (a^3*d^2 + (a*b^2*d
^2 + 3*b^3)*x^4 + 3*a^2*b + 2*(a^2*b*d^2 + 3*a*b^2)*x^2)*sqrt(a*d^2/b))*Ei(
I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + (3*a*b^2*d^2*x^4 + 6*a^2*b
*d^2*x^2 + 3*a^3*d^2 + (a^3*d^2 + (a*b^2*d^2 + 3*b^3)*x^4 + 3*a^2*b + 2*(a^
2*b*d^2 + 3*a*b^2)*x^2)*sqrt(a*d^2/b))*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - s
qrt(a*d^2/b)) + (3*a*b^2*d^2*x^4 + 6*a^2*b*d^2*x^2 + 3*a^3*d^2 - (a^3*d^2 +
 (a*b^2*d^2 + 3*b^3)*x^4 + 3*a^2*b + 2*(a^2*b*d^2 + 3*a*b^2)*x^2)*sqrt(a*d^
2/b))*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) + (3*a*b^2*d^2*x^
4 + 6*a^2*b*d^2*x^2 + 3*a^3*d^2 + (a^3*d^2 + (a*b^2*d^2 + 3*b^3)*x^4 + 3*a^
2*b + 2*(a^2*b*d^2 + 3*a*b^2)*x^2)*sqrt(a*d^2/b))*Ei(-I*d*x + sqrt(a*d^2/b)
)*e^(-I*c - sqrt(a*d^2/b)) - 4*(a^2*b*d^2*x^2 + a^3*d^2)*cos(d*x + c) - 4*(
3*a*b^2*d*x^3 + 5*a^2*b*d*x)*sin(d*x + c))/(a^3*b^3*d*x^4 + 2*a^4*b^2*d*x^2
 + a^5*b*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{(a+bx^2)^3}\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{(a+bx^2)^3}\,dx = \int \frac{\sin(dx+c)}{(bx^2+a)^3}\,dx$$

```
[In] integrate(sin(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")
[Out] integrate(sin(d*x + c)/(b*x^2 + a)^3, x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{(a+bx^2)^3}\,dx = \int \frac{\sin(dx+c)}{(bx^2+a)^3}\,dx$$

```
[In] integrate(sin(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")
[Out] integrate(sin(d*x + c)/(b*x^2 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{(a+bx^2)^3}\,dx = \int \frac{\sin(c+d\,x)}{(b\,x^2+a)^3}\,dx$$

```
[In] int(sin(c + d*x)/(a + b*x^2)^3,x)
[Out] int(sin(c + d*x)/(a + b*x^2)^3, x)
```

3.76 $\int \frac{\sin(c+dx)}{x(a+bx^2)^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 730

$$\int \frac{\sin(c+dx)}{x(a+bx^2)^3}\,dx = \frac{d\cos(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{d\cos(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{5d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{5/2}\sqrt{b}} + \frac{5d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{5/2}\sqrt{b}} + \frac{\text{CosIntegral}(dx)\sin(c)}{a^3} - \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^3} - \frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^2b} - \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^3} - \frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^2b} + \frac{\sin(c+dx)}{4a(a+bx^2)^2} + \frac{\sin(c+dx)}{2a^2(a+bx^2)} + \frac{\cos(c)\text{Si}(dx)}{a^3} + \frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^3} + \frac{d^2\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^2b} - \frac{5d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{5/2}\sqrt{b}} - \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^3} - \frac{d^2\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16a^2b} - \frac{5d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{5/2}\sqrt{b}}$$

```
[Out] cos(c)*Si(d*x)/a^3-1/2*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b^(1
/2))/a^3-1/16*d^2*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b^(1/2))/
a^2/b-1/2*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))/a^3-1/16
*d^2*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))/a^2/b+Ci(d*x)
*sin(c)/a^3+1/4*sin(d*x+c)/a/(b*x^2+a)^2+1/2*sin(d*x+c)/a^2/(b*x^2+a)-1/2*C
i(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))/a^3-1/16*d^2*Ci(d*x
+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))/a^2/b-1/2*Ci(-d*x+d*(-a)
^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/a^3-1/16*d^2*Ci(-d*x+d*(-a)^(1/
```

```
2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/a^2/b+5/16*d*Ci(d*x+d*(-a)^(1/2)/b^
(1/2))*cos(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)-5/16*d*Ci(-d*x+d*(-a)
^(1/2)/b^(1/2))*cos(c+d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)-5/16*d*Si(d*
x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)+5/16
*d*Si(d*x-d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1
/2)+1/16*d*cos(d*x+c)/a^2/b^(1/2)/((-a)^(1/2)-x*b^(1/2))-1/16*d*cos(d*x+c)/
a^2/b^(1/2)/((-a)^(1/2)+x*b^(1/2))
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 730, normalized size of antiderivative = 1.00, number of steps used = 41, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used

$= \{3426, 3384, 3380, 3383, 3422, 3415, 3378\}$

$$\int \frac{\sin(c+dx)}{x\left(a+bx^2\right)^3}\,dx = -\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^3} - \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^3} + \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^3} - \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^3} + \frac{\sin(c)\,\text{CosIntegral}(dx)}{a^3} + \frac{\cos(c)\text{Si}(dx)}{a^3} - \frac{d^2\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^2b} - \frac{d^2\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^2b} + \frac{d^2\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^2b} - \frac{d^2\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^2b} + \frac{\sin(c+dx)}{2a^2\left(a+bx^2\right)} + \frac{d\cos(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{d\cos(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{5d\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{5/2}\sqrt{b}} + \frac{5d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{5/2}\sqrt{b}} - \frac{5d\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{5/2}\sqrt{b}} - \frac{5d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{5/2}\sqrt{b}} + \frac{\sin(c+dx)}{4a\left(a+bx^2\right)^2}$$

```
[In] Int[Sin[c + d*x]/(x*(a + b*x^2)^3),x]
```

```
[Out] (d*Cos[c + d*x])/(16*a^2*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)) - (d*Cos[c + d*x])
/(16*a^2*Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)) - (5*d*Cos[c + (Sqrt[-a]*d)/Sqrt[b
]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(5/2)*Sqrt[b]) + (5*d*
Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*
(-a)^(5/2)*Sqrt[b]) + (CosIntegral[d*x]*Sin[c])/a^3 - (CosIntegral[(Sqrt[-a
```

```
]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(2*a^3) - (d^2*CosIntegr
al[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*a^2*b) -
(CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*
a^3) - (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sq
rt[b]])/(16*a^2*b) + Sin[c + d*x]/(4*a*(a + b*x^2)^2) + Sin[c + d*x]/(2*a^2
*(a + b*x^2)) + (Cos[c]*SinIntegral[d*x])/a^3 + (Cos[c + (Sqrt[-a]*d)/Sqrt[
b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^3) + (d^2*Cos[c + (Sqrt[-
a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^2*b) - (5*d*S
in[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(
-a)^(5/2)*Sqrt[b]) - (Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d
)/Sqrt[b] + d*x])/(2*a^3) - (d^2*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[
(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^2*b) - (5*d*Sin[c - (Sqrt[-a]*d)/Sqrt[b]
]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(5/2)*Sqrt[b])
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3415

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3422

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))),
x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\sin(c+dx)}{a^3x} - \frac{bx\sin(c+dx)}{a\left(a+bx^2\right)^3} - \frac{bx\sin(c+dx)}{a^2\left(a+bx^2\right)^2} - \frac{bx\sin(c+dx)}{a^3\left(a+bx^2\right)}\right)dx \\
&= \frac{\int \frac{\sin(c+dx)}{x}\,dx}{a^3} - \frac{b\int \frac{x\sin(c+dx)}{a+bx^2}\,dx}{a^3} - \frac{b\int \frac{x\sin(c+dx)}{\left(a+bx^2\right)^2}\,dx}{a^2} - \frac{b\int \frac{x\sin(c+dx)}{\left(a+bx^2\right)^3}\,dx}{a} \\
&= \frac{\sin(c+dx)}{4a\left(a+bx^2\right)^2} + \frac{\sin(c+dx)}{2a^2\left(a+bx^2\right)} - \frac{b\int \left(-\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{a^3} \\
&\quad - \frac{d\int \frac{\cos(c+dx)}{a+bx^2}\,dx}{2a^2} - \frac{d\int \frac{\cos(c+dx)}{\left(a+bx^2\right)^2}\,dx}{4a} + \frac{\cos(c)\int \frac{\sin(dx)}{x}\,dx}{a^3} + \frac{\sin(c)\int \frac{\cos(dx)}{x}\,dx}{a^3} \\
&= \frac{\text{CosIntegral}(dx)\sin(c)}{a^3} + \frac{\sin(c+dx)}{4a\left(a+bx^2\right)^2} + \frac{\sin(c+dx)}{2a^2\left(a+bx^2\right)} + \frac{\cos(c)\text{Si}(dx)}{a^3} \\
&\quad + \frac{\sqrt{b}\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2a^3} - \frac{\sqrt{b}\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2a^3} - \frac{d\int \left(\frac{\sqrt{-a}\cos(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{-a}\cos(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{2a^2} \\
&\quad - \frac{d\int \left(-\frac{b\cos(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}-bx\right)^2} - \frac{b\cos(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}+bx\right)^2} - \frac{b\cos(c+dx)}{2a\left(-ab-b^2x^2\right)}\right)dx}{4a}
\end{aligned}$$

$$= \frac{\text{CosIntegral}(dx)\sin(c)}{a^3} + \frac{\sin(c+dx)}{4a\left(a+bx^2\right)^2} + \frac{\sin(c+dx)}{2a^2\left(a+bx^2\right)} + \frac{\cos(c)\text{Si}(dx)}{a^3} + \frac{d\int\frac{\cos(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4(-a)^{5/2}}$$
$$+ \frac{d\int\frac{\cos(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4(-a)^{5/2}} + \frac{(bd)\int\frac{\cos(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}\,dx}{16a^2} + \frac{(bd)\int\frac{\cos(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^2}\,dx}{16a^2} + \frac{(bd)\int\frac{\cos(c+dx)}{-ab-b^2x^2}\,dx}{8a^2}$$
$$- \frac{\left(\sqrt{b}\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2a^3} - \frac{\left(\sqrt{b}\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2a^3}$$
$$- \frac{\left(\sqrt{b}\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2a^3} + \frac{\left(\sqrt{b}\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2a^3}$$

$$= \frac{d\cos(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{d\cos(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}$$
$$+ \frac{\text{CosIntegral}(dx)\sin(c)}{a^3} - \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^3}$$
$$- \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^3} + \frac{\sin(c+dx)}{4a\left(a+bx^2\right)^2}$$
$$+ \frac{\sin(c+dx)}{2a^2\left(a+bx^2\right)} + \frac{\cos(c)\text{Si}(dx)}{a^3} + \frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^3}$$
$$- \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^3} + \frac{(bd)\int\left(-\frac{\sqrt{-a}\cos(c+dx)}{2ab\left(\sqrt{-a}-\sqrt{b}x\right)}-\frac{\sqrt{-a}\cos(c+dx)}{2ab\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{8a^2}$$
$$+ \frac{d^2\int\frac{\sin(c+dx)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16a^2} - \frac{d^2\int\frac{\sin(c+dx)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16a^2} + \frac{\left(d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4(-a)^{5/2}}$$
$$+ \frac{\left(d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4(-a)^{5/2}} - \frac{\left(d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4(-a)^{5/2}}$$
$$+ \frac{\left(d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4(-a)^{5/2}}$$

$$= \frac{d\cos(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{d\cos(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}$$
$$- \frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{5/2}\sqrt{b}}$$
$$+ \frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{5/2}\sqrt{b}}$$
$$+ \frac{\text{CosIntegral}(dx)\sin(c)}{a^3} - \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^3}$$
$$- \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^3} + \frac{\sin(c+dx)}{4a\left(a+bx^2\right)^2}$$
$$+ \frac{\sin(c+dx)}{2a^2\left(a+bx^2\right)} + \frac{\cos(c)\text{Si}(dx)}{a^3} + \frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^3}$$
$$- \frac{d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{5/2}\sqrt{b}} - \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^3}$$
$$- \frac{d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{5/2}\sqrt{b}} + \frac{d\int\frac{\cos(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16(-a)^{5/2}} + \frac{d\int\frac{\cos(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16(-a)^{5/2}}$$
$$- \frac{\left(d^2\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16a^2} - \frac{\left(d^2\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16a^2}$$
$$- \frac{\left(d^2\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16a^2} + \frac{\left(d^2\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16a^2}$$

$$= \frac{d\cos(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{d\cos(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}$$
$$- \frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{5/2}\sqrt{b}}$$
$$+ \frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{5/2}\sqrt{b}}$$
$$+ \frac{\text{CosIntegral}(dx)\sin(c)}{a^3} - \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^3}$$
$$- \frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^2b}$$
$$- \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^3}$$
$$- \frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^2b}$$
$$+ \frac{\sin(c+dx)}{4a\left(a+bx^2\right)^2} + \frac{\sin(c+dx)}{2a^2\left(a+bx^2\right)} + \frac{\cos(c)\text{Si}(dx)}{a^3}$$
$$+ \frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^3} + \frac{d^2\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^2b}$$
$$- \frac{d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{5/2}\sqrt{b}} - \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^3}$$
$$- \frac{d^2\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16a^2b} - \frac{d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{5/2}\sqrt{b}}$$
$$+ \frac{\left(d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16(-a)^{5/2}} + \frac{\left(d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16(-a)^{5/2}}$$
$$- \frac{\left(d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16(-a)^{5/2}} + \frac{\left(d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16(-a)^{5/2}}$$

$$
\begin{aligned}
&= \frac{d\cos(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{d\cos(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} \\
&\quad - \frac{5d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{5/2}\sqrt{b}} \\
&\quad + \frac{5d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{5/2}\sqrt{b}} \\
&\quad + \frac{\text{CosIntegral}(dx)\sin(c)}{a^3} - \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^3} \\
&\quad - \frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^2b} \\
&\quad - \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^3} \\
&\quad - \frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^2b} \\
&\quad + \frac{\sin(c+dx)}{4a\left(a+bx^2\right)^2} + \frac{\sin(c+dx)}{2a^2\left(a+bx^2\right)} + \frac{\cos(c)\text{Si}(dx)}{a^3} \\
&\quad + \frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^3} + \frac{d^2\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^2b} \\
&\quad - \frac{5d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{5/2}\sqrt{b}} - \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^3} \\
&\quad - \frac{d^2\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16a^2b} - \frac{5d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{5/2}\sqrt{b}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.36 (sec) , antiderivative size = 672, normalized size of antiderivative = 0.92

$$
\int \frac{\sin(c+dx)}{x\left(a+bx^2\right)^3}\,dx
$$

$$
= \frac{-\frac{4adx\cos(c+dx)}{a+bx^2} + \frac{4i\sqrt{a}de^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)-\text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)\right)}{\sqrt{b}} - 8ie^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpI}\right.}{}
$$

```
[In] Integrate[Sin[c + d*x]/(x*(a + b*x^2)^3),x]
```

```
[Out] ((-4*a*d*x*Cos[c + d*x])/(a + b*x^2) + ((4*I)*Sqrt[a]*d*E^((-I)*c - (Sqrt[a
]*d)/Sqrt[b])*(E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b
]) - I*d*x] - ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] - I*d*x]))/Sqrt[b] - (8*I)*
E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*(E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-
((Sqrt[a]*d)/Sqrt[b]) - I*d*x] + ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] - I*d*x]
) - (I*Sqrt[a]*d*E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*(-((Sqrt[b] - Sqrt[a]*d)*
E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x]) +
(Sqrt[b] + Sqrt[a]*d)*ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] - I*d*x]))/b - ((4*
I)*Sqrt[a]*d*E^(I*c - (Sqrt[a]*d)/Sqrt[b])*(E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIn
tegralEi[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] - ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b
] + I*d*x]))/Sqrt[b] + (8*I)*E^(I*c - (Sqrt[a]*d)/Sqrt[b])*(E^((2*Sqrt[a]*d
)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] + ExpIntegralEi[(S
qrt[a]*d)/Sqrt[b] + I*d*x]) + (I*Sqrt[a]*d*E^(I*c - (Sqrt[a]*d)/Sqrt[b])*(-
((Sqrt[b] - Sqrt[a]*d)*E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d
)/Sqrt[b]) + I*d*x]) + (Sqrt[b] + Sqrt[a]*d)*ExpIntegralEi[(Sqrt[a]*d)/Sqrt
[b] + I*d*x]))/b + 32*CosIntegral[d*x]*Sin[c] + (8*a*(3*a + 2*b*x^2)*Sin[c
+ d*x])/(a + b*x^2)^2 + 32*Cos[c]*SinIntegral[d*x])/(32*a^3)
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 580, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{\sin(dx+c)d^2\left(3a\,d^2+2c^2b-4bc(dx+c)+2b(dx+c)^2\right)}{4a^2\left(a\,d^2+c^2b-2bc(dx+c)+b(dx+c)^2\right)^2}-\frac{\cos(dx+c)d^3x}{8a^2\left(a\,d^2+c^2b-2bc(dx+c)+b(dx+c)^2\right)}+\frac{\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)}{a^3}$
default	$\frac{\sin(dx+c)d^2\left(3a\,d^2+2c^2b-4bc(dx+c)+2b(dx+c)^2\right)}{4a^2\left(a\,d^2+c^2b-2bc(dx+c)+b(dx+c)^2\right)^2}-\frac{\cos(dx+c)d^3x}{8a^2\left(a\,d^2+c^2b-2bc(dx+c)+b(dx+c)^2\right)}+\frac{\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)}{a^3}$
risch	$\frac{i\mathrm{e}^{-\frac{icb-d\sqrt{ab}}{b}}\,\mathrm{Ei}_1\left(-\frac{icb-d\sqrt{ab}-b(idx+ic)}{b}\right)}{4a^3}+\frac{i\mathrm{e}^{-\frac{icb+d\sqrt{ab}}{b}}\,\mathrm{Ei}_1\left(-\frac{icb+d\sqrt{ab}-b(idx+ic)}{b}\right)}{4a^3}-\frac{i\mathrm{e}^{\frac{icb+d\sqrt{ab}}{b}}\,\mathrm{Ei}_1\left(\frac{icb+d\sqrt{ab}-b}{b}\right.}{32b\,a^2}$

```
[In] int(sin(d*x+c)/x/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sin(d*x+c)*d^2*(3*a*d^2+2*c^2*b-4*b*c*(d*x+c)+2*b*(d*x+c)^2)/a^2/(a*d^2
+c^2*b-2*b*c*(d*x+c)+b*(d*x+c)^2)^2-1/8*cos(d*x+c)*d^3*x/a^2/(a*d^2+c^2*b-2
*b*c*(d*x+c)+b*(d*x+c)^2)+1/a^3*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))-1/16*(a*d^2
+8*b)/b/a^3*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+C
i(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))-1/16*(a*d^2+8*
b)/b/a^3*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d
*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))+5/16*d^2/a^2/b/(-
(d*(-a*b)^(1/2)+c*b)/b+c)*(-Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^
(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))
+5/16*d^2/a^2/b/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b
)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b
)^(1/2)-c*b)/b))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 637, normalized size of antiderivative = 0.87

$$\int \frac{\sin(c+dx)}{x\,(a+bx^2)^3}\,dx =$$

$$-\frac{\left(-i\,a^3d^2 - i\,(ab^2d^2+8\,b^3)x^4 - 8i\,a^2b - 2i\,(a^2bd^2+8\,ab^2)x^2 + 5\,(i\,b^3x^4+2i\,ab^2x^2+i\,a^2b)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\Big(i}{}$$

```
[In] integrate(sin(d*x+c)/x/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/32*((-I*a^3*d^2 - I*(a*b^2*d^2 + 8*b^3)*x^4 - 8*I*a^2*b - 2*I*(a^2*b*d^2
 + 8*a*b^2)*x^2 + 5*(I*b^3*x^4 + 2*I*a*b^2*x^2 + I*a^2*b)*sqrt(a*d^2/b))*Ei
(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + (-I*a^3*d^2 - I*(a*b^2*d^
2 + 8*b^3)*x^4 - 8*I*a^2*b - 2*I*(a^2*b*d^2 + 8*a*b^2)*x^2 + 5*(-I*b^3*x^4
- 2*I*a*b^2*x^2 - I*a^2*b)*sqrt(a*d^2/b))*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c
- sqrt(a*d^2/b)) + (I*a^3*d^2 + I*(a*b^2*d^2 + 8*b^3)*x^4 + 8*I*a^2*b + 2*I
*(a^2*b*d^2 + 8*a*b^2)*x^2 + 5*(-I*b^3*x^4 - 2*I*a*b^2*x^2 - I*a^2*b)*sqrt(
a*d^2/b))*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) + (I*a^3*d^2
+ I*(a*b^2*d^2 + 8*b^3)*x^4 + 8*I*a^2*b + 2*I*(a^2*b*d^2 + 8*a*b^2)*x^2 + 5
*(I*b^3*x^4 + 2*I*a*b^2*x^2 + I*a^2*b)*sqrt(a*d^2/b))*Ei(-I*d*x + sqrt(a*d^
2/b))*e^(-I*c - sqrt(a*d^2/b)) - 32*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cos_int
egral(d*x)*sin(c) - 32*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cos(c)*sin_integral(
d*x) + 4*(a*b^2*d*x^3 + a^2*b*d*x)*cos(d*x + c) - 8*(2*a*b^2*x^2 + 3*a^2*b)
*sin(d*x + c))/(a^3*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{x\,(a+bx^2)^3}\,dx = \int \frac{\sin{(c+dx)}}{x\,(a+bx^2)^3}\,dx$$

```
[In] integrate(sin(d*x+c)/x/(b*x**2+a)**3,x)
```

```
[Out] Integral(sin(c + d*x)/(x*(a + b*x**2)**3), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x\left(a+bx^2\right)^3}\,dx = \int \frac{\sin\left(dx+c\right)}{\left(bx^2+a\right)^3 x}\,dx$$

```
[In] integrate(sin(d*x+c)/x/(b*x^2+a)^3,x, algorithm="maxima")
[Out] integrate(sin(d*x + c)/((b*x^2 + a)^3*x), x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{x\left(a+bx^2\right)^3}\,dx = \int \frac{\sin\left(dx+c\right)}{\left(bx^2+a\right)^3 x}\,dx$$

```
[In] integrate(sin(d*x+c)/x/(b*x^2+a)^3,x, algorithm="giac")
[Out] integrate(sin(d*x + c)/((b*x^2 + a)^3*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x\left(a+bx^2\right)^3}\,dx = \int \frac{\sin\left(c+d\,x\right)}{x\,\left(b\,x^2+a\right)^3}\,dx$$

```
[In] int(sin(c + d*x)/(x*(a + b*x^2)^3),x)
[Out] int(sin(c + d*x)/(x*(a + b*x^2)^3), x)
```

3.77 $\int \frac{\sin(c+dx)}{x^2(a+bx^2)^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 875

$$\int \frac{\sin(c+dx)}{x^2(a+bx^2)^3}\,dx = \frac{d\cos(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{d\cos(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}+\sqrt{b}x\right)}$$
$$+ \frac{d\cos(c)\operatorname{CosIntegral}(dx)}{a^3}$$
$$+ \frac{7d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^3}$$
$$+ \frac{7d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16a^3}$$
$$- \frac{15\sqrt{b}\operatorname{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{7/2}}$$
$$+ \frac{d^2\operatorname{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{5/2}\sqrt{b}}$$
$$+ \frac{15\sqrt{b}\operatorname{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{7/2}}$$
$$- \frac{d^2\operatorname{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{5/2}\sqrt{b}} - \frac{\sin(c+dx)}{a^3x}$$
$$- \frac{\sqrt{b}\sin(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}-\sqrt{b}x\right)^2} + \frac{7\sqrt{b}\sin(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}x\right)}$$
$$+ \frac{\sqrt{b}\sin(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}+\sqrt{b}x\right)^2} - \frac{7\sqrt{b}\sin(c+dx)}{16a^3\left(\sqrt{-a}+\sqrt{b}x\right)}$$
$$- \frac{d\sin(c)\operatorname{Si}(dx)}{a^3} - \frac{15\sqrt{b}\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{7/2}}$$
$$+ \frac{d^2\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{5/2}\sqrt{b}}$$
$$+ \frac{7d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^3}$$
$$- \frac{15\sqrt{b}\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{7/2}}$$
$$+ \frac{d^2\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{5/2}\sqrt{b}}$$
$$- \frac{7d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16a^3}$$

```
[Out] d*Ci(d*x)*cos(c)/a^3+7/16*d*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*cos(c-d*(-a)^(1/2)
/b^(1/2))/a^3+7/16*d*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*cos(c+d*(-a)^(1/2)/b^(1/
2))/a^3-d*Si(d*x)*sin(c)/a^3-sin(d*x+c)/a^3/x-7/16*d*Si(d*x+d*(-a)^(1/2)/b^
(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))/a^3-7/16*d*Si(d*x-d*(-a)^(1/2)/b^(1/2))*
sin(c+d*(-a)^(1/2)/b^(1/2))/a^3-1/16*d^2*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x
-d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)+1/16*d^2*cos(c-d*(-a)^(1/2)/b^(1/
2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)+1/16*d^2*Ci(d*x+d*(-a)^
(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)-1/16*d^2*Ci(-
d*x+d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)+15
/16*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(
7/2)-15/16*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))*b^(1/2)
/(-a)^(7/2)-15/16*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))*
b^(1/2)/(-a)^(7/2)+15/16*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b
^(1/2))*b^(1/2)/(-a)^(7/2)-1/16*sin(d*x+c)*b^(1/2)/(-a)^(5/2)/((-a)^(1/2)-x
*b^(1/2))^2+1/16*d*cos(d*x+c)/(-a)^(5/2)/((-a)^(1/2)-x*b^(1/2))+7/16*sin(d*
x+c)*b^(1/2)/a^3/((-a)^(1/2)-x*b^(1/2))+1/16*sin(d*x+c)*b^(1/2)/(-a)^(5/2)/
((-a)^(1/2)+x*b^(1/2))^2+1/16*d*cos(d*x+c)/(-a)^(5/2)/((-a)^(1/2)+x*b^(1/2)
)-7/16*sin(d*x+c)*b^(1/2)/a^3/((-a)^(1/2)+x*b^(1/2))
```

Rubi [A] (verified)

Time = 1.70 (sec) , antiderivative size = 875, normalized size of antiderivative = 1.00, number of steps used = 60, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used

$= \{3426, 3378, 3384, 3380, 3383, 3414\}$

$$\int \frac{\sin(c+dx)}{x^2\left(a+bx^2\right)^3}\,dx = \frac{\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)d^2}{16(-a)^{5/2}\sqrt{b}} - \frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)d^2}{16(-a)^{5/2}\sqrt{b}} + \frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)d^2}{16(-a)^{5/2}\sqrt{b}} + \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)d^2}{16(-a)^{5/2}\sqrt{b}} + \frac{\cos(c+dx)d}{16(-a)^{5/2}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cos(c+dx)d}{16(-a)^{5/2}\left(\sqrt{b}x+\sqrt{-a}\right)} + \frac{\cos(c)\,\text{CosIntegral}(dx)d}{a^3} + \frac{7\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)d}{16a^3} + \frac{7\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)d}{16a^3} - \frac{\sin(c)\text{Si}(dx)d}{a^3} + \frac{7\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)d}{16a^3} - \frac{7\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)d}{16a^3} - \frac{15\sqrt{b}\,\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{7/2}} + \frac{15\sqrt{b}\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{7/2}} - \frac{\sin(c+dx)}{a^3x} + \frac{7\sqrt{b}\sin(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{7\sqrt{b}\sin(c+dx)}{16a^3\left(\sqrt{b}x+\sqrt{-a}\right)} - \frac{\sqrt{b}\sin(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}-\sqrt{b}x\right)^2} + \frac{\sqrt{b}\sin(c+dx)}{16(-a)^{5/2}\left(\sqrt{b}x+\sqrt{-a}\right)^2} - \frac{15\sqrt{b}\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{7/2}} - \frac{15\sqrt{b}\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{7/2}}$$

```
[In] Int[Sin[c + d*x]/(x^2*(a + b*x^2)^3),x]
```

```
[Out] (d*Cos[c + d*x])/(16*(-a)^(5/2)*(Sqrt[-a] - Sqrt[b]*x)) + (d*Cos[c + d*x])/
(16*(-a)^(5/2)*(Sqrt[-a] + Sqrt[b]*x)) + (d*Cos[c]*CosIntegral[d*x])/a^3 +
(7*d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])
/(16*a^3) + (7*d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqr
t[b] + d*x])/(16*a^3) - (15*Sqrt[b]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]
*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(7/2)) + (d^2*CosIntegral[(Sqrt[-a
]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(5/2)*Sqrt[b])
+ (15*Sqrt[b]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/
Sqrt[b]])/(16*(-a)^(7/2)) - (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Si
n[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(5/2)*Sqrt[b]) - Sin[c + d*x]/(a^3*x)
 - (Sqrt[b]*Sin[c + d*x])/(16*(-a)^(5/2)*(Sqrt[-a] - Sqrt[b]*x)^2) + (7*Sqr
t[b]*Sin[c + d*x])/(16*a^3*(Sqrt[-a] - Sqrt[b]*x)) + (Sqrt[b]*Sin[c + d*x])
/(16*(-a)^(5/2)*(Sqrt[-a] + Sqrt[b]*x)^2) - (7*Sqrt[b]*Sin[c + d*x])/(16*a^
3*(Sqrt[-a] + Sqrt[b]*x)) - (d*Sin[c]*SinIntegral[d*x])/a^3 - (15*Sqrt[b]*C
os[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(
-a)^(7/2)) + (d^2*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sq
rt[b] - d*x])/(16*(-a)^(5/2)*Sqrt[b]) + (7*d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*
SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^3) - (15*Sqrt[b]*Cos[c - (Sq
rt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(7/2))
 + (d^2*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*
x])/(16*(-a)^(5/2)*Sqrt[b]) - (7*d*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegra
l[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^3)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\sin(c+dx)}{a^3x^2} - \frac{b\sin(c+dx)}{a\left(a+bx^2\right)^3} - \frac{b\sin(c+dx)}{a^2\left(a+bx^2\right)^2} - \frac{b\sin(c+dx)}{a^3\left(a+bx^2\right)}\right) dx$$

$$= \frac{\int \frac{\sin(c+dx)}{x^2}\,dx}{a^3} - \frac{b\int \frac{\sin(c+dx)}{a+bx^2}\,dx}{a^3} - \frac{b\int \frac{\sin(c+dx)}{\left(a+bx^2\right)^2}\,dx}{a^2} - \frac{b\int \frac{\sin(c+dx)}{\left(a+bx^2\right)^3}\,dx}{a}$$

$$= -\frac{\sin(c+dx)}{a^3x} - \frac{b\int \left(\frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right) dx}{a^3}$$
$$- \frac{b\int \left(-\frac{b\sin(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}-bx\right)^2} - \frac{b\sin(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}+bx\right)^2} - \frac{b\sin(c+dx)}{2a\left(-ab-b^2x^2\right)}\right) dx}{a^2}$$
$$- \frac{b\int \left(-\frac{b^{3/2}\sin(c+dx)}{8(-a)^{3/2}\left(\sqrt{-a}\sqrt{b}-bx\right)^3} - \frac{3b\sin(c+dx)}{16a^2\left(\sqrt{-a}\sqrt{b}-bx\right)^2} - \frac{b^{3/2}\sin(c+dx)}{8(-a)^{3/2}\left(\sqrt{-a}\sqrt{b}+bx\right)^3} - \frac{3b\sin(c+dx)}{16a^2\left(\sqrt{-a}\sqrt{b}+bx\right)^2} - \frac{3b\sin(c+}{8a^2(-ab-b}\right.}{a}$$
$$+ \frac{d\int \frac{\cos(c+dx)}{x}\,dx}{a^3}$$

$$= -\frac{\sin(c+dx)}{a^3x} - \frac{b\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2(-a)^{7/2}} - \frac{b\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2(-a)^{7/2}} + \frac{(3b^2)\int \frac{\sin(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}\,dx}{16a^3}$$
$$+ \frac{(3b^2)\int \frac{\sin(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^2}\,dx}{16a^3} + \frac{b^2\int \frac{\sin(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}\,dx}{4a^3} + \frac{b^2\int \frac{\sin(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^2}\,dx}{4a^3}$$
$$+ \frac{(3b^2)\int \frac{\sin(c+dx)}{-ab-b^2x^2}\,dx}{8a^3} + \frac{b^2\int \frac{\sin(c+dx)}{-ab-b^2x^2}\,dx}{2a^3} - \frac{b^{5/2}\int \frac{\sin(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^3}\,dx}{8(-a)^{5/2}}$$
$$- \frac{b^{5/2}\int \frac{\sin(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^3}\,dx}{8(-a)^{5/2}} + \frac{(d\cos(c))\int \frac{\cos(dx)}{x}\,dx}{a^3} - \frac{(d\sin(c))\int \frac{\sin(dx)}{x}\,dx}{a^3}$$

$$= \frac{d\cos(c)\,\text{CosIntegral}(dx)}{a^3} - \frac{\sin(c+dx)}{a^3x} - \frac{\sqrt{b}\sin(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}-\sqrt{b}x\right)^2}$$
$$+ \frac{7\sqrt{b}\sin(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{b}\sin(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}+\sqrt{b}x\right)^2} - \frac{7\sqrt{b}\sin(c+dx)}{16a^3\left(\sqrt{-a}+\sqrt{b}x\right)}$$
$$- \frac{d\sin(c)\text{Si}(dx)}{a^3} + \frac{(3b^2)\int \left(-\frac{\sqrt{-a}\sin(c+dx)}{2ab\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{\sqrt{-a}\sin(c+dx)}{2ab\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{8a^3}$$
$$+ \frac{b^2\int \left(-\frac{\sqrt{-a}\sin(c+dx)}{2ab\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{\sqrt{-a}\sin(c+dx)}{2ab\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{2a^3} - \frac{(3bd)\int \frac{\cos(c+dx)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16a^3}$$
$$+ \frac{(3bd)\int \frac{\cos(c+dx)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16a^3} - \frac{(bd)\int \frac{\cos(c+dx)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{4a^3} + \frac{(bd)\int \frac{\cos(c+dx)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{4a^3}$$
$$+ \frac{\left(b^{3/2}d\right)\int \frac{\cos(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}\,dx}{16(-a)^{5/2}} - \frac{\left(b^{3/2}d\right)\int \frac{\cos(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^2}\,dx}{16(-a)^{5/2}}$$
$$- \frac{\left(b\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2(-a)^{7/2}} + \frac{\left(b\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2(-a)^{7/2}}$$
$$- \frac{\left(b\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2(-a)^{7/2}} - \frac{\left(b\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2(-a)^{7/2}}$$

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Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.04 (sec) , antiderivative size = 593, normalized size of antiderivative = 0.68

$$\int \frac{\sin(c+dx)}{x^2\,(a+bx^2)^3}\,dx$$

$$= \frac{8\sqrt{b}e^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(-e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\,\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)+\text{ExpIntegralEi}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)\right)+\frac{e^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(-\left(\left(7b-7\sqrt{a}\sqrt{\right.\right.\right.}{}}{}$$

```
[In] Integrate[Sin[c + d*x]/(x^2*(a + b*x^2)^3),x]
```

```
[Out] (8*Sqrt[b]*E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*(-(E^((2*Sqrt[a]*d)/Sqrt[b])*Ex
pIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x]) + ExpIntegralEi[(Sqrt[a]*d)/Sq
rt[b] - I*d*x]) + (E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*(-((7*b - 7*Sqrt[a]*Sqr
t[b]*d + a*d^2)*E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[
b]) - I*d*x]) + (7*b + 7*Sqrt[a]*Sqrt[b]*d + a*d^2)*ExpIntegralEi[(Sqrt[a]*
d)/Sqrt[b] - I*d*x]))/Sqrt[b] + 8*Sqrt[b]*E^(I*c - (Sqrt[a]*d)/Sqrt[b])*(-(
E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]) +
ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] + I*d*x]) + (E^(I*c - (Sqrt[a]*d)/Sqrt[b]
)*(-((7*b - 7*Sqrt[a]*Sqrt[b]*d + a*d^2)*E^((2*Sqrt[a]*d)/Sqrt[b])*ExpInteg
ralEi[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]) + (7*b + 7*Sqrt[a]*Sqrt[b]*d + a*d^2
)*ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] + I*d*x]))/Sqrt[b] - (4*Sqrt[a]*Cos[d*x
]*(a*d*x*(a + b*x^2)*Cos[c] + (8*a^2 + 25*a*b*x^2 + 15*b^2*x^4)*Sin[c]))/(x
*(a + b*x^2)^2) + (4*Sqrt[a]*(-((8*a^2 + 25*a*b*x^2 + 15*b^2*x^4)*Cos[c]) +
 a*d*x*(a + b*x^2)*Sin[c])*Sin[d*x])/(x*(a + b*x^2)^2) + 32*Sqrt[a]*d*(Cos[
c]*CosIntegral[d*x] - Sin[c]*SinIntegral[d*x]))/(32*a^(7/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 910, normalized size of antiderivative = 1.04

method	result
risch	$\frac{d^2 e^{\frac{icb+d\sqrt{ab}}{b}}\,\text{Ei}_1\left(\frac{icb+d\sqrt{ab}-b(idx+ic)}{b}\right)}{32a^2\sqrt{ab}}-\frac{d^2 e^{\frac{icb-d\sqrt{ab}}{b}}\,\text{Ei}_1\left(-\frac{-icb+d\sqrt{ab}+b(idx+ic)}{b}\right)}{32a^2\sqrt{ab}}-\frac{7d\,e^{\frac{icb+d\sqrt{ab}}{b}}\,\text{Ei}_1\left(\frac{icb+d\sqrt{ab}-}{b}\right.}{32a^3}$
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int(sin(d*x+c)/x^2/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/a^2*d^2/(a*b)^(1/2)*exp((I*c*b+d*(a*b)^(1/2))/b)*Ei(1,(I*c*b+d*(a*b)^(
1/2)-b*(I*d*x+I*c))/b)-1/32/a^2*d^2/(a*b)^(1/2)*exp((I*c*b-d*(a*b)^(1/2))/b
)*Ei(1,-(-I*c*b+d*(a*b)^(1/2)+b*(I*d*x+I*c))/b)-7/32*d/a^3*exp((I*c*b+d*(a*
b)^(1/2))/b)*Ei(1,(I*c*b+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)-7/32*d/a^3*exp((I*
c*b-d*(a*b)^(1/2))/b)*Ei(1,-(-I*c*b+d*(a*b)^(1/2)+b*(I*d*x+I*c))/b)+15/32/a
^3/(a*b)^(1/2)*exp((I*c*b+d*(a*b)^(1/2))/b)*Ei(1,(I*c*b+d*(a*b)^(1/2)-b*(I*
d*x+I*c))/b)*b-15/32/a^3/(a*b)^(1/2)*exp((I*c*b-d*(a*b)^(1/2))/b)*Ei(1,-(-I
*c*b+d*(a*b)^(1/2)+b*(I*d*x+I*c))/b)*b-1/2*d/a^3*Ei(1,-I*d*x)*exp(I*c)-1/32
/a^2*d^2/(a*b)^(1/2)*exp(-(I*c*b+d*(a*b)^(1/2))/b)*Ei(1,-(I*c*b+d*(a*b)^(1/
2)-b*(I*d*x+I*c))/b)+1/32/a^2*d^2/(a*b)^(1/2)*exp(-(I*c*b-d*(a*b)^(1/2))/b)
*Ei(1,(-I*c*b+d*(a*b)^(1/2)+b*(I*d*x+I*c))/b)-7/32*d/a^3*exp(-(I*c*b+d*(a*b
)^(1/2))/b)*Ei(1,-(I*c*b+d*(a*b)^(1/2)-b*(I*d*x+I*c))/b)-7/32*d/a^3*exp(-(I
*c*b-d*(a*b)^(1/2))/b)*Ei(1,(-I*c*b+d*(a*b)^(1/2)+b*(I*d*x+I*c))/b)-15/32/a
^3/(a*b)^(1/2)*exp(-(I*c*b+d*(a*b)^(1/2))/b)*Ei(1,-(I*c*b+d*(a*b)^(1/2)-b*(
I*d*x+I*c))/b)*b+15/32/a^3/(a*b)^(1/2)*exp(-(I*c*b-d*(a*b)^(1/2))/b)*Ei(1,(
-I*c*b+d*(a*b)^(1/2)+b*(I*d*x+I*c))/b)*b-1/2*d/a^3*Ei(1,I*d*x)*exp(-I*c)+1/
8/a^2*d^2*(b*d^3*x^3+a*d^3*x)/x/(-b^2*d^4*x^4-2*a*b*d^4*x^2-a^2*d^4)*cos(d*
x+c)-1/8*(-15*b^2*d^4*x^4-25*a*b*d^4*x^2-8*a^2*d^4)/a^3/x/(-b^2*d^4*x^4-2*a
*b*d^4*x^2-a^2*d^4)*sin(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 714, normalized size of antiderivative = 0.82

$$\int \frac{\sin(c+dx)}{x^2(a+bx^2)^3}\,dx$$
$$= \frac{32\,(ab^2d^2x^5+2\,a^2bd^2x^3+a^3d^2x)\cos(c)\,\mathrm{Ci}\,(dx)+\Big(7\,ab^2d^2x^5+14\,a^2bd^2x^3+7\,a^3d^2x-((ab^2d^2+15\,b^3)x}{}$$

```
[In] integrate(sin(d*x+c)/x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(32*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*cos(c)*cos_integral(
d*x) + (7*a*b^2*d^2*x^5 + 14*a^2*b*d^2*x^3 + 7*a^3*d^2*x - ((a*b^2*d^2 + 15
*b^3)*x^5 + 2*(a^2*b*d^2 + 15*a*b^2)*x^3 + (a^3*d^2 + 15*a^2*b)*x)*sqrt(a*d
^2/b))*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + (7*a*b^2*d^2*x^5
 + 14*a^2*b*d^2*x^3 + 7*a^3*d^2*x + ((a*b^2*d^2 + 15*b^3)*x^5 + 2*(a^2*b*d^
2 + 15*a*b^2)*x^3 + (a^3*d^2 + 15*a^2*b)*x)*sqrt(a*d^2/b))*Ei(I*d*x + sqrt(
a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + (7*a*b^2*d^2*x^5 + 14*a^2*b*d^2*x^3 + 7
*a^3*d^2*x - ((a*b^2*d^2 + 15*b^3)*x^5 + 2*(a^2*b*d^2 + 15*a*b^2)*x^3 + (a^
3*d^2 + 15*a^2*b)*x)*sqrt(a*d^2/b))*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sq
rt(a*d^2/b)) + (7*a*b^2*d^2*x^5 + 14*a^2*b*d^2*x^3 + 7*a^3*d^2*x + ((a*b^2*
d^2 + 15*b^3)*x^5 + 2*(a^2*b*d^2 + 15*a*b^2)*x^3 + (a^3*d^2 + 15*a^2*b)*x)*
```

```
sqrt(a*d^2/b))*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) - 32*(a*
b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sin(c)*sin_integral(d*x) - 4*(a^
2*b*d^2*x^3 + a^3*d^2*x)*cos(d*x + c) - 4*(15*a*b^2*d*x^4 + 25*a^2*b*d*x^2
+ 8*a^3*d)*sin(d*x + c))/(a^4*b^2*d*x^5 + 2*a^5*b*d*x^3 + a^6*d*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x^2\left(a+bx^2\right)^3}\,dx=\text{Timed out}$$

```
[In] integrate(sin(d*x+c)/x**2/(b*x**2+a)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x^2\left(a+bx^2\right)^3}\,dx=\int \frac{\sin\left(dx+c\right)}{\left(bx^2+a\right)^3x^2}\,dx$$

```
[In] integrate(sin(d*x+c)/x^2/(b*x^2+a)^3,x, algorithm="maxima")
[Out] integrate(sin(d*x + c)/((b*x^2 + a)^3*x^2), x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{x^2\left(a+bx^2\right)^3}\,dx=\int \frac{\sin\left(dx+c\right)}{\left(bx^2+a\right)^3x^2}\,dx$$

```
[In] integrate(sin(d*x+c)/x^2/(b*x^2+a)^3,x, algorithm="giac")
[Out] integrate(sin(d*x + c)/((b*x^2 + a)^3*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x^2\left(a+bx^2\right)^3}\,dx=\int \frac{\sin\left(c+d\,x\right)}{x^2\,\left(b\,x^2+a\right)^3}\,dx$$

```
[In] int(sin(c + d*x)/(x^2*(a + b*x^2)^3),x)
[Out] int(sin(c + d*x)/(x^2*(a + b*x^2)^3), x)
```

3.78 $\int \frac{\sin(c+dx)}{x^3(a+bx^2)^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 791

$$\int \frac{\sin(c+dx)}{x^3(a+bx^2)^3}\,dx = -\frac{d\cos(c+dx)}{2a^3x} - \frac{\sqrt{b}d\cos(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{b}d\cos(c+dx)}{16a^3\left(\sqrt{-a}+\sqrt{b}x\right)}$$
$$-\frac{9\sqrt{b}d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{7/2}}$$
$$+\frac{9\sqrt{b}d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{7/2}}$$
$$-\frac{3b\,\text{CosIntegral}(dx)\sin(c)}{a^4} - \frac{d^2\,\text{CosIntegral}(dx)\sin(c)}{2a^3}$$
$$+\frac{3b\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^4}$$
$$+\frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^3}$$
$$+\frac{3b\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^4}$$
$$+\frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^3} - \frac{\sin(c+dx)}{2a^3x^2}$$
$$-\frac{b\sin(c+dx)}{4a^2(a+bx^2)^2} - \frac{b\sin(c+dx)}{a^3(a+bx^2)} - \frac{3b\cos(c)\text{Si}(dx)}{a^4}$$
$$-\frac{d^2\cos(c)\text{Si}(dx)}{2a^3} - \frac{3b\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^4}$$
$$-\frac{d^2\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^3}$$
$$-\frac{9\sqrt{b}d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{7/2}}$$
$$+\frac{3b\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^4}$$
$$+\frac{d^2\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16a^3}$$
$$-\frac{9\sqrt{b}d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{7/2}}$$

```
[Out] -1/2*d*cos(d*x+c)/a^3/x-3*b*cos(c)*Si(d*x)/a^4-1/2*d^2*cos(c)*Si(d*x)/a^3+3
/2*b*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b^(1/2))/a^4+1/16*d^2*
```

```
cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b^(1/2))/a^3+3/2*b*cos(c-d*
(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))/a^4+1/16*d^2*cos(c-d*(-a)^
(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))/a^3-3*b*Ci(d*x)*sin(c)/a^4-1/2*
d^2*Ci(d*x)*sin(c)/a^3-1/2*sin(d*x+c)/a^3/x^2-1/4*b*sin(d*x+c)/a^2/(b*x^2+a
)^2-b*sin(d*x+c)/a^3/(b*x^2+a)+3/2*b*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(
-a)^(1/2)/b^(1/2))/a^4+1/16*d^2*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(
1/2)/b^(1/2))/a^3+3/2*b*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^
(1/2))/a^4+1/16*d^2*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2
))/a^3+9/16*d*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*cos(c-d*(-a)^(1/2)/b^(1/2))*b^(1
/2)/(-a)^(7/2)-9/16*d*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*cos(c+d*(-a)^(1/2)/b^(1
/2))*b^(1/2)/(-a)^(7/2)-9/16*d*Si(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1
/2)/b^(1/2))*b^(1/2)/(-a)^(7/2)+9/16*d*Si(d*x-d*(-a)^(1/2)/b^(1/2))*sin(c+d
*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(7/2)-1/16*d*cos(d*x+c)*b^(1/2)/a^3/((-a)
^(1/2)-x*b^(1/2))+1/16*d*cos(d*x+c)*b^(1/2)/a^3/((-a)^(1/2)+x*b^(1/2))
```

Rubi [A] (verified)

Time = 2.03 (sec) , antiderivative size = 791, normalized size of antiderivative = 1.00, number of steps used = 46, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used

$= \{3426, 3378, 3384, 3380, 3383, 3422, 3415\}$

$$\int \frac{\sin(c+dx)}{x^3\left(a+bx^2\right)^3}\,dx = -\frac{3b\sin(c)\,\text{CosIntegral}(dx)}{a^4}$$
$$+\frac{3b\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^4}$$
$$+\frac{3b\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^4}$$
$$-\frac{3b\cos(c)\text{Si}(dx)}{a^4}-\frac{3b\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^4}$$
$$+\frac{3b\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^4}$$
$$+\frac{d^2\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^3}$$
$$+\frac{d^2\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^3}$$
$$-\frac{d^2\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^3}$$
$$+\frac{d^2\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^3}-\frac{b\sin(c+dx)}{a^3\left(a+bx^2\right)}$$
$$-\frac{\sqrt{b}d\cos(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{b}d\cos(c+dx)}{16a^3\left(\sqrt{-a}+\sqrt{b}x\right)}$$
$$-\frac{d^2\sin(c)\,\text{CosIntegral}(dx)}{2a^3}-\frac{d^2\cos(c)\text{Si}(dx)}{2a^3}$$
$$-\frac{\sin(c+dx)}{2a^3x^2}-\frac{d\cos(c+dx)}{2a^3x}-\frac{b\sin(c+dx)}{4a^2\left(a+bx^2\right)^2}$$
$$-\frac{9\sqrt{b}d\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{7/2}}$$
$$+\frac{9\sqrt{b}d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{7/2}}$$
$$-\frac{9\sqrt{b}d\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{7/2}}$$
$$-\frac{9\sqrt{b}d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{7/2}}$$

```
[In] Int[Sin[c + d*x]/(x^3*(a + b*x^2)^3),x]
```

```
[Out] -1/2*(d*Cos[c + d*x])/(a^3*x) - (Sqrt[b]*d*Cos[c + d*x])/(16*a^3*(Sqrt[-a]
- Sqrt[b]*x)) + (Sqrt[b]*d*Cos[c + d*x])/(16*a^3*(Sqrt[-a] + Sqrt[b]*x)) -
(9*Sqrt[b]*d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b]
 - d*x])/(16*(-a)^(7/2)) + (9*Sqrt[b]*d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIn
tegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(7/2)) - (3*b*CosIntegral[d*x]
*Sin[c])/a^4 - (d^2*CosIntegral[d*x]*Sin[c])/(2*a^3) + (3*b*CosIntegral[(Sq
rt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(2*a^4) + (d^2*CosI
ntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*a^3)
 + (3*b*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b
]])/(2*a^4) + (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a
]*d)/Sqrt[b]])/(16*a^3) - Sin[c + d*x]/(2*a^3*x^2) - (b*Sin[c + d*x])/(4*a^
2*(a + b*x^2)^2) - (b*Sin[c + d*x])/(a^3*(a + b*x^2)) - (3*b*Cos[c]*SinInte
gral[d*x])/a^4 - (d^2*Cos[c]*SinIntegral[d*x])/(2*a^3) - (3*b*Cos[c + (Sqrt
[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^4) - (d^2*Co
s[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^
3) - (9*Sqrt[b]*d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sq
rt[b] - d*x])/(16*(-a)^(7/2)) + (3*b*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinInteg
ral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^4) + (d^2*Cos[c - (Sqrt[-a]*d)/Sqrt[b
]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^3) - (9*Sqrt[b]*d*Sin[c -
 (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(7
/2))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3415

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3422

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))),
x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\sin(c+dx)}{a^3x^3} - \frac{3b\sin(c+dx)}{a^4x} + \frac{b^2x\sin(c+dx)}{a^2(a+bx^2)^3} + \frac{2b^2x\sin(c+dx)}{a^3(a+bx^2)^2} + \frac{3b^2x\sin(c+dx)}{a^4(a+bx^2)}\right)dx$$

$$= \frac{\int \frac{\sin(c+dx)}{x^3}\,dx}{a^3} - \frac{(3b)\int \frac{\sin(c+dx)}{x}\,dx}{a^4} + \frac{(3b^2)\int \frac{x\sin(c+dx)}{a+bx^2}\,dx}{a^4} + \frac{(2b^2)\int \frac{x\sin(c+dx)}{(a+bx^2)^2}\,dx}{a^3} + \frac{b^2\int \frac{x\sin(c+dx)}{(a+bx^2)^3}\,dx}{a^2}$$

$$= -\frac{\sin(c+dx)}{2a^3x^2} - \frac{b\sin(c+dx)}{4a^2(a+bx^2)^2} - \frac{b\sin(c+dx)}{a^3(a+bx^2)} + \frac{(3b^2)\int\left(-\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{a^4} + \frac{d\int \frac{\cos(c+dx)}{x^2}\,dx}{2a^3} + \frac{(bd)\int \frac{\cos(c+dx)}{a+bx^2}\,dx}{a^3} + \frac{(bd)\int \frac{\cos(c+dx)}{(a+bx^2)^2}\,dx}{4a^2} - \frac{(3b\cos(c))\int \frac{\sin(dx)}{x}\,dx}{a^4} - \frac{(3b\sin(c))\int \frac{\cos(dx)}{x}\,dx}{a^4}$$

$$= -\frac{d\cos(c+dx)}{2a^3x} - \frac{3b\,\text{CosIntegral}(dx)\sin(c)}{a^4} - \frac{\sin(c+dx)}{2a^3x^2} - \frac{b\sin(c+dx)}{4a^2\left(a+bx^2\right)^2} - \frac{b\sin(c+dx)}{a^3\left(a+bx^2\right)} - \frac{3b\cos(c)\text{Si}(dx)}{a^4} - \frac{\left(3b^{3/2}\right)\int\frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2a^4} + \frac{\left(3b^{3/2}\right)\int\frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2a^4} + \frac{(bd)\int\left(\frac{\sqrt{-a}\cos(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{-a}\cos(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{a^3} + \frac{(bd)\int\left(-\frac{b\cos(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}-bx\right)^2} - \frac{b\cos(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}+bx\right)^2} - \frac{b\cos(c+dx)}{2a(-ab-b^2x^2)}\right)dx}{4a^2} - \frac{d^2\int\frac{\sin(c+dx)}{x}\,dx}{2a^3}$$

$$= -\frac{d\cos(c+dx)}{2a^3x} - \frac{3b\,\text{CosIntegral}(dx)\sin(c)}{a^4} - \frac{\sin(c+dx)}{2a^3x^2} - \frac{b\sin(c+dx)}{4a^2\left(a+bx^2\right)^2} - \frac{b\sin(c+dx)}{a^3\left(a+bx^2\right)} - \frac{3b\cos(c)\text{Si}(dx)}{a^4} + \frac{(bd)\int\frac{\cos(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2(-a)^{7/2}} + \frac{(bd)\int\frac{\cos(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2(-a)^{7/2}} - \frac{(b^2d)\int\frac{\cos(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}\,dx}{16a^3} - \frac{(b^2d)\int\frac{\cos(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^2}\,dx}{16a^3} - \frac{(b^2d)\int\frac{\cos(c+dx)}{-ab-b^2x^2}\,dx}{8a^3} - \frac{(d^2\cos(c))\int\frac{\sin(dx)}{x}\,dx}{2a^3} + \frac{\left(3b^{3/2}\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2a^4} + \frac{\left(3b^{3/2}\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2a^4} - \frac{(d^2\sin(c))\int\frac{\cos(dx)}{x}\,dx}{2a^3} + \frac{\left(3b^{3/2}\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2a^4} - \frac{\left(3b^{3/2}\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2a^4}$$

$$
\begin{aligned}
&= -\frac{d\cos(c+dx)}{2a^3x} - \frac{\sqrt{b}d\cos(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{b}d\cos(c+dx)}{16a^3\left(\sqrt{-a}+\sqrt{b}x\right)} \\
&\quad - \frac{3b\,\text{CosIntegral}(dx)\sin(c)}{a^4} - \frac{d^2\,\text{CosIntegral}(dx)\sin(c)}{2a^3} \\
&\quad + \frac{3b\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^4} \\
&\quad + \frac{3b\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^4} - \frac{\sin(c+dx)}{2a^3x^2} \\
&\quad - \frac{b\sin(c+dx)}{4a^2\left(a+bx^2\right)^2} - \frac{b\sin(c+dx)}{a^3\left(a+bx^2\right)} - \frac{3b\cos(c)\text{Si}(dx)}{a^4} - \frac{d^2\cos(c)\text{Si}(dx)}{2a^3} \\
&\quad - \frac{3b\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^4} + \frac{3b\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^4} \\
&\quad - \frac{(b^2d)\int\left(-\frac{\sqrt{-a}\cos(c+dx)}{2ab\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{\sqrt{-a}\cos(c+dx)}{2ab\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{8a^3} - \frac{(bd^2)\int\frac{\sin(c+dx)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16a^3} \\
&\quad + \frac{(bd^2)\int\frac{\sin(c+dx)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16a^3} + \frac{\left(bd\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2(-a)^{7/2}} \\
&\quad + \frac{\left(bd\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2(-a)^{7/2}} - \frac{\left(bd\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2(-a)^{7/2}} \\
&\quad + \frac{\left(bd\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2(-a)^{7/2}}
\end{aligned}
$$

$$= -\frac{d\cos(c+dx)}{2a^3x} - \frac{\sqrt{b}d\cos(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{b}d\cos(c+dx)}{16a^3\left(\sqrt{-a}+\sqrt{b}x\right)}$$
$$- \frac{\sqrt{b}d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2(-a)^{7/2}}$$
$$+ \frac{\sqrt{b}d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2(-a)^{7/2}} - \frac{3b\,\text{CosIntegral}(dx)\sin(c)}{a^4}$$
$$- \frac{d^2\,\text{CosIntegral}(dx)\sin(c)}{2a^3} + \frac{3b\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^4}$$
$$+ \frac{3b\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^4} - \frac{\sin(c+dx)}{2a^3x^2}$$
$$- \frac{b\sin(c+dx)}{4a^2\left(a+bx^2\right)^2} - \frac{b\sin(c+dx)}{a^3\left(a+bx^2\right)} - \frac{3b\cos(c)\text{Si}(dx)}{a^4}$$
$$- \frac{d^2\cos(c)\text{Si}(dx)}{2a^3} - \frac{3b\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^4}$$
$$- \frac{\sqrt{b}d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2(-a)^{7/2}} + \frac{3b\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^4}$$
$$- \frac{\sqrt{b}d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2(-a)^{7/2}} + \frac{(bd)\int\frac{\cos(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16(-a)^{7/2}}$$
$$+ \frac{(bd)\int\frac{\cos(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16(-a)^{7/2}} + \frac{\left(bd^2\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16a^3}$$
$$+ \frac{\left(bd^2\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16a^3}$$
$$+ \frac{\left(bd^2\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16a^3}$$
$$- \frac{\left(bd^2\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16a^3}$$

$$
\begin{aligned}
&= -\frac{d\cos(c+dx)}{2a^3x} - \frac{\sqrt{b}d\cos(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{b}d\cos(c+dx)}{16a^3\left(\sqrt{-a}+\sqrt{b}x\right)} \\
&- \frac{\sqrt{b}d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2(-a)^{7/2}} \\
&+ \frac{\sqrt{b}d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2(-a)^{7/2}} - \frac{3b\,\text{CosIntegral}(dx)\sin(c)}{a^4} \\
&- \frac{d^2\,\text{CosIntegral}(dx)\sin(c)}{2a^3} + \frac{3b\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^4} \\
&+ \frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^3} \\
&+ \frac{3b\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^4} \\
&+ \frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^3} - \frac{\sin(c+dx)}{2a^3x^2} \\
&- \frac{b\sin(c+dx)}{4a^2\left(a+bx^2\right)^2} - \frac{b\sin(c+dx)}{a^3\left(a+bx^2\right)} - \frac{3b\cos(c)\text{Si}(dx)}{a^4} - \frac{d^2\cos(c)\text{Si}(dx)}{2a^3} \\
&- \frac{3b\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^4} - \frac{d^2\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^3} \\
&- \frac{\sqrt{b}d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2(-a)^{7/2}} + \frac{3b\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^4} \\
&+ \frac{d^2\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16a^3} - \frac{\sqrt{b}d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2(-a)^{7/2}} \\
&+ \frac{\left(bd\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16(-a)^{7/2}} + \frac{\left(bd\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16(-a)^{7/2}} \\
&- \frac{\left(bd\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16(-a)^{7/2}} + \frac{\left(bd\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16(-a)^{7/2}}
\end{aligned}
$$

$$= -\frac{d\cos(c+dx)}{2a^3x} - \frac{\sqrt{b}d\cos(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{b}d\cos(c+dx)}{16a^3\left(\sqrt{-a}+\sqrt{b}x\right)}$$
$$- \frac{9\sqrt{b}d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{7/2}}$$
$$+ \frac{9\sqrt{b}d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{7/2}} - \frac{3b\,\text{CosIntegral}(dx)\sin(c)}{a^4}$$
$$- \frac{d^2\,\text{CosIntegral}(dx)\sin(c)}{2a^3} + \frac{3b\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^4}$$
$$+ \frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^3}$$
$$+ \frac{3b\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^4}$$
$$+ \frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^3} - \frac{\sin(c+dx)}{2a^3x^2}$$
$$- \frac{b\sin(c+dx)}{4a^2(a+bx^2)^2} - \frac{b\sin(c+dx)}{a^3(a+bx^2)} - \frac{3b\cos(c)\text{Si}(dx)}{a^4} - \frac{d^2\cos(c)\text{Si}(dx)}{2a^3}$$
$$- \frac{3b\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^4} - \frac{d^2\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^3}$$
$$- \frac{9\sqrt{b}d\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{7/2}} + \frac{3b\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^4}$$
$$+ \frac{d^2\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16a^3} - \frac{9\sqrt{b}d\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{7/2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.86 (sec) , antiderivative size = 432, normalized size of antiderivative = 0.55

$$\int \frac{\sin(c+dx)}{x^3(a+bx^2)^3}\,dx$$
$$= \frac{ie^{-ic-\frac{\sqrt{a}d}{\sqrt{b}}}\left(\left(24b-9\sqrt{a}\sqrt{b}d+ad^2\right)e^{\frac{2\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)+\left(24b+9\sqrt{a}\sqrt{b}d+ad^2\right)\text{ExpInt}}{}$$

```
[In] Integrate[Sin[c + d*x]/(x^3*(a + b*x^2)^3),x]
```

```
[Out] (I*E^((-I)*c - (Sqrt[a]*d)/Sqrt[b])*((24*b - 9*Sqrt[a]*Sqrt[b]*d + a*d^2)*E
^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) - I*d*x] + (2
```

```
4*b + 9*Sqrt[a]*Sqrt[b]*d + a*d^2)*ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] - I*d*
x]) - I*E^(I*c - (Sqrt[a]*d)/Sqrt[b])*((24*b - 9*Sqrt[a]*Sqrt[b]*d + a*d^2)
*E^((2*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] +
(24*b + 9*Sqrt[a]*Sqrt[b]*d + a*d^2)*ExpIntegralEi[(Sqrt[a]*d)/Sqrt[b] + I*
d*x]) - (4*a*Cos[d*x]*(d*x*(4*a^2 + 7*a*b*x^2 + 3*b^2*x^4)*Cos[c] + 2*(2*a^
2 + 9*a*b*x^2 + 6*b^2*x^4)*Sin[c]))/(x^2*(a + b*x^2)^2) + (4*a*(-2*(2*a^2 +
 9*a*b*x^2 + 6*b^2*x^4)*Cos[c] + d*x*(4*a^2 + 7*a*b*x^2 + 3*b^2*x^4)*Sin[c]
)*Sin[d*x])/(x^2*(a + b*x^2)^2) - 16*(6*b + a*d^2)*(CosIntegral[d*x]*Sin[c]
 + Cos[c]*SinIntegral[d*x]))/(32*a^4)
```

Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 697, normalized size of antiderivative = 0.88

method	result
derivativedivides	$d^2 \left(-\frac{\sin(dx+c)\left(2a^2d^4+9ab\,c^2d^2-18abc\,d^2(dx+c)+9ab\,d^2(dx+c)^2+6b^2c^4-24b^2c^3(dx+c)+36b^2c^2(dx+c)^2-24b^2c(dx\right.}{4a^3d^2x^2\left(a\,d^2+c^2b-2bc(dx+c)+b(dx+c)^2\right)^2} \right.$
default	$d^2 \left(-\frac{\sin(dx+c)\left(2a^2d^4+9ab\,c^2d^2-18abc\,d^2(dx+c)+9ab\,d^2(dx+c)^2+6b^2c^4-24b^2c^3(dx+c)+36b^2c^2(dx+c)^2-24b^2c(dx\right.}{4a^3d^2x^2\left(a\,d^2+c^2b-2bc(dx+c)+b(dx+c)^2\right)^2} \right.$
risch	$-\frac{id^2\mathrm{e}^{-\frac{icb-d\sqrt{ab}}{b}}\,\mathrm{Ei}_1\left(-\frac{icb-d\sqrt{ab}-b(idx+ic)}{b}\right)}{32a^3}-\frac{9id\,\mathrm{e}^{\frac{icb+d\sqrt{ab}}{b}}\,\mathrm{Ei}_1\left(\frac{icb+d\sqrt{ab}-b(idx+ic)}{b}\right)b}{32a^3\sqrt{ab}}+\frac{9id\,\mathrm{e}^{\frac{icb-d\sqrt{ab}}{b}}\,\mathrm{Ei}_1\left(\frac{icb-\right.}{32a^3\sqrt{}}$

```
[In] int(sin(d*x+c)/x^3/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^2*(-1/4*sin(d*x+c)*(2*a^2*d^4+9*a*b*c^2*d^2-18*a*b*c*d^2*(d*x+c)+9*a*b*d^
2*(d*x+c)^2+6*b^2*c^4-24*b^2*c^3*(d*x+c)+36*b^2*c^2*(d*x+c)^2-24*b^2*c*(d*x
+c)^3+6*b^2*(d*x+c)^4)/a^3/d^2/x^2/(a*d^2+c^2*b-2*b*c*(d*x+c)+b*(d*x+c)^2)^
2-1/8*cos(d*x+c)*(4*a*d^2+3*c^2*b-6*b*c*(d*x+c)+3*b*(d*x+c)^2)/a^3/d/x/(a*d
^2+c^2*b-2*b*c*(d*x+c)+b*(d*x+c)^2)+1/16*(a*d^2+24*b)/d^2/a^4*(Si(d*x+c-(d*
(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c
*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))+1/16*(a*d^2+24*b)/d^2/a^4*(Si(d*x+c+(d*
(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c
*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-1/2/a^4*(a*d^2+6*b)/d^2*(Si(d*x)*cos(c)
+Ci(d*x)*sin(c))-9/16/a^3/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(-Si(d*x+c-(d*(-a*b)^
(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*
cos((d*(-a*b)^(1/2)+c*b)/b))-9/16/a^3/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+
(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2
)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 758, normalized size of antiderivative = 0.96

$$\int \frac{\sin(c+dx)}{x^3(a+bx^2)^3}\,dx =$$

$$-\frac{\left(i\,(ab^2d^2+24\,b^3)x^6+2i\,(a^2bd^2+24\,ab^2)x^4+i\,(a^3d^2+24\,a^2b)x^2+9\,(-i\,b^3x^6-2i\,ab^2x^4-i\,a^2bx^2)\sqrt{\cdot}\right.}{}$$

```
[In] integrate(sin(d*x+c)/x^3/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/32*((I*(a*b^2*d^2 + 24*b^3)*x^6 + 2*I*(a^2*b*d^2 + 24*a*b^2)*x^4 + I*(a^
3*d^2 + 24*a^2*b)*x^2 + 9*(-I*b^3*x^6 - 2*I*a*b^2*x^4 - I*a^2*b*x^2)*sqrt(a
*d^2/b))*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + (I*(a*b^2*d^2
+ 24*b^3)*x^6 + 2*I*(a^2*b*d^2 + 24*a*b^2)*x^4 + I*(a^3*d^2 + 24*a^2*b)*x^2
 + 9*(I*b^3*x^6 + 2*I*a*b^2*x^4 + I*a^2*b*x^2)*sqrt(a*d^2/b))*Ei(I*d*x + sq
rt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + (-I*(a*b^2*d^2 + 24*b^3)*x^6 - 2*I*(
a^2*b*d^2 + 24*a*b^2)*x^4 - I*(a^3*d^2 + 24*a^2*b)*x^2 + 9*(I*b^3*x^6 + 2*I
*a*b^2*x^4 + I*a^2*b*x^2)*sqrt(a*d^2/b))*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c
 + sqrt(a*d^2/b)) + (-I*(a*b^2*d^2 + 24*b^3)*x^6 - 2*I*(a^2*b*d^2 + 24*a*b^
2)*x^4 - I*(a^3*d^2 + 24*a^2*b)*x^2 + 9*(-I*b^3*x^6 - 2*I*a*b^2*x^4 - I*a^2
*b*x^2)*sqrt(a*d^2/b))*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b))
+ 16*((a*b^2*d^2 + 6*b^3)*x^6 + 2*(a^2*b*d^2 + 6*a*b^2)*x^4 + (a^3*d^2 + 6*
a^2*b)*x^2)*cos_integral(d*x)*sin(c) + 16*((a*b^2*d^2 + 6*b^3)*x^6 + 2*(a^2
*b*d^2 + 6*a*b^2)*x^4 + (a^3*d^2 + 6*a^2*b)*x^2)*cos(c)*sin_integral(d*x) +
 4*(3*a*b^2*d*x^5 + 7*a^2*b*d*x^3 + 4*a^3*d*x)*cos(d*x + c) + 8*(6*a*b^2*x^
4 + 9*a^2*b*x^2 + 2*a^3)*sin(d*x + c))/(a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x^3(a+bx^2)^3}\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)/x**3/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x^3(a+bx^2)^3}\,dx = \int \frac{\sin(dx+c)}{(bx^2+a)^3x^3}\,dx$$

```
[In] integrate(sin(d*x+c)/x^3/(b*x^2+a)^3,x, algorithm="maxima")
[Out] integrate(sin(d*x + c)/((b*x^2 + a)^3*x^3), x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{x^3(a+bx^2)^3}\,dx = \int \frac{\sin(dx+c)}{(bx^2+a)^3x^3}\,dx$$

```
[In] integrate(sin(d*x+c)/x^3/(b*x^2+a)^3,x, algorithm="giac")
[Out] integrate(sin(d*x + c)/((b*x^2 + a)^3*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x^3(a+bx^2)^3}\,dx = \int \frac{\sin(c+d\,x)}{x^3\,(b\,x^2+a)^3}\,dx$$

```
[In] int(sin(c + d*x)/(x^3*(a + b*x^2)^3),x)
[Out] int(sin(c + d*x)/(x^3*(a + b*x^2)^3), x)
```

3.79 $\int x^3(a+bx^3)\sin(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 156

$$\int x^3(a+bx^3)\sin(c+dx)\,dx = \frac{720b\cos(c+dx)}{d^7}+\frac{6ax\cos(c+dx)}{d^3}-\frac{360bx^2\cos(c+dx)}{d^5} - \frac{ax^3\cos(c+dx)}{d}+\frac{30bx^4\cos(c+dx)}{d^3}-\frac{bx^6\cos(c+dx)}{d} - \frac{6a\sin(c+dx)}{d^4}+\frac{720bx\sin(c+dx)}{d^6}+\frac{3ax^2\sin(c+dx)}{d^2} - \frac{120bx^3\sin(c+dx)}{d^4}+\frac{6bx^5\sin(c+dx)}{d^2}$$

```
[Out] 720*b*cos(d*x+c)/d^7+6*a*x*cos(d*x+c)/d^3-360*b*x^2*cos(d*x+c)/d^5-a*x^3*co
s(d*x+c)/d+30*b*x^4*cos(d*x+c)/d^3-b*x^6*cos(d*x+c)/d-6*a*sin(d*x+c)/d^4+72
0*b*x*sin(d*x+c)/d^6+3*a*x^2*sin(d*x+c)/d^2-120*b*x^3*sin(d*x+c)/d^4+6*b*x^
5*sin(d*x+c)/d^2
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3420, 3377, 2717, 2718}

$$\int x^3(a+bx^3)\sin(c+dx)\,dx = -\frac{6a\sin(c+dx)}{d^4}+\frac{6ax\cos(c+dx)}{d^3}+\frac{3ax^2\sin(c+dx)}{d^2} - \frac{ax^3\cos(c+dx)}{d}+\frac{720b\cos(c+dx)}{d^7}+\frac{720bx\sin(c+dx)}{d^6} - \frac{360bx^2\cos(c+dx)}{d^5}-\frac{120bx^3\sin(c+dx)}{d^4} + \frac{30bx^4\cos(c+dx)}{d^3}+\frac{6bx^5\sin(c+dx)}{d^2}-\frac{bx^6\cos(c+dx)}{d}$$

```
[In] Int[x^3*(a + b*x^3)*Sin[c + d*x],x]
```

```
[Out] (720*b*Cos[c + d*x])/d^7 + (6*a*x*Cos[c + d*x])/d^3 - (360*b*x^2*Cos[c + d*
x])/d^5 - (a*x^3*Cos[c + d*x])/d + (30*b*x^4*Cos[c + d*x])/d^3 - (b*x^6*Cos
[c + d*x])/d - (6*a*Sin[c + d*x])/d^4 + (720*b*x*Sin[c + d*x])/d^6 + (3*a*x
^2*Sin[c + d*x])/d^2 - (120*b*x^3*Sin[c + d*x])/d^4 + (6*b*x^5*Sin[c + d*x]
)/d^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(ax^3\sin(c+dx)+bx^6\sin(c+dx)\right)\,dx\\
&= a\int x^3\sin(c+dx)\,dx+b\int x^6\sin(c+dx)\,dx\\
&= -\frac{ax^3\cos(c+dx)}{d}-\frac{bx^6\cos(c+dx)}{d}+\frac{(3a)\int x^2\cos(c+dx)\,dx}{d}+\frac{(6b)\int x^5\cos(c+dx)\,dx}{d}\\
&= -\frac{ax^3\cos(c+dx)}{d}-\frac{bx^6\cos(c+dx)}{d}+\frac{3ax^2\sin(c+dx)}{d^2}\\
&\quad+\frac{6bx^5\sin(c+dx)}{d^2}-\frac{(6a)\int x\sin(c+dx)\,dx}{d^2}-\frac{(30b)\int x^4\sin(c+dx)\,dx}{d^2}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{6ax\cos(c+dx)}{d^3} - \frac{ax^3\cos(c+dx)}{d} + \frac{30bx^4\cos(c+dx)}{d^3} \\
&\quad - \frac{bx^6\cos(c+dx)}{d} + \frac{3ax^2\sin(c+dx)}{d^2} + \frac{6bx^5\sin(c+dx)}{d^2} \\
&\quad - \frac{(6a)\int\cos(c+dx)\,dx}{d^3} - \frac{(120b)\int x^3\cos(c+dx)\,dx}{d^3} \\
&= \frac{6ax\cos(c+dx)}{d^3} - \frac{ax^3\cos(c+dx)}{d} + \frac{30bx^4\cos(c+dx)}{d^3} - \frac{bx^6\cos(c+dx)}{d} - \frac{6a\sin(c+dx)}{d^4} \\
&\quad + \frac{3ax^2\sin(c+dx)}{d^2} - \frac{120bx^3\sin(c+dx)}{d^4} + \frac{6bx^5\sin(c+dx)}{d^2} + \frac{(360b)\int x^2\sin(c+dx)\,dx}{d^4} \\
&= \frac{6ax\cos(c+dx)}{d^3} - \frac{360bx^2\cos(c+dx)}{d^5} - \frac{ax^3\cos(c+dx)}{d} \\
&\quad + \frac{30bx^4\cos(c+dx)}{d^3} - \frac{bx^6\cos(c+dx)}{d} - \frac{6a\sin(c+dx)}{d^4} + \frac{3ax^2\sin(c+dx)}{d^2} \\
&\quad - \frac{120bx^3\sin(c+dx)}{d^4} + \frac{6bx^5\sin(c+dx)}{d^2} + \frac{(720b)\int x\cos(c+dx)\,dx}{d^5} \\
&= \frac{6ax\cos(c+dx)}{d^3} - \frac{360bx^2\cos(c+dx)}{d^5} - \frac{ax^3\cos(c+dx)}{d} + \frac{30bx^4\cos(c+dx)}{d^3} \\
&\quad - \frac{bx^6\cos(c+dx)}{d} - \frac{6a\sin(c+dx)}{d^4} + \frac{720bx\sin(c+dx)}{d^6} + \frac{3ax^2\sin(c+dx)}{d^2} \\
&\quad - \frac{120bx^3\sin(c+dx)}{d^4} + \frac{6bx^5\sin(c+dx)}{d^2} - \frac{(720b)\int\sin(c+dx)\,dx}{d^6} \\
&= \frac{720b\cos(c+dx)}{d^7} + \frac{6ax\cos(c+dx)}{d^3} - \frac{360bx^2\cos(c+dx)}{d^5} - \frac{ax^3\cos(c+dx)}{d} \\
&\quad + \frac{30bx^4\cos(c+dx)}{d^3} - \frac{bx^6\cos(c+dx)}{d} - \frac{6a\sin(c+dx)}{d^4} + \frac{720bx\sin(c+dx)}{d^6} \\
&\quad + \frac{3ax^2\sin(c+dx)}{d^2} - \frac{120bx^3\sin(c+dx)}{d^4} + \frac{6bx^5\sin(c+dx)}{d^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.65

$$
\int x^3(a+bx^3)\sin(c+dx)\,dx
$$

$$
= \frac{-((ad^4x(-6+d^2x^2)+b(-720+360d^2x^2-30d^4x^4+d^6x^6))\cos(c+dx))+3d(ad^2(-2+d^2x^2)+2bx(12}{d^7}
$$

```
[In] Integrate[x^3*(a + b*x^3)*Sin[c + d*x],x]
```

```
[Out] (-((a*d^4*x*(-6 + d^2*x^2) + b*(-720 + 360*d^2*x^2 - 30*d^4*x^4 + d^6*x^6))
*Cos[c + d*x]) + 3*d*(a*d^2*(-2 + d^2*x^2) + 2*b*x*(120 - 20*d^2*x^2 + d^4*
x^4))*Sin[c + d*x])/d^7
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{\left(b x^{6} d^{6}+a d^{6} x^{3}-30 b x^{4} d^{4}-6 a d^{4} x+360 d^{2} x^{2} b-720 b\right) \cos \left(d x+c\right)}{d^{7}}+\frac{3\left(2 b d^{4} x^{5}+a d^{4} x^{2}-40 b d^{2} x^{3}-2 a d^{2}+240 b x\right) \sin(d}{d^{6}}$
parallelrisch	$\frac{\left(x^{3}\left(b x^{3}+a\right) d^{6}-6 x\left(5 b x^{3}+a\right) d^{4}+360 d^{2} x^{2} b-1440 b\right)\left(\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)+6\left(x^{2}\left(2 b x^{3}+a\right) d^{4}+\left(-40 b x^{3}-2 a\right) d^{2}+240 b x\right) d \tan(}{d^{7}\left(1+\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}$
norman	$\frac{\frac{1440 b}{d^{7}}+\frac{a x^{3}\left(\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{d}+\frac{b x^{6}\left(\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{d}-\frac{12 a \tan\left(\frac{d x}{2}+\frac{c}{2}\right)}{d^{4}}+\frac{6 a x}{d^{3}}-\frac{a x^{3}}{d}-\frac{360 b x^{2}}{d^{5}}+\frac{30 b x^{4}}{d^{3}}-\frac{b x^{6}}{d}-\frac{6 a x\left(\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{d^{3}}}{1+\mathrm{tan}}$
meijerg	$\frac{64 b \sqrt{\pi}\, \sin(c)\left(\frac{x\left(d^{2}\right)^{\frac{7}{2}}\left(\frac{21}{8} d^{4} x^{4}-\frac{105}{2} d^{2} x^{2}+315\right) \cos(d x)}{28 \sqrt{\pi}\, d^{6}}-\frac{\left(d^{2}\right)^{\frac{7}{2}}\left(-\frac{7}{16} d^{6} x^{6}+\frac{105}{8} d^{4} x^{4}-\frac{315}{2} d^{2} x^{2}+315\right) \sin(d x)}{28 \sqrt{\pi}\, d^{7}}\right)}{d^{6} \sqrt{d^{2}}}+\frac{64 b \sqrt{\pi}\, \mathrm{c}}{}$
parts	$-\frac{b x^{6} \cos(d x+c)}{d}-\frac{a x^{3} \cos(d x+c)}{d}+\frac{\frac{3 a c^{2} \sin(d x+c)}{d^{2}}-\frac{6 a c(\cos(d x+c)+(d x+c) \sin(d x+c))}{d^{2}}+\frac{3 a\left((d x+c)^{2} \sin(d x+c)-2 \sin(d x+\right.}{d^{2}}}{}$
derivativedivides	$\frac{a c^{3} \cos(d x+c)+3 a c^{2}(\sin(d x+c)-\cos(d x+c)(d x+c))-3 a c\left(-(d x+c)^{2} \cos(d x+c)+2 \cos(d x+c)+2(d x+c) \sin(d x+c)\right)+a(}{}$
default	$\frac{a c^{3} \cos(d x+c)+3 a c^{2}(\sin(d x+c)-\cos(d x+c)(d x+c))-3 a c\left(-(d x+c)^{2} \cos(d x+c)+2 \cos(d x+c)+2(d x+c) \sin(d x+c)\right)+a(}{}$

```
[In] int(x^3*(b*x^3+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*d^6*x^6+a*d^6*x^3-30*b*d^4*x^4-6*a*d^4*x+360*b*d^2*x^2-720*b)/d^7*cos(d
*x+c)+3/d^6*(2*b*d^4*x^5+a*d^4*x^2-40*b*d^2*x^3-2*a*d^2+240*b*x)*sin(d*x+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.67

$$\int x^{3}\left(a+bx^{3}\right)\sin(c+dx)\,dx = -\frac{\left(bd^{6}x^{6}+ad^{6}x^{3}-30\,bd^{4}x^{4}-6\,ad^{4}x+360\,bd^{2}x^{2}-720\,b\right)\cos\left(dx+c\right)-3\left(2\,bd^{5}x^{5}+ad^{5}x^{2}-40\,bd^{3}x^{3}-\right.}{d^{7}}$$

```
[In] integrate(x^3*(b*x^3+a)*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b*d^6*x^6 + a*d^6*x^3 - 30*b*d^4*x^4 - 6*a*d^4*x + 360*b*d^2*x^2 - 720*b
)*cos(d*x + c) - 3*(2*b*d^5*x^5 + a*d^5*x^2 - 40*b*d^3*x^3 - 2*a*d^3 + 240*
b*d*x)*sin(d*x + c))/d^7
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.19

$$\int x^3(a+bx^3)\sin(c+dx)\,dx$$
$$=\begin{cases} -\frac{ax^3\cos(c+dx)}{d}+\frac{3ax^2\sin(c+dx)}{d^2}+\frac{6ax\cos(c+dx)}{d^3}-\frac{6a\sin(c+dx)}{d^4}-\frac{bx^6\cos(c+dx)}{d}+\frac{6bx^5\sin(c+dx)}{d^2}+\frac{30bx^4\cos(c+dx)}{d^3}- \\ \left(\frac{ax^4}{4}+\frac{bx^7}{7}\right)\sin(c) \end{cases}$$

```
[In] integrate(x**3*(b*x**3+a)*sin(d*x+c),x)
```

```
[Out] Piecewise((-a*x**3*cos(c + d*x)/d + 3*a*x**2*sin(c + d*x)/d**2 + 6*a*x*cos(
c + d*x)/d**3 - 6*a*sin(c + d*x)/d**4 - b*x**6*cos(c + d*x)/d + 6*b*x**5*si
n(c + d*x)/d**2 + 30*b*x**4*cos(c + d*x)/d**3 - 120*b*x**3*sin(c + d*x)/d**
4 - 360*b*x**2*cos(c + d*x)/d**5 + 720*b*x*sin(c + d*x)/d**6 + 720*b*cos(c
+ d*x)/d**7, Ne(d, 0)), ((a*x**4/4 + b*x**7/7)*sin(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(156) = 312$.

Time = 0.22 (sec) , antiderivative size = 449, normalized size of antiderivative = 2.88

$$\int x^3(a+bx^3)\sin(c+dx)\,dx$$
$$=\frac{ac^3\cos(dx+c)-\frac{bc^6\cos(dx+c)}{d^3}-3\left((dx+c)\cos(dx+c)-\sin(dx+c)\right)ac^2+\frac{6\left((dx+c)\cos(dx+c)-\sin(dx+c)\right)bc^5}{d^3}-}{}$$

```
[In] integrate(x^3*(b*x^3+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] (a*c^3*cos(d*x + c) - b*c^6*cos(d*x + c)/d^3 - 3*((d*x + c)*cos(d*x + c) -
sin(d*x + c))*a*c^2 + 6*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c^5/d^3 +
 3*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a*c - 15*(((
d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b*c^4/d^3 - (((d*x
 + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*a +
 20*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x
 + c))*b*c^3/d^3 - 15*(((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d*x + c) - 4
*((d*x + c)^3 - 6*d*x - 6*c)*sin(d*x + c))*b*c^2/d^3 + 6*(((d*x + c)^5 - 20
*(d*x + c)^3 + 120*d*x + 120*c)*cos(d*x + c) - 5*((d*x + c)^4 - 12*(d*x + c
)^2 + 24)*sin(d*x + c))*b*c/d^3 - (((d*x + c)^6 - 30*(d*x + c)^4 + 360*(d*x
 + c)^2 - 720)*cos(d*x + c) - 6*((d*x + c)^5 - 20*(d*x + c)^3 + 120*d*x + 1
20*c)*sin(d*x + c))*b/d^3)/d^4
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.68

$$\int x^3\left(a+bx^3\right)\sin(c+dx)\,dx = -\frac{\left(bd^6x^6+ad^6x^3-30\,bd^4x^4-6\,ad^4x+360\,bd^2x^2-720\,b\right)\cos\left(dx+c\right)}{d^7} + \frac{3\left(2\,bd^5x^5+ad^5x^2-40\,bd^3x^3-2\,ad^3+240\,bdx\right)\sin\left(dx+c\right)}{d^7}$$

```
[In] integrate(x^3*(b*x^3+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b*d^6*x^6 + a*d^6*x^3 - 30*b*d^4*x^4 - 6*a*d^4*x + 360*b*d^2*x^2 - 720*b)
*cos(d*x + c)/d^7 + 3*(2*b*d^5*x^5 + a*d^5*x^2 - 40*b*d^3*x^3 - 2*a*d^3 + 2
40*b*d*x)*sin(d*x + c)/d^7
```

Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.97

$$\int x^3\left(a+bx^3\right)\sin(c+dx)\,dx = \frac{d^4\left(6\,a\,x\,\cos\left(c+d\,x\right)+30\,b\,x^4\,\cos\left(c+d\,x\right)\right)+720\,b\,\cos\left(c+d\,x\right)-d^6\left(a\,x^3\,\cos\left(c+d\,x\right)+b\,x^6\,\cos\left(c+d\right.\right.}{}$$

```
[In] int(x^3*sin(c + d*x)*(a + b*x^3),x)
```

```
[Out] (d^4*(6*a*x*cos(c + d*x) + 30*b*x^4*cos(c + d*x)) + 720*b*cos(c + d*x) - d^
6*(a*x^3*cos(c + d*x) + b*x^6*cos(c + d*x)) + d^5*(3*a*x^2*sin(c + d*x) + 6
*b*x^5*sin(c + d*x)) - d^3*(6*a*sin(c + d*x) + 120*b*x^3*sin(c + d*x)) + 72
0*b*d*x*sin(c + d*x) - 360*b*d^2*x^2*cos(c + d*x))/d^7
```

3.80 $\int x^2(a+bx^3)\sin(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 126

$$\int x^2(a+bx^3)\sin(c+dx)\,dx = \frac{2a\cos(c+dx)}{d^3} - \frac{120bx\cos(c+dx)}{d^5} - \frac{ax^2\cos(c+dx)}{d} + \frac{20bx^3\cos(c+dx)}{d^3} - \frac{bx^5\cos(c+dx)}{d} + \frac{120b\sin(c+dx)}{d^6} + \frac{2ax\sin(c+dx)}{d^2} - \frac{60bx^2\sin(c+dx)}{d^4} + \frac{5bx^4\sin(c+dx)}{d^2}$$

```
[Out] 2*a*cos(d*x+c)/d^3-120*b*x*cos(d*x+c)/d^5-a*x^2*cos(d*x+c)/d+20*b*x^3*cos(d
*x+c)/d^3-b*x^5*cos(d*x+c)/d+120*b*sin(d*x+c)/d^6+2*a*x*sin(d*x+c)/d^2-60*b
*x^2*sin(d*x+c)/d^4+5*b*x^4*sin(d*x+c)/d^2
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3420, 3377, 2718, 2717}

$$\int x^2(a+bx^3)\sin(c+dx)\,dx = \frac{2a\cos(c+dx)}{d^3} + \frac{2ax\sin(c+dx)}{d^2} - \frac{ax^2\cos(c+dx)}{d} + \frac{120b\sin(c+dx)}{d^6} - \frac{120bx\cos(c+dx)}{d^5} - \frac{60bx^2\sin(c+dx)}{d^4} + \frac{20bx^3\cos(c+dx)}{d^3} + \frac{5bx^4\sin(c+dx)}{d^2} - \frac{bx^5\cos(c+dx)}{d}$$

```
[In] Int[x^2*(a + b*x^3)*Sin[c + d*x],x]
```

```
[Out] (2*a*Cos[c + d*x])/d^3 - (120*b*x*Cos[c + d*x])/d^5 - (a*x^2*Cos[c + d*x])/
d + (20*b*x^3*Cos[c + d*x])/d^3 - (b*x^5*Cos[c + d*x])/d + (120*b*Sin[c + d
```

```
*x])/d^6 + (2*a*x*Sin[c + d*x])/d^2 - (60*b*x^2*Sin[c + d*x])/d^4 + (5*b*x^
4*Sin[c + d*x])/d^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(ax^2\sin(c+dx)+bx^5\sin(c+dx)\right)\,dx \\
&= a\int x^2\sin(c+dx)\,dx+b\int x^5\sin(c+dx)\,dx \\
&= -\frac{ax^2\cos(c+dx)}{d}-\frac{bx^5\cos(c+dx)}{d}+\frac{(2a)\int x\cos(c+dx)\,dx}{d}+\frac{(5b)\int x^4\cos(c+dx)\,dx}{d} \\
&= -\frac{ax^2\cos(c+dx)}{d}-\frac{bx^5\cos(c+dx)}{d}+\frac{2ax\sin(c+dx)}{d^2} \\
&\quad +\frac{5bx^4\sin(c+dx)}{d^2}-\frac{(2a)\int\sin(c+dx)\,dx}{d^2}-\frac{(20b)\int x^3\sin(c+dx)\,dx}{d^2} \\
&= \frac{2a\cos(c+dx)}{d^3}-\frac{ax^2\cos(c+dx)}{d}+\frac{20bx^3\cos(c+dx)}{d^3}-\frac{bx^5\cos(c+dx)}{d} \\
&\quad +\frac{2ax\sin(c+dx)}{d^2}+\frac{5bx^4\sin(c+dx)}{d^2}-\frac{(60b)\int x^2\cos(c+dx)\,dx}{d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2a\cos(c+dx)}{d^3} - \frac{ax^2\cos(c+dx)}{d} + \frac{20bx^3\cos(c+dx)}{d^3} \\
&\quad - \frac{bx^5\cos(c+dx)}{d} + \frac{2ax\sin(c+dx)}{d^2} - \frac{60bx^2\sin(c+dx)}{d^4} \\
&\quad + \frac{5bx^4\sin(c+dx)}{d^2} + \frac{(120b)\int x\sin(c+dx)\,dx}{d^4} \\
&= \frac{2a\cos(c+dx)}{d^3} - \frac{120bx\cos(c+dx)}{d^5} - \frac{ax^2\cos(c+dx)}{d} + \frac{20bx^3\cos(c+dx)}{d^3} - \frac{bx^5\cos(c+dx)}{d} \\
&\quad + \frac{2ax\sin(c+dx)}{d^2} - \frac{60bx^2\sin(c+dx)}{d^4} + \frac{5bx^4\sin(c+dx)}{d^2} + \frac{(120b)\int\cos(c+dx)\,dx}{d^5} \\
&= \frac{2a\cos(c+dx)}{d^3} - \frac{120bx\cos(c+dx)}{d^5} - \frac{ax^2\cos(c+dx)}{d} + \frac{20bx^3\cos(c+dx)}{d^3} \\
&\quad - \frac{bx^5\cos(c+dx)}{d} + \frac{120b\sin(c+dx)}{d^6} + \frac{2ax\sin(c+dx)}{d^2} - \frac{60bx^2\sin(c+dx)}{d^4} \\
&\quad + \frac{5bx^4\sin(c+dx)}{d^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.67

$$
\int x^2(a+bx^3)\sin(c+dx)\,dx = \frac{-d(ad^2(-2+d^2x^2)+bx(120-20d^2x^2+d^4x^4))\cos(c+dx)+(2ad^4x+5b(24-12d^2x^2+d^4x^4))\sin(c+
}{d^6}
$$

```
[In] Integrate[x^2*(a + b*x^3)*Sin[c + d*x],x]
[Out] (-(d*(a*d^2*(-2 + d^2*x^2) + b*x*(120 - 20*d^2*x^2 + d^4*x^4))*Cos[c + d*x]
) + (2*a*d^4*x + 5*b*(24 - 12*d^2*x^2 + d^4*x^4))*Sin[c + d*x])/d^6
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{(b\,d^4x^5+a\,d^4x^2-20b\,d^2x^3-2a\,d^2+120bx)\cos(dx+c)}{d^5}+\frac{(5b\,x^4d^4+2a\,d^4x-60d^2x^2b+120b)\sin(dx+c)}{d^6}$
parallelrisch	$\frac{(x(b\,x^3+a)\,d^4-20d^2x^2b+120b)\,xd\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+((10b\,x^4+4ax)\,d^4-120d^2x^2b+240b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-(x^2(b\,x^3+a)\,d^4+(}{d^6\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$
norman	$\frac{\frac{4a}{d^3}+\frac{a\,x^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{b\,x^5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a\,x^2}{d}+\frac{240b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^6}-\frac{120bx}{d^5}+\frac{20b\,x^3}{d^3}-\frac{b\,x^5}{d}+\frac{4ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^2}+\frac{120bx\left(\tan^2\right.}{d}}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$
meijerg	$\frac{32b\sqrt{\pi}\,\sin(c)\left(-\frac{15}{4\sqrt{\pi}}+\frac{\left(\frac{15}{8}d^4x^4-\frac{45}{2}d^2x^2+45\right)\cos(dx)}{12\sqrt{\pi}}+\frac{xd\left(\frac{3}{8}d^4x^4-\frac{15}{2}d^2x^2+45\right)\sin(dx)}{12\sqrt{\pi}}\right)}{d^6}+\frac{32b\sqrt{\pi}\,\cos(c)\left(-\frac{xd\left(\frac{7}{8}d^4x^4-\right.}{}\right.}{}$
parts	$-\frac{b\,x^5\cos(dx+c)}{d}-\frac{a\,x^2\cos(dx+c)}{d}+\frac{-\frac{2ac\sin(dx+c)}{d}+\frac{2a(\cos(dx+c)+(dx+c)\sin(dx+c))}{d}+\frac{5b\,c^4\sin(dx+c)}{d^4}-\frac{20b\,c^3(\cos(dx+c}{}}{}$
derivativedivides	$\frac{-a\,c^2\cos(dx+c)-2ac(\sin(dx+c)-\cos(dx+c)(dx+c))+a\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)+\frac{b\,c^5\,c}{}}{}$
default	$\frac{-a\,c^2\cos(dx+c)-2ac(\sin(dx+c)-\cos(dx+c)(dx+c))+a\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)+\frac{b\,c^5\,c}{}}{}$

```
[In] int(x^2*(b*x^3+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^5*(b*d^4*x^5+a*d^4*x^2-20*b*d^2*x^3-2*a*d^2+120*b*x)*cos(d*x+c)+(5*b*d
^4*x^4+2*a*d^4*x-60*b*d^2*x^2+120*b)/d^6*sin(d*x+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.69

$$\int x^2(a+bx^3)\sin(c+dx)\,dx = -\frac{(bd^5x^5+ad^5x^2-20\,bd^3x^3-2\,ad^3+120\,bdx)\cos(dx+c)-(5\,bd^4x^4+2\,ad^4x-60\,bd^2x^2+120\,b)\sin(d}{d^6}$$

```
[In] integrate(x^2*(b*x^3+a)*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b*d^5*x^5 + a*d^5*x^2 - 20*b*d^3*x^3 - 2*a*d^3 + 120*b*d*x)*cos(d*x + c)
 - (5*b*d^4*x^4 + 2*a*d^4*x - 60*b*d^2*x^2 + 120*b)*sin(d*x + c))/d^6
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.20

$$\int x^2(a+bx^3)\sin(c+dx)\,dx$$
$$=\begin{cases} -\frac{ax^2\cos(c+dx)}{d}+\frac{2ax\sin(c+dx)}{d^2}+\frac{2a\cos(c+dx)}{d^3}-\frac{bx^5\cos(c+dx)}{d}+\frac{5bx^4\sin(c+dx)}{d^2}+\frac{20bx^3\cos(c+dx)}{d^3}-\frac{60bx^2\sin(c+dx)}{d^4}- \\ \left(\frac{ax^3}{3}+\frac{bx^6}{6}\right)\sin(c) \end{cases}$$

```
[In] integrate(x**2*(b*x**3+a)*sin(d*x+c),x)
```

```
[Out] Piecewise((-a*x**2*cos(c + d*x)/d + 2*a*x*sin(c + d*x)/d**2 + 2*a*cos(c + d
*x)/d**3 - b*x**5*cos(c + d*x)/d + 5*b*x**4*sin(c + d*x)/d**2 + 20*b*x**3*c
os(c + d*x)/d**3 - 60*b*x**2*sin(c + d*x)/d**4 - 120*b*x*cos(c + d*x)/d**5
+ 120*b*sin(c + d*x)/d**6, Ne(d, 0)), ((a*x**3/3 + b*x**6/6)*sin(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(126) = 252.

Time = 0.21 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.59

$$\int x^2(a+bx^3)\sin(c+dx)\,dx =$$

$$-\frac{ac^2\cos(dx+c)-\frac{bc^5\cos(dx+c)}{d^3}-2\left((dx+c)\cos(dx+c)-\sin(dx+c)\right)ac+\frac{5\left((dx+c)\cos(dx+c)-\sin(dx+c)\right)bc^4}{d^3}}{}$$

```
[In] integrate(x^2*(b*x^3+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a*c^2*cos(d*x + c) - b*c^5*cos(d*x + c)/d^3 - 2*((d*x + c)*cos(d*x + c) -
 sin(d*x + c))*a*c + 5*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c^4/d^3 +
(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a - 10*(((d*x +
 c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b*c^3/d^3 + 10*(((d*x +
 c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*b*c^2
/d^3 - 5*(((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d*x + c) - 4*((d*x + c)^3
 - 6*d*x - 6*c)*sin(d*x + c))*b*c/d^3 + (((d*x + c)^5 - 20*(d*x + c)^3 + 12
0*d*x + 120*c)*cos(d*x + c) - 5*((d*x + c)^4 - 12*(d*x + c)^2 + 24)*sin(d*x
 + c))*b/d^3)/d^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.70

$$\int x^2(a+bx^3)\sin(c+dx)\,dx = -\frac{(bd^5x^5+ad^5x^2-20\,bd^3x^3-2\,ad^3+120\,bdx)\cos(dx+c)}{d^6} + \frac{(5\,bd^4x^4+2\,ad^4x-60\,bd^2x^2+120\,b)\sin(dx+c)}{d^6}$$

```
[In] integrate(x^2*(b*x^3+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b*d^5*x^5 + a*d^5*x^2 - 20*b*d^3*x^3 - 2*a*d^3 + 120*b*d*x)*cos(d*x + c)/
d^6 + (5*b*d^4*x^4 + 2*a*d^4*x - 60*b*d^2*x^2 + 120*b)*sin(d*x + c)/d^6
```

Mupad [B] (verification not implemented)

Time = 6.37 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.96

$$\int x^2(a+bx^3)\sin(c+dx)\,dx$$
$$= \frac{120\,b\,\sin(c+d\,x)+d^4\,(5\,b\,x^4\,\sin(c+d\,x)+2\,a\,x\,\sin(c+d\,x))-d^5\,(a\,x^2\,\cos(c+d\,x)+b\,x^5\,\cos(c+d\,x)}{d^6}$$

```
[In] int(x^2*sin(c + d*x)*(a + b*x^3),x)
```

```
[Out] (120*b*sin(c + d*x) + d^4*(5*b*x^4*sin(c + d*x) + 2*a*x*sin(c + d*x)) - d^5
*(a*x^2*cos(c + d*x) + b*x^5*cos(c + d*x)) + d^3*(2*a*cos(c + d*x) + 20*b*x
^3*cos(c + d*x)) - 60*b*d^2*x^2*sin(c + d*x) - 120*b*d*x*cos(c + d*x))/d^6
```

3.81 $\int x(a+bx^3)\sin(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 95

$$\int x(a+bx^3)\sin(c+dx)\,dx = -\frac{24b\cos(c+dx)}{d^5}-\frac{ax\cos(c+dx)}{d}+\frac{12bx^2\cos(c+dx)}{d^3}-\frac{bx^4\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d^2}-\frac{24bx\sin(c+dx)}{d^4}+\frac{4bx^3\sin(c+dx)}{d^2}$$

```
[Out] -24*b*cos(d*x+c)/d^5-a*x*cos(d*x+c)/d+12*b*x^2*cos(d*x+c)/d^3-b*x^4*cos(d*x
+c)/d+a*sin(d*x+c)/d^2-24*b*x*sin(d*x+c)/d^4+4*b*x^3*sin(d*x+c)/d^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3420, 3377, 2717, 2718}

$$\int x(a+bx^3)\sin(c+dx)\,dx = \frac{a\sin(c+dx)}{d^2}-\frac{ax\cos(c+dx)}{d}-\frac{24b\cos(c+dx)}{d^5}-\frac{24bx\sin(c+dx)}{d^4}+\frac{12bx^2\cos(c+dx)}{d^3}+\frac{4bx^3\sin(c+dx)}{d^2}-\frac{bx^4\cos(c+dx)}{d}$$

```
[In] Int[x*(a + b*x^3)*Sin[c + d*x],x]
```

```
[Out] (-24*b*Cos[c + d*x])/d^5 - (a*x*Cos[c + d*x])/d + (12*b*x^2*Cos[c + d*x])/d
^3 - (b*x^4*Cos[c + d*x])/d + (a*Sin[c + d*x])/d^2 - (24*b*x*Sin[c + d*x])/
d^4 + (4*b*x^3*Sin[c + d*x])/d^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(ax\sin(c+dx)+bx^4\sin(c+dx)\right)\,dx \\
&= a\int x\sin(c+dx)\,dx+b\int x^4\sin(c+dx)\,dx \\
&= -\frac{ax\cos(c+dx)}{d}-\frac{bx^4\cos(c+dx)}{d}+\frac{a\int\cos(c+dx)\,dx}{d}+\frac{(4b)\int x^3\cos(c+dx)\,dx}{d} \\
&= -\frac{ax\cos(c+dx)}{d}-\frac{bx^4\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d^2} \\
&\quad+\frac{4bx^3\sin(c+dx)}{d^2}-\frac{(12b)\int x^2\sin(c+dx)\,dx}{d^2} \\
&= -\frac{ax\cos(c+dx)}{d}+\frac{12bx^2\cos(c+dx)}{d^3}-\frac{bx^4\cos(c+dx)}{d} \\
&\quad+\frac{a\sin(c+dx)}{d^2}+\frac{4bx^3\sin(c+dx)}{d^2}-\frac{(24b)\int x\cos(c+dx)\,dx}{d^3} \\
&= -\frac{ax\cos(c+dx)}{d}+\frac{12bx^2\cos(c+dx)}{d^3}-\frac{bx^4\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d^2} \\
&\quad-\frac{24bx\sin(c+dx)}{d^4}+\frac{4bx^3\sin(c+dx)}{d^2}+\frac{(24b)\int\sin(c+dx)\,dx}{d^4}
\end{aligned}
$$

$$= -\frac{24b\cos(c+dx)}{d^5} - \frac{ax\cos(c+dx)}{d} + \frac{12bx^2\cos(c+dx)}{d^3} - \frac{bx^4\cos(c+dx)}{d} + \frac{a\sin(c+dx)}{d^2} - \frac{24bx\sin(c+dx)}{d^4} + \frac{4bx^3\sin(c+dx)}{d^2}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.69

$$\int x(a+bx^3)\sin(c+dx)\,dx = \frac{-((ad^4x+b(24-12d^2x^2+d^4x^4))\cos(c+dx))+d(ad^2+4bx(-6+d^2x^2))\sin(c+dx)}{d^5}$$

```
[In] Integrate[x*(a + b*x^3)*Sin[c + d*x],x]
```

```
[Out] (-((a*d^4*x + b*(24 - 12*d^2*x^2 + d^4*x^4))*Cos[c + d*x]) + d*(a*d^2 + 4*b
*x*(-6 + d^2*x^2))*Sin[c + d*x])/d^5
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{(b\,x^4d^4+a\,d^4x-12d^2x^2b+24b)\cos(dx+c)}{d^5}+\frac{(4b\,d^2x^3+a\,d^2-24bx)\sin(dx+c)}{d^4}$
parallelrisch	$\frac{((b\,x^3+a)d^2-12bx)x\,d^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2((4b\,x^3+a)d^2-24bx)d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+(-b\,x^4-ax)d^4+12d^2x^2b-48b}{d^5\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$
norman	$\frac{\frac{ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{b\,x^4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{48b}{d^5}+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^2}-\frac{ax}{d}+\frac{12b\,x^2}{d^3}-\frac{b\,x^4}{d}-\frac{48bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^4}-\frac{12b\,x^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d^3}}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$
parts	$-\frac{b\,x^4\cos(dx+c)}{d}-\frac{ax\cos(dx+c)}{d}+\frac{a\sin(dx+c)-\frac{4b\,c^3\sin(dx+c)}{d^3}+\frac{12b\,c^2(\cos(dx+c)+(dx+c)\sin(dx+c))}{d^3}-\frac{12bc\left((dx+c)^2 s\right.}{}}{}$
meijerg	$\frac{16b\sqrt{\pi}\,\sin(c)\left(-\frac{x\left(d^2\right)^{\frac{5}{2}}\left(-\frac{5d^2x^2}{2}+15\right)\cos(dx)}{10\sqrt{\pi}\,d^4}+\frac{\left(d^2\right)^{\frac{5}{2}}\left(\frac{5}{8}d^4x^4-\frac{15}{2}d^2x^2+15\right)\sin(dx)}{10\sqrt{\pi}\,d^5}\right)}{d^4\sqrt{d^2}}+\frac{16b\sqrt{\pi}\,\cos(c)\left(\frac{3}{2\sqrt{\pi}}-\frac{\left(\frac{3}{8}d^4x^4\right.}{}\right.}{}$
derivativedivides	$\frac{ac\cos(dx+c)+a(\sin(dx+c)-\cos(dx+c)(dx+c))-\frac{b\,c^4\cos(dx+c)}{d^3}-\frac{4b\,c^3(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^3}+\frac{6b\,c^2\left(-(dx+c)^2\cos(d\right.}{}}{}$
default	$\frac{ac\cos(dx+c)+a(\sin(dx+c)-\cos(dx+c)(dx+c))-\frac{b\,c^4\cos(dx+c)}{d^3}-\frac{4b\,c^3(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^3}+\frac{6b\,c^2\left(-(dx+c)^2\cos(d\right.}{}}{}$

```
[In] int(x*(b*x^3+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*d^4*x^4+a*d^4*x-12*b*d^2*x^2+24*b)/d^5*cos(d*x+c)+1/d^4*(4*b*d^2*x^3+a*
d^2-24*b*x)*sin(d*x+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.72

$$\int x(a+bx^3)\sin(c+dx)\,dx = -\frac{(bd^4x^4+ad^4x-12\,bd^2x^2+24\,b)\cos(dx+c)-(4\,bd^3x^3+ad^3-24\,bdx)\sin(dx+c)}{d^5}$$

```
[In] integrate(x*(b*x^3+a)*sin(d*x+c),x, algorithm="fricas")
[Out] -((b*d^4*x^4 + a*d^4*x - 12*b*d^2*x^2 + 24*b)*cos(d*x + c) - (4*b*d^3*x^3 +
 a*d^3 - 24*b*d*x)*sin(d*x + c))/d^5
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.22

$$\int x(a+bx^3)\sin(c+dx)\,dx = \begin{cases} -\frac{ax\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d^2}-\frac{bx^4\cos(c+dx)}{d}+\frac{4bx^3\sin(c+dx)}{d^2}+\frac{12bx^2\cos(c+dx)}{d^3}-\frac{24bx\sin(c+dx)}{d^4}-\frac{24b\cos(c+dx)}{d^5} & \text{for } d \\ \left(\frac{ax^2}{2}+\frac{bx^5}{5}\right)\sin(c) & \text{other} \end{cases}$$

```
[In] integrate(x*(b*x**3+a)*sin(d*x+c),x)
[Out] Piecewise((-a*x*cos(c + d*x)/d + a*sin(c + d*x)/d**2 - b*x**4*cos(c + d*x)/
d + 4*b*x**3*sin(c + d*x)/d**2 + 12*b*x**2*cos(c + d*x)/d**3 - 24*b*x*sin(c
 + d*x)/d**4 - 24*b*cos(c + d*x)/d**5, Ne(d, 0)), ((a*x**2/2 + b*x**5/5)*si
n(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(95) = 190.

Time = 0.19 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.36

$$\int x(a+bx^3)\sin(c+dx)\,dx = \frac{ac\cos(dx+c)-\frac{bc^4\cos(dx+c)}{d^3}-((dx+c)\cos(dx+c)-\sin(dx+c))a+\frac{4((dx+c)\cos(dx+c)-\sin(dx+c))bc^3}{d^3}-\frac{6(((}{}}{}$$

```
[In] integrate(x*(b*x^3+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] (a*c*cos(d*x + c) - b*c^4*cos(d*x + c)/d^3 - ((d*x + c)*cos(d*x + c) - sin(
d*x + c))*a + 4*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c^3/d^3 - 6*(((d*
x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b*c^2/d^3 + 4*(((d*x
 + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*b*c
/d^3 - (((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d*x + c) - 4*((d*x + c)^3 -
 6*d*x - 6*c)*sin(d*x + c))*b/d^3)/d^2
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.73

$$\int x(a+bx^3)\sin(c+dx)\,dx = -\frac{(bd^4x^4 + ad^4x - 12\,bd^2x^2 + 24\,b)\cos{(dx+c)}}{d^5} + \frac{(4\,bd^3x^3 + ad^3 - 24\,bdx)\sin{(dx+c)}}{d^5}$$

```
[In] integrate(x*(b*x^3+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b*d^4*x^4 + a*d^4*x - 12*b*d^2*x^2 + 24*b)*cos(d*x + c)/d^5 + (4*b*d^3*x^
3 + a*d^3 - 24*b*d*x)*sin(d*x + c)/d^5
```

Mupad [B] (verification not implemented)

Time = 6.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97

$$\int x(a+bx^3)\sin(c+dx)\,dx = -\frac{d^4\,(a\,x\,\cos{(c+d\,x)} + b\,x^4\,\cos{(c+d\,x)}) + 24\,b\,\cos{(c+d\,x)} - d^3\,(a\,\sin{(c+d\,x)} + 4\,b\,x^3\,\sin{(c+d\,x)})}{d^5}$$

```
[In] int(x*sin(c + d*x)*(a + b*x^3),x)
```

```
[Out] -(d^4*(a*x*cos(c + d*x) + b*x^4*cos(c + d*x)) + 24*b*cos(c + d*x) - d^3*(a*
sin(c + d*x) + 4*b*x^3*sin(c + d*x)) + 24*b*d*x*sin(c + d*x) - 12*b*d^2*x^2
*cos(c + d*x))/d^5
```

3.82 $\int (a+bx^3)\sin(c+dx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 68

$$\int (a+bx^3)\sin(c+dx)\,dx = -\frac{a\cos(c+dx)}{d}+\frac{6bx\cos(c+dx)}{d^3}-\frac{bx^3\cos(c+dx)}{d}-\frac{6b\sin(c+dx)}{d^4}+\frac{3bx^2\sin(c+dx)}{d^2}$$

```
[Out] -a*cos(d*x+c)/d+6*b*x*cos(d*x+c)/d^3-b*x^3*cos(d*x+c)/d-6*b*sin(d*x+c)/d^4+
3*b*x^2*sin(d*x+c)/d^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3410, 2718, 3377, 2717}

$$\int (a+bx^3)\sin(c+dx)\,dx = -\frac{a\cos(c+dx)}{d}-\frac{6b\sin(c+dx)}{d^4}+\frac{6bx\cos(c+dx)}{d^3}+\frac{3bx^2\sin(c+dx)}{d^2}-\frac{bx^3\cos(c+dx)}{d}$$

```
[In] Int[(a + b*x^3)*Sin[c + d*x],x]
```

```
[Out] -((a*Cos[c + d*x])/d) + (6*b*x*Cos[c + d*x])/d^3 - (b*x^3*Cos[c + d*x])/d -
 (6*b*Sin[c + d*x])/d^4 + (3*b*x^2*Sin[c + d*x])/d^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3410

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d,
 n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a\sin(c+dx)+bx^3\sin(c+dx)\right)\,dx \\
&= a\int \sin(c+dx)\,dx+b\int x^3\sin(c+dx)\,dx \\
&= -\frac{a\cos(c+dx)}{d}-\frac{bx^3\cos(c+dx)}{d}+\frac{(3b)\int x^2\cos(c+dx)\,dx}{d} \\
&= -\frac{a\cos(c+dx)}{d}-\frac{bx^3\cos(c+dx)}{d}+\frac{3bx^2\sin(c+dx)}{d^2}-\frac{(6b)\int x\sin(c+dx)\,dx}{d^2} \\
&= -\frac{a\cos(c+dx)}{d}+\frac{6bx\cos(c+dx)}{d^3}-\frac{bx^3\cos(c+dx)}{d} \\
&\quad +\frac{3bx^2\sin(c+dx)}{d^2}-\frac{(6b)\int\cos(c+dx)\,dx}{d^3} \\
&= -\frac{a\cos(c+dx)}{d}+\frac{6bx\cos(c+dx)}{d^3}-\frac{bx^3\cos(c+dx)}{d}-\frac{6b\sin(c+dx)}{d^4}+\frac{3bx^2\sin(c+dx)}{d^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.74

$$
\int \left(a+bx^3\right)\sin(c+dx)\,dx=\frac{-d(ad^2+bx(-6+d^2x^2))\cos(c+dx)+3b(-2+d^2x^2)\sin(c+dx)}{d^4}
$$

```
[In] Integrate[(a + b*x^3)*Sin[c + d*x],x]
[Out] (-(d*(a*d^2 + b*x*(-6 + d^2*x^2))*Cos[c + d*x]) + 3*b*(-2 + d^2*x^2)*Sin[c
+ d*x])/d^4
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{\left(b\,d^{2} x^{3}+a\,d^{2}-6 b x \right) \cos \left(d x +c \right)}{d^{3}}+\frac{3 b \left(d^{2} x^{2}-2\right) \sin \left(d x +c \right)}{d^{4}}$
parallelrisch	$\frac{d x b \left(d^{2} x^{2}-6\right) \left(\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)+6 b \left(d^{2} x^{2}-2\right) \tan \left(\frac{d x}{2}+\frac{c}{2}\right)+\left(-b\,x^{3}-2 a \right) d^{3}+6 d x b}{d^{4} \left(1+\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}$
parts	$-\frac{b\,x^{3} \cos \left(d x +c \right)}{d}-\frac{a \cos \left(d x +c \right)}{d}+\frac{3 b \left(c^{2} \sin \left(d x +c \right)-2 c \left(\cos \left(d x +c \right)+\left(d x +c \right) \sin \left(d x +c \right)\right)+\left(d x +c \right)^{2} \sin \left(d x +c \right)-2 \sin (dx\right.}{d^{4}}$
norman	$\frac{\frac{2 a \left(\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{d}+\frac{b\,x^{3} \left(\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{d}-\frac{12 b \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{d^{4}}+\frac{6 b x}{d^{3}}-\frac{b\,x^{3}}{d}-\frac{6 b x \left(\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{d^{3}}+\frac{6 b\,x^{2} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{d^{2}}}{1+\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)}$
meijerg	$\frac{8 b \sqrt{\pi}\, \sin \left(c \right) \left(\frac{3}{4 \sqrt{\pi}}-\frac{\left(-\frac{3 d^{2} x^{2}}{2}+3\right) \cos \left(d x \right)}{4 \sqrt{\pi}}-\frac{d x \left(-\frac{d^{2} x^{2}}{2}+3\right) \sin \left(d x \right)}{4 \sqrt{\pi}}\right)}{d^{4}}+\frac{8 b \sqrt{\pi}\, \cos \left(c \right) \left(\frac{x d \left(-\frac{5 d^{2} x^{2}}{2}+15\right) \cos \left(d x \right)}{20 \sqrt{\pi}}-\frac{\left(-\frac{15 d^{2} x}{2}\right.}{}\right.}{d^{4}}$
derivativedivides	$\frac{-\cos \left(d x +c \right) a +\frac{b\,c^{3} \cos \left(d x +c \right)}{d^{3}}+\frac{3 b\,c^{2} \left(\sin \left(d x +c \right)-\cos \left(d x +c \right) \left(d x +c \right)\right)}{d^{3}}-\frac{3 b c \left(-\left(d x +c \right)^{2} \cos \left(d x +c \right)+2 \cos \left(d x +c \right)+2 \left(d x +c \right) \sin \left(d x +c \right)\right)}{d^{3}}+}{d}$
default	$\frac{-\cos \left(d x +c \right) a +\frac{b\,c^{3} \cos \left(d x +c \right)}{d^{3}}+\frac{3 b\,c^{2} \left(\sin \left(d x +c \right)-\cos \left(d x +c \right) \left(d x +c \right)\right)}{d^{3}}-\frac{3 b c \left(-\left(d x +c \right)^{2} \cos \left(d x +c \right)+2 \cos \left(d x +c \right)+2 \left(d x +c \right) \sin \left(d x +c \right)\right)}{d^{3}}+}{d}$

```
[In] int((b*x^3+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
[Out] -1/d^3*(b*d^2*x^3+a*d^2-6*b*x)*cos(d*x+c)+3*b*(d^2*x^2-2)/d^4*sin(d*x+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int \left(a+bx^3\right)\sin(c+dx)\,dx = -\frac{\left(bd^3x^3+ad^3-6\,bdx\right)\cos\left(dx+c\right)-3\left(bd^2x^2-2\,b\right)\sin\left(dx+c\right)}{d^4}$$

```
[In] integrate((b*x^3+a)*sin(d*x+c),x, algorithm="fricas")
[Out] -((b*d^3*x^3 + a*d^3 - 6*b*d*x)*cos(d*x + c) - 3*(b*d^2*x^2 - 2*b)*sin(d*x
+ c))/d^4
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.21

$$\int (a+bx^3)\sin(c+dx)\,dx = \begin{cases} -\frac{a\cos(c+dx)}{d} - \frac{bx^3\cos(c+dx)}{d} + \frac{3bx^2\sin(c+dx)}{d^2} + \frac{6bx\cos(c+dx)}{d^3} - \frac{6b\sin(c+dx)}{d^4} & \text{for } d \neq 0 \\ \left(ax + \frac{bx^4}{4}\right)\sin(c) & \text{otherwise} \end{cases}$$

```
[In] integrate((b*x**3+a)*sin(d*x+c),x)
```

```
[Out] Piecewise((-a*cos(c + d*x)/d - b*x**3*cos(c + d*x)/d + 3*b*x**2*sin(c + d*x
)/d**2 + 6*b*x*cos(c + d*x)/d**3 - 6*b*sin(c + d*x)/d**4, Ne(d, 0)), ((a*x
+ b*x**4/4)*sin(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(68) = 136.

Time = 0.19 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.07

$$\int (a+bx^3)\sin(c+dx)\,dx = -\frac{a\cos(dx+c) - \frac{bc^3\cos(dx+c)}{d^3} + \frac{3((dx+c)\cos(dx+c)-\sin(dx+c))bc^2}{d^3} - \frac{3\left(\left((dx+c)^2-2\right)\cos(dx+c)-2(dx+c)\sin(dx+c)\right)bc}{d^3} + \cdots}{d}$$

```
[In] integrate((b*x^3+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a*cos(d*x + c) - b*c^3*cos(d*x + c)/d^3 + 3*((d*x + c)*cos(d*x + c) - sin
(d*x + c))*b*c^2/d^3 - 3*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(
d*x + c))*b*c/d^3 + (((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c
)^2 - 2)*sin(d*x + c))*b/d^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int (a+bx^3)\sin(c+dx)\,dx = -\frac{(bd^3x^3 + ad^3 - 6\,bdx)\cos(dx+c)}{d^4} + \frac{3\,(bd^2x^2 - 2\,b)\sin(dx+c)}{d^4}$$

```
[In] integrate((b*x^3+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b*d^3*x^3 + a*d^3 - 6*b*d*x)*cos(d*x + c)/d^4 + 3*(b*d^2*x^2 - 2*b)*sin(d
*x + c)/d^4
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int (a+bx^3)\sin(c+dx)\,dx = -\frac{6\,b\,\sin\left(c+d\,x\right)+d^3\,\left(a\,\cos\left(c+d\,x\right)+b\,x^3\,\cos\left(c+d\,x\right)\right)-3\,b\,d^2\,x^2\,\sin\left(c+d\,x\right)-6\,b\,d\,x\,\cos\left(c+d\,x\right)}{d^4}$$

```
[In] int(sin(c + d*x)*(a + b*x^3),x)
```

```
[Out] -(6*b*sin(c + d*x) + d^3*(a*cos(c + d*x) + b*x^3*cos(c + d*x)) - 3*b*d^2*x^
2*sin(c + d*x) - 6*b*d*x*cos(c + d*x))/d^4
```

3.83 $\int \frac{(a+bx^3)\sin(c+dx)}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 57

$$\int \frac{(a+bx^3)\sin(c+dx)}{x}\,dx = \frac{2b\cos(c+dx)}{d^3} - \frac{bx^2\cos(c+dx)}{d} + a\,\text{CosIntegral}(dx)\sin(c) + \frac{2bx\sin(c+dx)}{d^2} + a\cos(c)\text{Si}(dx)$$

```
[Out] 2*b*cos(d*x+c)/d^3-b*x^2*cos(d*x+c)/d+a*cos(c)*Si(d*x)+a*Ci(d*x)*sin(c)+2*b
*x*sin(d*x+c)/d^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3420, 3384, 3380, 3383, 3377, 2718}

$$\int \frac{(a+bx^3)\sin(c+dx)}{x}\,dx = a\sin(c)\,\text{CosIntegral}(dx) + a\cos(c)\text{Si}(dx) + \frac{2b\cos(c+dx)}{d^3} + \frac{2bx\sin(c+dx)}{d^2} - \frac{bx^2\cos(c+dx)}{d}$$

```
[In] Int[((a + b*x^3)*Sin[c + d*x])/x,x]
```

```
[Out] (2*b*Cos[c + d*x])/d^3 - (b*x^2*Cos[c + d*x])/d + a*CosIntegral[d*x]*Sin[c]
 + (2*b*x*Sin[c + d*x])/d^2 + a*Cos[c]*SinIntegral[d*x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a\sin(c+dx)}{x} + bx^2\sin(c+dx)\right) dx \\
&= a\int \frac{\sin(c+dx)}{x}\,dx + b\int x^2\sin(c+dx)\,dx \\
&= -\frac{bx^2\cos(c+dx)}{d} + \frac{(2b)\int x\cos(c+dx)\,dx}{d} \\
&\quad + (a\cos(c))\int \frac{\sin(dx)}{x}\,dx + (a\sin(c))\int \frac{\cos(dx)}{x}\,dx \\
&= -\frac{bx^2\cos(c+dx)}{d} + a\,\text{CosIntegral}(dx)\sin(c) \\
&\quad + \frac{2bx\sin(c+dx)}{d^2} + a\cos(c)\text{Si}(dx) - \frac{(2b)\int \sin(c+dx)\,dx}{d^2}
\end{aligned}
$$

$$= \frac{2b\cos(c+dx)}{d^3} - \frac{bx^2\cos(c+dx)}{d} + a\,\text{CosIntegral}(dx)\sin(c) + \frac{2bx\sin(c+dx)}{d^2} + a\cos(c)\text{Si}(dx)$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)\sin(c+dx)}{x}\,dx = a\,\text{CosIntegral}(dx)\sin(c) + \frac{b((2-d^2x^2)\cos(c+dx)+2dx\sin(c+dx))}{d^3} + a\cos(c)\text{Si}(dx)$$

```
[In] Integrate[((a + b*x^3)*Sin[c + d*x])/x,x]
[Out] a*CosIntegral[d*x]*Sin[c] + (b*((2 - d^2*x^2)*Cos[c + d*x] + 2*d*x*Sin[c + 
d*x]))/d^3 + a*Cos[c]*SinIntegral[d*x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.18 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.72

method	result
risch	$-\frac{e^{-ic}\pi\ \text{csgn}(dx)a}{2} - \frac{ie^{-ic}\ \text{Ei}_1(-idx)a}{2} + \frac{ia\,e^{ic}\ \text{Ei}_1(-idx)}{2} - \frac{b\,x^2\cos(dx+c)}{d} + e^{-ic}\ \text{Si}\,(dx)\,a + \frac{2bx\sin(dx+}{d^2}$
derivativedivides	$a(\text{Si}\,(dx)\cos\,(c) + \text{Ci}\,(dx)\sin\,(c)) - \frac{3b\,c^2\cos(dx+c)}{d^3} - \frac{3bc(c+1)(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^3} +$
default	$a(\text{Si}\,(dx)\cos\,(c) + \text{Ci}\,(dx)\sin\,(c)) - \frac{3b\,c^2\cos(dx+c)}{d^3} - \frac{3bc(c+1)(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^3} +$
meijerg	$\frac{4b\sqrt{\pi}\ \sin(c)\left(\frac{x\left(d^2\right)^{\frac{3}{2}}\cos(dx)}{2\sqrt{\pi}\,d^2} - \frac{\left(d^2\right)^{\frac{3}{2}}\left(-\frac{3d^2x^2}{2}+3\right)\sin(dx)}{6\sqrt{\pi}\,d^3}\right)}{d^2\sqrt{d^2}} + \frac{4b\sqrt{\pi}\ \cos(c)\left(-\frac{1}{2\sqrt{\pi}} + \frac{\left(-\frac{d^2x^2}{2}+1\right)\cos(dx)}{2\sqrt{\pi}} + \frac{dx\sin(dx)}{2\sqrt{\pi}}\right.}{d^3}$

```
[In] int((b*x^3+a)*sin(d*x+c)/x,x,method=_RETURNVERBOSE)
[Out] -1/2*exp(-I*c)*Pi*csgn(d*x)*a-1/2*I*exp(-I*c)*Ei(1,-I*d*x)*a+1/2*I*a*exp(I*
c)*Ei(1,-I*d*x)-b*x^2*cos(d*x+c)/d+exp(-I*c)*Si(d*x)*a+2*b*x*sin(d*x+c)/d^2
+2*b*cos(d*x+c)/d^3
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)\sin(c+dx)}{x}\,dx = \frac{ad^3\operatorname{Ci}(dx)\sin(c)+ad^3\cos(c)\operatorname{Si}(dx)+2\,bdx\sin(dx+c)-(bd^2x^2-2\,b)\cos(dx+c)}{d^3}$$

```
[In] integrate((b*x^3+a)*sin(d*x+c)/x,x, algorithm="fricas")
```

```
[Out] (a*d^3*cos_integral(d*x)*sin(c) + a*d^3*cos(c)*sin_integral(d*x) + 2*b*d*x*
sin(d*x + c) - (b*d^2*x^2 - 2*b)*cos(d*x + c))/d^3
```

Sympy [A] (verification not implemented)

Time = 2.75 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.46

$$\int \frac{(a+bx^3)\sin(c+dx)}{x}\,dx = a\sin(c)\operatorname{Ci}(dx)+a\cos(c)\operatorname{Si}(dx) + bx^2\left(\begin{cases} x\sin(c) & \text{for}\ d=0 \\ -\frac{\cos(c+dx)}{d} & \text{otherwise} \end{cases}\right) - 2b\left(\begin{cases} \frac{x^3\sin(c)}{3} & \text{for}\ d=0 \\ -\frac{\begin{cases} \frac{x\sin(c+dx)}{d}+\frac{\cos(c+dx)}{d^2} & \text{for}\ d\neq 0 \\ \frac{x^2\cos(c)}{2} & \text{otherwise} \end{cases}}{d} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((b*x**3+a)*sin(d*x+c)/x,x)
```

```
[Out] a*sin(c)*Ci(d*x) + a*cos(c)*Si(d*x) + b*x**2*Piecewise((x*sin(c), Eq(d, 0))
, (-cos(c + d*x)/d, True)) - 2*b*Piecewise((x**3*sin(c)/3, Eq(d, 0)), (-Pie
cewise((x*sin(c + d*x)/d + cos(c + d*x)/d**2, Ne(d, 0)), (x**2*cos(c)/2, Tr
ue))/d, True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx^3)\sin(c+dx)}{x}\,dx$$
$$= \frac{(a(-i\,\mathrm{Ei}(i\,dx)+i\,\mathrm{Ei}(-i\,dx))\cos{(c)}+a(\mathrm{Ei}(i\,dx)+\mathrm{Ei}(-i\,dx))\sin{(c)})d^3+4\,bdx\sin{(dx+c)}-2\,(bd^2x^2-}{2\,d^3}$$

```
[In] integrate((b*x^3+a)*sin(d*x+c)/x,x, algorithm="maxima")
```

```
[Out] 1/2*((a*(-I*Ei(I*d*x) + I*Ei(-I*d*x))*cos(c) + a*(Ei(I*d*x) + Ei(-I*d*x))*s
in(c))*d^3 + 4*b*d*x*sin(d*x + c) - 2*(b*d^2*x^2 - 2*b)*cos(d*x + c))/d^3
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.32 (sec) , antiderivative size = 510, normalized size of antiderivative = 8.95

$$\int \frac{(a+bx^3)\sin(c+dx)}{x}\,dx =$$
$$-\frac{2\,bd^2x^2\tan\left(\frac{1}{2}\,dx\right)^2\tan\left(\frac{1}{2}\,c\right)^2+ad^3\Im(\mathrm{Ci}\,(dx))\tan\left(\frac{1}{2}\,dx\right)^2\tan\left(\frac{1}{2}\,c\right)^2-ad^3\Im(\mathrm{Ci}\,(-dx))\tan\left(\frac{1}{2}\,dx\right)^2\tan}{}$$

```
[In] integrate((b*x^3+a)*sin(d*x+c)/x,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*d^2*x^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + a*d^3*imag_part(cos_integra
l(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d^3*imag_part(cos_integral(-d*x))*t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d^3*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1
/2*c)^2 - 2*a*d^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) -
2*a*d^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*b*d^2*x
^2*tan(1/2*d*x)^2 - a*d^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 + a*d
^3*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 2*a*d^3*sin_integral(d*x)
*tan(1/2*d*x)^2 - 8*b*d^2*x^2*tan(1/2*d*x)*tan(1/2*c) - 2*b*d^2*x^2*tan(1/2
*c)^2 + a*d^3*imag_part(cos_integral(d*x))*tan(1/2*c)^2 - a*d^3*imag_part(c
os_integral(-d*x))*tan(1/2*c)^2 + 2*a*d^3*sin_integral(d*x)*tan(1/2*c)^2 -
2*a*d^3*real_part(cos_integral(d*x))*tan(1/2*c) - 2*a*d^3*real_part(cos_int
egral(-d*x))*tan(1/2*c) + 8*b*d*x*tan(1/2*d*x)^2*tan(1/2*c) + 8*b*d*x*tan(1
/2*d*x)*tan(1/2*c)^2 + 2*b*d^2*x^2 - a*d^3*imag_part(cos_integral(d*x)) + a
*d^3*imag_part(cos_integral(-d*x)) - 2*a*d^3*sin_integral(d*x) - 4*b*tan(1/
2*d*x)^2*tan(1/2*c)^2 - 8*b*d*x*tan(1/2*d*x) - 8*b*d*x*tan(1/2*c) + 4*b*tan
(1/2*d*x)^2 + 16*b*tan(1/2*d*x)*tan(1/2*c) + 4*b*tan(1/2*c)^2 - 4*b)/(d^3*t
an(1/2*d*x)^2*tan(1/2*c)^2 + d^3*tan(1/2*d*x)^2 + d^3*tan(1/2*c)^2 + d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)\sin(c+dx)}{x}\,dx = a\,\mathrm{cosint}(d\,x)\,\sin(c) + a\,\mathrm{sinint}(d\,x)\,\cos(c) + \frac{b\,(2\,\cos(c+d\,x) - d^2\,x^2\,\cos(c+d\,x) + 2\,d\,x\,\sin(c+d\,x))}{d^3}$$

```
[In] int((sin(c + d*x)*(a + b*x^3))/x,x)
```

```
[Out] a*cosint(d*x)*sin(c) + a*sinint(d*x)*cos(c) + (b*(2*cos(c + d*x) - d^2*x^2*
cos(c + d*x) + 2*d*x*sin(c + d*x)))/d^3
```

3.84 $\int \frac{(a+bx^3)\sin(c+dx)}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 56

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^2}\,dx = -\frac{bx\cos(c+dx)}{d} + ad\cos(c)\,\text{CosIntegral}(dx) + \frac{b\sin(c+dx)}{d^2} - \frac{a\sin(c+dx)}{x} - ad\sin(c)\text{Si}(dx)$$

```
[Out] a*d*Ci(d*x)*cos(c)-b*x*cos(d*x+c)/d-a*d*Si(d*x)*sin(c)+b*sin(d*x+c)/d^2-a*s
in(d*x+c)/x
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3420, 3378, 3384, 3380, 3383, 3377, 2717}

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^2}\,dx = ad\cos(c)\,\text{CosIntegral}(dx) - ad\sin(c)\text{Si}(dx) - \frac{a\sin(c+dx)}{x} + \frac{b\sin(c+dx)}{d^2} - \frac{bx\cos(c+dx)}{d}$$

```
[In] Int[((a + b*x^3)*Sin[c + d*x])/x^2,x]
```

```
[Out] -((b*x*Cos[c + d*x])/d) + a*d*Cos[c]*CosIntegral[d*x] + (b*Sin[c + d*x])/d^
2 - (a*Sin[c + d*x])/x - a*d*Sin[c]*SinIntegral[d*x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a\sin(c+dx)}{x^2}+bx\sin(c+dx)\right)dx \\
&= a\int \frac{\sin(c+dx)}{x^2}\,dx+b\int x\sin(c+dx)\,dx \\
&= -\frac{bx\cos(c+dx)}{d}-\frac{a\sin(c+dx)}{x}+\frac{b\int \cos(c+dx)\,dx}{d}+(ad)\int \frac{\cos(c+dx)}{x}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{bx\cos(c+dx)}{d} + \frac{b\sin(c+dx)}{d^2} - \frac{a\sin(c+dx)}{x} \\
&\quad + (ad\cos(c))\int \frac{\cos(dx)}{x}\,dx - (ad\sin(c))\int \frac{\sin(dx)}{x}\,dx \\
&= -\frac{bx\cos(c+dx)}{d} + ad\cos(c)\,\text{CosIntegral}(dx) \\
&\quad + \frac{b\sin(c+dx)}{d^2} - \frac{a\sin(c+dx)}{x} - ad\sin(c)\text{Si}(dx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$
\begin{aligned}
\int \frac{(a+bx^3)\sin(c+dx)}{x^2}\,dx = &-\frac{bx\cos(c+dx)}{d} + ad\cos(c)\,\text{CosIntegral}(dx) \\
&+ \frac{b\sin(c+dx)}{d^2} - \frac{a\sin(c+dx)}{x} - ad\sin(c)\text{Si}(dx)
\end{aligned}
$$

```
[In] Integrate[((a + b*x^3)*Sin[c + d*x])/x^2,x]
```

```
[Out] -((b*x*Cos[c + d*x])/d) + a*d*Cos[c]*CosIntegral[d*x] + (b*Sin[c + d*x])/d^
2 - (a*Sin[c + d*x])/x - a*d*Sin[c]*SinIntegral[d*x]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.41

method	result
derivativedivides	$d(a(-\frac{\sin(dx+c)}{dx} - \text{Si}(dx)\sin(c) + \text{Ci}(dx)\cos(c)) + \frac{3bc\cos(dx+c)}{d^3} + \frac{(2c+1)b(\sin(dx+c)-\cos(dx}{d^3}$
default	$d(a(-\frac{\sin(dx+c)}{dx} - \text{Si}(dx)\sin(c) + \text{Ci}(dx)\cos(c)) + \frac{3bc\cos(dx+c)}{d^3} + \frac{(2c+1)b(\sin(dx+c)-\cos(dx}{d^3}$
risch	$-\frac{-i\,\text{Ei}_1(idx)\sin(c)a\,d^3x+i\sin(c)\,\text{Ei}_1(-idx)a\,d^3x+\text{Ei}_1(idx)\cos(c)a\,d^3x+\cos(c)\,\text{Ei}_1(-idx)a\,d^3x+2\cos(dx+c)bd\,x^2+2}{2d^2x}$
meijerg	$\frac{2b\sqrt{\pi}\,\sin(c)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(dx)}{2\sqrt{\pi}}+\frac{dx\sin(dx)}{2\sqrt{\pi}}\right)}{d^2} + \frac{2b\sqrt{\pi}\,\cos(c)\left(-\frac{dx\cos(dx)}{2\sqrt{\pi}}+\frac{\sin(dx)}{2\sqrt{\pi}}\right)}{d^2} + \frac{a\sqrt{\pi}\,\sin(c)d^2(-\frac{4d^2\cos\left(x\sqrt{d^2}\right)}{x\left(d^2\right)^{\frac{3}{2}}\sqrt{\pi}}}{4\sqrt{d^2}}$

```
[In] int((b*x^3+a)*sin(d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] d*(a*(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c))+3*b/d^3*c*cos(d*x+c)+(
2*c+1)/d^3*b*(sin(d*x+c)-cos(d*x+c)*(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^2}\,dx = \frac{ad^3x\cos{(c)}\operatorname{Ci}{(dx)} - ad^3x\sin{(c)}\operatorname{Si}{(dx)} - bdx^2\cos{(dx+c)} - (ad^2-bx)\sin{(dx+c)}}{d^2x}$$

```
[In] integrate((b*x^3+a)*sin(d*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] (a*d^3*x*cos(c)*cos_integral(d*x) - a*d^3*x*sin(c)*sin_integral(d*x) - b*d*
x^2*cos(d*x + c) - (a*d^2 - b*x)*sin(d*x + c))/(d^2*x)
```

Sympy [F]

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^2}\,dx = \int \frac{(a+bx^3)\sin{(c+dx)}}{x^2}\,dx$$

```
[In] integrate((b*x**3+a)*sin(d*x+c)/x**2,x)
```

```
[Out] Integral((a + b*x**3)*sin(c + d*x)/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.23

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^2}\,dx = \frac{(a(\Gamma(-1,i\,dx)+\Gamma(-1,-i\,dx))\cos{(c)} + a(-i\,\Gamma(-1,i\,dx)+i\,\Gamma(-1,-i\,dx))\sin{(c)})d^3 - 2\,bdx\cos{(dx+c)}}{2\,d^2}$$

```
[In] integrate((b*x^3+a)*sin(d*x+c)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*((a*(gamma(-1, I*d*x) + gamma(-1, -I*d*x))*cos(c) + a*(-I*gamma(-1, I*d
*x) + I*gamma(-1, -I*d*x))*sin(c))*d^3 - 2*b*d*x*cos(d*x + c) + 2*b*sin(d*x
 + c))/d^2
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.30 (sec) , antiderivative size = 489, normalized size of antiderivative = 8.73

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^2}\,dx = -\frac{ad^3x\Re(\mathrm{Ci}\,(dx))\tan\left(\frac{1}{2}\,dx\right)^2\tan\left(\frac{1}{2}\,c\right)^2 + ad^3x\Re(\mathrm{Ci}\,(-dx))\tan\left(\frac{1}{2}\,dx\right)^2\tan\left(\frac{1}{2}\,c\right)^2 + 2\,ad^3x\Im(\mathrm{Ci}\,(dx))\tan}{}$$

```
[In] integrate((b*x^3+a)*sin(d*x+c)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(a*d^3*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + a*
d^3*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d^3*x
*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^3*x*imag_pa
rt(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a*d^3*x*sin_integral(d
*x)*tan(1/2*d*x)^2*tan(1/2*c) - a*d^3*x*real_part(cos_integral(d*x))*tan(1/
2*d*x)^2 - a*d^3*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2 + a*d^3*x*r
eal_part(cos_integral(d*x))*tan(1/2*c)^2 + a*d^3*x*real_part(cos_integral(-
d*x))*tan(1/2*c)^2 + 2*b*d*x^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d^3*x*imag
_part(cos_integral(d*x))*tan(1/2*c) - 2*a*d^3*x*imag_part(cos_integral(-d*x
))*tan(1/2*c) + 4*a*d^3*x*sin_integral(d*x)*tan(1/2*c) - a*d^3*x*real_part(
cos_integral(d*x)) - a*d^3*x*real_part(cos_integral(-d*x)) - 2*b*d*x^2*tan(
1/2*d*x)^2 - 8*b*d*x^2*tan(1/2*d*x)*tan(1/2*c) - 4*a*d^2*tan(1/2*d*x)^2*tan
(1/2*c) - 2*b*d*x^2*tan(1/2*c)^2 - 4*a*d^2*tan(1/2*d*x)*tan(1/2*c)^2 + 4*b*
x*tan(1/2*d*x)^2*tan(1/2*c) + 4*b*x*tan(1/2*d*x)*tan(1/2*c)^2 + 2*b*d*x^2 +
 4*a*d^2*tan(1/2*d*x) + 4*a*d^2*tan(1/2*c) - 4*b*x*tan(1/2*d*x) - 4*b*x*tan
(1/2*c))/(d^2*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + d^2*x*tan(1/2*d*x)^2 + d^2*x*
tan(1/2*c)^2 + d^2*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^2}\,dx = \int \frac{\sin\left(c+d\,x\right)\,\left(b\,x^3+a\right)}{x^2}\,dx$$

```
[In] int((sin(c + d*x)*(a + b*x^3))/x^2,x)
```

```
[Out] int((sin(c + d*x)*(a + b*x^3))/x^2, x)
```

3.85 $\int \frac{(a+bx^3)\sin(c+dx)}{x^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^3}\,dx = -\frac{b\cos(c+dx)}{d} - \frac{ad\cos(c+dx)}{2x} - \frac{1}{2}ad^2\,\mathrm{CosIntegral}(dx)\sin(c) - \frac{a\sin(c+dx)}{2x^2} - \frac{1}{2}ad^2\cos(c)\mathrm{Si}(dx)$$

```
[Out] -b*cos(d*x+c)/d-1/2*a*d*cos(d*x+c)/x-1/2*a*d^2*cos(c)*Si(d*x)-1/2*a*d^2*Ci(
d*x)*sin(c)-1/2*a*sin(d*x+c)/x^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3420, 2718, 3378, 3384, 3380, 3383}

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^3}\,dx = -\frac{1}{2}ad^2\sin(c)\,\mathrm{CosIntegral}(dx) - \frac{1}{2}ad^2\cos(c)\mathrm{Si}(dx) - \frac{a\sin(c+dx)}{2x^2} - \frac{ad\cos(c+dx)}{2x} - \frac{b\cos(c+dx)}{d}$$

```
[In] Int[((a + b*x^3)*Sin[c + d*x])/x^3,x]
```

```
[Out] -((b*Cos[c + d*x])/d) - (a*d*Cos[c + d*x])/(2*x) - (a*d^2*CosIntegral[d*x]*
Sin[c])/2 - (a*Sin[c + d*x])/(2*x^2) - (a*d^2*Cos[c]*SinIntegral[d*x])/2
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(b\sin(c+dx)+\frac{a\sin(c+dx)}{x^3}\right)dx\\
&= a\int \frac{\sin(c+dx)}{x^3}\,dx+b\int \sin(c+dx)\,dx\\
&= -\frac{b\cos(c+dx)}{d}-\frac{a\sin(c+dx)}{2x^2}+\frac{1}{2}(ad)\int \frac{\cos(c+dx)}{x^2}\,dx\\
&= -\frac{b\cos(c+dx)}{d}-\frac{ad\cos(c+dx)}{2x}-\frac{a\sin(c+dx)}{2x^2}-\frac{1}{2}\left(ad^2\right)\int \frac{\sin(c+dx)}{x}\,dx\\
&= -\frac{b\cos(c+dx)}{d}-\frac{ad\cos(c+dx)}{2x}-\frac{a\sin(c+dx)}{2x^2}\\
&\quad -\frac{1}{2}\left(ad^2\cos(c)\right)\int \frac{\sin(dx)}{x}\,dx-\frac{1}{2}\left(ad^2\sin(c)\right)\int \frac{\cos(dx)}{x}\,dx
\end{aligned}
$$

$$
= -\frac{b\cos(c+dx)}{d} - \frac{ad\cos(c+dx)}{2x} - \frac{1}{2}ad^2\,\text{CosIntegral}(dx)\sin(c) - \frac{a\sin(c+dx)}{2x^2} - \frac{1}{2}ad^2\cos(c)\text{Si}(dx)
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$
\int \frac{(a+bx^3)\sin(c+dx)}{x^3}\,dx = \frac{1}{2}\left(-\frac{2b\cos(c+dx)}{d} - \frac{ad\cos(c+dx)}{x} - ad^2\,\text{CosIntegral}(dx)\sin(c) - \frac{a\sin(c+dx)}{x^2} - ad^2\cos(c)\text{Si}(dx)\right)
$$

```
[In] Integrate[((a + b*x^3)*Sin[c + d*x])/x^3,x]
```

```
[Out] ((-2*b*Cos[c + d*x])/d - (a*d*Cos[c + d*x])/x - a*d^2*CosIntegral[d*x]*Sin[
c] - (a*Sin[c + d*x])/x^2 - a*d^2*Cos[c]*SinIntegral[d*x])/2
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

method	result
derivativedivides	$d^2\left(a\left(-\frac{\sin(dx+c)}{2d^2x^2} - \frac{\cos(dx+c)}{2dx} - \frac{\text{Si}(dx)\cos(c)}{2} - \frac{\text{Ci}(dx)\sin(c)}{2}\right) - \frac{b\cos(dx+c)}{d^3}\right)$
default	$d^2\left(a\left(-\frac{\sin(dx+c)}{2d^2x^2} - \frac{\cos(dx+c)}{2dx} - \frac{\text{Si}(dx)\cos(c)}{2} - \frac{\text{Ci}(dx)\sin(c)}{2}\right) - \frac{b\cos(dx+c)}{d^3}\right)$
risch	$-\frac{id^2\cos(c)a\ \text{Ei}_1(-idx)}{4} + \frac{id^2\cos(c)a\ \text{Ei}_1(idx)}{4} + \frac{d^2\sin(c)a\ \text{Ei}_1(-idx)}{4} + \frac{d^2\sin(c)a\ \text{Ei}_1(idx)}{4} - \frac{i(-2ia\,d^6x^3-4ib}{4d^5}$
meijerg	$\frac{b\sin(c)\sin(dx)}{d} + \frac{b\sqrt{\pi}\,\cos(c)\left(\frac{1}{\sqrt{\pi}} - \frac{\cos(dx)}{\sqrt{\pi}}\right)}{d} + \frac{a\sqrt{\pi}\,\sin(c)d^2\left(-\frac{4}{\sqrt{\pi}\,x^2d^2} - \frac{2\left(2\gamma-3+2\ln(x)+\ln\left(d^2\right)\right)}{\sqrt{\pi}} + \frac{-6d^2x^2+4}{\sqrt{\pi}\,x^2d^2} + \frac{4\gamma}{\sqrt{\pi}} +\right.}{8}$

```
[In] int((b*x^3+a)*sin(d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^2*(a*(-1/2*sin(d*x+c)/d^2/x^2-1/2*cos(d*x+c)/d/x-1/2*Si(d*x)*cos(c)-1/2*C
i(d*x)*sin(c))-b*cos(d*x+c)/d^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^3}\,dx = -\frac{ad^3x^2\,\mathrm{Ci}\,(dx)\sin\,(c)+ad^3x^2\cos\,(c)\,\mathrm{Si}\,(dx)+ad\sin\,(dx+c)+(ad^2x+2\,bx^2)\cos\,(dx+c)}{2\,dx^2}$$

```
[In] integrate((b*x^3+a)*sin(d*x+c)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a*d^3*x^2*cos_integral(d*x)*sin(c) + a*d^3*x^2*cos(c)*sin_integral(d*
x) + a*d*sin(d*x + c) + (a*d^2*x + 2*b*x^2)*cos(d*x + c))/(d*x^2)
```

Sympy [F]

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^3}\,dx = \int \frac{(a+bx^3)\sin\,(c+dx)}{x^3}\,dx$$

```
[In] integrate((b*x**3+a)*sin(d*x+c)/x**3,x)
```

```
[Out] Integral((a + b*x**3)*sin(c + d*x)/x**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 1146, normalized size of antiderivative = 16.37

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^3}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x^3+a)*sin(d*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(((I*exp_integral_e(3, I*d*x) - I*exp_integral_e(3, -I*d*x))*cos(c)^3 +
 (I*exp_integral_e(3, I*d*x) - I*exp_integral_e(3, -I*d*x))*cos(c)*sin(c)^2
 + (exp_integral_e(3, I*d*x) + exp_integral_e(3, -I*d*x))*sin(c)^3 + (I*exp
_integral_e(3, I*d*x) - I*exp_integral_e(3, -I*d*x))*cos(c) + ((exp_integra
l_e(3, I*d*x) + exp_integral_e(3, -I*d*x))*cos(c)^2 + exp_integral_e(3, I*d
*x) + exp_integral_e(3, -I*d*x))*sin(c))*b*c^3/((d*x + c)^2*(cos(c)^2 + sin
(c)^2)*d^3 - 2*(c*cos(c)^2 + c*sin(c)^2)*(d*x + c)*d^3 + (c^2*cos(c)^2 + c^
2*sin(c)^2)*d^3) - ((I*exp_integral_e(3, I*d*x) - I*exp_integral_e(3, -I*d*
x))*cos(c)^3 + (I*exp_integral_e(3, I*d*x) - I*exp_integral_e(3, -I*d*x))*c
```

```
os(c)*sin(c)^2 + (exp_integral_e(3, I*d*x) + exp_integral_e(3, -I*d*x))*sin
(c)^3 + (I*exp_integral_e(3, I*d*x) - I*exp_integral_e(3, -I*d*x))*cos(c) +
 ((exp_integral_e(3, I*d*x) + exp_integral_e(3, -I*d*x))*cos(c)^2 + exp_int
egral_e(3, I*d*x) + exp_integral_e(3, -I*d*x))*sin(c))*a/(c^2*cos(c)^2 + c^
2*sin(c)^2 + (d*x + c)^2*(cos(c)^2 + sin(c)^2) - 2*(c*cos(c)^2 + c*sin(c)^2
)*(d*x + c)) - (2*((b*cos(c)^2 + b*sin(c)^2)*(d*x + c)^3 - 3*(b*c*cos(c)^2
+ b*c*sin(c)^2)*(d*x + c)^2 + 3*(b*c^2*cos(c)^2 + b*c^2*sin(c)^2)*(d*x + c)
)*cos(d*x + c)^3 - 3*(b*c^3*(exp_integral_e(4, I*d*x) + exp_integral_e(4, -
I*d*x))*cos(c)^3 + b*c^3*(exp_integral_e(4, I*d*x) + exp_integral_e(4, -I*d
*x))*cos(c)*sin(c)^2 + b*c^3*(-I*exp_integral_e(4, I*d*x) + I*exp_integral_
e(4, -I*d*x))*sin(c)^3 + b*c^3*(exp_integral_e(4, I*d*x) + exp_integral_e(4
, -I*d*x))*cos(c) + (b*c^3*(-I*exp_integral_e(4, I*d*x) + I*exp_integral_e(
4, -I*d*x))*cos(c)^2 + b*c^3*(-I*exp_integral_e(4, I*d*x) + I*exp_integral_
e(4, -I*d*x)))*sin(c))*cos(d*x + c)^2 - (3*b*c^3*(exp_integral_e(4, I*d*x)
+ exp_integral_e(4, -I*d*x))*cos(c)^3 + 3*b*c^3*(exp_integral_e(4, I*d*x) +
 exp_integral_e(4, -I*d*x))*cos(c)*sin(c)^2 + 3*b*c^3*(-I*exp_integral_e(4,
 I*d*x) + I*exp_integral_e(4, -I*d*x))*sin(c)^3 + 3*b*c^3*(exp_integral_e(4
, I*d*x) + exp_integral_e(4, -I*d*x))*cos(c) - 2*((b*cos(c)^2 + b*sin(c)^2)
*(d*x + c)^3 - 3*(b*c*cos(c)^2 + b*c*sin(c)^2)*(d*x + c)^2 + 3*(b*c^2*cos(c
)^2 + b*c^2*sin(c)^2)*(d*x + c))*cos(d*x + c) + 3*(b*c^3*(-I*exp_integral_e
(4, I*d*x) + I*exp_integral_e(4, -I*d*x))*cos(c)^2 + b*c^3*(-I*exp_integral
_e(4, I*d*x) + I*exp_integral_e(4, -I*d*x)))*sin(c))*sin(d*x + c)^2 + 2*((b
*cos(c)^2 + b*sin(c)^2)*(d*x + c)^3 - 3*(b*c*cos(c)^2 + b*c*sin(c)^2)*(d*x
+ c)^2 + 3*(b*c^2*cos(c)^2 + b*c^2*sin(c)^2)*(d*x + c))*cos(d*x + c))/(((d*
x + c)^3*(cos(c)^2 + sin(c)^2)*d^3 - 3*(c*cos(c)^2 + c*sin(c)^2)*(d*x + c)^
2*d^3 + 3*(c^2*cos(c)^2 + c^2*sin(c)^2)*(d*x + c)*d^3 - (c^3*cos(c)^2 + c^3
*sin(c)^2)*d^3)*cos(d*x + c)^2 + ((d*x + c)^3*(cos(c)^2 + sin(c)^2)*d^3 - 3
*(c*cos(c)^2 + c*sin(c)^2)*(d*x + c)^2*d^3 + 3*(c^2*cos(c)^2 + c^2*sin(c)^2
)*(d*x + c)*d^3 - (c^3*cos(c)^2 + c^3*sin(c)^2)*d^3)*sin(d*x + c)^2))*d^2
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.32 (sec) , antiderivative size = 564, normalized size of antiderivative = 8.06

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^3}\,dx$$

$$= \frac{ad^3x^2\Im(\mathrm{Ci}\,(dx))\tan\left(\frac{1}{2}\,dx\right)^2\tan\left(\frac{1}{2}\,c\right)^2 - ad^3x^2\Im(\mathrm{Ci}\,(-dx))\tan\left(\frac{1}{2}\,dx\right)^2\tan\left(\frac{1}{2}\,c\right)^2 + 2\,ad^3x^2\,\mathrm{Si}\,(dx)\tan\left(\frac{1}{2}\,c\right.}{}$$

```
[In] integrate((b*x^3+a)*sin(d*x+c)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(a*d^3*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a
*d^3*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d^
```

```
3*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*d^3*x^2*real_part
(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^3*x^2*real_part(cos_i
ntegral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - a*d^3*x^2*imag_part(cos_integral
(d*x))*tan(1/2*d*x)^2 + a*d^3*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x
)^2 - 2*a*d^3*x^2*sin_integral(d*x)*tan(1/2*d*x)^2 + a*d^3*x^2*imag_part(co
s_integral(d*x))*tan(1/2*c)^2 - a*d^3*x^2*imag_part(cos_integral(-d*x))*tan
(1/2*c)^2 + 2*a*d^3*x^2*sin_integral(d*x)*tan(1/2*c)^2 - 2*a*d^3*x^2*real_p
art(cos_integral(d*x))*tan(1/2*c) - 2*a*d^3*x^2*real_part(cos_integral(-d*x
))*tan(1/2*c) - 2*a*d^2*x*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d^3*x^2*imag_part
(cos_integral(d*x)) + a*d^3*x^2*imag_part(cos_integral(-d*x)) - 2*a*d^3*x^2
*sin_integral(d*x) - 4*b*x^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d^2*x*tan(1/
2*d*x)^2 + 8*a*d^2*x*tan(1/2*d*x)*tan(1/2*c) + 2*a*d^2*x*tan(1/2*c)^2 + 4*b
*x^2*tan(1/2*d*x)^2 + 16*b*x^2*tan(1/2*d*x)*tan(1/2*c) + 4*a*d*tan(1/2*d*x)
^2*tan(1/2*c) + 4*b*x^2*tan(1/2*c)^2 + 4*a*d*tan(1/2*d*x)*tan(1/2*c)^2 - 2*
a*d^2*x - 4*b*x^2 - 4*a*d*tan(1/2*d*x) - 4*a*d*tan(1/2*c))/(d*x^2*tan(1/2*d
*x)^2*tan(1/2*c)^2 + d*x^2*tan(1/2*d*x)^2 + d*x^2*tan(1/2*c)^2 + d*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^3}\,dx = \int \frac{\sin\left(c+d\,x\right)\,\left(b\,x^3+a\right)}{x^3}\,dx$$

```
[In] int((sin(c + d*x)*(a + b*x^3))/x^3,x)
[Out] int((sin(c + d*x)*(a + b*x^3))/x^3, x)
```

3.86 $\int \frac{(a+bx^3)\sin(c+dx)}{x^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 91

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^4}\,dx = -\frac{ad\cos(c+dx)}{6x^2} - \frac{1}{6}ad^3\cos(c)\,\text{CosIntegral}(dx) + b\,\text{CosIntegral}(dx)\sin(c) - \frac{a\sin(c+dx)}{3x^3} + \frac{ad^2\sin(c+dx)}{6x} + b\cos(c)\text{Si}(dx) + \frac{1}{6}ad^3\sin(c)\text{Si}(dx)$$

```
[Out] -1/6*a*d^3*Ci(d*x)*cos(c)-1/6*a*d*cos(d*x+c)/x^2+b*cos(c)*Si(d*x)+b*Ci(d*x)
*sin(c)+1/6*a*d^3*Si(d*x)*sin(c)-1/3*a*sin(d*x+c)/x^3+1/6*a*d^2*sin(d*x+c)/
x
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3420, 3378, 3384, 3380, 3383}

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^4}\,dx = -\frac{1}{6}ad^3\cos(c)\,\text{CosIntegral}(dx) + \frac{1}{6}ad^3\sin(c)\text{Si}(dx) + \frac{ad^2\sin(c+dx)}{6x} - \frac{a\sin(c+dx)}{3x^3} - \frac{ad\cos(c+dx)}{6x^2} + b\sin(c)\,\text{CosIntegral}(dx) + b\cos(c)\text{Si}(dx)$$

```
[In] Int[((a + b*x^3)*Sin[c + d*x])/x^4,x]
```

```
[Out] -1/6*(a*d*Cos[c + d*x])/x^2 - (a*d^3*Cos[c]*CosIntegral[d*x])/6 + b*CosInte
gral[d*x]*Sin[c] - (a*Sin[c + d*x])/(3*x^3) + (a*d^2*Sin[c + d*x])/(6*x) +
b*Cos[c]*SinIntegral[d*x] + (a*d^3*Sin[c]*SinIntegral[d*x])/6
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a\sin(c+dx)}{x^4}+\frac{b\sin(c+dx)}{x}\right)\,dx \\
&= a\int \frac{\sin(c+dx)}{x^4}\,dx+b\int \frac{\sin(c+dx)}{x}\,dx \\
&= -\frac{a\sin(c+dx)}{3x^3}+\frac{1}{3}(ad)\int \frac{\cos(c+dx)}{x^3}\,dx+(b\cos(c))\int \frac{\sin(dx)}{x}\,dx+(b\sin(c))\int \frac{\cos(dx)}{x}\,dx \\
&= -\frac{ad\cos(c+dx)}{6x^2}+b\,\text{CosIntegral}(dx)\sin(c)-\frac{a\sin(c+dx)}{3x^3} \\
&\quad +b\cos(c)\text{Si}(dx)-\frac{1}{6}\left(ad^2\right)\int \frac{\sin(c+dx)}{x^2}\,dx
\end{aligned}$$

$$= -\frac{ad\cos(c+dx)}{6x^2} + b\,\mathrm{CosIntegral}(dx)\sin(c) - \frac{a\sin(c+dx)}{3x^3} + \frac{ad^2\sin(c+dx)}{6x} + b\cos(c)\mathrm{Si}(dx) - \frac{1}{6}(ad^3)\int\frac{\cos(c+dx)}{x}\,dx$$

$$= -\frac{ad\cos(c+dx)}{6x^2} + b\,\mathrm{CosIntegral}(dx)\sin(c) - \frac{a\sin(c+dx)}{3x^3} + \frac{ad^2\sin(c+dx)}{6x} + b\cos(c)\mathrm{Si}(dx) - \frac{1}{6}(ad^3\cos(c))\int\frac{\cos(dx)}{x}\,dx + \frac{1}{6}(ad^3\sin(c))\int\frac{\sin(dx)}{x}\,dx$$

$$= -\frac{ad\cos(c+dx)}{6x^2} - \frac{1}{6}ad^3\cos(c)\,\mathrm{CosIntegral}(dx) + b\,\mathrm{CosIntegral}(dx)\sin(c) - \frac{a\sin(c+dx)}{3x^3} + \frac{ad^2\sin(c+dx)}{6x} + b\cos(c)\mathrm{Si}(dx) + \frac{1}{6}ad^3\sin(c)\mathrm{Si}(dx)$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14

$$\int\frac{(a+bx^3)\sin(c+dx)}{x^4}\,dx = b\,\mathrm{CosIntegral}(dx)\sin(c) + \frac{a\cos(dx)\left(-dx\cos(c)-2\sin(c)+d^2x^2\sin(c)\right)}{6x^3} + \frac{a(-2\cos(c)+d^2x^2\cos(c)+dx\sin(c))\sin(dx)}{6x^3} + b\cos(c)\mathrm{Si}(dx) - \frac{1}{6}ad^3(\cos(c)\,\mathrm{CosIntegral}(dx) - \sin(c)\mathrm{Si}(dx))$$

```
[In] Integrate[((a + b*x^3)*Sin[c + d*x])/x^4,x]
```

```
[Out] b*CosIntegral[d*x]*Sin[c] + (a*Cos[d*x]*(-(d*x*Cos[c]) - 2*Sin[c] + d^2*x^2
*Sin[c]))/(6*x^3) + (a*(-2*Cos[c] + d^2*x^2*Cos[c] + d*x*Sin[c])*Sin[d*x])/
(6*x^3) + b*Cos[c]*SinIntegral[d*x] - (a*d^3*(Cos[c]*CosIntegral[d*x] - Sin
[c]*SinIntegral[d*x]))/6
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96

method	result
derivativedivides	$d^3\left(a\left(-\frac{\sin(dx+c)}{3d^3x^3}-\frac{\cos(dx+c)}{6d^2x^2}+\frac{\sin(dx+c)}{6dx}+\frac{\mathrm{Si}(dx)\sin(c)}{6}-\frac{\mathrm{Ci}(dx)\cos(c)}{6}\right)+\frac{b(\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin}{d^3}\right.$
default	$d^3\left(a\left(-\frac{\sin(dx+c)}{3d^3x^3}-\frac{\cos(dx+c)}{6d^2x^2}+\frac{\sin(dx+c)}{6dx}+\frac{\mathrm{Si}(dx)\sin(c)}{6}-\frac{\mathrm{Ci}(dx)\cos(c)}{6}\right)+\frac{b(\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin}{d^3}\right.$
risch	$\frac{\cos(c)\,\mathrm{Ei}_1(-idx)a\,d^3}{12}-\frac{i\cos(c)\,\mathrm{Ei}_1(idx)b}{2}+\frac{\cos(c)\,\mathrm{Ei}_1(idx)a\,d^3}{12}+\frac{i\cos(c)\,\mathrm{Ei}_1(-idx)b}{2}+\frac{i\sin(c)\,\mathrm{Ei}_1(-idx)a\,d^3}{12}-$
meijerg	$\frac{b\sqrt{\pi}\,\sin(c)\left(\frac{2\gamma+2\ln(x)+\ln(d^2)}{\sqrt{\pi}}-\frac{2\gamma}{\sqrt{\pi}}-\frac{2\ln(2)}{\sqrt{\pi}}-\frac{2\ln\left(\frac{dx}{2}\right)}{\sqrt{\pi}}+\frac{2\,\mathrm{Ci}(dx)}{\sqrt{\pi}}\right)}{2}+b\cos(c)\,\mathrm{Si}(dx)+\frac{a\sqrt{\pi}\,\sin(c)d^4\left(-\frac{8(-d}{}\right.}{}$

[In] int((b*x^3+a)*sin(d*x+c)/x^4,x,method=_RETURNVERBOSE)

[Out] d^3*(a*(-1/3*sin(d*x+c)/d^3/x^3-1/6*cos(d*x+c)/d^2/x^2+1/6*sin(d*x+c)/d/x+1/6*Si(d*x)*sin(c)-1/6*Ci(d*x)*cos(c))+1/d^3*b*(Si(d*x)*cos(c)+Ci(d*x)*sin(c)))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^4}\,dx = -\frac{adx\cos(dx+c)+(ad^3x^3\,\mathrm{Ci}(dx)-6\,bx^3\,\mathrm{Si}(dx))\cos(c)-(ad^2x^2-2\,a)\sin(dx+c)-(ad^3x^3\,\mathrm{Si}(dx)+}{6\,x^3}$$

[In] integrate((b*x^3+a)*sin(d*x+c)/x^4,x, algorithm="fricas")

[Out] -1/6*(a*d*x*cos(d*x + c) + (a*d^3*x^3*cos_integral(d*x) - 6*b*x^3*sin_integral(d*x))*cos(c) - (a*d^2*x^2 - 2*a)*sin(d*x + c) - (a*d^3*x^3*sin_integral(d*x) + 6*b*x^3*cos_integral(d*x))*sin(c))/x^3

Sympy [F]

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^4}\,dx = \int \frac{(a+bx^3)\sin(c+dx)}{x^4}\,dx$$

[In] integrate((b*x**3+a)*sin(d*x+c)/x**4,x)

[Out] Integral((a + b*x**3)*sin(c + d*x)/x**4, x)

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.45

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^4}\,dx = -\frac{((a(\Gamma(-3,i\,dx)+\Gamma(-3,-i\,dx))\cos{(c)}+a(-i\,\Gamma(-3,i\,dx)+i\,\Gamma(-3,-i\,dx))\sin{(c)})d^6-6\,(b(i\,\Gamma(-3,i\,d}{}$$

```
[In] integrate((b*x^3+a)*sin(d*x+c)/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*(((a*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*cos(c) + a*(-I*gamma(-3, I
*d*x) + I*gamma(-3, -I*d*x))*sin(c))*d^6 - 6*(b*(I*gamma(-3, I*d*x) - I*gam
ma(-3, -I*d*x))*cos(c) + b*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*sin(c))*d
^3)*x^3 + 2*b*d*x*sin(d*x + c) + 2*(b*d^2*x^2 - 2*b)*cos(d*x + c))/(d^3*x^3
)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.30 (sec) , antiderivative size = 796, normalized size of antiderivative = 8.75

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^4}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x^3+a)*sin(d*x+c)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*(a*d^3*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
a*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d
^3*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^3*x^3
*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a*d^3*x^3*sin_
integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) - a*d^3*x^3*real_part(cos_integral(
d*x))*tan(1/2*d*x)^2 - a*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)
^2 + a*d^3*x^3*real_part(cos_integral(d*x))*tan(1/2*c)^2 + a*d^3*x^3*real_p
art(cos_integral(-d*x))*tan(1/2*c)^2 + 2*a*d^3*x^3*imag_part(cos_integral(d
*x))*tan(1/2*c) - 2*a*d^3*x^3*imag_part(cos_integral(-d*x))*tan(1/2*c) + 4*
a*d^3*x^3*sin_integral(d*x)*tan(1/2*c) - 6*b*x^3*imag_part(cos_integral(d*x
))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 6*b*x^3*imag_part(cos_integral(-d*x))*tan(
1/2*d*x)^2*tan(1/2*c)^2 - 12*b*x^3*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2
*c)^2 - a*d^3*x^3*real_part(cos_integral(d*x)) - a*d^3*x^3*real_part(cos_in
tegral(-d*x)) - 4*a*d^2*x^2*tan(1/2*d*x)^2*tan(1/2*c) + 12*b*x^3*real_part(
cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 12*b*x^3*real_part(cos_integ
ral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 4*a*d^2*x^2*tan(1/2*d*x)*tan(1/2*c)^
```

```
2 + 6*b*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 - 6*b*x^3*imag_part
(cos_integral(-d*x))*tan(1/2*d*x)^2 + 12*b*x^3*sin_integral(d*x)*tan(1/2*d*
x)^2 - 6*b*x^3*imag_part(cos_integral(d*x))*tan(1/2*c)^2 + 6*b*x^3*imag_par
t(cos_integral(-d*x))*tan(1/2*c)^2 - 12*b*x^3*sin_integral(d*x)*tan(1/2*c)^
2 - 2*a*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a*d^2*x^2*tan(1/2*d*x) + 4*a*d^
2*x^2*tan(1/2*c) + 12*b*x^3*real_part(cos_integral(d*x))*tan(1/2*c) + 12*b*
x^3*real_part(cos_integral(-d*x))*tan(1/2*c) + 6*b*x^3*imag_part(cos_integr
al(d*x)) - 6*b*x^3*imag_part(cos_integral(-d*x)) + 12*b*x^3*sin_integral(d*
x) + 2*a*d*x*tan(1/2*d*x)^2 + 8*a*d*x*tan(1/2*d*x)*tan(1/2*c) + 2*a*d*x*tan
(1/2*c)^2 + 8*a*tan(1/2*d*x)^2*tan(1/2*c) + 8*a*tan(1/2*d*x)*tan(1/2*c)^2 -
 2*a*d*x - 8*a*tan(1/2*d*x) - 8*a*tan(1/2*c))/(x^3*tan(1/2*d*x)^2*tan(1/2*c
)^2 + x^3*tan(1/2*d*x)^2 + x^3*tan(1/2*c)^2 + x^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^4}\,dx = \int \frac{\sin\left(c+d\,x\right)\,\left(b\,x^3+a\right)}{x^4}\,dx$$

```
[In] int((sin(c + d*x)*(a + b*x^3))/x^4,x)
[Out] int((sin(c + d*x)*(a + b*x^3))/x^4, x)
```

3.87 $\int x(a+bx^3)^2 \sin(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 235

$$\begin{aligned}\int x(a+bx^3)^2 \sin(c+dx)\,dx = & -\frac{48ab\cos(c+dx)}{d^5}+\frac{5040b^2x\cos(c+dx)}{d^7}-\frac{a^2x\cos(c+dx)}{d} \\ & +\frac{24abx^2\cos(c+dx)}{d^3}-\frac{840b^2x^3\cos(c+dx)}{d^5} \\ & -\frac{2abx^4\cos(c+dx)}{d}+\frac{42b^2x^5\cos(c+dx)}{d^3}-\frac{b^2x^7\cos(c+dx)}{d} \\ & -\frac{5040b^2\sin(c+dx)}{d^8}+\frac{a^2\sin(c+dx)}{d^2}-\frac{48abx\sin(c+dx)}{d^4} \\ & +\frac{2520b^2x^2\sin(c+dx)}{d^6}+\frac{8abx^3\sin(c+dx)}{d^2} \\ & -\frac{210b^2x^4\sin(c+dx)}{d^4}+\frac{7b^2x^6\sin(c+dx)}{d^2}\end{aligned}$$

```
[Out] -48*a*b*cos(d*x+c)/d^5+5040*b^2*x*cos(d*x+c)/d^7-a^2*x*cos(d*x+c)/d+24*a*b*
x^2*cos(d*x+c)/d^3-840*b^2*x^3*cos(d*x+c)/d^5-2*a*b*x^4*cos(d*x+c)/d+42*b^2
*x^5*cos(d*x+c)/d^3-b^2*x^7*cos(d*x+c)/d-5040*b^2*sin(d*x+c)/d^8+a^2*sin(d*
x+c)/d^2-48*a*b*x*sin(d*x+c)/d^4+2520*b^2*x^2*sin(d*x+c)/d^6+8*a*b*x^3*sin(
d*x+c)/d^2-210*b^2*x^4*sin(d*x+c)/d^4+7*b^2*x^6*sin(d*x+c)/d^2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3420, 3377, 2717, 2718}

$$\begin{aligned}\int x(a+bx^3)^2\sin(c+dx)\,dx = &\frac{a^2\sin(c+dx)}{d^2}-\frac{a^2x\cos(c+dx)}{d}-\frac{48ab\cos(c+dx)}{d^5}\\&-\frac{48abx\sin(c+dx)}{d^4}+\frac{24abx^2\cos(c+dx)}{d^3}\\&+\frac{8abx^3\sin(c+dx)}{d^2}-\frac{2abx^4\cos(c+dx)}{d}\\&-\frac{5040b^2\sin(c+dx)}{d^8}+\frac{5040b^2x\cos(c+dx)}{d^7}\\&+\frac{2520b^2x^2\sin(c+dx)}{d^6}-\frac{840b^2x^3\cos(c+dx)}{d^5}\\&-\frac{210b^2x^4\sin(c+dx)}{d^4}+\frac{42b^2x^5\cos(c+dx)}{d^3}\\&+\frac{7b^2x^6\sin(c+dx)}{d^2}-\frac{b^2x^7\cos(c+dx)}{d}\end{aligned}$$

```
[In] Int[x*(a + b*x^3)^2*Sin[c + d*x],x]
```

```
[Out] (-48*a*b*Cos[c + d*x])/d^5 + (5040*b^2*x*Cos[c + d*x])/d^7 - (a^2*x*Cos[c +
 d*x])/d + (24*a*b*x^2*Cos[c + d*x])/d^3 - (840*b^2*x^3*Cos[c + d*x])/d^5 -
 (2*a*b*x^4*Cos[c + d*x])/d + (42*b^2*x^5*Cos[c + d*x])/d^3 - (b^2*x^7*Cos[
c + d*x])/d - (5040*b^2*Sin[c + d*x])/d^8 + (a^2*Sin[c + d*x])/d^2 - (48*a*
b*x*Sin[c + d*x])/d^4 + (2520*b^2*x^2*Sin[c + d*x])/d^6 + (8*a*b*x^3*Sin[c
+ d*x])/d^2 - (210*b^2*x^4*Sin[c + d*x])/d^4 + (7*b^2*x^6*Sin[c + d*x])/d^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a^2 x \sin(c+dx) + 2abx^4 \sin(c+dx) + b^2 x^7 \sin(c+dx)\right) dx \\
&= a^2 \int x \sin(c+dx)\,dx + (2ab) \int x^4 \sin(c+dx)\,dx + b^2 \int x^7 \sin(c+dx)\,dx \\
&= -\frac{a^2 x \cos(c+dx)}{d} - \frac{2abx^4 \cos(c+dx)}{d} - \frac{b^2 x^7 \cos(c+dx)}{d} \\
&\quad + \frac{a^2 \int \cos(c+dx)\,dx}{d} + \frac{(8ab) \int x^3 \cos(c+dx)\,dx}{d} + \frac{(7b^2) \int x^6 \cos(c+dx)\,dx}{d} \\
&= -\frac{a^2 x \cos(c+dx)}{d} - \frac{2abx^4 \cos(c+dx)}{d} - \frac{b^2 x^7 \cos(c+dx)}{d} \\
&\quad + \frac{a^2 \sin(c+dx)}{d^2} + \frac{8abx^3 \sin(c+dx)}{d^2} + \frac{7b^2 x^6 \sin(c+dx)}{d^2} \\
&\quad - \frac{(24ab) \int x^2 \sin(c+dx)\,dx}{d^2} - \frac{(42b^2) \int x^5 \sin(c+dx)\,dx}{d^2} \\
&= -\frac{a^2 x \cos(c+dx)}{d} + \frac{24abx^2 \cos(c+dx)}{d^3} - \frac{2abx^4 \cos(c+dx)}{d} \\
&\quad + \frac{42b^2 x^5 \cos(c+dx)}{d^3} - \frac{b^2 x^7 \cos(c+dx)}{d} + \frac{a^2 \sin(c+dx)}{d^2} + \frac{8abx^3 \sin(c+dx)}{d^2} \\
&\quad + \frac{7b^2 x^6 \sin(c+dx)}{d^2} - \frac{(48ab) \int x \cos(c+dx)\,dx}{d^3} - \frac{(210b^2) \int x^4 \cos(c+dx)\,dx}{d^3} \\
&= -\frac{a^2 x \cos(c+dx)}{d} + \frac{24abx^2 \cos(c+dx)}{d^3} - \frac{2abx^4 \cos(c+dx)}{d} \\
&\quad + \frac{42b^2 x^5 \cos(c+dx)}{d^3} - \frac{b^2 x^7 \cos(c+dx)}{d} + \frac{a^2 \sin(c+dx)}{d^2} \\
&\quad - \frac{48abx \sin(c+dx)}{d^4} + \frac{8abx^3 \sin(c+dx)}{d^2} - \frac{210b^2 x^4 \sin(c+dx)}{d^4} \\
&\quad + \frac{7b^2 x^6 \sin(c+dx)}{d^2} + \frac{(48ab) \int \sin(c+dx)\,dx}{d^4} + \frac{(840b^2) \int x^3 \sin(c+dx)\,dx}{d^4} \\
&= -\frac{48ab \cos(c+dx)}{d^5} - \frac{a^2 x \cos(c+dx)}{d} + \frac{24abx^2 \cos(c+dx)}{d^3} \\
&\quad - \frac{840b^2 x^3 \cos(c+dx)}{d^5} - \frac{2abx^4 \cos(c+dx)}{d} + \frac{42b^2 x^5 \cos(c+dx)}{d^3} \\
&\quad - \frac{b^2 x^7 \cos(c+dx)}{d} + \frac{a^2 \sin(c+dx)}{d^2} - \frac{48abx \sin(c+dx)}{d^4} + \frac{8abx^3 \sin(c+dx)}{d^2} \\
&\quad - \frac{210b^2 x^4 \sin(c+dx)}{d^4} + \frac{7b^2 x^6 \sin(c+dx)}{d^2} + \frac{(2520b^2) \int x^2 \cos(c+dx)\,dx}{d^5}
\end{aligned}
$$

$$= -\frac{48ab\cos(c+dx)}{d^5} - \frac{a^2x\cos(c+dx)}{d} + \frac{24abx^2\cos(c+dx)}{d^3} - \frac{840b^2x^3\cos(c+dx)}{d^5} - \frac{2abx^4\cos(c+dx)}{d} + \frac{42b^2x^5\cos(c+dx)}{d^3} - \frac{b^2x^7\cos(c+dx)}{d} + \frac{a^2\sin(c+dx)}{d^2} - \frac{48abx\sin(c+dx)}{d^4} + \frac{2520b^2x^2\sin(c+dx)}{d^6} + \frac{8abx^3\sin(c+dx)}{d^2} - \frac{210b^2x^4\sin(c+dx)}{d^4} + \frac{7b^2x^6\sin(c+dx)}{d^2} - \frac{(5040b^2)\int x\sin(c+dx)\,dx}{d^6}$$

$$= -\frac{48ab\cos(c+dx)}{d^5} + \frac{5040b^2x\cos(c+dx)}{d^7} - \frac{a^2x\cos(c+dx)}{d} + \frac{24abx^2\cos(c+dx)}{d^3} - \frac{840b^2x^3\cos(c+dx)}{d^5} - \frac{2abx^4\cos(c+dx)}{d} + \frac{42b^2x^5\cos(c+dx)}{d^3} - \frac{b^2x^7\cos(c+dx)}{d} + \frac{a^2\sin(c+dx)}{d^2} - \frac{48abx\sin(c+dx)}{d^4} + \frac{2520b^2x^2\sin(c+dx)}{d^6} + \frac{8abx^3\sin(c+dx)}{d^2} - \frac{210b^2x^4\sin(c+dx)}{d^4} + \frac{7b^2x^6\sin(c+dx)}{d^2} - \frac{(5040b^2)\int\cos(c+dx)\,dx}{d^7}$$

$$= -\frac{48ab\cos(c+dx)}{d^5} + \frac{5040b^2x\cos(c+dx)}{d^7} - \frac{a^2x\cos(c+dx)}{d} + \frac{24abx^2\cos(c+dx)}{d^3} - \frac{840b^2x^3\cos(c+dx)}{d^5} - \frac{2abx^4\cos(c+dx)}{d} + \frac{42b^2x^5\cos(c+dx)}{d^3} - \frac{b^2x^7\cos(c+dx)}{d} - \frac{5040b^2\sin(c+dx)}{d^8} + \frac{a^2\sin(c+dx)}{d^2} - \frac{48abx\sin(c+dx)}{d^4} + \frac{2520b^2x^2\sin(c+dx)}{d^6} + \frac{8abx^3\sin(c+dx)}{d^2} - \frac{210b^2x^4\sin(c+dx)}{d^4} + \frac{7b^2x^6\sin(c+dx)}{d^2}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.59

$$\int x(a+bx^3)^2\sin(c+dx)\,dx = \frac{-d(a^2d^6x+2abd^2(24-12d^2x^2+d^4x^4)+b^2x(-5040+840d^2x^2-42d^4x^4+d^6x^6))\cos(c+dx)+(a^2d^6+}{d^8}$$

```
[In] Integrate[x*(a + b*x^3)^2*Sin[c + d*x],x]
```

```
[Out] (-(d*(a^2*d^6*x + 2*a*b*d^2*(24 - 12*d^2*x^2 + d^4*x^4) + b^2*x*(-5040 + 84
0*d^2*x^2 - 42*d^4*x^4 + d^6*x^6))*Cos[c + d*x]) + (a^2*d^6 + 8*a*b*d^4*x*(
-6 + d^2*x^2) + 7*b^2*(-720 + 360*d^2*x^2 - 30*d^4*x^4 + d^6*x^6))*Sin[c +
d*x])/d^8
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{(b^2d^6x^7+2ab\,d^6x^4-42b^2d^4x^5+a^2d^6x-24ab\,d^4x^2+840b^2d^2x^3+48ab\,d^2-5040b^2x)\cos(dx+c)}{d^7}+\frac{(7b^2x^6d^6+8ab\,d^6x^3-\cdots}{\cdots}$
parallelrisch	$\frac{\left(x(b\,x^3+a)^2d^6+(-42b^2x^5-24ab\,x^2)d^4+(840b^2x^3+96ab)d^2-5040b^2x\right)d\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+((14b^2x^6+16ab\,x^3+2a^2)d^6+\cdots}{d^8\left(1+\tan\cdots\right.}$
norman	$\frac{\frac{96ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d^5}+\frac{b^2x^7\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{(a^2d^6-5040b^2)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d^7}-\frac{840b^2x^3}{d^5}+\frac{42b^2x^5}{d^3}-\frac{b^2x^7}{d}-\frac{(a^2d^6-5040b^2)x}{d^7}+\cdots}{\cdots}$
meijerg	$\frac{128b^2\sqrt{\pi}\,\sin(c)\left(\frac{315}{8\sqrt{\pi}}-\frac{\left(-\frac{7}{16}d^6x^6+\frac{105}{8}d^4x^4-\frac{315}{2}d^2x^2+315\right)\cos(dx)}{8\sqrt{\pi}}-\frac{xd\left(-\frac{1}{16}d^6x^6+\frac{21}{8}d^4x^4-\frac{105}{2}d^2x^2+315\right)\sin(dx)}{8\sqrt{\pi}}\right)}{d^8}+\cdots$
parts	$-\frac{b^2x^7\cos(dx+c)}{d}-\frac{2ab\,x^4\cos(dx+c)}{d}-\frac{a^2x\cos(dx+c)}{d}+\frac{a^2\sin(dx+c)-\frac{8ab\,c^3\sin(dx+c)}{d^3}+\frac{24ab\,c^2(\cos(dx+c)+(dx+c)\sin\cdots}{d^3}}{\cdots}$
derivativedivides	$\frac{a^2c\cos(dx+c)+a^2(\sin(dx+c)-\cos(dx+c)(dx+c))-\frac{2ab\,c^4\cos(dx+c)}{d^3}-\frac{8ab\,c^3(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^3}+\frac{12ab\,c^2\left(-(dx+c)^2\cdots\right.}{\cdots}}{\cdots}$
default	$\frac{a^2c\cos(dx+c)+a^2(\sin(dx+c)-\cos(dx+c)(dx+c))-\frac{2ab\,c^4\cos(dx+c)}{d^3}-\frac{8ab\,c^3(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^3}+\frac{12ab\,c^2\left(-(dx+c)^2\cdots\right.}{\cdots}}{\cdots}$

```
[In] int(x*(b*x^3+a)^2*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^7*(b^2*d^6*x^7+2*a*b*d^6*x^4-42*b^2*d^4*x^5+a^2*d^6*x-24*a*b*d^4*x^2+8
40*b^2*d^2*x^3+48*a*b*d^2-5040*b^2*x)*cos(d*x+c)+(7*b^2*d^6*x^6+8*a*b*d^6*x
^3-210*b^2*d^4*x^4+a^2*d^6-48*a*b*d^4*x+2520*b^2*d^2*x^2-5040*b^2)/d^8*sin(
d*x+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.69

$$\int x\left(a+bx^3\right)^2\sin(c+dx)\,dx = -\frac{(b^2d^7x^7+2\,abd^7x^4-42\,b^2d^5x^5-24\,abd^5x^2+840\,b^2d^3x^3+48\,abd^3+(a^2d^7-5040\,b^2d)x)\cos(dx+c)-\cdots}{d^8}$$

```
[In] integrate(x*(b*x^3+a)^2*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b^2*d^7*x^7 + 2*a*b*d^7*x^4 - 42*b^2*d^5*x^5 - 24*a*b*d^5*x^2 + 840*b^2*
d^3*x^3 + 48*a*b*d^3 + (a^2*d^7 - 5040*b^2*d)*x)*cos(d*x + c) - (7*b^2*d^6*
x^6 + 8*a*b*d^6*x^3 - 210*b^2*d^4*x^4 + a^2*d^6 - 48*a*b*d^4*x + 2520*b^2*d
^2*x^2 - 5040*b^2)*sin(d*x + c))/d^8
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.21

$$\int x(a+bx^3)^2\sin(c+dx)\,dx$$
$$=\begin{cases}-\frac{a^2x\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)}{d^2}-\frac{2abx^4\cos(c+dx)}{d}+\frac{8abx^3\sin(c+dx)}{d^2}+\frac{24abx^2\cos(c+dx)}{d^3}-\frac{48abx\sin(c+dx)}{d^4}-\frac{48ab\cos(c+dx)}{d^5} \\ \left(\frac{a^2x^2}{2}+\frac{2abx^5}{5}+\frac{b^2x^8}{8}\right)\sin(c)\end{cases}$$

```
[In] integrate(x*(b*x**3+a)**2*sin(d*x+c),x)
```

```
[Out] Piecewise((-a**2*x*cos(c + d*x)/d + a**2*sin(c + d*x)/d**2 - 2*a*b*x**4*cos
(c + d*x)/d + 8*a*b*x**3*sin(c + d*x)/d**2 + 24*a*b*x**2*cos(c + d*x)/d**3
- 48*a*b*x*sin(c + d*x)/d**4 - 48*a*b*cos(c + d*x)/d**5 - b**2*x**7*cos(c +
 d*x)/d + 7*b**2*x**6*sin(c + d*x)/d**2 + 42*b**2*x**5*cos(c + d*x)/d**3 -
210*b**2*x**4*sin(c + d*x)/d**4 - 840*b**2*x**3*cos(c + d*x)/d**5 + 2520*b*
*2*x**2*sin(c + d*x)/d**6 + 5040*b**2*x*cos(c + d*x)/d**7 - 5040*b**2*sin(c
 + d*x)/d**8, Ne(d, 0)), ((a**2*x**2/2 + 2*a*b*x**5/5 + b**2*x**8/8)*sin(c)
, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 662 vs. 2(235) = 470.

Time = 0.24 (sec) , antiderivative size = 662, normalized size of antiderivative = 2.82

$$\int x(a+bx^3)^2\sin(c+dx)\,dx$$
$$=\frac{a^2c\cos(dx+c)+\frac{b^2c^7\cos(dx+c)}{d^6}-\frac{2abc^4\cos(dx+c)}{d^3}-((dx+c)\cos(dx+c)-\sin(dx+c))a^2-\frac{7((dx+c)\cos(dx+c}{d^6}}{}$$

```
[In] integrate(x*(b*x^3+a)^2*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] (a^2*c*cos(d*x + c) + b^2*c^7*cos(d*x + c)/d^6 - 2*a*b*c^4*cos(d*x + c)/d^3
 - ((d*x + c)*cos(d*x + c) - sin(d*x + c))*a^2 - 7*((d*x + c)*cos(d*x + c)
- sin(d*x + c))*b^2*c^6/d^6 + 8*((d*x + c)*cos(d*x + c) - sin(d*x + c))*a*b
*c^3/d^3 + 21*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b
^2*c^5/d^6 - 12*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))
*a*b*c^2/d^3 - 35*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^
2 - 2)*sin(d*x + c))*b^2*c^4/d^6 + 8*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x +
 c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*a*b*c/d^3 + 35*(((d*x + c)^4 - 12*(
d*x + c)^2 + 24)*cos(d*x + c) - 4*((d*x + c)^3 - 6*d*x - 6*c)*sin(d*x + c))
*b^2*c^3/d^6 - 2*(((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d*x + c) - 4*((d*
x + c)^3 - 6*d*x - 6*c)*sin(d*x + c))*a*b/d^3 - 21*(((d*x + c)^5 - 20*(d*x
```

```
+ c)^3 + 120*d*x + 120*c)*cos(d*x + c) - 5*((d*x + c)^4 - 12*(d*x + c)^2 +
24)*sin(d*x + c))*b^2*c^2/d^6 + 7*(((d*x + c)^6 - 30*(d*x + c)^4 + 360*(d*x
 + c)^2 - 720)*cos(d*x + c) - 6*((d*x + c)^5 - 20*(d*x + c)^3 + 120*d*x + 1
20*c)*sin(d*x + c))*b^2*c/d^6 - (((d*x + c)^7 - 42*(d*x + c)^5 + 840*(d*x +
 c)^3 - 5040*d*x - 5040*c)*cos(d*x + c) - 7*((d*x + c)^6 - 30*(d*x + c)^4 +
 360*(d*x + c)^2 - 720)*sin(d*x + c))*b^2/d^6)/d^2
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.69

$$\int x(a+bx^3)^2\sin(c+dx)\,dx = -\frac{(b^2d^7x^7+2\,abd^7x^4-42\,b^2d^5x^5+a^2d^7x-24\,abd^5x^2+840\,b^2d^3x^3+48\,abd^3-5040\,b^2dx)\cos{(dx+c)}}{d^8} + \frac{(7\,b^2d^6x^6+8\,abd^6x^3-210\,b^2d^4x^4+a^2d^6-48\,abd^4x+2520\,b^2d^2x^2-5040\,b^2)\sin{(dx+c)}}{d^8}$$

```
[In] integrate(x*(b*x^3+a)^2*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b^2*d^7*x^7 + 2*a*b*d^7*x^4 - 42*b^2*d^5*x^5 + a^2*d^7*x - 24*a*b*d^5*x^2
 + 840*b^2*d^3*x^3 + 48*a*b*d^3 - 5040*b^2*d*x)*cos(d*x + c)/d^8 + (7*b^2*d
^6*x^6 + 8*a*b*d^6*x^3 - 210*b^2*d^4*x^4 + a^2*d^6 - 48*a*b*d^4*x + 2520*b^
2*d^2*x^2 - 5040*b^2)*sin(d*x + c)/d^8
```

Mupad [B] (verification not implemented)

Time = 6.45 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.96

$$\begin{aligned}\int x(a+bx^3)^2\sin(c+dx)\,dx &= \frac{42\,b^2\,x^5\,\cos(c+d\,x)+24\,a\,b\,x^2\,\cos(c+d\,x)}{d^3} \\ &\quad - \frac{b^2\,x^7\,\cos(c+d\,x)+a^2\,x\,\cos(c+d\,x)+2\,a\,b\,x^4\,\cos(c+d\,x)}{d} \\ &\quad - \frac{840\,b^2\,x^3\,\cos(c+d\,x)+48\,a\,b\,\cos(c+d\,x)}{d^5} \\ &\quad + \frac{a^2\,\sin(c+d\,x)+7\,b^2\,x^6\,\sin(c+d\,x)+8\,a\,b\,x^3\,\sin(c+d\,x)}{d^2} \\ &\quad - \frac{210\,b^2\,x^4\,\sin(c+d\,x)+48\,a\,b\,x\,\sin(c+d\,x)}{d^4} - \frac{5040\,b^2\,\sin(c+d\,x)}{d^8} \\ &\quad + \frac{2520\,b^2\,x^2\,\sin(c+d\,x)}{d^6} + \frac{5040\,b^2\,x\,\cos(c+d\,x)}{d^7}\end{aligned}$$

```
[In] int(x*sin(c + d*x)*(a + b*x^3)^2,x)
```

```
[Out] (42*b^2*x^5*cos(c + d*x) + 24*a*b*x^2*cos(c + d*x))/d^3 - (b^2*x^7*cos(c +
d*x) + a^2*x*cos(c + d*x) + 2*a*b*x^4*cos(c + d*x))/d - (840*b^2*x^3*cos(c
+ d*x) + 48*a*b*cos(c + d*x))/d^5 + (a^2*sin(c + d*x) + 7*b^2*x^6*sin(c + d
*x) + 8*a*b*x^3*sin(c + d*x))/d^2 - (210*b^2*x^4*sin(c + d*x) + 48*a*b*x*si
n(c + d*x))/d^4 - (5040*b^2*sin(c + d*x))/d^8 + (2520*b^2*x^2*sin(c + d*x))
/d^6 + (5040*b^2*x*cos(c + d*x))/d^7
```

3.88 $\int (a+bx^3)^2 \sin(c+dx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 188

$$\int (a+bx^3)^2 \sin(c+dx)\,dx = \frac{720b^2\cos(c+dx)}{d^7} - \frac{a^2\cos(c+dx)}{d} + \frac{12abx\cos(c+dx)}{d^3} - \frac{360b^2x^2\cos(c+dx)}{d^5} - \frac{2abx^3\cos(c+dx)}{d} + \frac{30b^2x^4\cos(c+dx)}{d^3} - \frac{b^2x^6\cos(c+dx)}{d} - \frac{12ab\sin(c+dx)}{d^4} + \frac{720b^2x\sin(c+dx)}{d^6} + \frac{6abx^2\sin(c+dx)}{d^2} - \frac{120b^2x^3\sin(c+dx)}{d^4} + \frac{6b^2x^5\sin(c+dx)}{d^2}$$

```
[Out] 720*b^2*cos(d*x+c)/d^7-a^2*cos(d*x+c)/d+12*a*b*x*cos(d*x+c)/d^3-360*b^2*x^2
*cos(d*x+c)/d^5-2*a*b*x^3*cos(d*x+c)/d+30*b^2*x^4*cos(d*x+c)/d^3-b^2*x^6*co
s(d*x+c)/d-12*a*b*sin(d*x+c)/d^4+720*b^2*x*sin(d*x+c)/d^6+6*a*b*x^2*sin(d*x
+c)/d^2-120*b^2*x^3*sin(d*x+c)/d^4+6*b^2*x^5*sin(d*x+c)/d^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used

$= \{3410, 2718, 3377, 2717\}$

$$\int \left(a+bx^3\right)^2 \sin(c+dx)\,dx = -\frac{a^2\cos(c+dx)}{d} - \frac{12ab\sin(c+dx)}{d^4} + \frac{12abx\cos(c+dx)}{d^3} + \frac{6abx^2\sin(c+dx)}{d^2} - \frac{2abx^3\cos(c+dx)}{d} + \frac{720b^2\cos(c+dx)}{d^7} + \frac{720b^2x\sin(c+dx)}{d^6} - \frac{360b^2x^2\cos(c+dx)}{d^5} - \frac{120b^2x^3\sin(c+dx)}{d^4} + \frac{30b^2x^4\cos(c+dx)}{d^3} + \frac{6b^2x^5\sin(c+dx)}{d^2} - \frac{b^2x^6\cos(c+dx)}{d}$$

```
[In] Int[(a + b*x^3)^2*Sin[c + d*x],x]
```

```
[Out] (720*b^2*Cos[c + d*x])/d^7 - (a^2*Cos[c + d*x])/d + (12*a*b*x*Cos[c + d*x])
/d^3 - (360*b^2*x^2*Cos[c + d*x])/d^5 - (2*a*b*x^3*Cos[c + d*x])/d + (30*b^
2*x^4*Cos[c + d*x])/d^3 - (b^2*x^6*Cos[c + d*x])/d - (12*a*b*Sin[c + d*x])/
d^4 + (720*b^2*x*Sin[c + d*x])/d^6 + (6*a*b*x^2*Sin[c + d*x])/d^2 - (120*b^
2*x^3*Sin[c + d*x])/d^4 + (6*b^2*x^5*Sin[c + d*x])/d^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3410

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d,
 n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^2 \sin(c+dx) + 2abx^3 \sin(c+dx) + b^2x^6 \sin(c+dx)\right) dx \\
&= a^2 \int \sin(c+dx)\,dx + (2ab) \int x^3 \sin(c+dx)\,dx + b^2 \int x^6 \sin(c+dx)\,dx \\
&= -\frac{a^2\cos(c+dx)}{d} - \frac{2abx^3\cos(c+dx)}{d} - \frac{b^2x^6\cos(c+dx)}{d} \\
&\quad + \frac{(6ab)\int x^2\cos(c+dx)\,dx}{d} + \frac{(6b^2)\int x^5\cos(c+dx)\,dx}{d} \\
&= -\frac{a^2\cos(c+dx)}{d} - \frac{2abx^3\cos(c+dx)}{d} - \frac{b^2x^6\cos(c+dx)}{d} + \frac{6abx^2\sin(c+dx)}{d^2} \\
&\quad + \frac{6b^2x^5\sin(c+dx)}{d^2} - \frac{(12ab)\int x\sin(c+dx)\,dx}{d^2} - \frac{(30b^2)\int x^4\sin(c+dx)\,dx}{d^2} \\
&= -\frac{a^2\cos(c+dx)}{d} + \frac{12abx\cos(c+dx)}{d^3} - \frac{2abx^3\cos(c+dx)}{d} \\
&\quad + \frac{30b^2x^4\cos(c+dx)}{d^3} - \frac{b^2x^6\cos(c+dx)}{d} + \frac{6abx^2\sin(c+dx)}{d^2} \\
&\quad + \frac{6b^2x^5\sin(c+dx)}{d^2} - \frac{(12ab)\int\cos(c+dx)\,dx}{d^3} - \frac{(120b^2)\int x^3\cos(c+dx)\,dx}{d^3} \\
&= -\frac{a^2\cos(c+dx)}{d} + \frac{12abx\cos(c+dx)}{d^3} - \frac{2abx^3\cos(c+dx)}{d} + \frac{30b^2x^4\cos(c+dx)}{d^3} \\
&\quad - \frac{b^2x^6\cos(c+dx)}{d} - \frac{12ab\sin(c+dx)}{d^4} + \frac{6abx^2\sin(c+dx)}{d^2} \\
&\quad - \frac{120b^2x^3\sin(c+dx)}{d^4} + \frac{6b^2x^5\sin(c+dx)}{d^2} + \frac{(360b^2)\int x^2\sin(c+dx)\,dx}{d^4} \\
&= -\frac{a^2\cos(c+dx)}{d} + \frac{12abx\cos(c+dx)}{d^3} - \frac{360b^2x^2\cos(c+dx)}{d^5} - \frac{2abx^3\cos(c+dx)}{d} \\
&\quad + \frac{30b^2x^4\cos(c+dx)}{d^3} - \frac{b^2x^6\cos(c+dx)}{d} - \frac{12ab\sin(c+dx)}{d^4} + \frac{6abx^2\sin(c+dx)}{d^2} \\
&\quad - \frac{120b^2x^3\sin(c+dx)}{d^4} + \frac{6b^2x^5\sin(c+dx)}{d^2} + \frac{(720b^2)\int x\cos(c+dx)\,dx}{d^5} \\
&= -\frac{a^2\cos(c+dx)}{d} + \frac{12abx\cos(c+dx)}{d^3} - \frac{360b^2x^2\cos(c+dx)}{d^5} - \frac{2abx^3\cos(c+dx)}{d} \\
&\quad + \frac{30b^2x^4\cos(c+dx)}{d^3} - \frac{b^2x^6\cos(c+dx)}{d} - \frac{12ab\sin(c+dx)}{d^4} + \frac{720b^2x\sin(c+dx)}{d^6} \\
&\quad + \frac{6abx^2\sin(c+dx)}{d^2} - \frac{120b^2x^3\sin(c+dx)}{d^4} + \frac{6b^2x^5\sin(c+dx)}{d^2} - \frac{(720b^2)\int\sin(c+dx)\,dx}{d^6}
\end{aligned}$$

$$=\frac{720b^2\cos(c+dx)}{d^7}-\frac{a^2\cos(c+dx)}{d}+\frac{12abx\cos(c+dx)}{d^3}-\frac{360b^2x^2\cos(c+dx)}{d^5}-\frac{2abx^3\cos(c+dx)}{d}+\frac{30b^2x^4\cos(c+dx)}{d^3}-\frac{b^2x^6\cos(c+dx)}{d}-\frac{12ab\sin(c+dx)}{d^4}+\frac{720b^2x\sin(c+dx)}{d^6}+\frac{6abx^2\sin(c+dx)}{d^2}-\frac{120b^2x^3\sin(c+dx)}{d^4}+\frac{6b^2x^5\sin(c+dx)}{d^2}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.60

$$\int (a+bx^3)^2\sin(c+dx)\,dx$$
$$=\frac{-((a^2d^6+2abd^4x(-6+d^2x^2)+b^2(-720+360d^2x^2-30d^4x^4+d^6x^6))\cos(c+dx))+6bd(ad^2(-2+d^2x}{d^7}$$

```
[In] Integrate[(a + b*x^3)^2*Sin[c + d*x],x]
```

```
[Out] (-((a^2*d^6 + 2*a*b*d^4*x*(-6 + d^2*x^2) + b^2*(-720 + 360*d^2*x^2 - 30*d^4
*x^4 + d^6*x^6))*Cos[c + d*x]) + 6*b*d*(a*d^2*(-2 + d^2*x^2) + b*x*(120 - 2
0*d^2*x^2 + d^4*x^4))*Sin[c + d*x])/d^7
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{(b^2x^6d^6+2ab\,d^6x^3-30b^2x^4d^4+a^2d^6-12ab\,d^4x+360d^2x^2b^2-720b^2)\cos(dx+c)}{d^7}+\frac{6b(b\,d^4x^5+a\,d^4x^2-20b\,d^2x^3-2a}{d^6}$
parallelrisch	$\frac{2\left(x^2\left(\frac{b\,x^3}{2}+a\right)d^4+(-15b\,x^3-6a)d^2+180bx\right)x\,d^2b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+12db(x^2(b\,x^3+a)d^4+(-20b\,x^3-2a)d^2+120bx)\tan}{d^7\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$
norman	$\frac{\frac{b^2x^6\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2a^2d^6-1440b^2}{d^7}-\frac{360b^2x^2}{d^5}+\frac{30b^2x^4}{d^3}-\frac{b^2x^6}{d}-\frac{24ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^4}+\frac{1440b^2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^6}+\frac{360b^2x^2\left(\tan^2\left(\frac{d}{2}\right.\right.}{d^5}}{}$
meijerg	$\frac{64b^2\sqrt{\pi}\sin(c)\left(\frac{x(d^2)^{\frac{7}{2}}\left(\frac{21}{8}d^4x^4-\frac{105}{2}d^2x^2+315\right)\cos(dx)}{28\sqrt{\pi}\,d^6}-\frac{(d^2)^{\frac{7}{2}}\left(-\frac{7}{16}d^6x^6+\frac{105}{8}d^4x^4-\frac{315}{2}d^2x^2+315\right)\sin(dx)}{28\sqrt{\pi}\,d^7}\right)}{d^6\sqrt{d^2}}+\frac{64b^2\sqrt{}}{}$
parts	$-\frac{b^2x^6\cos(dx+c)}{d}-\frac{2ab\,x^3\cos(dx+c)}{d}-\frac{a^2\cos(dx+c)}{d}+\frac{6b\left(a\,c^2\sin(dx+c)-2ac(\cos(dx+c)+(dx+c)\sin(dx+c))+\right.}{}$
derivativedivides	$\frac{-a^2\cos(dx+c)+\frac{2ab\,c^3\cos(dx+c)}{d^3}+\frac{6ab\,c^2(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^3}-\frac{6abc\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(d\right.}{d^3}}{}$
default	$\frac{-a^2\cos(dx+c)+\frac{2ab\,c^3\cos(dx+c)}{d^3}+\frac{6ab\,c^2(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^3}-\frac{6abc\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(d\right.}{d^3}}{}$

```
[In] int((b*x^3+a)^2*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -(b^2*d^6*x^6+2*a*b*d^6*x^3-30*b^2*d^4*x^4+a^2*d^6-12*a*b*d^4*x+360*b^2*d^2
*x^2-720*b^2)/d^7*cos(d*x+c)+6*b/d^6*(b*d^4*x^5+a*d^4*x^2-20*b*d^2*x^3-2*a*
d^2+120*b*x)*sin(d*x+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.69

$$\int \left(a+bx^3\right)^2 \sin(c+dx)\,dx = -\frac{\left(b^2d^6x^6+2\,abd^6x^3-30\,b^2d^4x^4+a^2d^6-12\,abd^4x+360\,b^2d^2x^2-720\,b^2\right)\cos\left(dx+c\right)-6\left(b^2d^5x^5+abd^5\right.}{d^7}$$

```
[In] integrate((b*x^3+a)^2*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b^2*d^6*x^6 + 2*a*b*d^6*x^3 - 30*b^2*d^4*x^4 + a^2*d^6 - 12*a*b*d^4*x +
360*b^2*d^2*x^2 - 720*b^2)*cos(d*x + c) - 6*(b^2*d^5*x^5 + a*b*d^5*x^2 - 20
*b^2*d^3*x^3 - 2*a*b*d^3 + 120*b^2*d*x)*sin(d*x + c))/d^7
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.20

$$\int \left(a+bx^3\right)^2 \sin(c+dx)\,dx = \begin{cases} -\frac{a^2\cos\left(c+dx\right)}{d}-\frac{2abx^3\cos\left(c+dx\right)}{d}+\frac{6abx^2\sin\left(c+dx\right)}{d^2}+\frac{12abx\cos\left(c+dx\right)}{d^3}-\frac{12ab\sin\left(c+dx\right)}{d^4}-\frac{b^2x^6\cos\left(c+dx\right)}{d}+\frac{6b^2x^5\sin\left(c+dx\right)}{d^2} \\ \left(a^2x+\frac{abx^4}{2}+\frac{b^2x^7}{7}\right)\sin\left(c\right) \end{cases}$$

```
[In] integrate((b*x**3+a)**2*sin(d*x+c),x)
```

```
[Out] Piecewise((-a**2*cos(c + d*x)/d - 2*a*b*x**3*cos(c + d*x)/d + 6*a*b*x**2*si
n(c + d*x)/d**2 + 12*a*b*x*cos(c + d*x)/d**3 - 12*a*b*sin(c + d*x)/d**4 - b
**2*x**6*cos(c + d*x)/d + 6*b**2*x**5*sin(c + d*x)/d**2 + 30*b**2*x**4*cos(
c + d*x)/d**3 - 120*b**2*x**3*sin(c + d*x)/d**4 - 360*b**2*x**2*cos(c + d*x
)/d**5 + 720*b**2*x*sin(c + d*x)/d**6 + 720*b**2*cos(c + d*x)/d**7, Ne(d, 0
)), ((a**2*x + a*b*x**4/2 + b**2*x**7/7)*sin(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(188) = 376$.

Time = 0.23 (sec) , antiderivative size = 489, normalized size of antiderivative = 2.60

$$\int \left(a+bx^3\right)^2 \sin(c+dx)\,dx = -\frac{a^2\cos\left(dx+c\right)+\frac{b^2c^6\cos(dx+c)}{d^6}-\frac{2\,abc^3\cos(dx+c)}{d^3}-\frac{6\,((dx+c)\cos(dx+c)-\sin(dx+c))b^2c^5}{d^6}+\frac{6\,((dx+c)\cos(dx+c)-\sin(dx+c))}{d^3}}{}$$

```
[In] integrate((b*x^3+a)^2*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a^2*cos(d*x + c) + b^2*c^6*cos(d*x + c)/d^6 - 2*a*b*c^3*cos(d*x + c)/d^3
- 6*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b^2*c^5/d^6 + 6*((d*x + c)*cos(
d*x + c) - sin(d*x + c))*a*b*c^2/d^3 + 15*(((d*x + c)^2 - 2)*cos(d*x + c) -
 2*(d*x + c)*sin(d*x + c))*b^2*c^4/d^6 - 6*(((d*x + c)^2 - 2)*cos(d*x + c)
- 2*(d*x + c)*sin(d*x + c))*a*b*c/d^3 - 20*(((d*x + c)^3 - 6*d*x - 6*c)*cos
(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*b^2*c^3/d^6 + 2*(((d*x + c)^3
 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*a*b/d^3 +
15*(((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d*x + c) - 4*((d*x + c)^3 - 6*d
*x - 6*c)*sin(d*x + c))*b^2*c^2/d^6 - 6*(((d*x + c)^5 - 20*(d*x + c)^3 + 12
0*d*x + 120*c)*cos(d*x + c) - 5*((d*x + c)^4 - 12*(d*x + c)^2 + 24)*sin(d*x
 + c))*b^2*c/d^6 + (((d*x + c)^6 - 30*(d*x + c)^4 + 360*(d*x + c)^2 - 720)*
cos(d*x + c) - 6*((d*x + c)^5 - 20*(d*x + c)^3 + 120*d*x + 120*c)*sin(d*x +
 c))*b^2/d^6)/d
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.70

$$\int \left(a+bx^3\right)^2 \sin(c+dx)\,dx = -\frac{\left(b^2d^6x^6+2\,abd^6x^3-30\,b^2d^4x^4+a^2d^6-12\,abd^4x+360\,b^2d^2x^2-720\,b^2\right)\cos\left(dx+c\right)}{d^7} + \frac{6\left(b^2d^5x^5+abd^5x^2-20\,b^2d^3x^3-2\,abd^3+120\,b^2dx\right)\sin\left(dx+c\right)}{d^7}$$

```
[In] integrate((b*x^3+a)^2*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b^2*d^6*x^6 + 2*a*b*d^6*x^3 - 30*b^2*d^4*x^4 + a^2*d^6 - 12*a*b*d^4*x + 3
60*b^2*d^2*x^2 - 720*b^2)*cos(d*x + c)/d^7 + 6*(b^2*d^5*x^5 + a*b*d^5*x^2 -
 20*b^2*d^3*x^3 - 2*a*b*d^3 + 120*b^2*d*x)*sin(d*x + c)/d^7
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.98

$$\int (a+bx^3)^2 \sin(c+dx)\,dx = \frac{\cos(c+d\,x)\,\left(720\,b^2-a^2\,d^6\right)}{d^7} - \frac{b^2\,x^6\,\cos(c+d\,x)}{d} + \frac{30\,b^2\,x^4\,\cos(c+d\,x)}{d^3} - \frac{360\,b^2\,x^2\,\cos(c+d\,x)}{d^5} + \frac{6\,b^2\,x^5\,\sin(c+d\,x)}{d^2} - \frac{120\,b^2\,x^3\,\sin(c+d\,x)}{d^4} - \frac{12\,a\,b\,\sin(c+d\,x)}{d^4} + \frac{720\,b^2\,x\,\sin(c+d\,x)}{d^6} - \frac{2\,a\,b\,x^3\,\cos(c+d\,x)}{d} + \frac{6\,a\,b\,x^2\,\sin(c+d\,x)}{d^2} + \frac{12\,a\,b\,x\,\cos(c+d\,x)}{d^3}$$

```
[In] int(sin(c + d*x)*(a + b*x^3)^2,x)
```

```
[Out] (cos(c + d*x)*(720*b^2 - a^2*d^6))/d^7 - (b^2*x^6*cos(c + d*x))/d + (30*b^2
*x^4*cos(c + d*x))/d^3 - (360*b^2*x^2*cos(c + d*x))/d^5 + (6*b^2*x^5*sin(c
+ d*x))/d^2 - (120*b^2*x^3*sin(c + d*x))/d^4 - (12*a*b*sin(c + d*x))/d^4 +
(720*b^2*x*sin(c + d*x))/d^6 - (2*a*b*x^3*cos(c + d*x))/d + (6*a*b*x^2*sin(
c + d*x))/d^2 + (12*a*b*x*cos(c + d*x))/d^3
```

3.89 $\int \frac{(a+bx^3)^2 \sin(c+dx)}{x} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 161

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x} \, dx = \frac{4ab\cos(c+dx)}{d^3} - \frac{120b^2x\cos(c+dx)}{d^5} - \frac{2abx^2\cos(c+dx)}{d} + \frac{20b^2x^3\cos(c+dx)}{d^3} - \frac{b^2x^5\cos(c+dx)}{d} + a^2\,\text{CosIntegral}(dx)\sin(c) + \frac{120b^2\sin(c+dx)}{d^6} + \frac{4abx\sin(c+dx)}{d^2} - \frac{60b^2x^2\sin(c+dx)}{d^4} + \frac{5b^2x^4\sin(c+dx)}{d^2} + a^2\cos(c)\text{Si}(dx)$$

```
[Out] 4*a*b*cos(d*x+c)/d^3-120*b^2*x*cos(d*x+c)/d^5-2*a*b*x^2*cos(d*x+c)/d+20*b^2
*x^3*cos(d*x+c)/d^3-b^2*x^5*cos(d*x+c)/d+a^2*cos(c)*Si(d*x)+a^2*Ci(d*x)*sin
(c)+120*b^2*sin(d*x+c)/d^6+4*a*b*x*sin(d*x+c)/d^2-60*b^2*x^2*sin(d*x+c)/d^4
+5*b^2*x^4*sin(d*x+c)/d^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used

$= \{3420, 3384, 3380, 3383, 3377, 2718, 2717\}$

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x} \, dx = a^2 \sin(c) \operatorname{CosIntegral}(dx) + a^2 \cos(c) \operatorname{Si}(dx) + \frac{4ab \cos(c+dx)}{d^3} + \frac{4abx \sin(c+dx)}{d^2} - \frac{2abx^2 \cos(c+dx)}{d} + \frac{120b^2 \sin(c+dx)}{d^6} - \frac{120b^2 x \cos(c+dx)}{d^5} - \frac{60b^2 x^2 \sin(c+dx)}{d^4} + \frac{20b^2 x^3 \cos(c+dx)}{d^3} + \frac{5b^2 x^4 \sin(c+dx)}{d^2} - \frac{b^2 x^5 \cos(c+dx)}{d}$$

```
[In] Int[((a + b*x^3)^2*Sin[c + d*x])/x,x]
```

```
[Out] (4*a*b*Cos[c + d*x])/d^3 - (120*b^2*x*Cos[c + d*x])/d^5 - (2*a*b*x^2*Cos[c
+ d*x])/d + (20*b^2*x^3*Cos[c + d*x])/d^3 - (b^2*x^5*Cos[c + d*x])/d + a^2*
CosIntegral[d*x]*Sin[c] + (120*b^2*Sin[c + d*x])/d^6 + (4*a*b*x*Sin[c + d*x
])/d^2 - (60*b^2*x^2*Sin[c + d*x])/d^4 + (5*b^2*x^4*Sin[c + d*x])/d^2 + a^2
*Cos[c]*SinIntegral[d*x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a^2\sin(c+dx)}{x}+2abx^2\sin(c+dx)+b^2x^5\sin(c+dx)\right)dx \\
&= a^2\int \frac{\sin(c+dx)}{x}\,dx+(2ab)\int x^2\sin(c+dx)\,dx+b^2\int x^5\sin(c+dx)\,dx \\
&= -\frac{2abx^2\cos(c+dx)}{d}-\frac{b^2x^5\cos(c+dx)}{d}+\frac{(4ab)\int x\cos(c+dx)\,dx}{d} \\
&\quad+\frac{(5b^2)\int x^4\cos(c+dx)\,dx}{d}+\left(a^2\cos(c)\right)\int \frac{\sin(dx)}{x}\,dx+\left(a^2\sin(c)\right)\int \frac{\cos(dx)}{x}\,dx \\
&= -\frac{2abx^2\cos(c+dx)}{d}-\frac{b^2x^5\cos(c+dx)}{d}+a^2\,\text{CosIntegral}(dx)\sin(c)+\frac{4abx\sin(c+dx)}{d^2} \\
&\quad+\frac{5b^2x^4\sin(c+dx)}{d^2}+a^2\cos(c)\text{Si}(dx)-\frac{(4ab)\int \sin(c+dx)\,dx}{d^2}-\frac{(20b^2)\int x^3\sin(c+dx)\,dx}{d^2} \\
&= \frac{4ab\cos(c+dx)}{d^3}-\frac{2abx^2\cos(c+dx)}{d}+\frac{20b^2x^3\cos(c+dx)}{d^3} \\
&\quad-\frac{b^2x^5\cos(c+dx)}{d}+a^2\,\text{CosIntegral}(dx)\sin(c)+\frac{4abx\sin(c+dx)}{d^2} \\
&\quad+\frac{5b^2x^4\sin(c+dx)}{d^2}+a^2\cos(c)\text{Si}(dx)-\frac{(60b^2)\int x^2\cos(c+dx)\,dx}{d^3} \\
&= \frac{4ab\cos(c+dx)}{d^3}-\frac{2abx^2\cos(c+dx)}{d}+\frac{20b^2x^3\cos(c+dx)}{d^3}-\frac{b^2x^5\cos(c+dx)}{d} \\
&\quad+a^2\,\text{CosIntegral}(dx)\sin(c)+\frac{4abx\sin(c+dx)}{d^2}-\frac{60b^2x^2\sin(c+dx)}{d^4} \\
&\quad+\frac{5b^2x^4\sin(c+dx)}{d^2}+a^2\cos(c)\text{Si}(dx)+\frac{(120b^2)\int x\sin(c+dx)\,dx}{d^4}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{4ab\cos(c+dx)}{d^3} - \frac{120b^2x\cos(c+dx)}{d^5} - \frac{2abx^2\cos(c+dx)}{d} \\
&\quad + \frac{20b^2x^3\cos(c+dx)}{d^3} - \frac{b^2x^5\cos(c+dx)}{d} + a^2\,\text{CosIntegral}(dx)\sin(c) \\
&\quad + \frac{4abx\sin(c+dx)}{d^2} - \frac{60b^2x^2\sin(c+dx)}{d^4} + \frac{5b^2x^4\sin(c+dx)}{d^2} \\
&\quad + a^2\cos(c)\text{Si}(dx) + \frac{(120b^2)\int\cos(c+dx)\,dx}{d^5} \\
&= \frac{4ab\cos(c+dx)}{d^3} - \frac{120b^2x\cos(c+dx)}{d^5} - \frac{2abx^2\cos(c+dx)}{d} + \frac{20b^2x^3\cos(c+dx)}{d^3} \\
&\quad - \frac{b^2x^5\cos(c+dx)}{d} + a^2\,\text{CosIntegral}(dx)\sin(c) + \frac{120b^2\sin(c+dx)}{d^6} \\
&\quad + \frac{4abx\sin(c+dx)}{d^2} - \frac{60b^2x^2\sin(c+dx)}{d^4} + \frac{5b^2x^4\sin(c+dx)}{d^2} + a^2\cos(c)\text{Si}(dx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.67

$$
\begin{aligned}
\int \frac{(a+bx^3)^2\sin(c+dx)}{x}\,dx &= -\frac{b(2ad^2(-2+d^2x^2)+bx(120-20d^2x^2+d^4x^4))\cos(c+dx)}{d^5} \\
&\quad + a^2\,\text{CosIntegral}(dx)\sin(c) \\
&\quad + \frac{b(4ad^4x+5b(24-12d^2x^2+d^4x^4))\sin(c+dx)}{d^6} \\
&\quad + a^2\cos(c)\text{Si}(dx)
\end{aligned}
$$

```
[In] Integrate[((a + b*x^3)^2*Sin[c + d*x])/x,x]
```

```
[Out] -((b*(2*a*d^2*(-2 + d^2*x^2) + b*x*(120 - 20*d^2*x^2 + d^4*x^4))*Cos[c + d*
x])/d^5) + a^2*CosIntegral[d*x]*Sin[c] + (b*(4*a*d^4*x + 5*b*(24 - 12*d^2*x
^2 + d^4*x^4))*Sin[c + d*x])/d^6 + a^2*Cos[c]*SinIntegral[d*x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.11

method	result
risch	$-\frac{b^2x^5\cos(dx+c)}{d}+\frac{5b^2x^4\sin(dx+c)}{d^2}-\frac{ia^2\mathrm{e}^{-ic}\operatorname{Ei}_1(idx)}{2}+\frac{ia^2\mathrm{e}^{ic}\operatorname{Ei}_1(-idx)}{2}-\frac{2ab\,x^2\cos(dx+c)}{d}+\frac{20b^2x^3\cos}{d^3}$
meijerg	$\frac{32b^2\sqrt{\pi}\,\sin(c)\left(-\frac{15}{4\sqrt{\pi}}+\frac{\left(\frac{15}{8}d^4x^4-\frac{45}{2}d^2x^2+45\right)\cos(dx)}{12\sqrt{\pi}}+\frac{xd\left(\frac{3}{8}d^4x^4-\frac{15}{2}d^2x^2+45\right)\sin(dx)}{12\sqrt{\pi}}\right)}{d^6}+\frac{32b^2\sqrt{\pi}\,\cos(c)\left(-\frac{xd\left(\frac{7}{8}d^4\right.}{}\right.}{}$
derivativedivides	$a^2(\operatorname{Si}(dx)\cos(c)+\operatorname{Ci}(dx)\sin(c))-\frac{6ab\,c^2\cos(dx+c)}{d^3}-\frac{6abc(c+1)(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^3}+$
default	$a^2(\operatorname{Si}(dx)\cos(c)+\operatorname{Ci}(dx)\sin(c))-\frac{6ab\,c^2\cos(dx+c)}{d^3}-\frac{6abc(c+1)(\sin(dx+c)-\cos(dx+c)(dx+c))}{d^3}+$

```
[In] int((b*x^3+a)^2*sin(d*x+c)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -b^2*x^5*cos(d*x+c)/d+5*b^2*x^4*sin(d*x+c)/d^2-1/2*I*a^2*exp(-I*c)*Ei(1,I*d
*x)+1/2*I*a^2*exp(I*c)*Ei(1,-I*d*x)-2*a*b*x^2*cos(d*x+c)/d+20*b^2*x^3*cos(d
*x+c)/d^3+4*a*b*x*sin(d*x+c)/d^2-60*b^2*x^2*sin(d*x+c)/d^4+4*a*b*cos(d*x+c)
/d^3-120*b^2*x*cos(d*x+c)/d^5+120*b^2*sin(d*x+c)/d^6
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx^3)^2\sin(c+dx)}{x}\,dx$$
$$=\frac{a^2d^6\operatorname{Ci}(dx)\sin(c)+a^2d^6\cos(c)\operatorname{Si}(dx)-(b^2d^5x^5+2\,abd^5x^2-20\,b^2d^3x^3-4\,abd^3+120\,b^2dx)\cos(dx+}{d^6}$$

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x,x, algorithm="fricas")
```

```
[Out] (a^2*d^6*cos_integral(d*x)*sin(c) + a^2*d^6*cos(c)*sin_integral(d*x) - (b^2
*d^5*x^5 + 2*a*b*d^5*x^2 - 20*b^2*d^3*x^3 - 4*a*b*d^3 + 120*b^2*d*x)*cos(d*
x + c) + (5*b^2*d^4*x^4 + 4*a*b*d^4*x - 60*b^2*d^2*x^2 + 120*b^2)*sin(d*x +
 c))/d^6
```

Sympy [A] (verification not implemented)

Time = 3.69 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.29

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x}\,dx$$

$$= a^2 \sin(c) \operatorname{Ci}(dx) + a^2 \cos(c) \operatorname{Si}(dx) + 2abx^2 \left(\begin{cases} x \sin(c) & \text{for } d = 0 \\ -\frac{\cos(c+dx)}{d} & \text{otherwise} \end{cases} \right)$$

$$- 4ab \left(\begin{cases} \frac{x^3 \sin(c)}{3} & \text{for } d = 0 \\ -\frac{\begin{cases} \frac{x \sin(c+dx)}{d} + \frac{\cos(c+dx)}{d^2} & \text{for } d \neq 0 \\ \frac{x^2 \cos(c)}{2} & \text{otherwise} \end{cases}}{d} & \text{otherwise} \end{cases} \right)$$

$$+ b^2x^5 \left(\begin{cases} x \sin(c) & \text{for } d = 0 \\ -\frac{\cos(c+dx)}{d} & \text{otherwise} \end{cases} \right)$$

$$-5b^2 \left(\begin{cases} \frac{x^6 \sin(c)}{6} & \text{for } d = 0 \\ -\frac{\begin{cases} \frac{x^4 \sin(c+dx)}{d} + \frac{4x^3 \cos(c+dx)}{d^2} - \frac{12x^2 \sin(c+dx)}{d^3} - \frac{24x \cos(c+dx)}{d^4} + \frac{24 \sin(c+dx)}{d^5} & \text{for } d \neq 0 \\ \frac{x^5 \cos(c)}{5} & \text{otherwise} \end{cases}}{d} & \text{otherwise} \end{cases} \right)$$

```
[In] integrate((b*x**3+a)**2*sin(d*x+c)/x,x)
```

```
[Out] a**2*sin(c)*Ci(d*x) + a**2*cos(c)*Si(d*x) + 2*a*b*x**2*Piecewise((x*sin(c),
 Eq(d, 0)), (-cos(c + d*x)/d, True)) - 4*a*b*Piecewise((x**3*sin(c)/3, Eq(d
, 0)), (-Piecewise((x*sin(c + d*x)/d + cos(c + d*x)/d**2, Ne(d, 0)), (x**2*
cos(c)/2, True))/d, True)) + b**2*x**5*Piecewise((x*sin(c), Eq(d, 0)), (-co
s(c + d*x)/d, True)) - 5*b**2*Piecewise((x**6*sin(c)/6, Eq(d, 0)), (-Piecew
ise((x**4*sin(c + d*x)/d + 4*x**3*cos(c + d*x)/d**2 - 12*x**2*sin(c + d*x)/
d**3 - 24*x*cos(c + d*x)/d**4 + 24*sin(c + d*x)/d**5, Ne(d, 0)), (x**5*cos(
c)/5, True))/d, True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.68 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x}\,dx$$

$$= \frac{(a^2(-i \operatorname{Ei}(i\,dx) + i \operatorname{Ei}(-i\,dx)) \cos(c) + a^2(\operatorname{Ei}(i\,dx) + \operatorname{Ei}(-i\,dx)) \sin(c))d^6 - 2\,(b^2d^5x^5 + 2\,abd^5x^2 - 20\,b^2d^3}{2\,d^6}$$

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2*(-I*Ei(I*d*x) + I*Ei(-I*d*x))*cos(c) + a^2*(Ei(I*d*x) + Ei(-I*d*x
))*sin(c))*d^6 - 2*(b^2*d^5*x^5 + 2*a*b*d^5*x^2 - 20*b^2*d^3*x^3 - 4*a*b*d^
3 + 120*b^2*d*x)*cos(d*x + c) + 2*(5*b^2*d^4*x^4 + 4*a*b*d^4*x - 60*b^2*d^2
*x^2 + 120*b^2)*sin(d*x + c))/d^6
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.34 (sec) , antiderivative size = 921, normalized size of antiderivative = 5.72

$$\int \frac{(a+bx^3)^2\sin(c+dx)}{x}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^2*d^5*x^5*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + 2*b^2*d^5*x^5*tan(
1/2*d*x + 1/2*c)^2 - 2*b^2*d^5*x^5*tan(1/2*c)^2 + 20*b^2*d^4*x^4*tan(1/2*d*
x + 1/2*c)*tan(1/2*c)^2 + 4*a*b*d^5*x^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2
 - a^2*d^6*imag_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2
 + a^2*d^6*imag_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^
2 - 2*a^2*d^6*sin_integral(d*x)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - 2*b^2
*d^5*x^5 + 2*a^2*d^6*real_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*ta
n(1/2*c) + 2*a^2*d^6*real_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*t
an(1/2*c) - 40*b^2*d^3*x^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + 20*b^2*d^4
*x^4*tan(1/2*d*x + 1/2*c) + 4*a*b*d^5*x^2*tan(1/2*d*x + 1/2*c)^2 + a^2*d^6*
imag_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2 - a^2*d^6*imag_part(cos
_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2 + 2*a^2*d^6*sin_integral(d*x)*tan(1
/2*d*x + 1/2*c)^2 - 4*a*b*d^5*x^2*tan(1/2*c)^2 - a^2*d^6*imag_part(cos_inte
gral(d*x))*tan(1/2*c)^2 + a^2*d^6*imag_part(cos_integral(-d*x))*tan(1/2*c)^
2 - 2*a^2*d^6*sin_integral(d*x)*tan(1/2*c)^2 - 40*b^2*d^3*x^3*tan(1/2*d*x +
 1/2*c)^2 + 2*a^2*d^6*real_part(cos_integral(d*x))*tan(1/2*c) + 2*a^2*d^6*r
eal_part(cos_integral(-d*x))*tan(1/2*c) + 40*b^2*d^3*x^3*tan(1/2*c)^2 + 16*
a*b*d^4*x*tan(1/2*d*x + 1/2*c)*tan(1/2*c)^2 - 4*a*b*d^5*x^2 + a^2*d^6*imag_
part(cos_integral(d*x)) - a^2*d^6*imag_part(cos_integral(-d*x)) + 2*a^2*d^6
*sin_integral(d*x) - 240*b^2*d^2*x^2*tan(1/2*d*x + 1/2*c)*tan(1/2*c)^2 - 8*
a*b*d^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + 40*b^2*d^3*x^3 + 16*a*b*d^4*x
*tan(1/2*d*x + 1/2*c) + 240*b^2*d*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - 2
40*b^2*d^2*x^2*tan(1/2*d*x + 1/2*c) - 8*a*b*d^3*tan(1/2*d*x + 1/2*c)^2 + 8*
a*b*d^3*tan(1/2*c)^2 + 240*b^2*d*x*tan(1/2*d*x + 1/2*c)^2 - 240*b^2*d*x*tan
(1/2*c)^2 + 8*a*b*d^3 + 480*b^2*tan(1/2*d*x + 1/2*c)*tan(1/2*c)^2 - 240*b^2
*d*x + 480*b^2*tan(1/2*d*x + 1/2*c))/(d^6*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)
^2 + d^6*tan(1/2*d*x + 1/2*c)^2 + d^6*tan(1/2*c)^2 + d^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x} \, dx = \int \frac{\sin(c+d\,x)\, \left(b\,x^3+a\right)^2}{x} \,dx$$

```
[In] int((sin(c + d*x)*(a + b*x^3)^2)/x,x)
[Out] int((sin(c + d*x)*(a + b*x^3)^2)/x, x)
```

3.90 $\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 145

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^2} dx = -\frac{24b^2\cos(c+dx)}{d^5} - \frac{2abx\cos(c+dx)}{d} + \frac{12b^2x^2\cos(c+dx)}{d^3} - \frac{b^2x^4\cos(c+dx)}{d} + a^2d\cos(c)\operatorname{CosIntegral}(dx) + \frac{2ab\sin(c+dx)}{d^2} - \frac{a^2\sin(c+dx)}{x} - \frac{24b^2x\sin(c+dx)}{d^4} + \frac{4b^2x^3\sin(c+dx)}{d^2} - a^2d\sin(c)\operatorname{Si}(dx)$$

```
[Out] a^2*d*Ci(d*x)*cos(c)-24*b^2*cos(d*x+c)/d^5-2*a*b*x*cos(d*x+c)/d+12*b^2*x^2*
cos(d*x+c)/d^3-b^2*x^4*cos(d*x+c)/d-a^2*d*Si(d*x)*sin(c)+2*a*b*sin(d*x+c)/d
^2-a^2*sin(d*x+c)/x-24*b^2*x*sin(d*x+c)/d^4+4*b^2*x^3*sin(d*x+c)/d^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {3420, 3378, 3384, 3380, 3383, 3377, 2717, 2718}

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^2} dx = a^2d\cos(c)\operatorname{CosIntegral}(dx) - a^2d\sin(c)\operatorname{Si}(dx) - \frac{a^2\sin(c+dx)}{x} + \frac{2ab\sin(c+dx)}{d^2} - \frac{2abx\cos(c+dx)}{d} - \frac{24b^2\cos(c+dx)}{d^5} - \frac{24b^2x\sin(c+dx)}{d^4} + \frac{12b^2x^2\cos(c+dx)}{d^3} + \frac{4b^2x^3\sin(c+dx)}{d^2} - \frac{b^2x^4\cos(c+dx)}{d}$$

```
[In] Int[((a + b*x^3)^2*Sin[c + d*x])/x^2,x]
```

```
[Out] (-24*b^2*Cos[c + d*x])/d^5 - (2*a*b*x*Cos[c + d*x])/d + (12*b^2*x^2*Cos[c +
 d*x])/d^3 - (b^2*x^4*Cos[c + d*x])/d + a^2*d*Cos[c]*CosIntegral[d*x] + (2*
a*b*Sin[c + d*x])/d^2 - (a^2*Sin[c + d*x])/x - (24*b^2*x*Sin[c + d*x])/d^4
+ (4*b^2*x^3*Sin[c + d*x])/d^2 - a^2*d*Sin[c]*SinIntegral[d*x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a^2\sin(c+dx)}{x^2} + 2abx\sin(c+dx) + b^2x^4\sin(c+dx)\right) dx \\
&= a^2\int \frac{\sin(c+dx)}{x^2}\,dx + (2ab)\int x\sin(c+dx)\,dx + b^2\int x^4\sin(c+dx)\,dx \\
&= -\frac{2abx\cos(c+dx)}{d} - \frac{b^2x^4\cos(c+dx)}{d} - \frac{a^2\sin(c+dx)}{x} \\
&\quad + \frac{(2ab)\int\cos(c+dx)\,dx}{d} + \frac{(4b^2)\int x^3\cos(c+dx)\,dx}{d} + \left(a^2d\right)\int\frac{\cos(c+dx)}{x}\,dx \\
&= -\frac{2abx\cos(c+dx)}{d} - \frac{b^2x^4\cos(c+dx)}{d} + \frac{2ab\sin(c+dx)}{d^2} - \frac{a^2\sin(c+dx)}{x} + \frac{4b^2x^3\sin(c+dx)}{d^2} \\
&\quad - \frac{(12b^2)\int x^2\sin(c+dx)\,dx}{d^2} + \left(a^2d\cos(c)\right)\int\frac{\cos(dx)}{x}\,dx - \left(a^2d\sin(c)\right)\int\frac{\sin(dx)}{x}\,dx \\
&= -\frac{2abx\cos(c+dx)}{d} + \frac{12b^2x^2\cos(c+dx)}{d^3} - \frac{b^2x^4\cos(c+dx)}{d} \\
&\quad + a^2d\cos(c)\,\text{CosIntegral}(dx) + \frac{2ab\sin(c+dx)}{d^2} - \frac{a^2\sin(c+dx)}{x} \\
&\quad + \frac{4b^2x^3\sin(c+dx)}{d^2} - a^2d\sin(c)\text{Si}(dx) - \frac{(24b^2)\int x\cos(c+dx)\,dx}{d^3} \\
&= -\frac{2abx\cos(c+dx)}{d} + \frac{12b^2x^2\cos(c+dx)}{d^3} - \frac{b^2x^4\cos(c+dx)}{d} \\
&\quad + a^2d\cos(c)\,\text{CosIntegral}(dx) + \frac{2ab\sin(c+dx)}{d^2} - \frac{a^2\sin(c+dx)}{x} - \frac{24b^2x\sin(c+dx)}{d^4} \\
&\quad + \frac{4b^2x^3\sin(c+dx)}{d^2} - a^2d\sin(c)\text{Si}(dx) + \frac{(24b^2)\int\sin(c+dx)\,dx}{d^4} \\
&= -\frac{24b^2\cos(c+dx)}{d^5} - \frac{2abx\cos(c+dx)}{d} + \frac{12b^2x^2\cos(c+dx)}{d^3} \\
&\quad - \frac{b^2x^4\cos(c+dx)}{d} + a^2d\cos(c)\,\text{CosIntegral}(dx) + \frac{2ab\sin(c+dx)}{d^2} \\
&\quad - \frac{a^2\sin(c+dx)}{x} - \frac{24b^2x\sin(c+dx)}{d^4} + \frac{4b^2x^3\sin(c+dx)}{d^2} - a^2d\sin(c)\text{Si}(dx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^2}\,dx = -\frac{24b^2\cos(c+dx)}{d^5} - \frac{2abx\cos(c+dx)}{d} + \frac{12b^2x^2\cos(c+dx)}{d^3} - \frac{b^2x^4\cos(c+dx)}{d} + a^2d\cos(c)\,\text{CosIntegral}(dx) + \frac{2ab\sin(c+dx)}{d^2} - \frac{a^2\sin(c+dx)}{x} - \frac{24b^2x\sin(c+dx)}{d^4} + \frac{4b^2x^3\sin(c+dx)}{d^2} - a^2d\sin(c)\text{Si}(dx)$$

```
[In] Integrate[((a + b*x^3)^2*Sin[c + d*x])/x^2,x]
```

```
[Out] (-24*b^2*Cos[c + d*x])/d^5 - (2*a*b*x*Cos[c + d*x])/d + (12*b^2*x^2*Cos[c +
 d*x])/d^3 - (b^2*x^4*Cos[c + d*x])/d + a^2*d*Cos[c]*CosIntegral[d*x] + (2*
a*b*Sin[c + d*x])/d^2 - (a^2*Sin[c + d*x])/x - (24*b^2*x*Sin[c + d*x])/d^4
+ (4*b^2*x^3*Sin[c + d*x])/d^2 - a^2*d*Sin[c]*SinIntegral[d*x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.49 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.48

method	result
risch	$-\frac{-\pi\ \text{csgn}(dx)\sin(c)a^2d^6x+2\ \text{Si}(dx)\sin(c)a^2d^6x-i\pi\ \text{csgn}(dx)\cos(c)a^2d^6x+2\cos(dx+c)b^2d^4x^5+2i\,\text{Si}(dx)\cos(c)a^2d^6x+}{}$
meijerg	$\frac{16b^2\sqrt{\pi}\ \sin(c)\left(-\frac{x\left(d^2\right)^{\frac{5}{2}}\left(-\frac{5d^2x^2}{2}+15\right)\cos(dx)}{10\sqrt{\pi}\,d^4}+\frac{\left(d^2\right)^{\frac{5}{2}}\left(\frac{5}{8}d^4x^4-\frac{15}{2}d^2x^2+15\right)\sin(dx)}{10\sqrt{\pi}\,d^5}\right)}{d^4\sqrt{d^2}}+\frac{16b^2\sqrt{\pi}\ \cos(c)\left(\frac{3}{2\sqrt{\pi}}-\frac{\left(\frac{3}{8}d^4x^4\right.}{}\right.}{}$
derivativedivides	$d\left(-\frac{15b^2c^4\cos(dx+c)}{d^6}+a^2\left(-\frac{\sin(dx+c)}{dx}-\text{Si}\left(dx\right)\sin\left(c\right)+\text{Ci}\left(dx\right)\cos\left(c\right)\right)+\frac{15(3c^2+2c+1)c^2b^2\left(-\right.}{}\right.$
default	$d\left(-\frac{15b^2c^4\cos(dx+c)}{d^6}+a^2\left(-\frac{\sin(dx+c)}{dx}-\text{Si}\left(dx\right)\sin\left(c\right)+\text{Ci}\left(dx\right)\cos\left(c\right)\right)+\frac{15(3c^2+2c+1)c^2b^2\left(-\right.}{}\right.$

```
[In] int((b*x^3+a)^2*sin(d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x/d^5*(-Pi*csgn(d*x)*sin(c)*a^2*d^6*x+2*Si(d*x)*sin(c)*a^2*d^6*x-I*Pi*
csgn(d*x)*cos(c)*a^2*d^6*x+2*cos(d*x+c)*b^2*d^4*x^5+2*I*Si(d*x)*cos(c)*a^2*
d^6*x+2*Ei(1,-I*d*x)*cos(c)*a^2*d^6*x-8*sin(d*x+c)*b^2*d^3*x^4+4*cos(d*x+c)
*a*b*d^4*x^2+2*sin(d*x+c)*a^2*d^5-24*cos(d*x+c)*b^2*d^2*x^3-4*sin(d*x+c)*a*
b*d^3*x+48*sin(d*x+c)*b^2*d*x^2+48*cos(d*x+c)*b^2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^2\sin(c+dx)}{x^2}\,dx = \frac{a^2d^6x\cos(c)\operatorname{Ci}(dx) - a^2d^6x\sin(c)\operatorname{Si}(dx) - (b^2d^4x^5 + 2\,abd^4x^2 - 12\,b^2d^2x^3 + 24\,b^2x)\cos(dx+c) + (4\,b^2d^3x^4 \cdots}{d^5x}$$

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] (a^2*d^6*x*cos(c)*cos_integral(d*x) - a^2*d^6*x*sin(c)*sin_integral(d*x) -
(b^2*d^4*x^5 + 2*a*b*d^4*x^2 - 12*b^2*d^2*x^3 + 24*b^2*x)*cos(d*x + c) + (4
*b^2*d^3*x^4 - a^2*d^5 + 2*a*b*d^3*x - 24*b^2*d*x^2)*sin(d*x + c))/(d^5*x)
```

Sympy [F]

$$\int \frac{(a+bx^3)^2\sin(c+dx)}{x^2}\,dx = \int \frac{(a+bx^3)^2\sin(c+dx)}{x^2}\,dx$$

```
[In] integrate((b*x**3+a)**2*sin(d*x+c)/x**2,x)
```

```
[Out] Integral((a + b*x**3)**2*sin(c + d*x)/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.47 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^3)^2\sin(c+dx)}{x^2}\,dx = \frac{(a^2(\Gamma(-1,i\,dx)+\Gamma(-1,-i\,dx))\cos(c) + a^2(-i\,\Gamma(-1,i\,dx)+i\,\Gamma(-1,-i\,dx))\sin(c))d^6 - 2\,(b^2d^4x^4 + 2\,a \cdots}{2\,d^5}$$

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2*(gamma(-1, I*d*x) + gamma(-1, -I*d*x))*cos(c) + a^2*(-I*gamma(-1,
 I*d*x) + I*gamma(-1, -I*d*x))*sin(c))*d^6 - 2*(b^2*d^4*x^4 + 2*a*b*d^4*x -
 12*b^2*d^2*x^2 + 24*b^2)*cos(d*x + c) + 4*(2*b^2*d^3*x^3 + a*b*d^3 - 12*b^
2*d*x)*sin(d*x + c))/d^5
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.34 (sec) , antiderivative size = 2038, normalized size of antiderivative = 14.06

$$\int \frac{(a+bx^3)^2\sin(c+dx)}{x^2}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^2*d^4*x^5*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2
*d^6*x*real_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*t
an(1/2*c)^2 - a^2*d^6*x*real_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^
2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*b^2*d^4*x^5*tan(1/2*d*x + 1/2*c)^2*tan(1/
2*d*x)^2 - 2*a^2*d^6*x*imag_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*
tan(1/2*d*x)^2*tan(1/2*c) + 2*a^2*d^6*x*imag_part(cos_integral(-d*x))*tan(1
/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c) - 4*a^2*d^6*x*sin_integral(d*x)
*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c) + 2*b^2*d^4*x^5*tan(1/2*d
*x + 1/2*c)^2*tan(1/2*c)^2 - 2*b^2*d^4*x^5*tan(1/2*d*x)^2*tan(1/2*c)^2 + a^
2*d^6*x*real_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2
+ a^2*d^6*x*real_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*
x)^2 - a^2*d^6*x*real_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/
2*c)^2 - a^2*d^6*x*real_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*tan
(1/2*c)^2 - a^2*d^6*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c
)^2 - a^2*d^6*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
 16*b^2*d^3*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a*b*d^
4*x^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*b^2*d^4*x^5*ta
n(1/2*d*x + 1/2*c)^2 - 2*b^2*d^4*x^5*tan(1/2*d*x)^2 - 2*a^2*d^6*x*imag_part
(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c) + 2*a^2*d^6*x*imag_pa
rt(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c) - 4*a^2*d^6*x*sin_
integral(d*x)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c) - 2*a^2*d^6*x*imag_part(cos
_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 2*a^2*d^6*x*imag_part(cos_integ
ral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 4*a^2*d^6*x*sin_integral(d*x)*tan(1/
2*d*x)^2*tan(1/2*c) - 2*b^2*d^4*x^5*tan(1/2*c)^2 - 24*b^2*d^2*x^3*tan(1/2*d
*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + a^2*d^6*x*real_part(cos_integra
l(d*x))*tan(1/2*d*x + 1/2*c)^2 + a^2*d^6*x*real_part(cos_integral(-d*x))*ta
n(1/2*d*x + 1/2*c)^2 + a^2*d^6*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^
2 + a^2*d^6*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2 + 16*b^2*d^3*x^4
*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x)^2 + 4*a*b*d^4*x^2*tan(1/2*d*x + 1/2*c)^2
*tan(1/2*d*x)^2 + 4*a^2*d^5*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c
) - a^2*d^6*x*real_part(cos_integral(d*x))*tan(1/2*c)^2 - a^2*d^6*x*real_pa
rt(cos_integral(-d*x))*tan(1/2*c)^2 + 16*b^2*d^3*x^4*tan(1/2*d*x + 1/2*c)*t
an(1/2*c)^2 + 4*a*b*d^4*x^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + 4*a^2*d^5
*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)*tan(1/2*c)^2 - 4*a*b*d^4*x^2*tan(1/2*d
*x)^2*tan(1/2*c)^2 - 2*b^2*d^4*x^5 - 24*b^2*d^2*x^3*tan(1/2*d*x + 1/2*c)^2*
```

```
tan(1/2*d*x)^2 - 2*a^2*d^6*x*imag_part(cos_integral(d*x))*tan(1/2*c) + 2*a^
2*d^6*x*imag_part(cos_integral(-d*x))*tan(1/2*c) - 4*a^2*d^6*x*sin_integral
(d*x)*tan(1/2*c) - 24*b^2*d^2*x^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + 24*
b^2*d^2*x^3*tan(1/2*d*x)^2*tan(1/2*c)^2 + 8*a*b*d^3*x*tan(1/2*d*x + 1/2*c)*
tan(1/2*d*x)^2*tan(1/2*c)^2 + a^2*d^6*x*real_part(cos_integral(d*x)) + a^2*
d^6*x*real_part(cos_integral(-d*x)) + 16*b^2*d^3*x^4*tan(1/2*d*x + 1/2*c) +
 4*a*b*d^4*x^2*tan(1/2*d*x + 1/2*c)^2 - 4*a^2*d^5*tan(1/2*d*x + 1/2*c)^2*ta
n(1/2*d*x) - 4*a*b*d^4*x^2*tan(1/2*d*x)^2 - 4*a^2*d^5*tan(1/2*d*x + 1/2*c)^
2*tan(1/2*c) + 4*a^2*d^5*tan(1/2*d*x)^2*tan(1/2*c) - 4*a*b*d^4*x^2*tan(1/2*
c)^2 + 4*a^2*d^5*tan(1/2*d*x)*tan(1/2*c)^2 - 96*b^2*d*x^2*tan(1/2*d*x + 1/2
*c)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 24*b^2*d^2*x^3*tan(1/2*d*x + 1/2*c)^2 + 2
4*b^2*d^2*x^3*tan(1/2*d*x)^2 + 8*a*b*d^3*x*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x
)^2 + 24*b^2*d^2*x^3*tan(1/2*c)^2 + 8*a*b*d^3*x*tan(1/2*d*x + 1/2*c)*tan(1/
2*c)^2 + 48*b^2*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a*
b*d^4*x^2 - 4*a^2*d^5*tan(1/2*d*x) - 96*b^2*d*x^2*tan(1/2*d*x + 1/2*c)*tan(
1/2*d*x)^2 - 4*a^2*d^5*tan(1/2*c) - 96*b^2*d*x^2*tan(1/2*d*x + 1/2*c)*tan(1
/2*c)^2 + 24*b^2*d^2*x^3 + 8*a*b*d^3*x*tan(1/2*d*x + 1/2*c) + 48*b^2*x*tan(
1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2 + 48*b^2*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2
*c)^2 - 48*b^2*x*tan(1/2*d*x)^2*tan(1/2*c)^2 - 96*b^2*d*x^2*tan(1/2*d*x + 1
/2*c) + 48*b^2*x*tan(1/2*d*x + 1/2*c)^2 - 48*b^2*x*tan(1/2*d*x)^2 - 48*b^2*
x*tan(1/2*c)^2 - 48*b^2*x)/(d^5*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan
(1/2*c)^2 + d^5*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2 + d^5*x*tan(1/2*d*x
 + 1/2*c)^2*tan(1/2*c)^2 + d^5*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + d^5*x*tan(1/
2*d*x + 1/2*c)^2 + d^5*x*tan(1/2*d*x)^2 + d^5*x*tan(1/2*c)^2 + d^5*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^2\sin(c+dx)}{x^2}\,dx=\int \frac{\sin\left(c+d\,x\right)\,\left(b\,x^3+a\right)^2}{x^2}\,dx$$

```
[In] int((sin(c + d*x)*(a + b*x^3)^2)/x^2,x)
[Out] int((sin(c + d*x)*(a + b*x^3)^2)/x^2, x)
```

3.91 $\int \frac{(a+bx^3)^2\sin(c+dx)}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 142

$$\begin{aligned}\int \frac{(a+bx^3)^2\sin(c+dx)}{x^3}\,dx = &-\frac{2ab\cos(c+dx)}{d}-\frac{a^2d\cos(c+dx)}{2x}\\&+\frac{6b^2x\cos(c+dx)}{d^3}-\frac{b^2x^3\cos(c+dx)}{d}\\&-\frac{1}{2}a^2d^2\,\text{CosIntegral}(dx)\sin(c)-\frac{6b^2\sin(c+dx)}{d^4}\\&-\frac{a^2\sin(c+dx)}{2x^2}+\frac{3b^2x^2\sin(c+dx)}{d^2}-\frac{1}{2}a^2d^2\cos(c)\text{Si}(dx)\end{aligned}$$

```
[Out] -2*a*b*cos(d*x+c)/d-1/2*a^2*d*cos(d*x+c)/x+6*b^2*x*cos(d*x+c)/d^3-b^2*x^3*c
os(d*x+c)/d-1/2*a^2*d^2*cos(c)*Si(d*x)-1/2*a^2*d^2*Ci(d*x)*sin(c)-6*b^2*sin
(d*x+c)/d^4-1/2*a^2*sin(d*x+c)/x^2+3*b^2*x^2*sin(d*x+c)/d^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {3420, 2718, 3378, 3384, 3380, 3383, 3377, 2717}

$$\begin{aligned}\int \frac{(a+bx^3)^2\sin(c+dx)}{x^3}\,dx = &-\frac{1}{2}a^2d^2\sin(c)\,\text{CosIntegral}(dx)-\frac{1}{2}a^2d^2\cos(c)\text{Si}(dx)\\&-\frac{a^2\sin(c+dx)}{2x^2}-\frac{a^2d\cos(c+dx)}{2x}-\frac{2ab\cos(c+dx)}{d}\\&-\frac{6b^2\sin(c+dx)}{d^4}+\frac{6b^2x\cos(c+dx)}{d^3}\\&+\frac{3b^2x^2\sin(c+dx)}{d^2}-\frac{b^2x^3\cos(c+dx)}{d}\end{aligned}$$

```
[In] Int[((a + b*x^3)^2*Sin[c + d*x])/x^3,x]
```

```
[Out] (-2*a*b*Cos[c + d*x])/d - (a^2*d*Cos[c + d*x])/(2*x) + (6*b^2*x*Cos[c + d*x
])/d^3 - (b^2*x^3*Cos[c + d*x])/d - (a^2*d^2*CosIntegral[d*x]*Sin[c])/2 - (
6*b^2*Sin[c + d*x])/d^4 - (a^2*Sin[c + d*x])/(2*x^2) + (3*b^2*x^2*Sin[c + d
*x])/d^2 - (a^2*d^2*Cos[c]*SinIntegral[d*x])/2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(2ab\sin(c+dx)+\frac{a^2\sin(c+dx)}{x^3}+b^2x^3\sin(c+dx)\right)dx\\
&= a^2\int \frac{\sin(c+dx)}{x^3}\,dx+(2ab)\int \sin(c+dx)\,dx+b^2\int x^3\sin(c+dx)\,dx\\
&= -\frac{2ab\cos(c+dx)}{d}-\frac{b^2x^3\cos(c+dx)}{d}-\frac{a^2\sin(c+dx)}{2x^2}\\
&\quad+\frac{(3b^2)\int x^2\cos(c+dx)\,dx}{d}+\frac{1}{2}\left(a^2d\right)\int \frac{\cos(c+dx)}{x^2}\,dx\\
&= -\frac{2ab\cos(c+dx)}{d}-\frac{a^2d\cos(c+dx)}{2x}-\frac{b^2x^3\cos(c+dx)}{d}-\frac{a^2\sin(c+dx)}{2x^2}\\
&\quad+\frac{3b^2x^2\sin(c+dx)}{d^2}-\frac{(6b^2)\int x\sin(c+dx)\,dx}{d^2}-\frac{1}{2}\left(a^2d^2\right)\int \frac{\sin(c+dx)}{x}\,dx\\
&= -\frac{2ab\cos(c+dx)}{d}-\frac{a^2d\cos(c+dx)}{2x}+\frac{6b^2x\cos(c+dx)}{d^3}-\frac{b^2x^3\cos(c+dx)}{d}\\
&\quad-\frac{a^2\sin(c+dx)}{2x^2}+\frac{3b^2x^2\sin(c+dx)}{d^2}-\frac{(6b^2)\int \cos(c+dx)\,dx}{d^3}\\
&\quad-\frac{1}{2}\left(a^2d^2\cos(c)\right)\int \frac{\sin(dx)}{x}\,dx-\frac{1}{2}\left(a^2d^2\sin(c)\right)\int \frac{\cos(dx)}{x}\,dx\\
&= -\frac{2ab\cos(c+dx)}{d}-\frac{a^2d\cos(c+dx)}{2x}+\frac{6b^2x\cos(c+dx)}{d^3}\\
&\quad-\frac{b^2x^3\cos(c+dx)}{d}-\frac{1}{2}a^2d^2\,\text{CosIntegral}(dx)\sin(c)-\frac{6b^2\sin(c+dx)}{d^4}\\
&\quad-\frac{a^2\sin(c+dx)}{2x^2}+\frac{3b^2x^2\sin(c+dx)}{d^2}-\frac{1}{2}a^2d^2\cos(c)\text{Si}(dx)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)^2\sin(c+dx)}{x^3}\,dx = \frac{1}{2}\left(-\frac{4ab\cos(c+dx)}{d} - \frac{a^2d\cos(c+dx)}{x} + \frac{12b^2x\cos(c+dx)}{d^3} - \frac{2b^2x^3\cos(c+dx)}{d} - a^2d^2\,\text{CosIntegral}(dx)\sin(c) - \frac{12b^2\sin(c+dx)}{d^4} - \frac{a^2\sin(c+dx)}{x^2} + \frac{6b^2x^2\sin(c+dx)}{d^2} - a^2d^2\cos(c)\text{Si}(dx)\right)$$

```
[In] Integrate[((a + b*x^3)^2*Sin[c + d*x])/x^3,x]
```

```
[Out] ((-4*a*b*Cos[c + d*x])/d - (a^2*d*Cos[c + d*x])/x + (12*b^2*x*Cos[c + d*x])
/d^3 - (2*b^2*x^3*Cos[c + d*x])/d - a^2*d^2*CosIntegral[d*x]*Sin[c] - (12*b
^2*Sin[c + d*x])/d^4 - (a^2*Sin[c + d*x])/x^2 + (6*b^2*x^2*Sin[c + d*x])/d^
2 - a^2*d^2*Cos[c]*SinIntegral[d*x])/2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.50 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.48

method	result
risch	$-\frac{-\pi\ \text{csgn}(dx)\cos(c)a^2d^6x^2+2\ \text{Si}(dx)\cos(c)a^2d^6x^2+i\pi\ \text{csgn}(dx)\sin(c)a^2d^6x^2-2i\,\text{Si}(dx)\sin(c)a^2d^6x^2-2\ \text{Ei}_1(-idx)\,\text{s}}{}$
derivativedivides	$d^2\left(\frac{20b^2c^3\cos(dx+c)}{d^6} - \frac{2ab\cos(dx+c)}{d^3} + a^2\left(-\frac{\sin(dx+c)}{2d^2x^2} - \frac{\cos(dx+c)}{2dx} - \frac{\text{Si}(dx)\cos(c)}{2} - \frac{\text{Ci}(dx)\sin(c)}{2}\right) +\right.$
default	$d^2\left(\frac{20b^2c^3\cos(dx+c)}{d^6} - \frac{2ab\cos(dx+c)}{d^3} + a^2\left(-\frac{\sin(dx+c)}{2d^2x^2} - \frac{\cos(dx+c)}{2dx} - \frac{\text{Si}(dx)\cos(c)}{2} - \frac{\text{Ci}(dx)\sin(c)}{2}\right) +\right.$
meijerg	$\frac{8b^2\sqrt{\pi}\sin(c)\left(\frac{3}{4\sqrt{\pi}} - \frac{\left(-\frac{3d^2x^2}{2}+3\right)\cos(dx)}{4\sqrt{\pi}} - \frac{dx\left(-\frac{d^2x^2}{2}+3\right)\sin(dx)}{4\sqrt{\pi}}\right)}{d^4} + \frac{8b^2\sqrt{\pi}\cos(c)\left(\frac{xd\left(-\frac{5d^2x^2}{2}+15\right)\cos(dx)}{20\sqrt{\pi}} - \frac{(-1}{}\right.}{d^4}$

```
[In] int((b*x^3+a)^2*sin(d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/x^2/d^4*(-Pi*csgn(d*x)*cos(c)*a^2*d^6*x^2+2*Si(d*x)*cos(c)*a^2*d^6*x^2
+I*Pi*csgn(d*x)*sin(c)*a^2*d^6*x^2-2*I*Si(d*x)*sin(c)*a^2*d^6*x^2-2*Ei(1,-I
*d*x)*sin(c)*a^2*d^6*x^2+4*cos(d*x+c)*b^2*d^3*x^5-12*sin(d*x+c)*b^2*d^2*x^4
+2*cos(d*x+c)*a^2*d^5*x+8*cos(d*x+c)*a*b*d^3*x^2+2*sin(d*x+c)*a^2*d^4-24*co
s(d*x+c)*b^2*d*x^3+24*sin(d*x+c)*b^2*x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^3}\,dx = -\frac{a^2d^6x^2 \operatorname{Ci}(dx)\sin(c) + a^2d^6x^2\cos(c)\operatorname{Si}(dx) + (2b^2d^3x^5 + a^2d^5x + 4abd^3x^2 - 12b^2dx^3)\cos(dx+c) - (}{2d^4x^2}$$

[In] integrate((b*x^3+a)^2*sin(d*x+c)/x^3,x, algorithm="fricas")

[Out] -1/2*(a^2*d^6*x^2*cos_integral(d*x)*sin(c) + a^2*d^6*x^2*cos(c)*sin_integra l(d*x) + (2*b^2*d^3*x^5 + a^2*d^5*x + 4*a*b*d^3*x^2 - 12*b^2*d*x^3)*cos(d*x + c) - (6*b^2*d^2*x^4 - a^2*d^4 - 12*b^2*x^2)*sin(d*x + c))/(d^4*x^2)

Sympy [F]

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^3}\,dx = \int \frac{(a+bx^3)^2 \sin(c+dx)}{x^3}\,dx$$

[In] integrate((b*x**3+a)**2*sin(d*x+c)/x**3,x)

[Out] Integral((a + b*x**3)**2*sin(c + d*x)/x**3, x)

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.19 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^3}\,dx = \frac{(a^2(i\,\Gamma(-2,i\,dx) - i\,\Gamma(-2,-i\,dx))\cos(c) + a^2(\Gamma(-2,i\,dx) + \Gamma(-2,-i\,dx))\sin(c))d^6 - 2\,(b^2d^3x^3 + 2\,abd^3}{2\,d^4}$$

[In] integrate((b*x^3+a)^2*sin(d*x+c)/x^3,x, algorithm="maxima")

[Out] 1/2*((a^2*(I*gamma(-2, I*d*x) - I*gamma(-2, -I*d*x))*cos(c) + a^2*(gamma(-2 , I*d*x) + gamma(-2, -I*d*x))*sin(c))*d^6 - 2*(b^2*d^3*x^3 + 2*a*b*d^3 - 6* b^2*d*x)*cos(d*x + c) + 6*(b^2*d^2*x^2 - 2*b^2)*sin(d*x + c))/d^4

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.32 (sec) , antiderivative size = 2171, normalized size of antiderivative = 15.29

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^3} \, dx = \text{Too large to display}$$

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x^3,x, algorithm="giac")
[Out] 1/4*(a^2*d^6*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/
2*d*x)^2*tan(1/2*c)^2 - a^2*d^6*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d
*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^2*d^6*x^2*sin_integral(d*x)
*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*d^6*x^2*real_pa
rt(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c) - 2*
a^2*d^6*x^2*real_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*
x)^2*tan(1/2*c) + 4*b^2*d^3*x^5*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1
/2*c)^2 - a^2*d^6*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*t
an(1/2*d*x)^2 + a^2*d^6*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2
*c)^2*tan(1/2*d*x)^2 - 2*a^2*d^6*x^2*sin_integral(d*x)*tan(1/2*d*x + 1/2*c)
^2*tan(1/2*d*x)^2 + a^2*d^6*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x +
1/2*c)^2*tan(1/2*c)^2 - a^2*d^6*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d
*x + 1/2*c)^2*tan(1/2*c)^2 + 2*a^2*d^6*x^2*sin_integral(d*x)*tan(1/2*d*x +
1/2*c)^2*tan(1/2*c)^2 + a^2*d^6*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*
x)^2*tan(1/2*c)^2 - a^2*d^6*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^
2*tan(1/2*c)^2 + 2*a^2*d^6*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^
2 + 4*b^2*d^3*x^5*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2 - 2*a^2*d^6*x^2*rea
l_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c) - 2*a^2*d^6*x^2
*real_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c) - 2*a^2*d^
6*x^2*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^6*x^
2*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*b^2*d^3*x^5*t
an(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - 4*b^2*d^3*x^5*tan(1/2*d*x)^2*tan(1/2*c
)^2 - 2*a^2*d^5*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*
d^6*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2 + a^2*d^6*x^2*i
mag_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2 - 2*a^2*d^6*x^2*sin_int
egral(d*x)*tan(1/2*d*x + 1/2*c)^2 - a^2*d^6*x^2*imag_part(cos_integral(d*x)
)*tan(1/2*d*x)^2 + a^2*d^6*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2
 - 2*a^2*d^6*x^2*sin_integral(d*x)*tan(1/2*d*x)^2 + a^2*d^6*x^2*imag_part(c
os_integral(d*x))*tan(1/2*c)^2 - a^2*d^6*x^2*imag_part(cos_integral(-d*x))*
tan(1/2*c)^2 + 2*a^2*d^6*x^2*sin_integral(d*x)*tan(1/2*c)^2 + 24*b^2*d^2*x^
4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 8*a*b*d^3*x^2*tan(1/2*
d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*b^2*d^3*x^5*tan(1/2*d*x + 1/
2*c)^2 - 4*b^2*d^3*x^5*tan(1/2*d*x)^2 + 2*a^2*d^5*x*tan(1/2*d*x + 1/2*c)^2*
tan(1/2*d*x)^2 - 2*a^2*d^6*x^2*real_part(cos_integral(d*x))*tan(1/2*c) - 2*
a^2*d^6*x^2*real_part(cos_integral(-d*x))*tan(1/2*c) + 8*a^2*d^5*x*tan(1/2*
```

```
d*x + 1/2*c)^2*tan(1/2*d*x)*tan(1/2*c) - 4*b^2*d^3*x^5*tan(1/2*c)^2 + 2*a^2
*d^5*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - 2*a^2*d^5*x*tan(1/2*d*x)^2*tan
(1/2*c)^2 - 24*b^2*d*x^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c)^2
 - a^2*d^6*x^2*imag_part(cos_integral(d*x)) + a^2*d^6*x^2*imag_part(cos_int
egral(-d*x)) - 2*a^2*d^6*x^2*sin_integral(d*x) + 24*b^2*d^2*x^4*tan(1/2*d*x
 + 1/2*c)*tan(1/2*d*x)^2 + 8*a*b*d^3*x^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x
)^2 + 4*a^2*d^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c) + 24*b^2*d
^2*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*c)^2 + 8*a*b*d^3*x^2*tan(1/2*d*x + 1/2*
c)^2*tan(1/2*c)^2 + 4*a^2*d^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)*tan(1/2*c
)^2 - 8*a*b*d^3*x^2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*b^2*d^3*x^5 - 2*a^2*d^5
*x*tan(1/2*d*x + 1/2*c)^2 + 2*a^2*d^5*x*tan(1/2*d*x)^2 - 24*b^2*d*x^3*tan(1
/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2 + 8*a^2*d^5*x*tan(1/2*d*x)*tan(1/2*c) + 2*
a^2*d^5*x*tan(1/2*c)^2 - 24*b^2*d*x^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 +
 24*b^2*d*x^3*tan(1/2*d*x)^2*tan(1/2*c)^2 + 24*b^2*d^2*x^4*tan(1/2*d*x + 1/
2*c) + 8*a*b*d^3*x^2*tan(1/2*d*x + 1/2*c)^2 - 4*a^2*d^4*tan(1/2*d*x + 1/2*c
)^2*tan(1/2*d*x) - 8*a*b*d^3*x^2*tan(1/2*d*x)^2 - 4*a^2*d^4*tan(1/2*d*x + 1
/2*c)^2*tan(1/2*c) + 4*a^2*d^4*tan(1/2*d*x)^2*tan(1/2*c) - 8*a*b*d^3*x^2*ta
n(1/2*c)^2 + 4*a^2*d^4*tan(1/2*d*x)*tan(1/2*c)^2 - 48*b^2*x^2*tan(1/2*d*x +
 1/2*c)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*d^5*x - 24*b^2*d*x^3*tan(1/2*d*
x + 1/2*c)^2 + 24*b^2*d*x^3*tan(1/2*d*x)^2 + 24*b^2*d*x^3*tan(1/2*c)^2 - 8*
a*b*d^3*x^2 - 4*a^2*d^4*tan(1/2*d*x) - 48*b^2*x^2*tan(1/2*d*x + 1/2*c)*tan(
1/2*d*x)^2 - 4*a^2*d^4*tan(1/2*c) - 48*b^2*x^2*tan(1/2*d*x + 1/2*c)*tan(1/2
*c)^2 + 24*b^2*d*x^3 - 48*b^2*x^2*tan(1/2*d*x + 1/2*c))/(d^4*x^2*tan(1/2*d*
x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + d^4*x^2*tan(1/2*d*x + 1/2*c)^2*t
an(1/2*d*x)^2 + d^4*x^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + d^4*x^2*tan(1
/2*d*x)^2*tan(1/2*c)^2 + d^4*x^2*tan(1/2*d*x + 1/2*c)^2 + d^4*x^2*tan(1/2*d
*x)^2 + d^4*x^2*tan(1/2*c)^2 + d^4*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^3}\,dx = \int \frac{\sin\left(c+d\,x\right)\,{\left(b\,x^3+a\right)}^2}{x^3}\,dx$$

```
[In] int((sin(c + d*x)*(a + b*x^3)^2)/x^3,x)
```

```
[Out] int((sin(c + d*x)*(a + b*x^3)^2)/x^3, x)
```

3.92 $\int \frac{(a+bx^3)^2\sin(c+dx)}{x^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 151

$$\begin{aligned}\int \frac{(a+bx^3)^2\sin(c+dx)}{x^4}\,dx &= \frac{2b^2\cos(c+dx)}{d^3}-\frac{a^2d\cos(c+dx)}{6x^2}-\frac{b^2x^2\cos(c+dx)}{d}\\ &\quad-\frac{1}{6}a^2d^3\cos(c)\,\text{CosIntegral}(dx)+2ab\,\text{CosIntegral}(dx)\sin(c)\\ &\quad-\frac{a^2\sin(c+dx)}{3x^3}+\frac{a^2d^2\sin(c+dx)}{6x}+\frac{2b^2x\sin(c+dx)}{d^2}\\ &\quad+2ab\cos(c)\text{Si}(dx)+\frac{1}{6}a^2d^3\sin(c)\text{Si}(dx)\end{aligned}$$

```
[Out] -1/6*a^2*d^3*Ci(d*x)*cos(c)+2*b^2*cos(d*x+c)/d^3-1/6*a^2*d*cos(d*x+c)/x^2-b
^2*x^2*cos(d*x+c)/d+2*a*b*cos(c)*Si(d*x)+2*a*b*Ci(d*x)*sin(c)+1/6*a^2*d^3*S
i(d*x)*sin(c)-1/3*a^2*sin(d*x+c)/x^3+1/6*a^2*d^2*sin(d*x+c)/x+2*b^2*x*sin(d
*x+c)/d^2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3420, 3378, 3384, 3380, 3383, 3377, 2718}

$$\begin{aligned}\int \frac{(a+bx^3)^2\sin(c+dx)}{x^4}\,dx &= -\frac{1}{6}a^2d^3\cos(c)\,\text{CosIntegral}(dx)+\frac{1}{6}a^2d^3\sin(c)\text{Si}(dx)\\ &\quad+\frac{a^2d^2\sin(c+dx)}{6x}-\frac{a^2\sin(c+dx)}{3x^3}-\frac{a^2d\cos(c+dx)}{6x^2}\\ &\quad+2ab\sin(c)\,\text{CosIntegral}(dx)+2ab\cos(c)\text{Si}(dx)\\ &\quad+\frac{2b^2\cos(c+dx)}{d^3}+\frac{2b^2x\sin(c+dx)}{d^2}-\frac{b^2x^2\cos(c+dx)}{d}\end{aligned}$$

```
[In] Int[((a + b*x^3)^2*Sin[c + d*x])/x^4,x]
```

```
[Out] (2*b^2*Cos[c + d*x])/d^3 - (a^2*d*Cos[c + d*x])/(6*x^2) - (b^2*x^2*Cos[c +
d*x])/d - (a^2*d^3*Cos[c]*CosIntegral[d*x])/6 + 2*a*b*CosIntegral[d*x]*Sin[
c] - (a^2*Sin[c + d*x])/(3*x^3) + (a^2*d^2*Sin[c + d*x])/(6*x) + (2*b^2*x*S
in[c + d*x])/d^2 + 2*a*b*Cos[c]*SinIntegral[d*x] + (a^2*d^3*Sin[c]*SinInteg
ral[d*x])/6
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a^2\sin(c+dx)}{x^4}+\frac{2ab\sin(c+dx)}{x}+b^2x^2\sin(c+dx)\right)dx\\
&= a^2\int \frac{\sin(c+dx)}{x^4}\,dx+(2ab)\int \frac{\sin(c+dx)}{x}\,dx+b^2\int x^2\sin(c+dx)\,dx\\
&= -\frac{b^2x^2\cos(c+dx)}{d}-\frac{a^2\sin(c+dx)}{3x^3}\\
&\quad+\frac{(2b^2)\int x\cos(c+dx)\,dx}{d}+\frac{1}{3}\left(a^2d\right)\int \frac{\cos(c+dx)}{x^3}\,dx\\
&\quad+(2ab\cos(c))\int \frac{\sin(dx)}{x}\,dx+(2ab\sin(c))\int \frac{\cos(dx)}{x}\,dx\\
&= -\frac{a^2d\cos(c+dx)}{6x^2}-\frac{b^2x^2\cos(c+dx)}{d}+2ab\,\text{CosIntegral}(dx)\sin(c)-\frac{a^2\sin(c+dx)}{3x^3}\\
&\quad+\frac{2b^2x\sin(c+dx)}{d^2}+2ab\cos(c)\text{Si}(dx)-\frac{(2b^2)\int \sin(c+dx)\,dx}{d^2}-\frac{1}{6}\left(a^2d^2\right)\int \frac{\sin(c+dx)}{x^2}\,dx\\
&= \frac{2b^2\cos(c+dx)}{d^3}-\frac{a^2d\cos(c+dx)}{6x^2}-\frac{b^2x^2\cos(c+dx)}{d}\\
&\quad+2ab\,\text{CosIntegral}(dx)\sin(c)-\frac{a^2\sin(c+dx)}{3x^3}+\frac{a^2d^2\sin(c+dx)}{6x}\\
&\quad+\frac{2b^2x\sin(c+dx)}{d^2}+2ab\cos(c)\text{Si}(dx)-\frac{1}{6}\left(a^2d^3\right)\int \frac{\cos(c+dx)}{x}\,dx\\
&= \frac{2b^2\cos(c+dx)}{d^3}-\frac{a^2d\cos(c+dx)}{6x^2}-\frac{b^2x^2\cos(c+dx)}{d}+2ab\,\text{CosIntegral}(dx)\sin(c)\\
&\quad-\frac{a^2\sin(c+dx)}{3x^3}+\frac{a^2d^2\sin(c+dx)}{6x}+\frac{2b^2x\sin(c+dx)}{d^2}+2ab\cos(c)\text{Si}(dx)\\
&\quad-\frac{1}{6}\left(a^2d^3\cos(c)\right)\int \frac{\cos(dx)}{x}\,dx+\frac{1}{6}\left(a^2d^3\sin(c)\right)\int \frac{\sin(dx)}{x}\,dx\\
&= \frac{2b^2\cos(c+dx)}{d^3}-\frac{a^2d\cos(c+dx)}{6x^2}-\frac{b^2x^2\cos(c+dx)}{d}-\frac{1}{6}a^2d^3\cos(c)\,\text{CosIntegral}(dx)\\
&\quad+2ab\,\text{CosIntegral}(dx)\sin(c)-\frac{a^2\sin(c+dx)}{3x^3}+\frac{a^2d^2\sin(c+dx)}{6x}\\
&\quad+\frac{2b^2x\sin(c+dx)}{d^2}+2ab\cos(c)\text{Si}(dx)+\frac{1}{6}a^2d^3\sin(c)\text{Si}(dx)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^4} dx = \frac{1}{6}\left(\frac{12b^2\cos(c+dx)}{d^3} - \frac{a^2 d\cos(c+dx)}{x^2} - \frac{6b^2x^2\cos(c+dx)}{d} - a\,\text{CosIntegral}(dx)\left(ad^3\cos(c) - 12b\sin(c)\right) - \frac{2a^2\sin(c+dx)}{x^3} + \frac{a^2d^2\sin(c+dx)}{x} + \frac{12b^2x\sin(c+dx)}{d^2} + a\left(12b\cos(c) + ad^3\sin(c)\right)\text{Si}(dx)\right)$$

```
[In] Integrate[((a + b*x^3)^2*Sin[c + d*x])/x^4,x]
```

```
[Out] ((12*b^2*Cos[c + d*x])/d^3 - (a^2*d*Cos[c + d*x])/x^2 - (6*b^2*x^2*Cos[c +
d*x])/d - a*CosIntegral[d*x]*(a*d^3*Cos[c] - 12*b*Sin[c]) - (2*a^2*Sin[c +
d*x])/x^3 + (a^2*d^2*Sin[c + d*x])/x + (12*b^2*x*Sin[c + d*x])/d^2 + a*(12*
b*Cos[c] + a*d^3*Sin[c])*SinIntegral[d*x])/6
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.30

method	result
derivativedivides	$d^3\left(\frac{2ab(\text{Si}(dx)\cos(c)+\text{Ci}(dx)\sin(c))}{d^3} - \frac{15b^2c^2\cos(dx+c)}{d^6} + a^2\left(-\frac{\sin(dx+c)}{3d^3x^3} - \frac{\cos(dx+c)}{6d^2x^2} + \frac{\sin(dx+c)}{6dx} + \text{Si}(\right.\right.$
default	$d^3\left(\frac{2ab(\text{Si}(dx)\cos(c)+\text{Ci}(dx)\sin(c))}{d^3} - \frac{15b^2c^2\cos(dx+c)}{d^6} + a^2\left(-\frac{\sin(dx+c)}{3d^3x^3} - \frac{\cos(dx+c)}{6d^2x^2} + \frac{\sin(dx+c)}{6dx} + \text{Si}(\right.\right.$
risch	$-\frac{\pi\ \text{csgn}(dx)\sin(c)a^2d^6x^3 - 2\ \text{Si}(dx)\sin(c)a^2d^6x^3 + i\pi\ \text{csgn}(dx)\cos(c)a^2d^6x^3 - 2i\,\text{Si}(dx)\cos(c)a^2d^6x^3 + 12\pi\ \text{csgn}(dx)\cos}{}$
meijerg	$\frac{4b^2\sqrt{\pi}\sin(c)\left(\frac{x\left(d^2\right)^{\frac{3}{2}}\cos(dx)}{2\sqrt{\pi}\,d^2} - \frac{\left(d^2\right)^{\frac{3}{2}}\left(-\frac{3d^2x^2}{2}+3\right)\sin(dx)}{6\sqrt{\pi}\,d^3}\right)}{d^2\sqrt{d^2}} + \frac{4b^2\sqrt{\pi}\cos(c)\left(-\frac{1}{2\sqrt{\pi}} + \frac{\left(-\frac{d^2x^2}{2}+1\right)\cos(dx)}{2\sqrt{\pi}} + \frac{dx\sin(dx)}{2\sqrt{\pi}}\right.}{d^3}$

```
[In] int((b*x^3+a)^2*sin(d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] d^3*(2/d^3*a*b*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))-15/d^6*b^2*c^2*cos(d*x+c)+a^
2*(-1/3*sin(d*x+c)/d^3/x^3-1/6*cos(d*x+c)/d^2/x^2+1/6*sin(d*x+c)/d/x+1/6*Si
(d*x)*sin(c)-1/6*Ci(d*x)*cos(c))+(10*c^2+4*c+1)/d^6*b^2*(-(d*x+c)^2*cos(d*x
+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))-6*c*b^2*(4*c+1)/d^6*(sin(d*x+c)-cos(
d*x+c)*(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^4} dx = -\frac{(6 b^2 d^2 x^5 + a^2 d^4 x - 12 b^2 x^3) \cos(dx+c) + (a^2 d^6 x^3 \operatorname{Ci}(dx) - 12 abd^3 x^3 \operatorname{Si}(dx)) \cos(c) - (a^2 d^5 x^2 + 12}{6 d^3 x^3}$$

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*((6*b^2*d^2*x^5 + a^2*d^4*x - 12*b^2*x^3)*cos(d*x + c) + (a^2*d^6*x^3*
cos_integral(d*x) - 12*a*b*d^3*x^3*sin_integral(d*x))*cos(c) - (a^2*d^5*x^2
 + 12*b^2*d*x^4 - 2*a^2*d^3)*sin(d*x + c) - (a^2*d^6*x^3*sin_integral(d*x)
+ 12*a*b*d^3*x^3*cos_integral(d*x))*sin(c))/(d^3*x^3)
```

Sympy [F]

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^4} dx = \int \frac{(a+bx^3)^2 \sin(c+dx)}{x^4} dx$$

```
[In] integrate((b*x**3+a)**2*sin(d*x+c)/x**4,x)
```

```
[Out] Integral((a + b*x**3)**2*sin(c + d*x)/x**4, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.11 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^4} dx = -\frac{((a^2(\Gamma(-3, i\,dx) + \Gamma(-3, -i\,dx)) \cos(c) - a^2(i\,\Gamma(-3, i\,dx) - i\,\Gamma(-3, -i\,dx)) \sin(c))d^6 + 12\,(ab(-i\,\Gamma(-}{}$$

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*(((a^2*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*cos(c) - a^2*(I*gamma(-3
, I*d*x) - I*gamma(-3, -I*d*x))*sin(c))*d^6 + 12*(a*b*(-I*gamma(-3, I*d*x)
+ I*gamma(-3, -I*d*x))*cos(c) - a*b*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*
sin(c))*d^3)*x^3 + 2*(b^2*d^2*x^5 + 2*a*b*d^2*x^2 - 2*b^2*x^3 - 4*a*b)*cos(
d*x + c) - 4*(b^2*d*x^4 - a*b*d*x)*sin(d*x + c))/(d^3*x^3)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.34 (sec) , antiderivative size = 1181, normalized size of antiderivative = 7.82

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^4}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*(a^2*d^6*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2
+ a^2*d^6*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2
*a^2*d^6*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2
*d^6*x^3*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^2*d^
6*x^3*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) - a^2*d^6*x^3*real_part(c
os_integral(d*x))*tan(1/2*d*x)^2 - a^2*d^6*x^3*real_part(cos_integral(-d*x)
)*tan(1/2*d*x)^2 + a^2*d^6*x^3*real_part(cos_integral(d*x))*tan(1/2*c)^2 +
a^2*d^6*x^3*real_part(cos_integral(-d*x))*tan(1/2*c)^2 + 2*a^2*d^6*x^3*imag
_part(cos_integral(d*x))*tan(1/2*c) - 2*a^2*d^6*x^3*imag_part(cos_integral(
-d*x))*tan(1/2*c) + 4*a^2*d^6*x^3*sin_integral(d*x)*tan(1/2*c) - 12*b^2*d^2
*x^5*tan(1/2*d*x)^2*tan(1/2*c)^2 - 12*a*b*d^3*x^3*imag_part(cos_integral(d*
x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 12*a*b*d^3*x^3*imag_part(cos_integral(-d*
x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 24*a*b*d^3*x^3*sin_integral(d*x)*tan(1/2*
d*x)^2*tan(1/2*c)^2 - a^2*d^6*x^3*real_part(cos_integral(d*x)) - a^2*d^6*x^
3*real_part(cos_integral(-d*x)) - 4*a^2*d^5*x^2*tan(1/2*d*x)^2*tan(1/2*c) +
 24*a*b*d^3*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 24
*a*b*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 4*a^
2*d^5*x^2*tan(1/2*d*x)*tan(1/2*c)^2 + 12*b^2*d^2*x^5*tan(1/2*d*x)^2 + 12*a*
b*d^3*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 - 12*a*b*d^3*x^3*imag
_part(cos_integral(-d*x))*tan(1/2*d*x)^2 + 24*a*b*d^3*x^3*sin_integral(d*x)
*tan(1/2*d*x)^2 + 48*b^2*d^2*x^5*tan(1/2*d*x)*tan(1/2*c) + 12*b^2*d^2*x^5*t
an(1/2*c)^2 - 12*a*b*d^3*x^3*imag_part(cos_integral(d*x))*tan(1/2*c)^2 + 12
*a*b*d^3*x^3*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 - 24*a*b*d^3*x^3*si
n_integral(d*x)*tan(1/2*c)^2 - 2*a^2*d^4*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*
a^2*d^5*x^2*tan(1/2*d*x) + 4*a^2*d^5*x^2*tan(1/2*c) + 24*a*b*d^3*x^3*real_p
art(cos_integral(d*x))*tan(1/2*c) + 24*a*b*d^3*x^3*real_part(cos_integral(-
d*x))*tan(1/2*c) - 48*b^2*d*x^4*tan(1/2*d*x)^2*tan(1/2*c) - 48*b^2*d*x^4*ta
n(1/2*d*x)*tan(1/2*c)^2 - 12*b^2*d^2*x^5 + 12*a*b*d^3*x^3*imag_part(cos_int
egral(d*x)) - 12*a*b*d^3*x^3*imag_part(cos_integral(-d*x)) + 24*a*b*d^3*x^3
*sin_integral(d*x) + 2*a^2*d^4*x*tan(1/2*d*x)^2 + 8*a^2*d^4*x*tan(1/2*d*x)*
tan(1/2*c) + 2*a^2*d^4*x*tan(1/2*c)^2 + 24*b^2*x^3*tan(1/2*d*x)^2*tan(1/2*c
)^2 + 48*b^2*d*x^4*tan(1/2*d*x) + 48*b^2*d*x^4*tan(1/2*c) + 8*a^2*d^3*tan(1
/2*d*x)^2*tan(1/2*c) + 8*a^2*d^3*tan(1/2*d*x)*tan(1/2*c)^2 - 2*a^2*d^4*x -
24*b^2*x^3*tan(1/2*d*x)^2 - 96*b^2*x^3*tan(1/2*d*x)*tan(1/2*c) - 24*b^2*x^3
*tan(1/2*c)^2 - 8*a^2*d^3*tan(1/2*d*x) - 8*a^2*d^3*tan(1/2*c) + 24*b^2*x^3)
```

```
/(d^3*x^3*tan(1/2*d*x)^2*tan(1/2*c)^2 + d^3*x^3*tan(1/2*d*x)^2 + d^3*x^3*ta
n(1/2*c)^2 + d^3*x^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^4} \, dx = \int \frac{\sin(c+d\,x)\, \left(b\,x^3+a\right)^2}{x^4} \, dx$$

```
[In] int((sin(c + d*x)*(a + b*x^3)^2)/x^4,x)
[Out] int((sin(c + d*x)*(a + b*x^3)^2)/x^4, x)
```

3.93 $\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^5} dx$

Optimal result

Integrand size = 19, antiderivative size = 167

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^5} dx = -\frac{a^2 d\cos(c+dx)}{12x^3} + \frac{a^2 d^3 \cos(c+dx)}{24x} - \frac{b^2 x\cos(c+dx)}{d} + 2abd\cos(c)\,\text{CosIntegral}(dx) + \frac{1}{24}a^2 d^4\,\text{CosIntegral}(dx)\sin(c) + \frac{b^2\sin(c+dx)}{d^2} - \frac{a^2\sin(c+dx)}{4x^4} + \frac{a^2 d^2 \sin(c+dx)}{24x^2} - \frac{2ab\sin(c+dx)}{x} + \frac{1}{24}a^2 d^4 \cos(c)\text{Si}(dx) - 2abd\sin(c)\text{Si}(dx)$$

```
[Out] 2*a*b*d*Ci(d*x)*cos(c)-1/12*a^2*d*cos(d*x+c)/x^3+1/24*a^2*d^3*cos(d*x+c)/x-
b^2*x*cos(d*x+c)/d+1/24*a^2*d^4*cos(c)*Si(d*x)+1/24*a^2*d^4*Ci(d*x)*sin(c)-
2*a*b*d*Si(d*x)*sin(c)+b^2*sin(d*x+c)/d^2-1/4*a^2*sin(d*x+c)/x^4+1/24*a^2*d
^2*sin(d*x+c)/x^2-2*a*b*sin(d*x+c)/x
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used

$= \{3420, 3378, 3384, 3380, 3383, 3377, 2717\}$

$$\int \frac{(a+bx^3)^2\sin(c+dx)}{x^5}\,dx = \frac{1}{24}a^2d^4\sin(c)\,\text{CosIntegral}(dx) + \frac{1}{24}a^2d^4\cos(c)\text{Si}(dx) + \frac{a^2d^3\cos(c+dx)}{24x} + \frac{a^2d^2\sin(c+dx)}{24x^2} - \frac{a^2\sin(c+dx)}{4x^4} - \frac{a^2d\cos(c+dx)}{12x^3} + 2abd\cos(c)\,\text{CosIntegral}(dx) - 2abd\sin(c)\text{Si}(dx) - \frac{2ab\sin(c+dx)}{x} + \frac{b^2\sin(c+dx)}{d^2} - \frac{b^2x\cos(c+dx)}{d}$$

```
[In] Int[((a + b*x^3)^2*Sin[c + d*x])/x^5,x]
```

```
[Out] -1/12*(a^2*d*Cos[c + d*x])/x^3 + (a^2*d^3*Cos[c + d*x])/(24*x) - (b^2*x*Cos
[c + d*x])/d + 2*a*b*d*Cos[c]*CosIntegral[d*x] + (a^2*d^4*CosIntegral[d*x]*
Sin[c])/24 + (b^2*Sin[c + d*x])/d^2 - (a^2*Sin[c + d*x])/(4*x^4) + (a^2*d^2
*Sin[c + d*x])/(24*x^2) - (2*a*b*Sin[c + d*x])/x + (a^2*d^4*Cos[c]*SinInteg
ral[d*x])/24 - 2*a*b*d*Sin[c]*SinIntegral[d*x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a^2\sin(c+dx)}{x^5}+\frac{2ab\sin(c+dx)}{x^2}+b^2x\sin(c+dx)\right)dx\\
&= a^2\int\frac{\sin(c+dx)}{x^5}\,dx+(2ab)\int\frac{\sin(c+dx)}{x^2}\,dx+b^2\int x\sin(c+dx)\,dx\\
&= -\frac{b^2x\cos(c+dx)}{d}-\frac{a^2\sin(c+dx)}{4x^4}-\frac{2ab\sin(c+dx)}{x}+\frac{b^2\int\cos(c+dx)\,dx}{d}\\
&\quad+\frac{1}{4}\left(a^2d\right)\int\frac{\cos(c+dx)}{x^4}\,dx+(2abd)\int\frac{\cos(c+dx)}{x}\,dx\\
&= -\frac{a^2d\cos(c+dx)}{12x^3}-\frac{b^2x\cos(c+dx)}{d}+\frac{b^2\sin(c+dx)}{d^2}-\frac{a^2\sin(c+dx)}{4x^4}-\frac{2ab\sin(c+dx)}{x}\\
&\quad-\frac{1}{12}\left(a^2d^2\right)\int\frac{\sin(c+dx)}{x^3}\,dx+(2abd\cos(c))\int\frac{\cos(dx)}{x}\,dx-(2abd\sin(c))\int\frac{\sin(dx)}{x}\,dx\\
&= -\frac{a^2d\cos(c+dx)}{12x^3}-\frac{b^2x\cos(c+dx)}{d}+2abd\cos(c)\,\text{CosIntegral}(dx)\\
&\quad+\frac{b^2\sin(c+dx)}{d^2}-\frac{a^2\sin(c+dx)}{4x^4}+\frac{a^2d^2\sin(c+dx)}{24x^2}\\
&\quad-\frac{2ab\sin(c+dx)}{x}-2abd\sin(c)\text{Si}(dx)-\frac{1}{24}\left(a^2d^3\right)\int\frac{\cos(c+dx)}{x^2}\,dx\\
&= -\frac{a^2d\cos(c+dx)}{12x^3}+\frac{a^2d^3\cos(c+dx)}{24x}-\frac{b^2x\cos(c+dx)}{d}\\
&\quad+2abd\cos(c)\,\text{CosIntegral}(dx)+\frac{b^2\sin(c+dx)}{d^2}-\frac{a^2\sin(c+dx)}{4x^4}+\frac{a^2d^2\sin(c+dx)}{24x^2}\\
&\quad-\frac{2ab\sin(c+dx)}{x}-2abd\sin(c)\text{Si}(dx)+\frac{1}{24}\left(a^2d^4\right)\int\frac{\sin(c+dx)}{x}\,dx
\end{aligned}
$$

$$= -\frac{a^2d\cos(c+dx)}{12x^3} + \frac{a^2d^3\cos(c+dx)}{24x} - \frac{b^2x\cos(c+dx)}{d} + 2abd\cos(c)\,\mathrm{CosIntegral}(dx) + \frac{b^2\sin(c+dx)}{d^2} - \frac{a^2\sin(c+dx)}{4x^4} + \frac{a^2d^2\sin(c+dx)}{24x^2} - \frac{2ab\sin(c+dx)}{x} - 2abd\sin(c)\mathrm{Si}(dx) + \frac{1}{24}\left(a^2d^4\cos(c)\right)\int\frac{\sin(dx)}{x}\,dx + \frac{1}{24}\left(a^2d^4\sin(c)\right)\int\frac{\cos(dx)}{x}\,dx$$

$$= -\frac{a^2d\cos(c+dx)}{12x^3} + \frac{a^2d^3\cos(c+dx)}{24x} - \frac{b^2x\cos(c+dx)}{d} + 2abd\cos(c)\,\mathrm{CosIntegral}(dx) + \frac{1}{24}a^2d^4\,\mathrm{CosIntegral}(dx)\sin(c) + \frac{b^2\sin(c+dx)}{d^2} - \frac{a^2\sin(c+dx)}{4x^4} + \frac{a^2d^2\sin(c+dx)}{24x^2} - \frac{2ab\sin(c+dx)}{x} + \frac{1}{24}a^2d^4\cos(c)\mathrm{Si}(dx) - 2abd\sin(c)\mathrm{Si}(dx)$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.89

$$\int\frac{(a+bx^3)^2\sin(c+dx)}{x^5}\,dx = \frac{1}{24}\left(-\frac{2a^2d\cos(c+dx)}{x^3} + \frac{a^2d^3\cos(c+dx)}{x} - \frac{24b^2x\cos(c+dx)}{d} + ad\,\mathrm{CosIntegral}(dx)\left(48b\cos(c) + ad^3\sin(c)\right) + \frac{24b^2\sin(c+dx)}{d^2} - \frac{6a^2\sin(c+dx)}{x^4} + \frac{a^2d^2\sin(c+dx)}{x^2} - \frac{48ab\sin(c+dx)}{x} + ad\left(ad^3\cos(c) - 48b\sin(c)\right)\mathrm{Si}(dx)\right)$$

```
[In] Integrate[((a + b*x^3)^2*Sin[c + d*x])/x^5,x]
```

```
[Out] ((-2*a^2*d*Cos[c + d*x])/x^3 + (a^2*d^3*Cos[c + d*x])/x - (24*b^2*x*Cos[c +
 d*x])/d + a*d*CosIntegral[d*x]*(48*b*Cos[c] + a*d^3*Sin[c]) + (24*b^2*Sin[
c + d*x])/d^2 - (6*a^2*Sin[c + d*x])/x^4 + (a^2*d^2*Sin[c + d*x])/x^2 - (48
*a*b*Sin[c + d*x])/x + a*d*(a*d^3*Cos[c] - 48*b*Sin[c])*SinIntegral[d*x])/2
4
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00

method	result
derivativedivides	$d^4\left(\frac{2ab\left(-\frac{\sin(dx+c)}{dx}-\mathrm{Si}(dx)\sin(c)+\mathrm{Ci}(dx)\cos(c)\right)}{d^3}+a^2\left(-\frac{\sin(dx+c)}{4d^4x^4}-\frac{\cos(dx+c)}{12d^3x^3}+\frac{\sin(dx+c)}{24d^2x^2}+\frac{\cos(dx+c)}{24dx}\right.\right.$
default	$d^4\left(\frac{2ab\left(-\frac{\sin(dx+c)}{dx}-\mathrm{Si}(dx)\sin(c)+\mathrm{Ci}(dx)\cos(c)\right)}{d^3}+a^2\left(-\frac{\sin(dx+c)}{4d^4x^4}-\frac{\cos(dx+c)}{12d^3x^3}+\frac{\sin(dx+c)}{24d^2x^2}+\frac{\cos(dx+c)}{24dx}\right.\right.$
risch	$\frac{-\pi\,\mathrm{csgn}(dx)\cos(c)a^2d^6x^4+2\,\mathrm{Si}(dx)\cos(c)a^2d^6x^4-2i\,\mathrm{Si}(dx)\sin(c)a^2d^6x^4-96i\,\mathrm{Si}(dx)\cos(c)ab\,d^3x^4+48\pi\,\mathrm{csgn}(dx)\sin(c)}{}$
meijerg	$\frac{2b^2\sqrt{\pi}\,\sin(c)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(dx)}{2\sqrt{\pi}}+\frac{dx\sin(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{2b^2\sqrt{\pi}\,\cos(c)\left(-\frac{dx\cos(dx)}{2\sqrt{\pi}}+\frac{\sin(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{d^2ab\sqrt{\pi}\,\sin(c)\left(-\frac{4d^2\cos\left(x\sqrt{d^2}\right.}{x\left(d^2\right)^{\frac{3}{2}}\sqrt{\pi}}\right.}{2\sqrt{d^2}}$

```
[In] int((b*x^3+a)^2*sin(d*x+c)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] d^4*(2/d^3*a*b*(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c))+a^2*(-1/4*si
n(d*x+c)/d^4/x^4-1/12*cos(d*x+c)/d^3/x^3+1/24*sin(d*x+c)/d^2/x^2+1/24*cos(d
*x+c)/d/x+1/24*Si(d*x)*cos(c)+1/24*Ci(d*x)*sin(c))+6/d^6*b^2*c*cos(d*x+c)+(
5*c+1)/d^6*b^2*(sin(d*x+c)-cos(d*x+c)*(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^2\sin(c+dx)}{x^5}\,dx$$
$$=\frac{(a^2d^5x^3-24\,b^2dx^5-2\,a^2d^3x)\cos{(dx+c)}+(a^2d^6x^4\,\mathrm{Si}\,(dx)+48\,abd^3x^4\,\mathrm{Ci}\,(dx))\cos{(c)}+(a^2d^4x^2-48\,ab}{24\,d^2x^4}$$

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*((a^2*d^5*x^3 - 24*b^2*d*x^5 - 2*a^2*d^3*x)*cos(d*x + c) + (a^2*d^6*x^
4*sin_integral(d*x) + 48*a*b*d^3*x^4*cos_integral(d*x))*cos(c) + (a^2*d^4*x
^2 - 48*a*b*d^2*x^3 + 24*b^2*x^4 - 6*a^2*d^2)*sin(d*x + c) + (a^2*d^6*x^4*c
os_integral(d*x) - 48*a*b*d^3*x^4*sin_integral(d*x))*sin(c))/(d^2*x^4)
```

Sympy [F]

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^5}\, dx = \int \frac{(a+bx^3)^2 \sin{(c+dx)}}{x^5}\, dx$$

```
[In] integrate((b*x**3+a)**2*sin(d*x+c)/x**5,x)
```

```
[Out] Integral((a + b*x**3)**2*sin(c + d*x)/x**5, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.95 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^5}\, dx = \frac{((a^2(-i\,\Gamma(-4,i\,dx)+i\,\Gamma(-4,-i\,dx))\cos{(c)}-a^2(\Gamma(-4,i\,dx)+\Gamma(-4,-i\,dx))\sin{(c)})d^7-48\,(ab(\Gamma(-4,i$$

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*(((a^2*(-I*gamma(-4, I*d*x) + I*gamma(-4, -I*d*x))*cos(c) - a^2*(gamma(
-4, I*d*x) + gamma(-4, -I*d*x))*sin(c))*d^7 - 48*(a*b*(gamma(-4, I*d*x) + g
amma(-4, -I*d*x))*cos(c) + a*b*(-I*gamma(-4, I*d*x) + I*gamma(-4, -I*d*x))*
sin(c))*d^4)*x^4 - 2*(b^2*d^2*x^5 + 2*a*b*d^2*x^2 - 12*a*b)*cos(d*x + c) +
2*(b^2*d*x^4 - 4*a*b*d*x)*sin(d*x + c))/(d^3*x^4)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.31 (sec) , antiderivative size = 1255, normalized size of antiderivative = 7.51

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^5}\, dx = \text{Too large to display}$$

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x^5,x, algorithm="giac")
```

```
[Out] -1/48*(a^2*d^6*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2
 - a^2*d^6*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
2*a^2*d^6*x^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*d^6*x^4
*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^6*x^4*rea
l_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - a^2*d^6*x^4*imag_par
t(cos_integral(d*x))*tan(1/2*d*x)^2 + a^2*d^6*x^4*imag_part(cos_integral(-d
```

```
*x))*tan(1/2*d*x)^2 - 2*a^2*d^6*x^4*sin_integral(d*x)*tan(1/2*d*x)^2 + a^2*
d^6*x^4*imag_part(cos_integral(d*x))*tan(1/2*c)^2 - a^2*d^6*x^4*imag_part(c
os_integral(-d*x))*tan(1/2*c)^2 + 2*a^2*d^6*x^4*sin_integral(d*x)*tan(1/2*c
)^2 - 2*a^2*d^6*x^4*real_part(cos_integral(d*x))*tan(1/2*c) - 2*a^2*d^6*x^4
*real_part(cos_integral(-d*x))*tan(1/2*c) - 2*a^2*d^5*x^3*tan(1/2*d*x)^2*ta
n(1/2*c)^2 + 48*a*b*d^3*x^4*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan
(1/2*c)^2 + 48*a*b*d^3*x^4*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan
(1/2*c)^2 - a^2*d^6*x^4*imag_part(cos_integral(d*x)) + a^2*d^6*x^4*imag_par
t(cos_integral(-d*x)) - 2*a^2*d^6*x^4*sin_integral(d*x) + 96*a*b*d^3*x^4*im
ag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 96*a*b*d^3*x^4*imag_
part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 192*a*b*d^3*x^4*sin_in
tegral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) + 2*a^2*d^5*x^3*tan(1/2*d*x)^2 - 48*a
*b*d^3*x^4*real_part(cos_integral(d*x))*tan(1/2*d*x)^2 - 48*a*b*d^3*x^4*rea
l_part(cos_integral(-d*x))*tan(1/2*d*x)^2 + 8*a^2*d^5*x^3*tan(1/2*d*x)*tan(
1/2*c) + 2*a^2*d^5*x^3*tan(1/2*c)^2 + 48*a*b*d^3*x^4*real_part(cos_integral
(d*x))*tan(1/2*c)^2 + 48*a*b*d^3*x^4*real_part(cos_integral(-d*x))*tan(1/2*
c)^2 + 48*b^2*d*x^5*tan(1/2*d*x)^2*tan(1/2*c)^2 + 96*a*b*d^3*x^4*imag_part(
cos_integral(d*x))*tan(1/2*c) - 96*a*b*d^3*x^4*imag_part(cos_integral(-d*x)
)*tan(1/2*c) + 192*a*b*d^3*x^4*sin_integral(d*x)*tan(1/2*c) + 4*a^2*d^4*x^2
*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^2*d^4*x^2*tan(1/2*d*x)*tan(1/2*c)^2 - 2*a^
2*d^5*x^3 - 48*a*b*d^3*x^4*real_part(cos_integral(d*x)) - 48*a*b*d^3*x^4*re
al_part(cos_integral(-d*x)) - 48*b^2*d*x^5*tan(1/2*d*x)^2 - 192*b^2*d*x^5*t
an(1/2*d*x)*tan(1/2*c) - 192*a*b*d^2*x^3*tan(1/2*d*x)^2*tan(1/2*c) - 48*b^2
*d*x^5*tan(1/2*c)^2 - 192*a*b*d^2*x^3*tan(1/2*d*x)*tan(1/2*c)^2 + 4*a^2*d^3
*x*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a^2*d^4*x^2*tan(1/2*d*x) - 4*a^2*d^4*x^2
*tan(1/2*c) + 96*b^2*x^4*tan(1/2*d*x)^2*tan(1/2*c) + 96*b^2*x^4*tan(1/2*d*x
)*tan(1/2*c)^2 + 48*b^2*d*x^5 + 192*a*b*d^2*x^3*tan(1/2*d*x) - 4*a^2*d^3*x*
tan(1/2*d*x)^2 + 192*a*b*d^2*x^3*tan(1/2*c) - 16*a^2*d^3*x*tan(1/2*d*x)*tan
(1/2*c) - 4*a^2*d^3*x*tan(1/2*c)^2 - 96*b^2*x^4*tan(1/2*d*x) - 96*b^2*x^4*t
an(1/2*c) - 24*a^2*d^2*tan(1/2*d*x)^2*tan(1/2*c) - 24*a^2*d^2*tan(1/2*d*x)*
tan(1/2*c)^2 + 4*a^2*d^3*x + 24*a^2*d^2*tan(1/2*d*x) + 24*a^2*d^2*tan(1/2*c
))/(d^2*x^4*tan(1/2*d*x)^2*tan(1/2*c)^2 + d^2*x^4*tan(1/2*d*x)^2 + d^2*x^4*
tan(1/2*c)^2 + d^2*x^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^2\sin(c+dx)}{x^5}\,dx = \int \frac{\sin(c+d\,x)\,\left(b\,x^3+a\right)^2}{x^5}\,dx$$

```
[In] int((sin(c + d*x)*(a + b*x^3)^2)/x^5,x)
[Out] int((sin(c + d*x)*(a + b*x^3)^2)/x^5, x)
```

3.94 $\int \frac{x^4 \sin(c+dx)}{a+bx^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 371

$$\begin{aligned}\int \frac{x^4 \sin(c+dx)}{a+bx^3} dx = &-\frac{x\cos(c+dx)}{bd} + \frac{a^{2/3}\operatorname{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{5/3}} \\ &+ \frac{(-1)^{2/3}a^{2/3}\operatorname{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{5/3}} \\ &- \frac{\sqrt[3]{-1}a^{2/3}\operatorname{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{5/3}} \\ &+ \frac{\sin(c+dx)}{bd^2} \\ &- \frac{(-1)^{2/3}a^{2/3}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{5/3}} \\ &+ \frac{a^{2/3}\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b^{5/3}} \\ &- \frac{\sqrt[3]{-1}a^{2/3}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b^{5/3}}\end{aligned}$$

```
[Out] -x*cos(d*x+c)/b/d+1/3*(-1)^(2/3)*a^(2/3)*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3)
)*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/b^(5/3)+1/3*a^(2/3)*cos(c-a^(1/3)*d
/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/b^(5/3)-1/3*(-1)^(1/3)*a^(2/3)*cos(c-(-
1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/b^(5/3)+1/
3*a^(2/3)*Ci(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/b^(5/3)+1/3*(-
1)^(2/3)*a^(2/3)*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1
```

```
/3)*d/b^(1/3))/b^(5/3)-1/3*(-1)^(1/3)*a^(2/3)*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/
3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/b^(5/3)+sin(d*x+c)/b/d^2
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3426, 3377, 2717, 3384, 3380, 3383}

$$\int \frac{x^4\sin(c+dx)}{a+bx^3}\,dx = \frac{a^{2/3}\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{5/3}} + \frac{(-1)^{2/3}a^{2/3}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{5/3}} - \frac{\sqrt[3]{-1}a^{2/3}\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{5/3}} - \frac{(-1)^{2/3}a^{2/3}\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{5/3}} + \frac{a^{2/3}\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{5/3}} - \frac{\sqrt[3]{-1}a^{2/3}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{5/3}} + \frac{\sin(c+dx)}{bd^2} - \frac{x\cos(c+dx)}{bd}$$

```
[In] Int[(x^4*Sin[c + d*x])/(a + b*x^3),x]
```

```
[Out] -((x*Cos[c + d*x])/(b*d)) + (a^(2/3)*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]
*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*b^(5/3)) + ((-1)^(2/3)*a^(2/3)*CosIntegra
l[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1
/3)])/(3*b^(5/3)) - ((-1)^(1/3)*a^(2/3)*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/
b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*b^(5/3)) + Sin[c
 + d*x]/(b*d^2) - ((-1)^(2/3)*a^(2/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3
)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*b^(5/3)) + (a^(2/3
)*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*b
^(5/3)) - ((-1)^(1/3)*a^(2/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIn
tegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*b^(5/3))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \int \left(\frac{x\sin(c+dx)}{b} - \frac{ax\sin(c+dx)}{b(a+bx^3)}\right) dx$$

$$= \frac{\int x\sin(c+dx)\,dx}{b} - \frac{a\int \frac{x\sin(c+dx)}{a+bx^3}\,dx}{b}$$

$$= -\frac{x\cos(c+dx)}{bd} - \frac{a\int\left(-\frac{\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{b} + \frac{\int\cos(c+dx)\,dx}{bd}$$

$$= -\frac{x\cos(c+dx)}{bd} + \frac{\sin(c+dx)}{bd^2} + \frac{a^{2/3}\int\frac{\sin(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{3b^{4/3}} - \frac{\left(\sqrt[3]{-1}a^{2/3}\right)\int\frac{\sin(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}dx}{3b^{4/3}} + \frac{\left((-1)^{2/3}a^{2/3}\right)\int\frac{\sin(c+dx)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}dx}{3b^{4/3}}$$

$$= -\frac{x\cos(c+dx)}{bd} + \frac{\sin(c+dx)}{bd^2} + \frac{\left(a^{2/3}\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{3b^{4/3}} + \frac{\left(\sqrt[3]{-1}a^{2/3}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}dx}{3b^{4/3}} + \frac{\left((-1)^{2/3}a^{2/3}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\sin\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}dx}{3b^{4/3}} + \frac{\left(a^{2/3}\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{3b^{4/3}} - \frac{\left(\sqrt[3]{-1}a^{2/3}\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}dx}{3b^{4/3}} + \frac{\left((-1)^{2/3}a^{2/3}\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\cos\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}dx}{3b^{4/3}}$$

$$= -\frac{x\cos(c+dx)}{bd} + \frac{a^{2/3}\operatorname{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{5/3}}$$
$$+ \frac{(-1)^{2/3}a^{2/3}\operatorname{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{5/3}}$$
$$- \frac{\sqrt[3]{-1}a^{2/3}\operatorname{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{5/3}}$$
$$+ \frac{\sin(c+dx)}{bd^2} - \frac{(-1)^{2/3}a^{2/3}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{5/3}}$$
$$+ \frac{a^{2/3}\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b^{5/3}}$$
$$- \frac{\sqrt[3]{-1}a^{2/3}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b^{5/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.14 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.62

$$\int \frac{x^4\sin(c+dx)}{a+bx^3}\,dx$$
$$= \frac{-iad^2\operatorname{RootSum}\left[a+b\#1^3\&, \frac{\cos(c+d\#1)\operatorname{CosIntegral}(d(x-\#1))-i\operatorname{CosIntegral}(d(x-\#1))\sin(c+d\#1)-i\cos(c+d\#1)\operatorname{Si}(d(x-}{\#1}\right.}{}$$

```
[In] Integrate[(x^4*Sin[c + d*x])/(a + b*x^3),x]
```

```
[Out] ((-I)*a*d^2*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] -
 I*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*SinIntegral[d*(x
 - #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1 & ] + I*a*d^2*RootSum[a
 + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*(x -
 #1)]*Sin[c + d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#
1]*SinIntegral[d*(x - #1)])/#1 & ] + 6*b*(-(d*x*Cos[c + d*x]) + Sin[c + d*x
]))/(6*b^2*d^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.40 (sec) , antiderivative size = 558, normalized size of antiderivative = 1.50

method	result
derivativedivides	$\frac{\frac{d^3c^4\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3\_Z^2bc+3c^2b\_Z+a\,d^3-c^3b\right)}\frac{-\mathrm{Si}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\mathrm{Ci}\left(dx-\_R1+c\right)\sin\left(\_R1\right)}{\_R1^2-2\_R1c+c^2}\right.}{3b}}{}$
default	$\frac{\frac{d^3c^4\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3\_Z^2bc+3c^2b\_Z+a\,d^3-c^3b\right)}\frac{-\mathrm{Si}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\mathrm{Ci}\left(dx-\_R1+c\right)\sin\left(\_R1\right)}{\_R1^2-2\_R1c+c^2}\right.}{3b}}{}$
risch	Expression too large to display

```
[In] int(x^4*sin(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^5*(1/3*d^3*c^4/b*sum(1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci
(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
-4/3*d^3*c^3/b*sum(_R1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x
-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+2*d
^3*c^2/b*sum(_R1^2/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1
+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+4*d^3*c
/b*cos(d*x+c)+4/3/b^2*d^3*c*sum((-3*_R1^2*b*c+3*_R1*b*c^2+a*d^3-b*c^3)/(_R1
^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootO
f(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+(-3*cos(d*x+c)*d^3*c+d^3*(sin(
d*x+c)-cos(d*x+c)*(d*x+c)))/b-1/3/b^2*d^3*sum((-6*_R1^2*b*c^2+_R1*a*d^3+8*_
R1*b*c^3+3*a*c*d^3-3*b*c^4)/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+C
i(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3)
))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.07

$$\int \frac{x^4\sin(c+dx)}{a+bx^3}\,dx$$

$$=\frac{\left(\frac{i\,ad^3}{b}\right)^{\frac{2}{3}}\left(\sqrt{3}+i\right)\mathrm{Ei}\left(-i\,dx+\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\,\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\,\sqrt{3}+1\right)-i\,c\right)}-\left(-\frac{i\,ad^3}{b}\right)^{\frac{2}{3}}\left(\sqrt{3}+i\right)\mathrm{Ei}\left(i\,d\right.}{}$$

```
[In] integrate(x^4*sin(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/12*((I*a*d^3/b)^(2/3)*(sqrt(3) + I)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I
*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) - (-I*a*d^3/
b)^(2/3)*(sqrt(3) + I)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*
e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) - (I*a*d^3/b)^(2/3)*(sqrt(
3) - I)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/
b)^(1/3)*(-I*sqrt(3) + 1) - I*c) + (-I*a*d^3/b)^(2/3)*(sqrt(3) - I)*Ei(I*d*
x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*s
qrt(3) + 1) + I*c) - 12*d*x*cos(d*x + c) + 2*I*(-I*a*d^3/b)^(2/3)*Ei(I*d*x
+ (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) - 2*I*(I*a*d^3/b)^(2/3)*
Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)) + 12*sin(d*x +
c))/(b*d^2)
```

Sympy [F]

$$\int \frac{x^4 \sin(c+dx)}{a+bx^3}\,dx = \int \frac{x^4 \sin{(c+dx)}}{a+bx^3}\,dx$$

```
[In] integrate(x**4*sin(d*x+c)/(b*x**3+a),x)
```

```
[Out] Integral(x**4*sin(c + d*x)/(a + b*x**3), x)
```

Maxima [F]

$$\int \frac{x^4 \sin(c+dx)}{a+bx^3}\,dx = \int \frac{x^4 \sin{(dx+c)}}{bx^3+a}\,dx$$

```
[In] integrate(x^4*sin(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/2*((cos(c)^2 + sin(c)^2)*d*x^4*cos(d*x + c) - (cos(c)^2 + sin(c)^2)*x^3*
sin(d*x + c) + ((d*x^4*cos(c) + x^3*sin(c))*cos(d*x + c)^2 + (d*x^4*cos(c)
+ x^3*sin(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b*cos(c)^2 + b*sin(c)^2
)*d^2*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b*cos(c)^2 +
b*sin(c)^2)*d^2*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d^2)*sin(d*x + c)^2)*integr
ate(-3/2*(a*d*x^3*cos(d*x + c) - a*x^2*sin(d*x + c))/(b^2*d^2*x^6 + 2*a*b*d
^2*x^3 + a^2*d^2), x) + 2*(((b*cos(c)^2 + b*sin(c)^2)*d^2*x^3 + (a*cos(c)^2
 + a*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b*cos(c)^2 + b*sin(c)^2)*d^2*x^3 + (
a*cos(c)^2 + a*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(-3/2*(a*d*x^3*cos(d
*x + c) - a*x^2*sin(d*x + c))/((b^2*d^2*x^6 + 2*a*b*d^2*x^3 + a^2*d^2)*cos(
d*x + c)^2 + (b^2*d^2*x^6 + 2*a*b*d^2*x^3 + a^2*d^2)*sin(d*x + c)^2), x) +
((d*x^4*sin(c) - x^3*cos(c))*cos(d*x + c)^2 + (d*x^4*sin(c) - x^3*cos(c))*s
in(d*x + c)^2)*sin(d*x + 2*c))/(((b*cos(c)^2 + b*sin(c)^2)*d^2*x^3 + (a*cos
(c)^2 + a*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b*cos(c)^2 + b*sin(c)^2)*d^2*x^
3 + (a*cos(c)^2 + a*sin(c)^2)*d^2)*sin(d*x + c)^2)
```

Giac [F]

$$\int \frac{x^4 \sin(c+dx)}{a+bx^3}\,dx = \int \frac{x^4 \sin\left(dx+c\right)}{bx^3+a}\,dx$$

```
[In] integrate(x^4*sin(d*x+c)/(b*x^3+a),x, algorithm="giac")
[Out] integrate(x^4*sin(d*x + c)/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \sin(c+dx)}{a+bx^3}\,dx = \int \frac{x^4 \sin\left(c+d\,x\right)}{b\,x^3+a}\,dx$$

```
[In] int((x^4*sin(c + d*x))/(a + b*x^3),x)
[Out] int((x^4*sin(c + d*x))/(a + b*x^3), x)
```

3.95 $\int \frac{x^3 \sin(c+dx)}{a+bx^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 357

$$\begin{aligned}\int \frac{x^3 \sin(c+dx)}{a+bx^3} \, dx = & -\frac{\cos(c+dx)}{bd} - \frac{\sqrt[3]{a}\,\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{4/3}} \\ & + \frac{\sqrt[3]{-1}\sqrt[3]{a}\,\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{4/3}} \\ & - \frac{(-1)^{2/3}\sqrt[3]{a}\,\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{4/3}} \\ & - \frac{\sqrt[3]{-1}\sqrt[3]{a}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{4/3}} \\ & - \frac{\sqrt[3]{a}\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b^{4/3}} \\ & - \frac{(-1)^{2/3}\sqrt[3]{a}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b^{4/3}}\end{aligned}$$

```
[Out] -cos(d*x+c)/b/d+1/3*(-1)^(1/3)*a^(1/3)*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*
Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/b^(4/3)-1/3*a^(1/3)*cos(c-a^(1/3)*d/b
^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/b^(4/3)-1/3*(-1)^(2/3)*a^(1/3)*cos(c-(-1)
^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/b^(4/3)-1/3*
a^(1/3)*Ci(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/b^(4/3)+1/3*(-1)
^(1/3)*a^(1/3)*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3
)*d/b^(1/3))/b^(4/3)-1/3*(-1)^(2/3)*a^(1/3)*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)
+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/b^(4/3)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3426, 2718, 3414, 3384, 3380, 3383}

$$\int \frac{x^3\sin(c+dx)}{a+bx^3}\,dx = -\frac{\sqrt[3]{a}\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{4/3}} + \frac{\sqrt[3]{-1}\sqrt[3]{a}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{4/3}} - \frac{(-1)^{2/3}\sqrt[3]{a}\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{4/3}} - \frac{\sqrt[3]{-1}\sqrt[3]{a}\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{4/3}} - \frac{\sqrt[3]{a}\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{4/3}} - \frac{(-1)^{2/3}\sqrt[3]{a}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{4/3}} - \frac{\cos(c+dx)}{bd}$$

```
[In] Int[(x^3*Sin[c + d*x])/(a + b*x^3),x]
```

```
[Out] -(Cos[c + d*x]/(b*d)) - (a^(1/3)*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin
[c - (a^(1/3)*d)/b^(1/3)])/(3*b^(4/3)) + ((-1)^(1/3)*a^(1/3)*CosIntegral[((
-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]
)/(3*b^(4/3)) - ((-1)^(2/3)*a^(1/3)*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1
/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*b^(4/3)) - ((-1)^(1/
3)*a^(1/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*
a^(1/3)*d)/b^(1/3) - d*x])/(3*b^(4/3)) - (a^(1/3)*Cos[c - (a^(1/3)*d)/b^(1/
3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*b^(4/3)) - ((-1)^(2/3)*a^(1/
3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*
d)/b^(1/3) + d*x])/(3*b^(4/3))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{\sin(c+dx)}{b} - \frac{a\sin(c+dx)}{b\left(a+bx^3\right)}\right) dx \\
&= \frac{\int \sin(c+dx)\,dx}{b} - \frac{a\int \frac{\sin(c+dx)}{a+bx^3}\,dx}{b} \\
&= -\frac{\cos(c+dx)}{bd} \\
&\quad - \frac{a\int \left(-\frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right) dx}{b} \\
&= -\frac{\cos(c+dx)}{bd} + \frac{\sqrt[3]{a}\int \frac{\sin(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3b} + \frac{\sqrt[3]{a}\int \frac{\sin(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3b} + \frac{\sqrt[3]{a}\int \frac{\sin(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3b}
\end{aligned}
$$

$$= -\frac{\cos(c+dx)}{bd} + \frac{\left(\sqrt[3]{a}\cos\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3b}$$

$$- \frac{\left(\sqrt[3]{a}\cos\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3b}$$

$$+ \frac{\left(\sqrt[3]{a}\cos\left(c - \frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3b}$$

$$+ \frac{\left(\sqrt[3]{a}\sin\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3b}$$

$$+ \frac{\left(\sqrt[3]{a}\sin\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3b}$$

$$+ \frac{\left(\sqrt[3]{a}\sin\left(c - \frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3b}$$

$$= -\frac{\cos(c+dx)}{bd} - \frac{\sqrt[3]{a}\,\mathrm{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{4/3}}$$

$$+ \frac{\sqrt[3]{-1}\sqrt[3]{a}\,\mathrm{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{4/3}}$$

$$- \frac{(-1)^{2/3}\sqrt[3]{a}\,\mathrm{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{4/3}}$$

$$- \frac{\sqrt[3]{-1}\sqrt[3]{a}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{4/3}}$$

$$- \frac{\sqrt[3]{a}\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b^{4/3}}$$

$$- \frac{(-1)^{2/3}\sqrt[3]{a}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b^{4/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.11 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.61

$$\int \frac{x^3 \sin(c+dx)}{a+bx^3} \, dx =$$
$$-\frac{6b\cos(c+dx) + iad\text{RootSum}\left[a+b\#1^3\&, \frac{\cos(c+d\#1)\,\text{CosIntegral}(d(x-\#1)) - i\,\text{CosIntegral}(d(x-\#1))\sin(c+d\#1) - i}{\#1^2}\right.}{}$$

```
[In] Integrate[(x^3*Sin[c + d*x])/(a + b*x^3),x]
```

```
[Out] -1/6*(6*b*Cos[c + d*x] + I*a*d*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosInt
egral[d*(x - #1)] - I*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#
1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1^2 &
] - I*a*d*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] + I
*CosIntegral[d*(x - #1)]*Sin[c + d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x -
 #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1^2 & ])/(b^2*d)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.37 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.10

method	result
derivativedivides	$-\frac{d^3c^3\left(\sum_{\_R1=\text{RootOf}\left(b\_Z^3-3\_Z^2bc+3c^2b\_Z+a\,d^3-c^3b\right)} \frac{-\text{Si}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\text{Ci}\left(dx-\_R1+c\right)\sin\left(\_R\right.}{\_R1^2-2\_R1c+c^2}\right.}{3b}$
default	$-\frac{d^3c^3\left(\sum_{\_R1=\text{RootOf}\left(b\_Z^3-3\_Z^2bc+3c^2b\_Z+a\,d^3-c^3b\right)} \frac{-\text{Si}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\text{Ci}\left(dx-\_R1+c\right)\sin\left(\_R\right.}{\_R1^2-2\_R1c+c^2}\right.}{3b}$
risch	$\frac{i\left(\sum_{\_R1=\text{RootOf}\left(-3i\_Z^2bc-id^3a+ib\,c^3+b\_Z^3-3c^2b\_Z\right)} \frac{e^{\_R1}\,\text{Ei}_1\left(-idx-ic+\_R1\right)}{-2ic\_R1+\_R1^2-c^2}\right)c^3}{6db} - \frac{i\left(\sum_{\_R1=\text{RootOf}\left(-3i\_Z^2bc\right.}\right.}{}$

```
[In] int(x^3*sin(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^4*(-1/3*d^3*c^3/b*sum(1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+C
i(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3)
)+d^3*c^2/b*sum(_R1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R
1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-d^3*c/
```

```
b*sum(_R1^2/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin
(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-d^3/b*cos(d*x+
c)-1/3/b^2*d^3*sum((-3*_R1^2*b*c+3*_R1*b*c^2+a*d^3-b*c^3)/(_R1^2-2*_R1*c+c^
2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z
^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.10

$$\int \frac{x^3 \sin(c+dx)}{a+bx^3}\,dx = \frac{\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}(-i\sqrt{3}-1)\mathrm{Ei}\left(-i\,dx+\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}(-i\sqrt{3}-1)\right)e^{\left(\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}+1\right)-i\,c\right)}+\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}(-i\sqrt{3}-1}{}$$

```
[In] integrate(x^3*sin(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/12*((I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*
(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) + (-I*a*d
^3/b)^(1/3)*(-I*sqrt(3) - 1)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3)
- 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) + (I*a*d^3/b)^(1/3)*
(I*sqrt(3) - 1)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(
I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) - I*c) + (-I*a*d^3/b)^(1/3)*(I*sqrt(3) -
1)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(
1/3)*(-I*sqrt(3) + 1) + I*c) + 2*(-I*a*d^3/b)^(1/3)*Ei(I*d*x + (-I*a*d^3/b)
^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) + 2*(I*a*d^3/b)^(1/3)*Ei(-I*d*x + (I*a
*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)) - 12*cos(d*x + c))/(b*d)
```

Sympy [F]

$$\int \frac{x^3 \sin(c+dx)}{a+bx^3}\,dx = \int \frac{x^3 \sin{(c+dx)}}{a+bx^3}\,dx$$

```
[In] integrate(x**3*sin(d*x+c)/(b*x**3+a),x)
```

```
[Out] Integral(x**3*sin(c + d*x)/(a + b*x**3), x)
```

Maxima [F]

$$\int \frac{x^3 \sin(c+dx)}{a+bx^3}\,dx = \int \frac{x^3 \sin{(dx+c)}}{bx^3+a}\,dx$$

```
[In] integrate(x^3*sin(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/2*((cos(c)^2 + sin(c)^2)*x^3*cos(d*x + c) + (x^3*cos(d*x + c)^2*cos(c) +
 x^3*cos(c)*sin(d*x + c)^2)*cos(d*x + 2*c) - 6*(((a*b*cos(c)^2 + a*b*sin(c)
^2)*d*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*cos(d*x + c)^2 + ((a*b*cos(c)^
2 + a*b*sin(c)^2)*d*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*sin(d*x + c)^2)*
integrate(1/2*x^2*cos(d*x + c)/(b^2*d*x^6 + 2*a*b*d*x^3 + a^2*d), x) - 6*((
(a*b*cos(c)^2 + a*b*sin(c)^2)*d*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*cos(
d*x + c)^2 + ((a*b*cos(c)^2 + a*b*sin(c)^2)*d*x^3 + (a^2*cos(c)^2 + a^2*sin
(c)^2)*d)*sin(d*x + c)^2)*integrate(1/2*x^2*cos(d*x + c)/((b^2*d*x^6 + 2*a*
b*d*x^3 + a^2*d)*cos(d*x + c)^2 + (b^2*d*x^6 + 2*a*b*d*x^3 + a^2*d)*sin(d*x
 + c)^2), x) + (x^3*cos(d*x + c)^2*sin(c) + x^3*sin(d*x + c)^2*sin(c))*sin(
d*x + 2*c))/(((b*cos(c)^2 + b*sin(c)^2)*d*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d
)*cos(d*x + c)^2 + ((b*cos(c)^2 + b*sin(c)^2)*d*x^3 + (a*cos(c)^2 + a*sin(c
)^2)*d)*sin(d*x + c)^2)
```

Giac [F]

$$\int \frac{x^3 \sin(c+dx)}{a+bx^3}\,dx = \int \frac{x^3 \sin{(dx+c)}}{bx^3+a}\,dx$$

```
[In] integrate(x^3*sin(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*sin(d*x + c)/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \sin(c+dx)}{a+bx^3}\,dx = \int \frac{x^3\,\sin{(c+d\,x)}}{b\,x^3+a}\,dx$$

```
[In] int((x^3*sin(c + d*x))/(a + b*x^3),x)
```

```
[Out] int((x^3*sin(c + d*x))/(a + b*x^3), x)
```

3.96 $\int \frac{x^2 \sin(c+dx)}{a+bx^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 281

$$\int \frac{x^2 \sin(c+dx)}{a+bx^3} dx = \frac{\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b} + \frac{\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b} + \frac{\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b} - \frac{\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b} + \frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b} + \frac{\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b}$$

```
[Out] 1/3*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*
x)/b+1/3*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/b+1/3*cos(c-(-1
)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/b+1/3*Ci(a^
(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/b+1/3*Ci((-1)^(1/3)*a^(1/3)*d
/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/b+1/3*Ci((-1)^(2/3)*a^(1/
3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/b
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3426, 3384, 3380, 3383}

$$\int \frac{x^2 \sin(c+dx)}{a+bx^3}\,dx = \frac{\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b} + \frac{\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b} + \frac{\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b} - \frac{\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b} + \frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b} + \frac{\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b}$$

```
[In] Int[(x^2*Sin[c + d*x])/(a + b*x^3),x]
```

```
[Out] (CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*b)
 + (CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a
^(1/3)*d)/b^(1/3)])/(3*b) + (CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d
*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*b) - (Cos[c + ((-1)^(1/3)*a
^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*b)
 + (Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3
*b) + (Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1
/3)*d)/b^(1/3) + d*x])/(3*b)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\sin(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sin(c+dx)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sin(c+dx)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)} \right) dx$$

$$= \frac{\int \frac{\sin(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}} + \frac{\int \frac{\sin(c+dx)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}} + \frac{\int \frac{\sin(c+dx)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}}$$

$$= \frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sin\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}} - \frac{\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}}$$

$$+ \frac{\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sin\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}}$$

$$+ \frac{\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\cos\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}}$$

$$+ \frac{\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}}$$

$$+ \frac{\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\cos\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}}$$

$$
\begin{aligned}
&= \frac{\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b} \\
&+ \frac{\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b} \\
&+ \frac{\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b} \\
&- \frac{\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b} + \frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b} \\
&+ \frac{\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 5.04 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.66

$$
\int \frac{x^2\sin(c+dx)}{a+bx^3}\,dx = \frac{i\left(\text{RootSum}\left[a+b\#1^3\&,\cos(c+d\#1)\,\text{CosIntegral}(d(x-\#1))-i\,\text{CosIntegral}(d(x-\#1))\sin(c+d\#1)\right.\right.}{}
$$

```
[In] Integrate[(x^2*Sin[c + d*x])/(a + b*x^3),x]
```

```
[Out] ((I/6)*(RootSum[a + b*#1^3 & , Cos[c + d*#1]*CosIntegral[d*(x - #1)] - I*Co
sIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*SinIntegral[d*(x - #1
)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)] & ] - RootSum[a + b*#1^3 & , Cos
[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*(x - #1)]*Sin[c + d*#1
] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinIntegral[d*(
x - #1)] & ]))/b
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.29 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{d^3c^2\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3\_Z^2bc+3c^2b\_Z+a\,d^3-c^3b\right)}\frac{-\mathrm{Si}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\mathrm{Ci}\left(dx-\_R1+c\right)\sin\left(\_R1\right)}{\_R1^2-2\_R1c+c^2}\right.}{3b}$
default	$\frac{d^3c^2\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3\_Z^2bc+3c^2b\_Z+a\,d^3-c^3b\right)}\frac{-\mathrm{Si}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\mathrm{Ci}\left(dx-\_R1+c\right)\sin\left(\_R1\right)}{\_R1^2-2\_R1c+c^2}\right.}{3b}$
risch	$-\frac{ic^2\left(\sum_{\_R1=\mathrm{RootOf}\left(-3i\_Z^2bc-id^3a+ibc^3+b\_Z^3-3c^2b\_Z\right)}\frac{e^{\_R1}\mathrm{Ei}_1\left(-idx-ic+\_R1\right)}{-2ic\_R1+\_R1^2-c^2}\right)}{6b}+\frac{ic^2\left(\sum_{\_R1=\mathrm{RootOf}\left(-3i\_Z\right.}\right.}{}$

[In] int(x^2*sin(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] 1/d^3*(1/3*d^3*c^2/b*sum(1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci
(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
-2/3*d^3*c/b*sum(_R1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_
R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/3*d
^3/b*sum(_R1^2/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*
sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3)))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.04

$$\int \frac{x^2\sin(c+dx)}{a+bx^3}\,dx$$
$$=\frac{i\,\mathrm{Ei}\left(-i\,dx+\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}+1\right)-i\,c\right)}-i\,\mathrm{Ei}\left(i\,dx+\frac{1}{2}\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\right.}}{}$$

[In] integrate(x^2*sin(d*x+c)/(b*x^3+a),x, algorithm="fricas")

[Out] 1/6*(I*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/
b)^(1/3)*(I*sqrt(3) + 1) - I*c) - I*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*s
qrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) + I*Ei(-I*d*x
 + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt
(3) + 1) - I*c) - I*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1
/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) - I*Ei(I*d*x + (-I*a*d^3/b)^(
1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) + I*Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I
*c - (I*a*d^3/b)^(1/3)))/b

Sympy [F]

$$\int \frac{x^2 \sin(c+dx)}{a+bx^3} \, dx = \int \frac{x^2 \sin{(c+dx)}}{a+bx^3} \, dx$$

```
[In] integrate(x**2*sin(d*x+c)/(b*x**3+a),x)
```

```
[Out] Integral(x**2*sin(c + d*x)/(a + b*x**3), x)
```

Maxima [F]

$$\int \frac{x^2 \sin(c+dx)}{a+bx^3} \, dx = \int \frac{x^2 \sin{(dx+c)}}{bx^3+a} \, dx$$

```
[In] integrate(x^2*sin(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/2*((cos(c)^2 + sin(c)^2)*d*x^2*cos(d*x + c) + (cos(c)^2 + sin(c)^2)*x*si
n(d*x + c) + ((d*x^2*cos(c) - x*sin(c))*cos(d*x + c)^2 + (d*x^2*cos(c) - x*
sin(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b*cos(c)^2 + b*sin(c)^2)*d^2*
x^3 + (a*cos(c)^2 + a*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b*cos(c)^2 + b*sin(
c)^2)*d^2*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(-1
/2*(3*a*d*x*cos(d*x + c) - (2*b*x^3 - a)*sin(d*x + c))/(b^2*d^2*x^6 + 2*a*b
*d^2*x^3 + a^2*d^2), x) + 2*(((b*cos(c)^2 + b*sin(c)^2)*d^2*x^3 + (a*cos(c)
^2 + a*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b*cos(c)^2 + b*sin(c)^2)*d^2*x^3 +
 (a*cos(c)^2 + a*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(-1/2*(3*a*d*x*cos
(d*x + c) - (2*b*x^3 - a)*sin(d*x + c))/((b^2*d^2*x^6 + 2*a*b*d^2*x^3 + a^2
*d^2)*cos(d*x + c)^2 + (b^2*d^2*x^6 + 2*a*b*d^2*x^3 + a^2*d^2)*sin(d*x + c)
^2), x) + ((d*x^2*sin(c) + x*cos(c))*cos(d*x + c)^2 + (d*x^2*sin(c) + x*cos
(c))*sin(d*x + c)^2)*sin(d*x + 2*c))/(((b*cos(c)^2 + b*sin(c)^2)*d^2*x^3 +
(a*cos(c)^2 + a*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b*cos(c)^2 + b*sin(c)^2)*
d^2*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d^2)*sin(d*x + c)^2)
```

Giac [F]

$$\int \frac{x^2 \sin(c+dx)}{a+bx^3} \, dx = \int \frac{x^2 \sin{(dx+c)}}{bx^3+a} \, dx$$

```
[In] integrate(x^2*sin(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*sin(d*x + c)/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \sin(c+dx)}{a+bx^3}\,dx = \int \frac{x^2\,\sin\left(c+d\,x\right)}{b\,x^3+a}\,dx$$

```
[In] int((x^2*sin(c + d*x))/(a + b*x^3),x)
[Out] int((x^2*sin(c + d*x))/(a + b*x^3), x)
```

3.97 $\int \frac{x\sin(c+dx)}{a+bx^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 343

$$\begin{aligned}\int \frac{x\sin(c+dx)}{a+bx^3}\,dx = &-\frac{\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}b^{2/3}} \\ &-\frac{(-1)^{2/3}\,\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}b^{2/3}} \\ &+\frac{\sqrt[3]{-1}\,\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}b^{2/3}} \\ &+\frac{(-1)^{2/3}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3\sqrt[3]{a}b^{2/3}} \\ &-\frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3\sqrt[3]{a}b^{2/3}} \\ &+\frac{\sqrt[3]{-1}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3\sqrt[3]{a}b^{2/3}}\end{aligned}$$

```
[Out] -1/3*(-1)^(2/3)*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*
d/b^(1/3)+d*x)/a^(1/3)/b^(2/3)-1/3*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^
(1/3)+d*x)/a^(1/3)/b^(2/3)+1/3*(-1)^(1/3)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3
))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(1/3)/b^(2/3)-1/3*Ci(a^(1/3)*d/b^
(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(1/3)/b^(2/3)-1/3*(-1)^(2/3)*Ci((-1)^
(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(1/3)/b^
```

```
(2/3)+1/3*(-1)^(1/3)*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*
a^(1/3)*d/b^(1/3))/a^(1/3)/b^(2/3)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3426, 3384, 3380, 3383}

$$\int \frac{x\sin(c+dx)}{a+bx^3}\,dx = -\frac{\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}b^{2/3}} - \frac{(-1)^{2/3}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3\sqrt[3]{a}b^{2/3}} + \frac{\sqrt[3]{-1}\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}b^{2/3}} + \frac{(-1)^{2/3}\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3\sqrt[3]{a}b^{2/3}} - \frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}b^{2/3}} + \frac{\sqrt[3]{-1}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}b^{2/3}}$$

```
[In] Int[(x*Sin[c + d*x])/(a + b*x^3),x]
```

```
[Out] -1/3*(CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/
(a^(1/3)*b^(2/3)) - ((-1)^(2/3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3)
- d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(1/3)*b^(2/3)) + ((-1)
^(1/3)*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3
)*a^(1/3)*d)/b^(1/3)])/(3*a^(1/3)*b^(2/3)) + ((-1)^(2/3)*Cos[c + ((-1)^(1/3
)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3
*a^(1/3)*b^(2/3)) - (Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b
^(1/3) + d*x])/(3*a^(1/3)*b^(2/3)) + ((-1)^(1/3)*Cos[c - ((-1)^(2/3)*a^(1/3
)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(1/3)
*b^(2/3))
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right) dx$$

$$= -\frac{\int \frac{\sin(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}} + \frac{\sqrt[3]{-1}\int \frac{\sin(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}} - \frac{(-1)^{2/3}\int \frac{\sin(c+dx)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}$$

$$= -\frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sin\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}$$

$$-\frac{\left(\sqrt[3]{-1}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}$$

$$-\frac{\left((-1)^{2/3}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}$$

$$-\frac{\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\cos\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}$$

$$+\frac{\left(\sqrt[3]{-1}\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}$$

$$-\frac{\left((-1)^{2/3}\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}$$

$$= -\frac{\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}b^{2/3}} - \frac{(-1)^{2/3}\,\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}b^{2/3}} + \frac{\sqrt[3]{-1}\,\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}b^{2/3}} + \frac{(-1)^{2/3}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3\sqrt[3]{a}b^{2/3}} - \frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3\sqrt[3]{a}b^{2/3}} + \frac{\sqrt[3]{-1}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3\sqrt[3]{a}b^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 5.05 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.57

$$\int \frac{x\sin(c+dx)}{a+bx^3}\,dx = \frac{i\Big(\text{RootSum}\Big[a+b\#1^3\&,\frac{\cos(c+d\#1)\,\text{CosIntegral}(d(x-\#1))-i\,\text{CosIntegral}(d(x-\#1))\sin(c+d\#1)-i\cos(c+d\#1)\text{Si}(d(x-\#1))}{\#1}}{}$$

```
[In] Integrate[(x*Sin[c + d*x])/(a + b*x^3),x]
```

```
[Out] ((I/6)*(RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] - I*C
osIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*SinIntegral[d*(x - #
1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1 & ] - RootSum[a + b*#1^3 &
, (Cos[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*(x - #1)]*Sin[c
+ d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinIntegr
al[d*(x - #1)])/#1 & ]))/b
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.28 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.51

method	result
derivativedivides	$\frac{d^3\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3\_Z^2bc+3c^2b\_Z+a\,d^3-c^3b\right)}\frac{\_R1\left(-\mathrm{Si}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\mathrm{Ci}\left(dx-\_R1+c\right)\sin\left(\_\right.\right.}{\_R1^2-2\_R1c+c^2}\right.}{3b}$
default	$\frac{d^3\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3\_Z^2bc+3c^2b\_Z+a\,d^3-c^3b\right)}\frac{\_R1\left(-\mathrm{Si}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\mathrm{Ci}\left(dx-\_R1+c\right)\sin\left(\_\right.\right.}{\_R1^2-2\_R1c+c^2}\right.}{3b}$
risch	$\frac{d\left(\sum_{\_R1=\mathrm{RootOf}\left(-3i\_Z^2bc-id^3a+ib\,c^3+b\_Z^3-3c^2b\_Z\right)}\frac{\_R1\,\mathrm{e}^{-\_R1}\,\mathrm{Ei}_1\left(idx+ic-\_R1\right)}{-2ic\_R1+\_R1^2-c^2}\right)}{6b}-\frac{idc\left(\sum_{\_R1=\mathrm{RootOf}\left(-3i\_\right.}\right.}{}$

```
[In] int(x*sin(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(1/3*d^3/b*sum(_R1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d
*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1
/3*d^3*c/b*sum(1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c
)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.10

$$\int \frac{x\sin(c+dx)}{a+bx^3}\,dx =$$

$$-\frac{\left(\frac{i\,ad^3}{b}\right)^{\frac{2}{3}}\left(\sqrt{3}+i\right)\mathrm{Ei}\left(-i\,dx+\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}+1\right)-i\,c\right)}-\left(-\frac{i\,ad^3}{b}\right)^{\frac{2}{3}}\left(\sqrt{3}+i\right)\mathrm{Ei}\left(\right.}{}$$

```
[In] integrate(x*sin(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/12*((I*a*d^3/b)^(2/3)*(sqrt(3) + I)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-
I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) - (-I*a*d^3
/b)^(2/3)*(sqrt(3) + I)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))
*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) - (I*a*d^3/b)^(2/3)*(sqrt
(3) - I)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3
/b)^(1/3)*(-I*sqrt(3) + 1) - I*c) + (-I*a*d^3/b)^(2/3)*(sqrt(3) - I)*Ei(I*d
*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*
```

```
sqrt(3) + 1) + I*c) + 2*I*(-I*a*d^3/b)^(2/3)*Ei(I*d*x + (-I*a*d^3/b)^(1/3))
*e^(I*c - (-I*a*d^3/b)^(1/3)) - 2*I*(I*a*d^3/b)^(2/3)*Ei(-I*d*x + (I*a*d^3/
b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)))/(a*d^2)
```

Sympy [F]

$$\int \frac{x\sin(c+dx)}{a+bx^3}\,dx = \int \frac{x\sin(c+dx)}{a+bx^3}\,dx$$

```
[In] integrate(x*sin(d*x+c)/(b*x**3+a),x)
[Out] Integral(x*sin(c + d*x)/(a + b*x**3), x)
```

Maxima [F]

$$\int \frac{x\sin(c+dx)}{a+bx^3}\,dx = \int \frac{x\sin(dx+c)}{bx^3+a}\,dx$$

```
[In] integrate(x*sin(d*x+c)/(b*x^3+a),x, algorithm="maxima")
[Out] -1/2*((cos(c)^2 + sin(c)^2)*x*cos(d*x + c) + (x*cos(d*x + c)^2*cos(c) + x*c
os(c)*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b*cos(c)^2 + b*sin(c)^2)*d*x^3
+ (a*cos(c)^2 + a*sin(c)^2)*d)*cos(d*x + c)^2 + ((b*cos(c)^2 + b*sin(c)^2)*
d*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d)*sin(d*x + c)^2)*integrate(1/2*(2*b*x^3
 - a)*cos(d*x + c)/(b^2*d*x^6 + 2*a*b*d*x^3 + a^2*d), x) + 2*(((b*cos(c)^2
+ b*sin(c)^2)*d*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d)*cos(d*x + c)^2 + ((b*cos
(c)^2 + b*sin(c)^2)*d*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d)*sin(d*x + c)^2)*in
tegrate(1/2*(2*b*x^3 - a)*cos(d*x + c)/((b^2*d*x^6 + 2*a*b*d*x^3 + a^2*d)*c
os(d*x + c)^2 + (b^2*d*x^6 + 2*a*b*d*x^3 + a^2*d)*sin(d*x + c)^2), x) + (x*
cos(d*x + c)^2*sin(c) + x*sin(d*x + c)^2*sin(c))*sin(d*x + 2*c))/(((b*cos(c
)^2 + b*sin(c)^2)*d*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d)*cos(d*x + c)^2 + ((b
*cos(c)^2 + b*sin(c)^2)*d*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d)*sin(d*x + c)^2
)
```

Giac [F]

$$\int \frac{x\sin(c+dx)}{a+bx^3}\,dx = \int \frac{x\sin(dx+c)}{bx^3+a}\,dx$$

```
[In] integrate(x*sin(d*x+c)/(b*x^3+a),x, algorithm="giac")
[Out] integrate(x*sin(d*x + c)/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sin(c+dx)}{a+bx^3}\,dx = \int \frac{x\,\sin\left(c+d\,x\right)}{b\,x^3+a}\,dx$$

```
[In] int((x*sin(c + d*x))/(a + b*x^3),x)
[Out] int((x*sin(c + d*x))/(a + b*x^3), x)
```

3.98 $\int \frac{\sin(c+dx)}{a+bx^3} dx$

Optimal result

Integrand size = 16, antiderivative size = 343

$$\begin{aligned}\int \frac{\sin(c+dx)}{a+bx^3} dx &= \frac{\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}} \\ &- \frac{\sqrt[3]{-1}\,\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}} \\ &+ \frac{(-1)^{2/3}\,\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}} \\ &+ \frac{\sqrt[3]{-1}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}} \\ &+ \frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{2/3}\sqrt[3]{b}} \\ &+ \frac{(-1)^{2/3}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{2/3}\sqrt[3]{b}}\end{aligned}$$

```
[Out] -1/3*(-1)^(1/3)*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*
d/b^(1/3)+d*x)/a^(2/3)/b^(1/3)+1/3*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^
(1/3)+d*x)/a^(2/3)/b^(1/3)+1/3*(-1)^(2/3)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3
))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(2/3)/b^(1/3)+1/3*Ci(a^(1/3)*d/b^
(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(2/3)/b^(1/3)-1/3*(-1)^(1/3)*Ci((-1)^
(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(2/3)/b^
```

```
(1/3)+1/3*(-1)^(2/3)*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*
a^(1/3)*d/b^(1/3))/a^(2/3)/b^(1/3)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3414, 3384, 3380, 3383}

$$\int \frac{\sin(c+dx)}{a+bx^3}\,dx = \frac{\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{\sqrt[3]{-1}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}} + \frac{(-1)^{2/3}\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}} + \frac{\sqrt[3]{-1}\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}} + \frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}} + \frac{(-1)^{2/3}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}$$

```
[In] Int[Sin[c + d*x]/(a + b*x^3),x]
```

```
[Out] (CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*a^
(2/3)*b^(1/3)) - ((-1)^(1/3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d
*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(2/3)*b^(1/3)) + ((-1)^(2
/3)*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a
^(1/3)*d)/b^(1/3)])/(3*a^(2/3)*b^(1/3)) + ((-1)^(1/3)*Cos[c + ((-1)^(1/3)*a
^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^
(2/3)*b^(1/3)) + (Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1
/3) + d*x])/(3*a^(2/3)*b^(1/3)) + ((-1)^(2/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d
)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(2/3)*b^
(1/3))
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)}-\frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}-\frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right) dx \\
&= -\frac{\int \frac{\sin(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3a^{2/3}}-\frac{\int \frac{\sin(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3a^{2/3}}-\frac{\int \frac{\sin(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3a^{2/3}}
\end{aligned}$$

$$= -\frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\sin\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3a^{2/3}}$$

$$+\frac{\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3a^{2/3}}$$

$$-\frac{\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\sin\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3a^{2/3}}$$

$$-\frac{\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\cos\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3a^{2/3}}$$

$$-\frac{\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3a^{2/3}}$$

$$-\frac{\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\cos\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3a^{2/3}}$$

$$=\frac{\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$-\frac{\sqrt[3]{-1}\,\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$+\frac{(-1)^{2/3}\,\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$+\frac{\sqrt[3]{-1}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$+\frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$+\frac{(-1)^{2/3}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{2/3}\sqrt[3]{b}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 5.05 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.57

$$\int \frac{\sin(c+dx)}{a+bx^3}\,dx = \frac{i\left(\text{RootSum}\left[a+b\#1^3\&,\frac{\cos(c+d\#1)\,\text{CosIntegral}(d(x-\#1))-i\,\text{CosIntegral}(d(x-\#1))\sin(c+d\#1)-i\cos(c+d\#1)\text{Si}(d(x-\#1)}{\#1^2}\right.\right.}{}$$

```
[In] Integrate[Sin[c + d*x]/(a + b*x^3),x]
```

```
[Out] ((I/6)*(RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] - I*C
osIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*SinIntegral[d*(x - #
1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1^2 & ] - RootSum[a + b*#1^3
& , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*(x - #1)]*Sin[
c + d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinInte
gral[d*(x - #1)])/#1^2 & ]))/b
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.25

method	result
derivativedivides	$\frac{d^2\left(\sum_{\_R1=\text{RootOf}\left(b\_Z^3-3\_Z^2bc+3c^2b\_Z+a\,d^3-c^3b\right)}\frac{-\text{Si}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\text{Ci}\left(dx-\_R1+c\right)\sin\left(\_R1\right)}{\_R1^2-2\_R1c+c^2}\right.}{3b}$
default	$\frac{d^2\left(\sum_{\_R1=\text{RootOf}\left(b\_Z^3-3\_Z^2bc+3c^2b\_Z+a\,d^3-c^3b\right)}\frac{-\text{Si}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\text{Ci}\left(dx-\_R1+c\right)\sin\left(\_R1\right)}{\_R1^2-2\_R1c+c^2}\right.}{3b}$
risch	$-\frac{id^2\left(\sum_{\_R1=\text{RootOf}\left(-3i\_Z^2bc-id^3a+ibc^3+b\_Z^3-3c^2b\_Z\right)}\frac{e^{\_R1}\,\text{Ei}_1\left(-idx-ic+\_R1\right)}{-2ic\_R1+\_R1^2-c^2}\right)}{6b}+\frac{id^2\left(\sum_{\_R1=\text{RootOf}\left(-3i\right.}\right.}{}$

```
[In] int(sin(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*d^2/b*sum(1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)
*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.12

$$\int \frac{\sin(c+dx)}{a+bx^3}\,dx$$
$$= \frac{\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}(i\sqrt{3}+1)\mathrm{Ei}\left(-i\,dx+\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}(-i\sqrt{3}-1)\right)e^{\left(\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}(i\sqrt{3}+1)-i\,c\right)}+\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}(i\sqrt{3}+1)\mathrm{Ei}}{}$$

```
[In] integrate(sin(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/12*((I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(
-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) + (-I*a*d^
3/b)^(1/3)*(I*sqrt(3) + 1)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) -
1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) + (I*a*d^3/b)^(1/3)*(-
I*sqrt(3) + 1)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I
*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) - I*c) + (-I*a*d^3/b)^(1/3)*(-I*sqrt(3) +
1)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(
1/3)*(-I*sqrt(3) + 1) + I*c) - 2*(-I*a*d^3/b)^(1/3)*Ei(I*d*x + (-I*a*d^3/b)
^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) - 2*(I*a*d^3/b)^(1/3)*Ei(-I*d*x + (I*a
*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)))/(a*d)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{a+bx^3}\,dx = \int \frac{\sin(c+dx)}{a+bx^3}\,dx$$

```
[In] integrate(sin(d*x+c)/(b*x**3+a),x)
```

```
[Out] Integral(sin(c + d*x)/(a + b*x**3), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{a+bx^3}\,dx = \int \frac{\sin(dx+c)}{bx^3+a}\,dx$$

```
[In] integrate(sin(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{a+bx^3}\,dx = \int \frac{\sin(dx+c)}{bx^3+a}\,dx$$

```
[In] integrate(sin(d*x+c)/(b*x^3+a),x, algorithm="giac")
[Out] integrate(sin(d*x + c)/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{a+bx^3}\,dx = \int \frac{\sin(c+d\,x)}{b\,x^3+a}\,dx$$

```
[In] int(sin(c + d*x)/(a + b*x^3),x)
[Out] int(sin(c + d*x)/(a + b*x^3), x)
```

3.99 $\int \frac{\sin(c+dx)}{x(a+bx^3)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 301

$$\begin{aligned}\int \frac{\sin(c+dx)}{x(a+bx^3)}\,dx &= \frac{\text{CosIntegral}(dx)\sin(c)}{a} - \frac{\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a} \\ &- \frac{\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a} \\ &- \frac{\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a} \\ &+ \frac{\cos(c)\text{Si}(dx)}{a} + \frac{\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a} \\ &- \frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a} \\ &- \frac{\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a}\end{aligned}$$

```
[Out] cos(c)*Si(d*x)/a-1/3*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(
1/3)*d/b^(1/3)+d*x)/a-1/3*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x
)/a-1/3*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)
+d*x)/a+Ci(d*x)*sin(c)/a-1/3*Ci(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1
/3))/a-1/3*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/
b^(1/3))/a-1/3*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3
)*d/b^(1/3))/a
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3426, 3384, 3380, 3383}

$$\int \frac{\sin(c+dx)}{x(a+bx^3)}\,dx = -\frac{\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a} - \frac{\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a} - \frac{\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a} + \frac{\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a} - \frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a} - \frac{\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a} + \frac{\sin(c)\text{CosIntegral}(dx)}{a} + \frac{\cos(c)\text{Si}(dx)}{a}$$

```
[In] Int[Sin[c + d*x]/(x*(a + b*x^3)),x]
```

```
[Out] (CosIntegral[d*x]*Sin[c])/a - (CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c
 - (a^(1/3)*d)/b^(1/3)])/(3*a) - (CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3
) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(3*a) - (CosIntegral[((-1
)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/
(3*a) + (Cos[c]*SinIntegral[d*x])/a + (Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/
3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a) - (Cos[c - (a^
(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a) - (Cos[c -
((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3)
+ d*x])/(3*a)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\sin(c+dx)}{ax} - \frac{bx^2\sin(c+dx)}{a(a+bx^3)}\right) dx \\
&= \frac{\int \frac{\sin(c+dx)}{x}\,dx}{a} - \frac{b\int \frac{x^2\sin(c+dx)}{a+bx^3}\,dx}{a} \\
&= -\frac{b\int \left(\frac{\sin(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sin(c+dx)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sin(c+dx)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right) dx}{a} \\
&\quad + \frac{\cos(c)\int \frac{\sin(dx)}{x}\,dx}{a} + \frac{\sin(c)\int \frac{\cos(dx)}{x}\,dx}{a} \\
&= \frac{\text{CosIntegral}(dx)\sin(c)}{a} + \frac{\cos(c)\text{Si}(dx)}{a} - \frac{\sqrt[3]{b}\int \frac{\sin(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a} \\
&\quad - \frac{\sqrt[3]{b}\int \frac{\sin(c+dx)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a} - \frac{\sqrt[3]{b}\int \frac{\sin(c+dx)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a}
\end{aligned}$$

$$= \frac{\text{CosIntegral}(dx)\sin(c)}{a} + \frac{\cos(c)\text{Si}(dx)}{a} - \frac{\left(\sqrt[3]{b}\cos\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a}$$

$$+ \frac{\left(\sqrt[3]{b}\cos\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a}$$

$$- \frac{\left(\sqrt[3]{b}\cos\left(c - \frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a}$$

$$- \frac{\left(\sqrt[3]{b}\sin\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a}$$

$$- \frac{\left(\sqrt[3]{b}\sin\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a}$$

$$- \frac{\left(\sqrt[3]{b}\sin\left(c - \frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a}$$

$$= \frac{\text{CosIntegral}(dx)\sin(c)}{a} - \frac{\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a}$$

$$- \frac{\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a}$$

$$- \frac{\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c - \frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a}$$

$$+ \frac{\cos(c)\text{Si}(dx)}{a} + \frac{\cos\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a}$$

$$- \frac{\cos\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a} - \frac{\cos\left(c - \frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.13 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.68

$$\int \frac{\sin(c+dx)}{x(a+bx^3)}\,dx = \frac{-i\text{RootSum}\left[a+b\#1^3\&,\cos(c+d\#1)\,\text{CosIntegral}(d(x-\#1))-i\,\text{CosIntegral}(d(x-\#1))\sin(c+d\#1)\right.}{}$$

```
[In] Integrate[Sin[c + d*x]/(x*(a + b*x^3)),x]
```

```
[Out] ((-I)*RootSum[a + b*#1^3 & , Cos[c + d*#1]*CosIntegral[d*(x - #1)] - I*CosI
ntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*SinIntegral[d*(x - #1)]
 - Sin[c + d*#1]*SinIntegral[d*(x - #1)] & ] + I*RootSum[a + b*#1^3 & , Cos
[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*(x - #1)]*Sin[c + d*#1
] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinIntegral[d*(
x - #1)] & ] + 6*CosIntegral[d*x]*Sin[c] + 6*Cos[c]*SinIntegral[d*x])/(6*a)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.29

method	result
derivativedivides	$\frac{\text{Si}(dx)\cos(c)+\text{Ci}(dx)\sin(c)}{a}-\frac{\sum_{\_R1=\text{RootOf}\left(b\_Z^3-3\_Z^2bc+3c^2b\_Z+a\,d^3-c^3b\right)}\left(-\text{Si}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\text{C}\right.}{3a}$
default	$\frac{\text{Si}(dx)\cos(c)+\text{Ci}(dx)\sin(c)}{a}-\frac{\sum_{\_R1=\text{RootOf}\left(b\_Z^3-3\_Z^2bc+3c^2b\_Z+a\,d^3-c^3b\right)}\left(-\text{Si}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\text{C}\right.}{3a}$
risch	$-\frac{i\left(\sum_{\_R1=\text{RootOf}\left(-3i\_Z^2bc-id^3a+ibc^3+b\_Z^3-3c^2b\_Z\right)}e^{\_R1}\,\text{Ei}_1\left(-idx-ic+\_R1\right)\right)}{6a}+\frac{ie^{ic}\,\text{Ei}_1(-idx)}{2a}+\frac{i\left(\_R1\right.}{}$

```
[In] int(sin(d*x+c)/x/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))-1/3/a*sum(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d
*x-_R1+c)*sin(_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.02

$$\int \frac{\sin(c+dx)}{x(a+bx^3)}\,dx$$

$$= \frac{-i\,\mathrm{Ei}\left(-i\,dx+\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}+1\right)-i\,c\right)}+i\,\mathrm{Ei}\left(i\,dx+\frac{1}{2}\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\right)}{}$$

```
[In] integrate(sin(d*x+c)/x/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(-I*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3
/b)^(1/3)*(I*sqrt(3) + 1) - I*c) + I*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*
sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) - I*Ei(-I*d*
x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(-I*sqr
t(3) + 1) - I*c) + I*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(
1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) + I*Ei(I*d*x + (-I*a*d^3/b)^
(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) - I*Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-
I*c - (I*a*d^3/b)^(1/3)) + 6*cos_integral(d*x)*sin(c) + 6*cos(c)*sin_integr
al(d*x))/a
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{x(a+bx^3)}\,dx = \int \frac{\sin(c+dx)}{x(a+bx^3)}\,dx$$

```
[In] integrate(sin(d*x+c)/x/(b*x**3+a),x)
```

```
[Out] Integral(sin(c + d*x)/(x*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x(a+bx^3)}\,dx = \int \frac{\sin(dx+c)}{(bx^3+a)x}\,dx$$

```
[In] integrate(sin(d*x+c)/x/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x^3 + a)*x), x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{x\,(a+bx^3)}\,dx = \int \frac{\sin\,(dx+c)}{(bx^3+a)x}\,dx$$

```
[In] integrate(sin(d*x+c)/x/(b*x^3+a),x, algorithm="giac")
[Out] integrate(sin(d*x + c)/((b*x^3 + a)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x\,(a+bx^3)}\,dx = \int \frac{\sin\,(c+d\,x)}{x\,\left(b\,x^3+a\right)}\,dx$$

```
[In] int(sin(c + d*x)/(x*(a + b*x^3)),x)
[Out] int(sin(c + d*x)/(x*(a + b*x^3)), x)
```

3.100 $\int \frac{\sin(c+dx)}{x^2(a+bx^3)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 380

$$
\begin{aligned}
\int \frac{\sin(c+dx)}{x^2(a+bx^3)}\,dx &= \frac{d\cos(c)\,\text{CosIntegral}(dx)}{a} \\
&+ \frac{\sqrt[3]{b}\,\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}} \\
&+ \frac{(-1)^{2/3}\sqrt[3]{b}\,\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}} \\
&- \frac{\sqrt[3]{-1}\sqrt[3]{b}\,\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}} \\
&- \frac{\sin(c+dx)}{ax} - \frac{d\sin(c)\text{Si}(dx)}{a} \\
&- \frac{(-1)^{2/3}\sqrt[3]{b}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}} \\
&+ \frac{\sqrt[3]{b}\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{4/3}} \\
&- \frac{\sqrt[3]{-1}\sqrt[3]{b}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{4/3}}
\end{aligned}
$$

```
[Out] d*Ci(d*x)*cos(c)/a+1/3*(-1)^(2/3)*b^(1/3)*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3
))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(4/3)+1/3*b^(1/3)*cos(c-a^(1/3)*
d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^(4/3)-1/3*(-1)^(1/3)*b^(1/3)*cos(c-(
```

```
-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(4/3)-d
*Si(d*x)*sin(c)/a+1/3*b^(1/3)*Ci(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(
1/3))/a^(4/3)+1/3*(-1)^(2/3)*b^(1/3)*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*s
in(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(4/3)-1/3*(-1)^(1/3)*b^(1/3)*Ci((-1)^(
2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(4/3)-sin
(d*x+c)/a/x
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3426, 3378, 3384, 3380, 3383}

$$\begin{aligned}\int \frac{\sin(c+dx)}{x^2(a+bx^3)}\,dx &= \frac{\sqrt[3]{b}\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}} \\ &+ \frac{(-1)^{2/3}\sqrt[3]{b}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}} \\ &- \frac{\sqrt[3]{-1}\sqrt[3]{b}\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}} \\ &- \frac{(-1)^{2/3}\sqrt[3]{b}\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}} \\ &+ \frac{\sqrt[3]{b}\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}} \\ &- \frac{\sqrt[3]{-1}\sqrt[3]{b}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}} \\ &+ \frac{d\cos(c)\,\text{CosIntegral}(dx)}{a} - \frac{d\sin(c)\text{Si}(dx)}{a} - \frac{\sin(c+dx)}{ax}\end{aligned}$$

```
[In] Int[Sin[c + d*x]/(x^2*(a + b*x^3)),x]
```

```
[Out] (d*Cos[c]*CosIntegral[d*x])/a + (b^(1/3)*CosIntegral[(a^(1/3)*d)/b^(1/3) +
d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*a^(4/3)) + ((-1)^(2/3)*b^(1/3)*CosInt
egral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/
b^(1/3)])/(3*a^(4/3)) - ((-1)^(1/3)*b^(1/3)*CosIntegral[((-1)^(2/3)*a^(1/3)
*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(4/3)) - S
in[c + d*x]/(a*x) - (d*Sin[c]*SinIntegral[d*x])/a - ((-1)^(2/3)*b^(1/3)*Cos
[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(
1/3) - d*x])/(3*a^(4/3)) + (b^(1/3)*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegra
```

```
l[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(4/3)) - ((-1)^(1/3)*b^(1/3)*Cos[c - ((-
1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d
*x])/(3*a^(4/3))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\sin(c+dx)}{ax^2} - \frac{bx\sin(c+dx)}{a\,(a+bx^3)}\right) dx$$

$$= \frac{\int \frac{\sin(c+dx)}{x^2}\,dx}{a} - \frac{b\int \frac{x\sin(c+dx)}{a+bx^3}\,dx}{a}$$

$$= -\frac{\sin(c+dx)}{ax} - \frac{b\int\left(-\frac{\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{a} + \frac{d\int\frac{\cos(c+dx)}{x}dx}{a}$$

$$= -\frac{\sin(c+dx)}{ax} + \frac{b^{2/3}\int\frac{\sin(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{3a^{4/3}} - \frac{\left(\sqrt[3]{-1}b^{2/3}\right)\int\frac{\sin(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}dx}{3a^{4/3}} + \frac{\left((-1)^{2/3}b^{2/3}\right)\int\frac{\sin(c+dx)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}dx}{3a^{4/3}} + \frac{(d\cos(c))\int\frac{\cos(dx)}{x}dx}{a} - \frac{(d\sin(c))\int\frac{\sin(dx)}{x}dx}{a}$$

$$= \frac{d\cos(c)\,\text{CosIntegral}(dx)}{a} - \frac{\sin(c+dx)}{ax} - \frac{d\sin(c)\text{Si}(dx)}{a} + \frac{\left(b^{2/3}\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{3a^{4/3}} + \frac{\left(\sqrt[3]{-1}b^{2/3}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}dx}{3a^{4/3}} + \frac{\left((-1)^{2/3}b^{2/3}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\sin\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}dx}{3a^{4/3}} + \frac{\left(b^{2/3}\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{3a^{4/3}} - \frac{\left(\sqrt[3]{-1}b^{2/3}\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}dx}{3a^{4/3}} + \frac{\left((-1)^{2/3}b^{2/3}\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\cos\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}dx}{3a^{4/3}}$$

$$= \frac{d\cos(c)\,\mathrm{CosIntegral}(dx)}{a} + \frac{\sqrt[3]{b}\,\mathrm{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}} + dx\right)\sin\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}$$
$$+ \frac{(-1)^{2/3}\sqrt[3]{b}\,\mathrm{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}} - dx\right)\sin\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}$$
$$- \frac{\sqrt[3]{-1}\sqrt[3]{b}\,\mathrm{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}} + dx\right)\sin\left(c - \frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}} - \frac{\sin(c + dx)}{ax}$$
$$- \frac{d\sin(c)\mathrm{Si}(dx)}{a} - \frac{(-1)^{2/3}\sqrt[3]{b}\cos\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}} - dx\right)}{3a^{4/3}}$$
$$+ \frac{\sqrt[3]{b}\cos\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}} + dx\right)}{3a^{4/3}}$$
$$- \frac{\sqrt[3]{-1}\sqrt[3]{b}\cos\left(c - \frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}} + dx\right)}{3a^{4/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.18 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.61

$$\int \frac{\sin(c + dx)}{x^2(a + bx^3)}\,dx$$
$$= \frac{6dx\cos(c)\,\mathrm{CosIntegral}(dx) - ix\mathrm{RootSum}\left[a + b\#1^3\&, \frac{\cos(c+d\#1)\,\mathrm{CosIntegral}(d(x-\#1)) - i\,\mathrm{CosIntegral}(d(x-\#1))\sin(}{}\right.}{}$$

```
[In] Integrate[Sin[c + d*x]/(x^2*(a + b*x^3)),x]
```

```
[Out] (6*d*x*Cos[c]*CosIntegral[d*x] - I*x*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*
CosIntegral[d*(x - #1)] - I*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c
 + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#
1 & ] + I*x*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] +
 I*CosIntegral[d*(x - #1)]*Sin[c + d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x
 - #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1 & ] - 6*Sin[c + d*x] -
6*d*x*Sin[c]*SinIntegral[d*x])/(6*a*x)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.41 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.31

method	result
derivativedivides	$d\left(-\frac{\sin(dx+c)}{adx}+\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3\_Z^2bc+3c^2b\_Z+a\,d^3-c^3b\right)}\frac{-\mathrm{Si}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\mathrm{Ci}\left(dx-\_R1+\right.}{-\_R1+c}}{3a}\right.$
default	$d\left(-\frac{\sin(dx+c)}{adx}+\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3\_Z^2bc+3c^2b\_Z+a\,d^3-c^3b\right)}\frac{-\mathrm{Si}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\mathrm{Ci}\left(dx-\_R1+\right.}{-\_R1+c}}{3a}\right.$
risch	$-\frac{d\,\mathrm{Ei}_1(-idx)\mathrm{e}^{ic}}{2a}+\frac{d\left(\sum_{\_R1=\mathrm{RootOf}\left(-3i\_Z^2bc-id^3a+ib\,c^3+b\_Z^3-3c^2b\_Z\right)}\frac{\mathrm{e}^{\_R1}\,\mathrm{Ei}_1\left(-idx-ic+\_R1\right)}{-ic+\_R1}\right)}{6a}-\frac{d\,\mathrm{Ei}_1(i}{2}$

```
[In] int(sin(d*x+c)/x^2/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] d*(-sin(d*x+c)/a/d/x+1/3/a*sum(1/(-_R1+c)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-
_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/a*
(-Si(d*x)*sin(c)+Ci(d*x)*cos(c)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.18

$$\int \frac{\sin(c+dx)}{x^2\left(a+bx^3\right)}\,dx$$

$$=\frac{12\,ad^3x\cos\left(c\right)\mathrm{Ci}\left(dx\right)-12\,ad^3x\sin\left(c\right)\mathrm{Si}\left(dx\right)+2i\left(-\frac{i\,ad^3}{b}\right)^{\frac{2}{3}}bx\mathrm{Ei}\left(i\,dx+\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\right)e^{\left(ic-\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\right)}-2}{}$$

```
[In] integrate(sin(d*x+c)/x^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/12*(12*a*d^3*x*cos(c)*cos_integral(d*x) - 12*a*d^3*x*sin(c)*sin_integral(
d*x) + 2*I*(-I*a*d^3/b)^(2/3)*b*x*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (
-I*a*d^3/b)^(1/3)) - 2*I*(I*a*d^3/b)^(2/3)*b*x*Ei(-I*d*x + (I*a*d^3/b)^(1/3
))*e^(-I*c - (I*a*d^3/b)^(1/3)) - 12*a*d^2*sin(d*x + c) + (I*a*d^3/b)^(2/3)
*(sqrt(3)*b*x + I*b*x)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*
```

```
e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) - (-I*a*d^3/b)^(2/3)*(sqrt(
3)*b*x + I*b*x)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*
(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) - (I*a*d^3/b)^(2/3)*(sqrt(3)*b*x
- I*b*x)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3
/b)^(1/3)*(-I*sqrt(3) + 1) - I*c) + (-I*a*d^3/b)^(2/3)*(sqrt(3)*b*x - I*b*x
)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1
/3)*(-I*sqrt(3) + 1) + I*c))/(a^2*d^2*x)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{x^2\,(a+bx^3)}\,dx = \int \frac{\sin\left(c+dx\right)}{x^2\,(a+bx^3)}\,dx$$

```
[In] integrate(sin(d*x+c)/x**2/(b*x**3+a),x)
[Out] Integral(sin(c + d*x)/(x**2*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x^2\,(a+bx^3)}\,dx = \int \frac{\sin\left(dx+c\right)}{(bx^3+a)x^2}\,dx$$

```
[In] integrate(sin(d*x+c)/x^2/(b*x^3+a),x, algorithm="maxima")
[Out] integrate(sin(d*x + c)/((b*x^3 + a)*x^2), x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{x^2\,(a+bx^3)}\,dx = \int \frac{\sin\left(dx+c\right)}{(bx^3+a)x^2}\,dx$$

```
[In] integrate(sin(d*x+c)/x^2/(b*x^3+a),x, algorithm="giac")
[Out] integrate(sin(d*x + c)/((b*x^3 + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x^2\left(a+bx^3\right)}\,dx=\int \frac{\sin\left(c+d\,x\right)}{x^2\,\left(b\,x^3+a\right)}\,dx$$

```
[In] int(sin(c + d*x)/(x^2*(a + b*x^3)),x)
[Out] int(sin(c + d*x)/(x^2*(a + b*x^3)), x)
```

3.101 $\int \frac{\sin(c+dx)}{x^3(a+bx^3)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 408

$$\begin{aligned}\int \frac{\sin(c+dx)}{x^3(a+bx^3)}\,dx =& -\frac{d\cos(c+dx)}{2ax} - \frac{d^2\,\text{CosIntegral}(dx)\sin(c)}{2a} \\ &- \frac{b^{2/3}\,\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}} \\ &+ \frac{\sqrt[3]{-1}b^{2/3}\,\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}} \\ &- \frac{(-1)^{2/3}b^{2/3}\,\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}} \\ &- \frac{\sin(c+dx)}{2ax^2} - \frac{d^2\cos(c)\text{Si}(dx)}{2a} \\ &- \frac{\sqrt[3]{-1}b^{2/3}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}} \\ &- \frac{b^{2/3}\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{5/3}} \\ &- \frac{(-1)^{2/3}b^{2/3}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{5/3}}\end{aligned}$$

```
[Out] -1/2*d*cos(d*x+c)/a/x-1/2*d^2*cos(c)*Si(d*x)/a+1/3*(-1)^(1/3)*b^(2/3)*cos(c
+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(5/3
)-1/3*b^(2/3)*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)-1/
```

```
3*(-1)^(2/3)*b^(2/3)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1
/3)*d/b^(1/3)+d*x)/a^(5/3)-1/2*d^2*Ci(d*x)*sin(c)/a-1/3*b^(2/3)*Ci(a^(1/3)*
d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(5/3)+1/3*(-1)^(1/3)*b^(2/3)*Ci((
-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(5/3
)-1/3*(-1)^(2/3)*b^(2/3)*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2
/3)*a^(1/3)*d/b^(1/3))/a^(5/3)-1/2*sin(d*x+c)/a/x^2
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3426, 3378, 3384, 3380, 3383, 3414}

$$\begin{aligned}\int \frac{\sin(c+dx)}{x^3(a+bx^3)}\,dx = &-\frac{b^{2/3}\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}}\\ &+\frac{\sqrt[3]{-1}b^{2/3}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}}\\ &-\frac{(-1)^{2/3}b^{2/3}\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}}\\ &-\frac{\sqrt[3]{-1}b^{2/3}\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}}\\ &-\frac{b^{2/3}\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}}\\ &-\frac{(-1)^{2/3}b^{2/3}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}}\\ &-\frac{d^2\sin(c)\,\text{CosIntegral}(dx)}{2a}-\frac{d^2\cos(c)\text{Si}(dx)}{2a}\\ &-\frac{\sin(c+dx)}{2ax^2}-\frac{d\cos(c+dx)}{2ax}\end{aligned}$$

```
[In] Int[Sin[c + d*x]/(x^3*(a + b*x^3)),x]
```

```
[Out] -1/2*(d*Cos[c + d*x])/(a*x) - (d^2*CosIntegral[d*x]*Sin[c])/(2*a) - (b^(2/3
)*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*a
^(5/3)) + ((-1)^(1/3)*b^(2/3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) -
d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(5/3)) - ((-1)^(2/3)*b^(
2/3)*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*
a^(1/3)*d)/b^(1/3)])/(3*a^(5/3)) - Sin[c + d*x]/(2*a*x^2) - (d^2*Cos[c]*Sin
```

```
Integral[d*x])/(2*a) - ((-1)^(1/3)*b^(2/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b
^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(5/3)) - (b
^(2/3)*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])
/(3*a^(5/3)) - ((-1)^(2/3)*b^(2/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*
SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(5/3))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\sin(c+dx)}{ax^3} - \frac{b\sin(c+dx)}{a\left(a+bx^3\right)} \right) dx$$

$$= \frac{\int \frac{\sin(c+dx)}{x^3}\,dx}{a} - \frac{b\int \frac{\sin(c+dx)}{a+bx^3}\,dx}{a}$$

$$= -\frac{\sin(c+dx)}{2ax^2} - \frac{b\int \left(-\frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)} \right) dx}{a} + \frac{d\int \frac{\cos(c+dx)}{x^2}\,dx}{2a}$$

$$= -\frac{d\cos(c+dx)}{2ax} - \frac{\sin(c+dx)}{2ax^2} + \frac{b\int \frac{\sin(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3a^{5/3}} + \frac{b\int \frac{\sin(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3a^{5/3}} + \frac{b\int \frac{\sin(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3a^{5/3}} - \frac{d^2\int \frac{\sin(c+dx)}{x}\,dx}{2a}$$

$$= -\frac{d\cos(c+dx)}{2ax} - \frac{\sin(c+dx)}{2ax^2} - \frac{(d^2\cos(c))\int\frac{\sin(dx)}{x}\,dx}{2a} + \frac{\left(b\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3a^{5/3}} - \frac{\left(b\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3a^{5/3}} + \frac{\left(b\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\sin\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3a^{5/3}} - \frac{(d^2\sin(c))\int\frac{\cos(dx)}{x}\,dx}{2a} + \frac{\left(b\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3a^{5/3}} + \frac{\left(b\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3a^{5/3}} + \frac{\left(b\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\cos\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3a^{5/3}}$$

$$= -\frac{d\cos(c+dx)}{2ax} - \frac{d^2\,\text{CosIntegral}(dx)\sin(c)}{2a} - \frac{b^{2/3}\,\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}} + \frac{\sqrt[3]{-1}b^{2/3}\,\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}} - \frac{(-1)^{2/3}b^{2/3}\,\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}} - \frac{\sin(c+dx)}{2ax^2} - \frac{d^2\cos(c)\text{Si}(dx)}{2a} - \frac{\sqrt[3]{-1}b^{2/3}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}} - \frac{b^{2/3}\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{5/3}} - \frac{(-1)^{2/3}b^{2/3}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{5/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.23 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.62

$$\int \frac{\sin(c+dx)}{x^3(a+bx^3)}\,dx$$

$$= \frac{-ix^2\text{RootSum}\left[a+b\#1^3\&, \frac{\cos(c+d\#1)\,\text{CosIntegral}(d(x-\#1))-i\,\text{CosIntegral}(d(x-\#1))\sin(c+d\#1)-i\cos(c+d\#1)\text{Si}(d(x-\#1}{\#1^2}\right.}{}$$

```
[In] Integrate[Sin[c + d*x]/(x^3*(a + b*x^3)),x]
```

```
[Out] ((-I)*x^2*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] - I
*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*SinIntegral[d*(x -
 #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1^2 & ] + I*x^2*RootSum[a +
 b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*(x - #
1)]*Sin[c + d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]
*SinIntegral[d*(x - #1)])/#1^2 & ] - 3*(d*x*Cos[c + d*x] + d^2*x^2*CosInteg
ral[d*x]*Sin[c] + Sin[c + d*x] + d^2*x^2*Cos[c]*SinIntegral[d*x]))/(6*a*x^2
)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.45 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.33

method	result
derivativedivides	$d^2\left(\frac{-\frac{\sin(dx+c)}{2d^2x^2}-\frac{\cos(dx+c)}{2dx}-\frac{\mathrm{Si}(dx)\cos(c)}{2}-\frac{\mathrm{Ci}(dx)\sin(c)}{2}}{a}-\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3\_Z^2bc+3c^2b\_Z+a\,d^3-c^3b\right)}\frac{-\mathrm{Si}\left(-dx\right.}{}}{3a}\right.$
default	$d^2\left(\frac{-\frac{\sin(dx+c)}{2d^2x^2}-\frac{\cos(dx+c)}{2dx}-\frac{\mathrm{Si}(dx)\cos(c)}{2}-\frac{\mathrm{Ci}(dx)\sin(c)}{2}}{a}-\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3\_Z^2bc+3c^2b\_Z+a\,d^3-c^3b\right)}\frac{-\mathrm{Si}\left(-dx\right.}{}}{3a}\right.$
risch	$-\frac{id^2\,\mathrm{Ei}_1(-idx)\mathrm{e}^{ic}}{4a}+\frac{id^2\left(\sum_{\_R1=\mathrm{RootOf}\left(-3i\_Z^2bc-id^3a+ibc^3+b\_Z^3-3c^2b\_Z\right)}\frac{\mathrm{e}^{\_R1}\,\mathrm{Ei}_1\left(-idx-ic+\_R1\right)}{-2ic\_R1+\_R1^2-c^2}\right)}{6a}+id$

```
[In] int(sin(d*x+c)/x^3/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] d^2*(1/a*(-1/2*sin(d*x+c)/d^2/x^2-1/2*cos(d*x+c)/d/x-1/2*Si(d*x)*cos(c)-1/2
*Ci(d*x)*sin(c))-1/3/a*sum(1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+
Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3
)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.20

$$\int \frac{\sin(c+dx)}{x^3(a+bx^3)}\,dx =$$

$$-\frac{6\,ad^3x^2\,\mathrm{Ci}\,(dx)\sin\,(c)+6\,ad^3x^2\cos\,(c)\,\mathrm{Si}\,(dx)-2\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}bx^2\mathrm{Ei}\left(i\,dx+\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\right)e^{\left(i\,c-\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\right)}}{}$$

```
[In] integrate(sin(d*x+c)/x^3/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/12*(6*a*d^3*x^2*cos_integral(d*x)*sin(c) + 6*a*d^3*x^2*cos(c)*sin_integr
al(d*x) - 2*(-I*a*d^3/b)^(1/3)*b*x^2*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c
- (-I*a*d^3/b)^(1/3)) - 2*(I*a*d^3/b)^(1/3)*b*x^2*Ei(-I*d*x + (I*a*d^3/b)^(
1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)) + 6*a*d^2*x*cos(d*x + c) - (-I*sqrt(3)*b
```

```
*x^2 - b*x^2)*(I*a*d^3/b)^(1/3)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(
3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) - (-I*sqrt(3)*b*x^
2 - b*x^2)*(-I*a*d^3/b)^(1/3)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3)
 - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) - (I*sqrt(3)*b*x^2
- b*x^2)*(I*a*d^3/b)^(1/3)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1
))*e^(1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) - I*c) - (I*sqrt(3)*b*x^2 - b*
x^2)*(-I*a*d^3/b)^(1/3)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*
e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) + 6*a*d*sin(d*x + c))/(a^
2*d*x^2)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{x^3(a+bx^3)}\,dx = \int \frac{\sin(c+dx)}{x^3(a+bx^3)}\,dx$$

```
[In] integrate(sin(d*x+c)/x**3/(b*x**3+a),x)
[Out] Integral(sin(c + d*x)/(x**3*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x^3(a+bx^3)}\,dx = \int \frac{\sin(dx+c)}{(bx^3+a)x^3}\,dx$$

```
[In] integrate(sin(d*x+c)/x^3/(b*x^3+a),x, algorithm="maxima")
[Out] integrate(sin(d*x + c)/((b*x^3 + a)*x^3), x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{x^3(a+bx^3)}\,dx = \int \frac{\sin(dx+c)}{(bx^3+a)x^3}\,dx$$

```
[In] integrate(sin(d*x+c)/x^3/(b*x^3+a),x, algorithm="giac")
[Out] integrate(sin(d*x + c)/((b*x^3 + a)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x^3\,(a+bx^3)}\,dx = \int \frac{\sin\left(c+d\,x\right)}{x^3\,\left(b\,x^3+a\right)}\,dx$$

```
[In] int(sin(c + d*x)/(x^3*(a + b*x^3)),x)
[Out] int(sin(c + d*x)/(x^3*(a + b*x^3)), x)
```

3.102 $\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 714

$$\int \frac{x^3\sin(c+dx)}{(a+bx^3)^2}\,dx = -\frac{(-1)^{2/3}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9\sqrt[3]{a}b^{5/3}} - \frac{d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9\sqrt[3]{a}b^{5/3}} + \frac{\sqrt[3]{-1}d\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9\sqrt[3]{a}b^{5/3}} + \frac{\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}} - \frac{\sqrt[3]{-1}\,\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}} + \frac{(-1)^{2/3}\,\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}} - \frac{x\sin(c+dx)}{3b(a+bx^3)} + \frac{\sqrt[3]{-1}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}} - \frac{(-1)^{2/3}d\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9\sqrt[3]{a}b^{5/3}} + \frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{2/3}b^{4/3}} + \frac{d\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9\sqrt[3]{a}b^{5/3}} + \frac{(-1)^{2/3}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{2/3}b^{4/3}} - \frac{\sqrt[3]{-1}d\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9\sqrt[3]{a}b^{5/3}}$$

```
[Out] -1/9*d*Ci(a^(1/3)*d/b^(1/3)+d*x)*cos(c-a^(1/3)*d/b^(1/3))/a^(1/3)/b^(5/3)-1
/9*(-1)^(2/3)*d*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cos(c+(-1)^(1/3)*a^(1/
```

```
3)*d/b^(1/3))/a^(1/3)/b^(5/3)+1/9*(-1)^(1/3)*d*Ci((-1)^(2/3)*a^(1/3)*d/b^(1
/3)+d*x)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(1/3)/b^(5/3)-1/9*(-1)^(1/3)
*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/
a^(2/3)/b^(4/3)+1/9*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^(2
/3)/b^(4/3)+1/9*(-1)^(2/3)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3
)*a^(1/3)*d/b^(1/3)+d*x)/a^(2/3)/b^(4/3)+1/9*Ci(a^(1/3)*d/b^(1/3)+d*x)*sin(
c-a^(1/3)*d/b^(1/3))/a^(2/3)/b^(4/3)+1/9*d*Si(a^(1/3)*d/b^(1/3)+d*x)*sin(c-
a^(1/3)*d/b^(1/3))/a^(1/3)/b^(5/3)-1/9*(-1)^(1/3)*Ci((-1)^(1/3)*a^(1/3)*d/b
^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(2/3)/b^(4/3)+1/9*(-1)^(2
/3)*d*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1
/3))/a^(1/3)/b^(5/3)+1/9*(-1)^(2/3)*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*si
n(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(2/3)/b^(4/3)-1/9*(-1)^(1/3)*d*Si((-1)^
(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(1/3)/b^
(5/3)-1/3*x*sin(d*x+c)/b/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 714, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used

$= \{3424, 3414, 3384, 3380, 3383, 3427\}$

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^2}\,dx = -\frac{\sqrt[3]{-1}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}}$$
$$+\frac{\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}$$
$$+\frac{(-1)^{2/3}\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}$$
$$+\frac{\sqrt[3]{-1}\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}}$$
$$+\frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}$$
$$+\frac{(-1)^{2/3}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}$$
$$-\frac{(-1)^{2/3}d\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9\sqrt[3]{a}b^{5/3}}$$
$$-\frac{d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9\sqrt[3]{a}b^{5/3}}$$
$$+\frac{\sqrt[3]{-1}d\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9\sqrt[3]{a}b^{5/3}}$$
$$-\frac{(-1)^{2/3}d\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9\sqrt[3]{a}b^{5/3}}$$
$$+\frac{d\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9\sqrt[3]{a}b^{5/3}}$$
$$-\frac{\sqrt[3]{-1}d\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9\sqrt[3]{a}b^{5/3}}-\frac{x\sin(c+dx)}{3b(a+bx^3)}$$

```
[In] Int[(x^3*Sin[c + d*x])/(a + b*x^3)^2,x]
```

```
[Out] -1/9*((-1)^(2/3)*d*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1
)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(a^(1/3)*b^(5/3)) - (d*Cos[c - (a^(1/3)*
```

```
d)/b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(1/3)*b^(5/3)) + (
(-1)^(1/3)*d*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3
)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(1/3)*b^(5/3)) + (CosIntegral[(a^(1/3)*d)
/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(9*a^(2/3)*b^(4/3)) - ((-1)^(
1/3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*
a^(1/3)*d)/b^(1/3)])/(9*a^(2/3)*b^(4/3)) + ((-1)^(2/3)*CosIntegral[((-1)^(2
/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(9*a
^(2/3)*b^(4/3)) - (x*Sin[c + d*x])/(3*b*(a + b*x^3)) + ((-1)^(1/3)*Cos[c +
((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3)
- d*x])/(9*a^(2/3)*b^(4/3)) - ((-1)^(2/3)*d*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/
b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(1/3)*b^(5
/3)) + (Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x]
)/(9*a^(2/3)*b^(4/3)) + (d*Sin[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3
)*d)/b^(1/3) + d*x])/(9*a^(1/3)*b^(5/3)) + ((-1)^(2/3)*Cos[c - ((-1)^(2/3)*
a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a
^(2/3)*b^(4/3)) - ((-1)^(1/3)*d*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*Sin
Integral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(1/3)*b^(5/3))
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3424

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
```

```
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3427

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\frac{x\sin(c+dx)}{3b(a+bx^3)} + \frac{\int \frac{\sin(c+dx)}{a+bx^3}\,dx}{3b} + \frac{d\int \frac{x\cos(c+dx)}{a+bx^3}\,dx}{3b}$$

$$= -\frac{x\sin(c+dx)}{3b(a+bx^3)} + \frac{\int \left(-\frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{3b} + \frac{d\int \left(-\frac{\cos(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}\cos(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}\cos(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{3b}$$

$$= -\frac{x\sin(c+dx)}{3b(a+bx^3)} - \frac{\int \frac{\sin(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{9a^{2/3}b} - \frac{\int \frac{\sin(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{9a^{2/3}b} - \frac{\int \frac{\sin(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{9a^{2/3}b} - \frac{d\int \frac{\cos(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9\sqrt[3]{a}b^{4/3}} + \frac{\left(\sqrt[3]{-1}d\right)\int \frac{\cos(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{9\sqrt[3]{a}b^{4/3}} - \frac{\left((-1)^{2/3}d\right)\int \frac{\cos(c+dx)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{9\sqrt[3]{a}b^{4/3}}$$

$$= -\frac{x\sin(c+dx)}{3b\left(a+bx^3\right)} - \frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sin\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{9a^{2/3}b}$$

$$-\frac{\left(d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9\sqrt[3]{a}b^{4/3}}$$

$$+\frac{\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{9a^{2/3}b}$$

$$+\frac{\left(\sqrt[3]{-1}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{9\sqrt[3]{a}b^{4/3}}$$

$$-\frac{\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sin\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{9a^{2/3}b}$$

$$-\frac{\left((-1)^{2/3}d\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{9\sqrt[3]{a}b^{4/3}}$$

$$-\frac{\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\cos\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{9a^{2/3}b}+\frac{\left(d\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9\sqrt[3]{a}b^{4/3}}$$

$$-\frac{\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{9a^{2/3}b}$$

$$+\frac{\left(\sqrt[3]{-1}d\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{9\sqrt[3]{a}b^{4/3}}$$

$$-\frac{\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\cos\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{9a^{2/3}b}$$

$$+\frac{\left((-1)^{2/3}d\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{9\sqrt[3]{a}b^{4/3}}$$

$$
\begin{aligned}
&= -\frac{(-1)^{2/3}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9\sqrt[3]{a}b^{5/3}} \\
&\quad -\frac{d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9\sqrt[3]{a}b^{5/3}} \\
&\quad +\frac{\sqrt[3]{-1}d\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9\sqrt[3]{a}b^{5/3}} \\
&\quad +\frac{\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}} \\
&\quad -\frac{\sqrt[3]{-1}\,\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}} \\
&\quad +\frac{(-1)^{2/3}\,\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}} \\
&\quad -\frac{x\sin(c+dx)}{3b\,(a+bx^3)}+\frac{\sqrt[3]{-1}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}} \\
&\quad -\frac{(-1)^{2/3}d\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9\sqrt[3]{a}b^{5/3}} \\
&\quad +\frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{2/3}b^{4/3}}+\frac{d\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9\sqrt[3]{a}b^{5/3}} \\
&\quad +\frac{(-1)^{2/3}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{2/3}b^{4/3}} \\
&\quad -\frac{\sqrt[3]{-1}d\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9\sqrt[3]{a}b^{5/3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.17 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.54

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^2}\,dx = \frac{\text{RootSum}\left[a+b\#1^3\&, \frac{i\cos(c+d\#1)\,\text{CosIntegral}(d(x-\#1))+\text{CosIntegral}(d(x-\#1))\sin(c+d\#1)+\cos(c+d\#1)\text{Si}(d(x-\#1))-i\sin}{}\right.}{}$$

```
[In] Integrate[(x^3*Sin[c + d*x])/(a + b*x^3)^2,x]
```

```
[Out] (RootSum[a + b*#1^3 & , (I*Cos[c + d*#1]*CosIntegral[d*(x - #1)] + CosInteg
ral[d*(x - #1)]*Sin[c + d*#1] + Cos[c + d*#1]*SinIntegral[d*(x - #1)] - I*S
in[c + d*#1]*SinIntegral[d*(x - #1)] + d*Cos[c + d*#1]*CosIntegral[d*(x - #
1)]*#1 - I*d*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1 - I*d*Cos[c + d*#1]*S
inIntegral[d*(x - #1)]*#1 - d*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1)/#1^
2 & ] + RootSum[a + b*#1^3 & , ((-I)*Cos[c + d*#1]*CosIntegral[d*(x - #1)]
+ CosIntegral[d*(x - #1)]*Sin[c + d*#1] + Cos[c + d*#1]*SinIntegral[d*(x -
#1)] + I*Sin[c + d*#1]*SinIntegral[d*(x - #1)] + d*Cos[c + d*#1]*CosIntegra
l[d*(x - #1)]*#1 + I*d*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1 + I*d*Cos[c
 + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c + d*#1]*SinIntegral[d*(x - #1
)]*#1)/#1^2 & ] - (6*b*x*Sin[c + d*x])/(a + b*x^3))/(18*b^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.93 (sec) , antiderivative size = 1184, normalized size of antiderivative = 1.66

method	result	size
derivativedivides	Expression too large to display	1184
default	Expression too large to display	1184
risch	Expression too large to display	1379

```
[In] int(x^3*sin(d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^4*(-d^6*c^3*(sin(d*x+c)*(1/3/a/d^3*(d*x+c)-1/3*c/a/d^3)/(a*d^3-c^3*b+3*
b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)+2/9/a/d^3/b*sum(1/(_R1^2-2*_R1*c
+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3
*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/9/a/d^3/b*sum(1/(-_RR1+c)*(Si(-d*x+_RR
1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z
*b*c^2+a*d^3-b*c^3)))+sin(d*x+c)*(c^2*d^3/a*(d*x+c)^2-c^3*d^3/a*(d*x+c))/(a
*d^3-c^3*b+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)+1/3*c^2*d^3/a/b*sum
```

```
((c+_R1)/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R
1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/3*c^2*d^3/a/b*s
um(_RR1/(-_RR1+c)*(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1
=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+sin(d*x+c)*(-2*c^2*d^3/a
*(d*x+c)^2+3*c^3*d^3/a*(d*x+c)+c*d^3*(a*d^3-b*c^3)/a/b)/(a*d^3-c^3*b+3*b*c^
2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)-2/3*c^2*d^3/a/b*sum(_R1/(_R1^2-2*_R1
*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b
-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/3*c*d^3/a/b^2*sum((-2*_RR1^2*b*c+3*_
RR1*b*c^2+a*d^3-b*c^3)/(_RR1^2-2*_RR1*c+c^2)*(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(
d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3
))+sin(d*x+c)*(c^2*d^3/a*(d*x+c)^2-1/3*d^3*(a*d^3+5*b*c^3)/a/b*(d*x+c)-2/3*
c*d^3*(a*d^3-b*c^3)/a/b)/(a*d^3-c^3*b+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*
x+c)^3)+1/9*d^3/a/b^2*sum((3*_R1*b*c^2+a*d^3-b*c^3)/(_R1^2-2*_R1*c+c^2)*(-S
i(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c
+3*_Z*b*c^2+a*d^3-b*c^3))+1/9*d^3/a/b^2*sum((-3*_RR1^2*b*c^2+_RR1*a*d^3+5*_
RR1*b*c^3+2*a*c*d^3-2*b*c^4)/(_RR1^2-2*_RR1*c+c^2)*(Si(-d*x+_RR1-c)*sin(_RR
1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3
-b*c^3)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 670, normalized size of antiderivative = 0.94

$$\int \frac{x^3\sin(c+dx)}{(a+bx^3)^2}\,dx =$$

$$-\frac{12\,adx\sin\left(dx+c\right)+\left(\left(bx^3-\sqrt{3}(i\,bx^3+i\,a)+a\right)\left(\frac{i\,ad^3}{b}\right)^{\frac{2}{3}}-\left(bx^3+\sqrt{3}(i\,bx^3+i\,a)+a\right)\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\right)\mathrm{Ei}\Big(}{}$$

```
[In] integrate(x^3*sin(d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/36*(12*a*d*x*sin(d*x + c) + ((b*x^3 - sqrt(3)*(I*b*x^3 + I*a) + a)*(I*a*
d^3/b)^(2/3) - (b*x^3 + sqrt(3)*(I*b*x^3 + I*a) + a)*(I*a*d^3/b)^(1/3))*Ei(
-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(
I*sqrt(3) + 1) - I*c) + ((b*x^3 - sqrt(3)*(I*b*x^3 + I*a) + a)*(-I*a*d^3/b)
^(2/3) - (b*x^3 + sqrt(3)*(I*b*x^3 + I*a) + a)*(-I*a*d^3/b)^(1/3))*Ei(I*d*x
 + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sq
rt(3) + 1) + I*c) + ((b*x^3 - sqrt(3)*(-I*b*x^3 - I*a) + a)*(I*a*d^3/b)^(2/
3) - (b*x^3 + sqrt(3)*(-I*b*x^3 - I*a) + a)*(I*a*d^3/b)^(1/3))*Ei(-I*d*x +
1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3)
 + 1) - I*c) + ((b*x^3 - sqrt(3)*(-I*b*x^3 - I*a) + a)*(-I*a*d^3/b)^(2/3) -
 (b*x^3 + sqrt(3)*(-I*b*x^3 - I*a) + a)*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*
(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) +
```

```
 1) + I*c) - 2*((b*x^3 + a)*(-I*a*d^3/b)^(2/3) - (b*x^3 + a)*(-I*a*d^3/b)^(
1/3))*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) - 2*((b*x
^3 + a)*(I*a*d^3/b)^(2/3) - (b*x^3 + a)*(I*a*d^3/b)^(1/3))*Ei(-I*d*x + (I*a
*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)))/(a*b^2*d*x^3 + a^2*b*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*sin(d*x+c)/(b*x**3+a)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^2}\,dx = \int \frac{x^3 \sin(dx+c)}{(bx^3+a)^2}\,dx$$

```
[In] integrate(x^3*sin(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
[Out] -1/2*(3*(cos(c)^2 + sin(c)^2)*d*x^2*sin(d*x + c) + ((d^2*x^3*cos(c) - 3*d*x
^2*sin(c) - 12*x*cos(c))*cos(d*x + c)^2 + (d^2*x^3*cos(c) - 3*d*x^2*sin(c)
- 12*x*cos(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + ((cos(c)^2 + sin(c)^2)*d^2*
x^3 - 12*(cos(c)^2 + sin(c)^2)*x)*cos(d*x + c) - 2*(((b^2*cos(c)^2 + b^2*si
n(c)^2)*d^3*x^6 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^3*x^3 + (a^2*cos(c)^2 +
 a^2*sin(c)^2)*d^3)*cos(d*x + c)^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^6
 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^3*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*
d^3)*sin(d*x + c)^2)*integrate(3*(3*a*d*x*sin(d*x + c) + (a*d^2*x^2 + 10*b*
x^3 - 2*a)*cos(d*x + c))/(b^3*d^3*x^9 + 3*a*b^2*d^3*x^6 + 3*a^2*b*d^3*x^3 +
 a^3*d^3), x) - 2*(((b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^6 + 2*(a*b*cos(c)^2
 + a*b*sin(c)^2)*d^3*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^3)*cos(d*x + c)^
2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^6 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2
)*d^3*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^3)*sin(d*x + c)^2)*integrate(3*
(3*a*d*x*sin(d*x + c) + (a*d^2*x^2 + 10*b*x^3 - 2*a)*cos(d*x + c))/((b^3*d^
3*x^9 + 3*a*b^2*d^3*x^6 + 3*a^2*b*d^3*x^3 + a^3*d^3)*cos(d*x + c)^2 + (b^3*
d^3*x^9 + 3*a*b^2*d^3*x^6 + 3*a^2*b*d^3*x^3 + a^3*d^3)*sin(d*x + c)^2), x)
+ ((d^2*x^3*sin(c) + 3*d*x^2*cos(c) - 12*x*sin(c))*cos(d*x + c)^2 + (d^2*x^
3*sin(c) + 3*d*x^2*cos(c) - 12*x*sin(c))*sin(d*x + c)^2)*sin(d*x + 2*c))/((
(b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^6 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^3
*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^3)*cos(d*x + c)^2 + ((b^2*cos(c)^2 +
 b^2*sin(c)^2)*d^3*x^6 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^3*x^3 + (a^2*cos
(c)^2 + a^2*sin(c)^2)*d^3)*sin(d*x + c)^2)
```

Giac [F]

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^2}\,dx = \int \frac{x^3 \sin(dx+c)}{(bx^3+a)^2}\,dx$$

```
[In] integrate(x^3*sin(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
[Out] integrate(x^3*sin(d*x + c)/(b*x^3 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^2}\,dx = \int \frac{x^3 \sin(c+d\,x)}{(b\,x^3+a)^2}\,dx$$

```
[In] int((x^3*sin(c + d*x))/(a + b*x^3)^2,x)
[Out] int((x^3*sin(c + d*x))/(a + b*x^3)^2, x)
```

3.103 $\int \frac{x^2 \sin(c+dx)}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 371

$$\begin{aligned}\int \frac{x^2\sin(c+dx)}{(a+bx^3)^2}\,dx = &-\frac{\sqrt[3]{-1}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}} \\ &+\frac{d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{2/3}b^{4/3}} \\ &+\frac{(-1)^{2/3}d\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{2/3}b^{4/3}} \\ &-\frac{\sin(c+dx)}{3b(a+bx^3)}-\frac{\sqrt[3]{-1}d\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}} \\ &-\frac{d\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{2/3}b^{4/3}} \\ &-\frac{(-1)^{2/3}d\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{2/3}b^{4/3}}\end{aligned}$$

```
[Out] 1/9*d*Ci(a^(1/3)*d/b^(1/3)+d*x)*cos(c-a^(1/3)*d/b^(1/3))/a^(2/3)/b^(4/3)-1/
9*(-1)^(1/3)*d*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cos(c+(-1)^(1/3)*a^(1/3
)*d/b^(1/3))/a^(2/3)/b^(4/3)+1/9*(-1)^(2/3)*d*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/
3)+d*x)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(2/3)/b^(4/3)-1/9*d*Si(a^(1/3
)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(2/3)/b^(4/3)+1/9*(-1)^(1/3)*d*
Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a
```

```
^(2/3)/b^(4/3)-1/9*(-1)^(2/3)*d*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-
(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(2/3)/b^(4/3)-1/3*sin(d*x+c)/b/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3422, 3415, 3384, 3380, 3383}

$$\begin{aligned}\int \frac{x^2\sin(c+dx)}{(a+bx^3)^2}\,dx = & -\frac{\sqrt[3]{-1}d\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}} \\ & +\frac{d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}} \\ & +\frac{(-1)^{2/3}d\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}} \\ & -\frac{\sqrt[3]{-1}d\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}} \\ & -\frac{d\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}} \\ & -\frac{(-1)^{2/3}d\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}} \\ & -\frac{\sin(c+dx)}{3b\,(a+bx^3)}\end{aligned}$$

```
[In] Int[(x^2*Sin[c + d*x])/(a + b*x^3)^2,x]
```

```
[Out] -1/9*((-1)^(1/3)*d*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1
)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(a^(2/3)*b^(4/3)) + (d*Cos[c - (a^(1/3)*
d)/b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(2/3)*b^(4/3)) + (
(-1)^(2/3)*d*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3
)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(2/3)*b^(4/3)) - Sin[c + d*x]/(3*b*(a + b
*x^3)) - ((-1)^(1/3)*d*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[
((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(2/3)*b^(4/3)) - (d*Sin[c - (a^
(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(2/3)*b^(4/3
)) - ((-1)^(2/3)*d*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1
)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(2/3)*b^(4/3))
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3415

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3422

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))),
x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sin(c+dx)}{3b\left(a+bx^3\right)}+\frac{d\int \frac{\cos(c+dx)}{a+bx^3}\,dx}{3b}\\
&= -\frac{\sin(c+dx)}{3b\left(a+bx^3\right)}\\
&\quad+\frac{d\int \left(-\frac{\cos(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)}-\frac{\cos(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}-\frac{\cos(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{3b}\\
&= -\frac{\sin(c+dx)}{3b\left(a+bx^3\right)}-\frac{d\int \frac{\cos(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{9a^{2/3}b}-\frac{d\int \frac{\cos(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{9a^{2/3}b}-\frac{d\int \frac{\cos(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{9a^{2/3}b}
\end{aligned}$$

$$= -\frac{\sin(c+dx)}{3b(a+bx^3)} - \frac{\left(d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}dx}{9a^{2/3}b}$$

$$-\frac{\left(d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}dx}{9a^{2/3}b}$$

$$-\frac{\left(d\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}dx}{9a^{2/3}b}$$

$$+\frac{\left(d\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}dx}{9a^{2/3}b}$$

$$-\frac{\left(d\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}dx}{9a^{2/3}b}$$

$$+\frac{\left(d\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}dx}{9a^{2/3}b}$$

$$= -\frac{\sqrt[3]{-1}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}}$$

$$+\frac{d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{2/3}b^{4/3}}$$

$$+\frac{(-1)^{2/3}d\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{2/3}b^{4/3}}$$

$$-\frac{\sin(c+dx)}{3b(a+bx^3)} - \frac{\sqrt[3]{-1}d\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}}$$

$$-\frac{d\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{2/3}b^{4/3}}$$

$$-\frac{(-1)^{2/3}d\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{2/3}b^{4/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.13 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.58

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^3)^2}\,dx = \frac{d\text{RootSum}\left[a+b\#1^3\&, \frac{\cos(c+d\#1)\text{CosIntegral}(d(x-\#1))-i\,\text{CosIntegral}(d(x-\#1))\sin(c+d\#1)-i\cos(c+d\#1)\text{Si}(d(x-\#1))-\text{s}}{\#1^2}\right.}{}$$

```
[In] Integrate[(x^2*Sin[c + d*x])/(a + b*x^3)^2,x]
```

```
[Out] (d*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] - I*CosInt
egral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] -
 Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1^2 & ] + d*RootSum[a + b*#1^3 & ,
 (Cos[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*(x - #1)]*Sin[c +
 d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinIntegra
l[d*(x - #1)])/#1^2 & ] - (6*b*Sin[c + d*x])/(a + b*x^3))/(18*b^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.74 (sec) , antiderivative size = 823, normalized size of antiderivative = 2.22

method	result	size
derivativedivides	Expression too large to display	823
default	Expression too large to display	823
risch	Expression too large to display	926

```
[In] int(x^2*sin(d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^3*(d^6*c^2*(sin(d*x+c)*(1/3/a/d^3*(d*x+c)-1/3*c/a/d^3)/(a*d^3-c^3*b+3*b
*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)+2/9/a/d^3/b*sum(1/(_R1^2-2*_R1*c+
c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*
_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/9/a/d^3/b*sum(1/(-_RR1+c)*(Si(-d*x+_RR1
-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*
b*c^2+a*d^3-b*c^3)))+sin(d*x+c)*(-2/3*c*d^3/a*(d*x+c)^2+2/3*c^2*d^3/a*(d*x+
c))/(a*d^3-c^3*b+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)-2/9*c*d^3/a/b
*sum((c+_R1)/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*si
n(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-2/9*c*d^3/a/b
*sum(_RR1/(-_RR1+c)*(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_R
R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+sin(d*x+c)*(2/3*c*d^3/
a*(d*x+c)^2-c^2*d^3/a*(d*x+c)-1/3*d^3*(a*d^3-b*c^3)/a/b)/(a*d^3-c^3*b+3*b*c
```

```
^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)+2/9*c*d^3/a/b*sum(_R1/(_R1^2-2*_R1*
c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-
3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/9*d^3/a/b^2*sum((-2*_RR1^2*b*c+3*_RR1
*b*c^2+a*d^3-b*c^3)/(_RR1^2-2*_RR1*c+c^2)*(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(d*x
-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.30

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^3)^2}\,dx$$
$$= \frac{\left(-i\,bx^3+\sqrt{3}(bx^3+a)-i\,a\right)\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\mathrm{Ei}\left(-i\,dx+\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}+1\right)-i\,c\right)}+\left(i\,b\right.}{}$$

```
[In] integrate(x^2*sin(d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*((-I*b*x^3 + sqrt(3)*(b*x^3 + a) - I*a)*(I*a*d^3/b)^(1/3)*Ei(-I*d*x +
1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3)
 + 1) - I*c) + (I*b*x^3 - sqrt(3)*(b*x^3 + a) + I*a)*(-I*a*d^3/b)^(1/3)*Ei(
I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*
(I*sqrt(3) + 1) + I*c) + (-I*b*x^3 - sqrt(3)*(b*x^3 + a) - I*a)*(I*a*d^3/b)
^(1/3)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b
)^(1/3)*(-I*sqrt(3) + 1) - I*c) + (I*b*x^3 + sqrt(3)*(b*x^3 + a) + I*a)*(-I
*a*d^3/b)^(1/3)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(
-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) - 2*(I*b*x^3 + I*a)*(-I*a*d^3/b)^
(1/3)*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) - 2*(-I*b
*x^3 - I*a)*(I*a*d^3/b)^(1/3)*Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I*c - (I*a
*d^3/b)^(1/3)) - 12*a*sin(d*x + c))/(a*b^2*x^3 + a^2*b)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*sin(d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^2\sin(c+dx)}{(a+bx^3)^2}\,dx = \int \frac{x^2\sin(dx+c)}{(bx^3+a)^2}\,dx$$

```
[In] integrate(x^2*sin(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((cos(c)^2 + sin(c)^2)*d*x^2*cos(d*x + c) + 4*(cos(c)^2 + sin(c)^2)*x*
sin(d*x + c) + ((d*x^2*cos(c) - 4*x*sin(c))*cos(d*x + c)^2 + (d*x^2*cos(c)
- 4*x*sin(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b^2*cos(c)^2 + b^2*sin(
c)^2)*d^2*x^6 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^2*x^3 + (a^2*cos(c)^2 + a
^2*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d^2*x^6 +
 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^2*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^
2)*sin(d*x + c)^2)*integrate(-(3*a*d*x*cos(d*x + c) - 2*(5*b*x^3 - a)*sin(d
*x + c))/(b^3*d^2*x^9 + 3*a*b^2*d^2*x^6 + 3*a^2*b*d^2*x^3 + a^3*d^2), x) +
2*(((b^2*cos(c)^2 + b^2*sin(c)^2)*d^2*x^6 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)
*d^2*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b^2*cos(c)
^2 + b^2*sin(c)^2)*d^2*x^6 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^2*x^3 + (a^2
*cos(c)^2 + a^2*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(-(3*a*d*x*cos(d*x
+ c) - 2*(5*b*x^3 - a)*sin(d*x + c))/((b^3*d^2*x^9 + 3*a*b^2*d^2*x^6 + 3*a^
2*b*d^2*x^3 + a^3*d^2)*cos(d*x + c)^2 + (b^3*d^2*x^9 + 3*a*b^2*d^2*x^6 + 3*
a^2*b*d^2*x^3 + a^3*d^2)*sin(d*x + c)^2), x) + ((d*x^2*sin(c) + 4*x*cos(c))
*cos(d*x + c)^2 + (d*x^2*sin(c) + 4*x*cos(c))*sin(d*x + c)^2)*sin(d*x + 2*c
))/(((b^2*cos(c)^2 + b^2*sin(c)^2)*d^2*x^6 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2
)*d^2*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b^2*cos(c
)^2 + b^2*sin(c)^2)*d^2*x^6 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^2*x^3 + (a^
2*cos(c)^2 + a^2*sin(c)^2)*d^2)*sin(d*x + c)^2)
```

Giac [F]

$$\int \frac{x^2\sin(c+dx)}{(a+bx^3)^2}\,dx = \int \frac{x^2\sin(dx+c)}{(bx^3+a)^2}\,dx$$

```
[In] integrate(x^2*sin(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*sin(d*x + c)/(b*x^3 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^3)^2} \, dx = \int \frac{x^2 \sin(c+d\,x)}{(b\,x^3+a)^2} \, dx$$

```
[In] int((x^2*sin(c + d*x))/(a + b*x^3)^2,x)
[Out] int((x^2*sin(c + d*x))/(a + b*x^3)^2, x)
```

3.104 $\int \frac{x \sin(c+dx)}{(a+bx^3)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 691

$$\int \frac{x\sin(c+dx)}{(a+bx^3)^2}\,dx = -\frac{d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9ab}$$
$$-\frac{d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9ab}$$
$$-\frac{d\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9ab}$$
$$-\frac{\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}}$$
$$-\frac{(-1)^{2/3}\,\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}}$$
$$+\frac{\sqrt[3]{-1}\,\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}}$$
$$+\frac{\sin(c+dx)}{3abx}-\frac{\sin(c+dx)}{3bx\,(a+bx^3)}$$
$$+\frac{(-1)^{2/3}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{4/3}b^{2/3}}$$
$$-\frac{d\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9ab}$$
$$-\frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{4/3}b^{2/3}}$$
$$+\frac{d\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9ab}$$
$$+\frac{\sqrt[3]{-1}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{4/3}b^{2/3}}$$
$$+\frac{d\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9ab}$$

```
[Out] -1/9*d*Ci(a^(1/3)*d/b^(1/3)+d*x)*cos(c-a^(1/3)*d/b^(1/3))/a/b-1/9*d*Ci((-1)
^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a/b-1/9*d
```

```
*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a
/b-1/9*(-1)^(2/3)*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3
)*d/b^(1/3)+d*x)/a^(4/3)/b^(2/3)-1/9*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/
b^(1/3)+d*x)/a^(4/3)/b^(2/3)+1/9*(-1)^(1/3)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1
/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(4/3)/b^(2/3)-1/9*Ci(a^(1/3)*d/
b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(4/3)/b^(2/3)+1/9*d*Si(a^(1/3)*d/b^
(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a/b-1/9*(-1)^(2/3)*Ci((-1)^(1/3)*a^(1/3
)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(4/3)/b^(2/3)+1/9*d*
Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a
/b+1/9*(-1)^(1/3)*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(
1/3)*d/b^(1/3))/a^(4/3)/b^(2/3)+1/9*d*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*
sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a/b+1/3*sin(d*x+c)/a/b/x-1/3*sin(d*x+c)
/b/x/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 691, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used

$= \{3424, 3426, 3378, 3384, 3380, 3383, 3427\}$

$$\int \frac{x\sin(c+dx)}{(a+bx^3)^2}\,dx = -\frac{(-1)^{2/3}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\operatorname{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{4/3}b^{2/3}}$$
$$-\frac{\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}}$$
$$+\frac{\sqrt[3]{-1}\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}}$$
$$+\frac{(-1)^{2/3}\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\operatorname{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{4/3}b^{2/3}}$$
$$-\frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}}$$
$$+\frac{\sqrt[3]{-1}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}}$$
$$-\frac{d\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\operatorname{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9ab}$$
$$-\frac{d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9ab}$$
$$-\frac{d\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9ab}$$
$$-\frac{d\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\operatorname{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9ab}$$
$$+\frac{d\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9ab}$$
$$+\frac{d\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9ab}$$
$$-\frac{\sin(c+dx)}{3bx\,(a+bx^3)}+\frac{\sin(c+dx)}{3abx}$$

```
[In] Int[(x*Sin[c + d*x])/(a + b*x^3)^2,x]
```

```
[Out] -1/9*(d*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(1/3)*a^(
1/3)*d)/b^(1/3) - d*x])/(a*b) - (d*Cos[c - (a^(1/3)*d)/b^(1/3)]*CosIntegral
[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a*b) - (d*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^
(1/3)]*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a*b) - (CosInt
egral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(9*a^(4/3)*b
^(2/3)) - ((-1)^(2/3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin
[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(4/3)*b^(2/3)) + ((-1)^(1/3)*Cos
Integral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*
d)/b^(1/3)])/(9*a^(4/3)*b^(2/3)) + Sin[c + d*x]/(3*a*b*x) - Sin[c + d*x]/(3
*b*x*(a + b*x^3)) + ((-1)^(2/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*Sin
Integral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(4/3)*b^(2/3)) - (d*Si
n[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^
(1/3) - d*x])/(9*a*b) - (Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*
d)/b^(1/3) + d*x])/(9*a^(4/3)*b^(2/3)) + (d*Sin[c - (a^(1/3)*d)/b^(1/3)]*Si
nIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a*b) + ((-1)^(1/3)*Cos[c - ((-1)^(
2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])
/(9*a^(4/3)*b^(2/3)) + (d*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegr
al[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a*b)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3424

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
```

```
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3427

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sin(c+dx)}{3bx\,(a+bx^3)} - \frac{\int \frac{\sin(c+dx)}{x^2(a+bx^3)}\,dx}{3b} + \frac{d\int \frac{\cos(c+dx)}{x(a+bx^3)}\,dx}{3b} \\
&= -\frac{\sin(c+dx)}{3bx\,(a+bx^3)} - \frac{\int \left(\frac{\sin(c+dx)}{ax^2} - \frac{bx\sin(c+dx)}{a(a+bx^3)}\right)dx}{3b} + \frac{d\int \left(\frac{\cos(c+dx)}{ax} - \frac{bx^2\cos(c+dx)}{a(a+bx^3)}\right)dx}{3b} \\
&= -\frac{\sin(c+dx)}{3bx\,(a+bx^3)} + \frac{\int \frac{x\sin(c+dx)}{a+bx^3}\,dx}{3a} - \frac{\int \frac{\sin(c+dx)}{x^2}\,dx}{3ab} - \frac{d\int \frac{x^2\cos(c+dx)}{a+bx^3}\,dx}{3a} + \frac{d\int \frac{\cos(c+dx)}{x}\,dx}{3ab} \\
&= \frac{\sin(c+dx)}{3abx} - \frac{\sin(c+dx)}{3bx\,(a+bx^3)} \\
&\quad + \frac{\int \left(-\frac{\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{3a} \\
&\quad - \frac{d\int \left(\frac{\cos(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\cos(c+dx)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\cos(c+dx)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right)dx}{3a} \\
&\quad - \frac{d\int \frac{\cos(c+dx)}{x}\,dx}{3ab} + \frac{(d\cos(c))\int \frac{\cos(dx)}{x}\,dx}{3ab} - \frac{(d\sin(c))\int \frac{\sin(dx)}{x}\,dx}{3ab}
\end{aligned}$$

$$= \frac{d\cos(c)\,\text{CosIntegral}(dx)}{3ab} + \frac{\sin(c+dx)}{3abx} - \frac{\sin(c+dx)}{3bx\,(a+bx^3)} - \frac{d\sin(c)\text{Si}(dx)}{3ab} - \frac{\int \frac{\sin(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{4/3}\sqrt[3]{b}} + \frac{\sqrt[3]{-1}\int \frac{\sin(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{9a^{4/3}\sqrt[3]{b}} - \frac{(-1)^{2/3}\int \frac{\sin(c+dx)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{9a^{4/3}\sqrt[3]{b}} - \frac{d\int \frac{\cos(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9ab^{2/3}} - \frac{d\int \frac{\cos(c+dx)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9ab^{2/3}} - \frac{d\int \frac{\cos(c+dx)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9ab^{2/3}} - \frac{(d\cos(c))\int \frac{\cos(dx)}{x}\,dx}{3ab} + \frac{(d\sin(c))\int \frac{\sin(dx)}{x}\,dx}{3ab}$$

$$= \frac{\sin(c+dx)}{3abx} - \frac{\sin(c+dx)}{3bx\,(a+bx^3)} - \frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\sin\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{4/3}\sqrt[3]{b}}$$

$$-\frac{\left(d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9ab^{2/3}}$$

$$-\frac{\left(\sqrt[3]{-1}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{9a^{4/3}\sqrt[3]{b}}$$

$$-\frac{\left(d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9ab^{2/3}}$$

$$-\frac{\left((-1)^{2/3}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\sin\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{9a^{4/3}\sqrt[3]{b}}$$

$$-\frac{\left(d\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\cos\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9ab^{2/3}}$$

$$-\frac{\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\cos\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{4/3}\sqrt[3]{b}} + \frac{\left(d\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9ab^{2/3}}$$

$$+\frac{\left(\sqrt[3]{-1}\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{9a^{4/3}\sqrt[3]{b}}$$

$$-\frac{\left(d\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9ab^{2/3}}$$

$$-\frac{\left((-1)^{2/3}\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\cos\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{9a^{4/3}\sqrt[3]{b}}$$

$$+\frac{\left(d\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\sin\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9ab^{2/3}}$$

$$= -\frac{d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9ab} - \frac{d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9ab} - \frac{d\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9ab} - \frac{\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}} - \frac{(-1)^{2/3}\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}} + \frac{\sqrt[3]{-1}\,\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}} + \frac{\sin(c+dx)}{3abx} - \frac{\sin(c+dx)}{3bx\,(a+bx^3)} + \frac{(-1)^{2/3}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{4/3}b^{2/3}} - \frac{d\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9ab} - \frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{4/3}b^{2/3}} + \frac{d\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9ab} + \frac{\sqrt[3]{-1}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{4/3}b^{2/3}} + \frac{d\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9ab}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 408, normalized size of antiderivative = 0.59

$$\int \frac{x\sin(c+dx)}{(a+bx^3)^2}\,dx = -\frac{(a+bx^3)\,\text{RootSum}\left[a+b\#1^3\&,\frac{-i\cos(c+d\#1)\,\text{CosIntegral}(d(x-\#1))-\text{CosIntegral}(d(x-\#1))\sin(c+d\#1)-\cos(c+d\#1)\text{Si}}{}\right.}{}$$

```
[In] Integrate[(x*Sin[c + d*x])/(a + b*x^3)^2,x]
```

```
[Out] -1/18*((a + b*x^3)*RootSum[a + b*#1^3 & , ((-I)*Cos[c + d*#1]*CosIntegral[d
*(x - #1)] - CosIntegral[d*(x - #1)]*Sin[c + d*#1] - Cos[c + d*#1]*SinInteg
ral[d*(x - #1)] + I*Sin[c + d*#1]*SinIntegral[d*(x - #1)] + d*Cos[c + d*#1]
*CosIntegral[d*(x - #1)]*#1 - I*d*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1
- I*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c + d*#1]*SinIntegra
l[d*(x - #1)]*#1)/#1 & ] + (a + b*x^3)*RootSum[a + b*#1^3 & , (I*Cos[c + d*
#1]*CosIntegral[d*(x - #1)] - CosIntegral[d*(x - #1)]*Sin[c + d*#1] - Cos[c
 + d*#1]*SinIntegral[d*(x - #1)] - I*Sin[c + d*#1]*SinIntegral[d*(x - #1)]
+ d*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1 + I*d*CosIntegral[d*(x - #1)]*
Sin[c + d*#1]*#1 + I*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c +
 d*#1]*SinIntegral[d*(x - #1)]*#1)/#1 & ] - 6*b*x^2*Sin[c + d*x])/(a*b*(a +
 b*x^3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.63 (sec) , antiderivative size = 508, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\dfrac{\dfrac{\sin(dx+c)\left(\frac{d^3(dx+c)^2}{3a}-\frac{c\,d^3(dx+c)}{3a}\right)}{a\,d^3-c^3b+3b\,c^2(dx+c)-3bc(dx+c)^2+b(dx+c)^3}+\dfrac{d^3\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3\_Z^2bc+3c^2b\_Z+a\,d^3-c^3b\right)}\frac{\left(c+\_R1\right)\left(-\mathrm{Si}\left(-d\right.\right.}{}\right.}{9ab}}{}$
default	$\dfrac{\dfrac{\sin(dx+c)\left(\frac{d^3(dx+c)^2}{3a}-\frac{c\,d^3(dx+c)}{3a}\right)}{a\,d^3-c^3b+3b\,c^2(dx+c)-3bc(dx+c)^2+b(dx+c)^3}+\dfrac{d^3\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3\_Z^2bc+3c^2b\_Z+a\,d^3-c^3b\right)}\frac{\left(c+\_R1\right)\left(-\mathrm{Si}\left(-d\right.\right.}{}\right.}{9ab}}{}$
risch	$\dfrac{dc\left(\sum_{\_R1=\mathrm{RootOf}\left(-3i\_Z^2bc-id^3a+ibc^3+b\_Z^3-3c^2b\_Z\right)}\frac{\left(i\_R1+c-2i\right)e^{\_R1}\,\mathrm{Ei}_1\left(-idx-ic+\_R1\right)}{2ic\_R1-\_R1^2+c^2}\right)}{18ab}+\dfrac{d\left(\sum_{\_R1=\mathrm{R}}\right.}{}$

```
[In] int(x*sin(d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(sin(d*x+c)*(1/3*d^3/a*(d*x+c)^2-1/3*c*d^3/a*(d*x+c))/(a*d^3-c^3*b+3*
b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)+1/9*d^3/a/b*sum((c+_R1)/(_R1^2-2
*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z
^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/9*d^3/a/b*sum(_RR1/(-_RR1+c)*(Si
(-d*x+_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2
*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-d^6*c*(sin(d*x+c)*(1/3/a/d^3*(d*x+c)-1/3*c/a/
d^3)/(a*d^3-c^3*b+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)+2/9/a/d^3/b*
sum(1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1))
```

```
,_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/9/a/d^3/b*sum(1/(-
_RR1+c)*(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z
^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 655, normalized size of antiderivative = 0.95

$$\int \frac{x\sin(c+dx)}{(a+bx^3)^2}\,dx$$

$$= \frac{12\,abd^2x^2\sin\left(dx+c\right)-\left(2\,abd^3x^3+2\,a^2d^3-\left(-i\,b^2x^3-i\,ab-\sqrt{3}(b^2x^3+ab)\right)\left(\frac{i\,ad^3}{b}\right)^{\frac{2}{3}}\right)\mathrm{Ei}\left(-i\,dx+\frac{1}{2}\left(\frac{i}{}\right.\right.}{}$$

```
[In] integrate(x*sin(d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(12*a*b*d^2*x^2*sin(d*x + c) - (2*a*b*d^3*x^3 + 2*a^2*d^3 - (-I*b^2*x^
3 - I*a*b - sqrt(3)*(b^2*x^3 + a*b))*(I*a*d^3/b)^(2/3))*Ei(-I*d*x + 1/2*(I*
a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) -
 I*c) - (2*a*b*d^3*x^3 + 2*a^2*d^3 - (I*b^2*x^3 + I*a*b + sqrt(3)*(b^2*x^3
+ a*b))*(-I*a*d^3/b)^(2/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) -
 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) - (2*a*b*d^3*x^3 + 2*
a^2*d^3 - (-I*b^2*x^3 - I*a*b + sqrt(3)*(b^2*x^3 + a*b))*(I*a*d^3/b)^(2/3))
*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3
)*(-I*sqrt(3) + 1) - I*c) - (2*a*b*d^3*x^3 + 2*a^2*d^3 - (I*b^2*x^3 + I*a*b
 - sqrt(3)*(b^2*x^3 + a*b))*(-I*a*d^3/b)^(2/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)
^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) -
 2*(a*b*d^3*x^3 + a^2*d^3 + (I*b^2*x^3 + I*a*b)*(-I*a*d^3/b)^(2/3))*Ei(I*d*
x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) - 2*(a*b*d^3*x^3 + a^2
*d^3 + (-I*b^2*x^3 - I*a*b)*(I*a*d^3/b)^(2/3))*Ei(-I*d*x + (I*a*d^3/b)^(1/3
))*e^(-I*c - (I*a*d^3/b)^(1/3)))/(a^2*b^2*d^2*x^3 + a^3*b*d^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x\sin(c+dx)}{(a+bx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x*sin(d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x\sin(c+dx)}{(a+bx^3)^2}\,dx = \int \frac{x\sin(dx+c)}{(bx^3+a)^2}\,dx$$

```
[In] integrate(x*sin(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((cos(c)^2 + sin(c)^2)*x*cos(d*x + c) + (x*cos(d*x + c)^2*cos(c) + x*c
os(c)*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b^2*cos(c)^2 + b^2*sin(c)^2)*d*
x^6 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)
*d)*cos(d*x + c)^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d*x^6 + 2*(a*b*cos(c)^2
 + a*b*sin(c)^2)*d*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*sin(d*x + c)^2)*i
ntegrate(1/2*(5*b*x^3 - a)*cos(d*x + c)/(b^3*d*x^9 + 3*a*b^2*d*x^6 + 3*a^2*
b*d*x^3 + a^3*d), x) + 2*(((b^2*cos(c)^2 + b^2*sin(c)^2)*d*x^6 + 2*(a*b*cos
(c)^2 + a*b*sin(c)^2)*d*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*cos(d*x + c)
^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d*x^6 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)
*d*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*sin(d*x + c)^2)*integrate(1/2*(5*
b*x^3 - a)*cos(d*x + c)/((b^3*d*x^9 + 3*a*b^2*d*x^6 + 3*a^2*b*d*x^3 + a^3*d
)*cos(d*x + c)^2 + (b^3*d*x^9 + 3*a*b^2*d*x^6 + 3*a^2*b*d*x^3 + a^3*d)*sin(
d*x + c)^2), x) + (x*cos(d*x + c)^2*sin(c) + x*sin(d*x + c)^2*sin(c))*sin(d
*x + 2*c))/(((b^2*cos(c)^2 + b^2*sin(c)^2)*d*x^6 + 2*(a*b*cos(c)^2 + a*b*si
n(c)^2)*d*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*cos(d*x + c)^2 + ((b^2*cos
(c)^2 + b^2*sin(c)^2)*d*x^6 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d*x^3 + (a^2*
cos(c)^2 + a^2*sin(c)^2)*d)*sin(d*x + c)^2)
```

Giac [F]

$$\int \frac{x\sin(c+dx)}{(a+bx^3)^2}\,dx = \int \frac{x\sin(dx+c)}{(bx^3+a)^2}\,dx$$

```
[In] integrate(x*sin(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*sin(d*x + c)/(b*x^3 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sin(c+dx)}{(a+bx^3)^2}\,dx = \int \frac{x\,\sin(c+d\,x)}{(b\,x^3+a)^2}\,dx$$

```
[In] int((x*sin(c + d*x))/(a + b*x^3)^2,x)
```

```
[Out] int((x*sin(c + d*x))/(a + b*x^3)^2, x)
```

3.105 $\int \frac{\sin(c+dx)}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 735

$$
\begin{aligned}
\int \frac{\sin(c+dx)}{(a+bx^3)^2}\,dx =& \frac{(-1)^{2/3}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{4/3}b^{2/3}} \\
&+\frac{d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{4/3}b^{2/3}} \\
&-\frac{\sqrt[3]{-1}d\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{4/3}b^{2/3}} \\
&+\frac{2\,\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}} \\
&-\frac{2\sqrt[3]{-1}\,\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}} \\
&+\frac{2(-1)^{2/3}\,\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}} \\
&+\frac{\sin(c+dx)}{3abx^2}-\frac{\sin(c+dx)}{3bx^2\left(a+bx^3\right)} \\
&+\frac{2\sqrt[3]{-1}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{5/3}\sqrt[3]{b}} \\
&+\frac{(-1)^{2/3}d\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{4/3}b^{2/3}} \\
&+\frac{2\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{5/3}\sqrt[3]{b}} \\
&-\frac{d\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{4/3}b^{2/3}} \\
&+\frac{2(-1)^{2/3}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{5/3}\sqrt[3]{b}} \\
&+\frac{\sqrt[3]{-1}d\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{4/3}b^{2/3}}
\end{aligned}
$$

```
[Out] 1/9*d*Ci(a^(1/3)*d/b^(1/3)+d*x)*cos(c-a^(1/3)*d/b^(1/3))/a^(4/3)/b^(2/3)+1/
9*(-1)^(2/3)*d*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cos(c+(-1)^(1/3)*a^(1/3
)*d/b^(1/3))/a^(4/3)/b^(2/3)-1/9*(-1)^(1/3)*d*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/
3)+d*x)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(4/3)/b^(2/3)-2/9*(-1)^(1/3)*
cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a
^(5/3)/b^(1/3)+2/9*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^(5/
3)/b^(1/3)+2/9*(-1)^(2/3)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)
*a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(1/3)+2/9*Ci(a^(1/3)*d/b^(1/3)+d*x)*sin(c
-a^(1/3)*d/b^(1/3))/a^(5/3)/b^(1/3)-1/9*d*Si(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a
^(1/3)*d/b^(1/3))/a^(4/3)/b^(2/3)-2/9*(-1)^(1/3)*Ci((-1)^(1/3)*a^(1/3)*d/b^
(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(1/3)-1/9*(-1)^(2/
3)*d*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/
3))/a^(4/3)/b^(2/3)+2/9*(-1)^(2/3)*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin
(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(1/3)+1/9*(-1)^(1/3)*d*Si((-1)^(
2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(4/3)/b^(
2/3)+1/3*sin(d*x+c)/a/b/x^2-1/3*sin(d*x+c)/b/x^2/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 735, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used

$= \{3412, 3426, 3378, 3384, 3380, 3383, 3414, 3427\}$

$$\int \frac{\sin(c+dx)}{(a+bx^3)^2}\,dx = \frac{(-1)^{2/3} d\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{4/3}b^{2/3}}$$
$$+\frac{d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}}$$
$$-\frac{\sqrt[3]{-1}d\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}}$$
$$+\frac{(-1)^{2/3} d\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{4/3}b^{2/3}}$$
$$-\frac{d\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}}$$
$$+\frac{\sqrt[3]{-1}d\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}}$$
$$-\frac{2\sqrt[3]{-1}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{5/3}\sqrt[3]{b}}$$
$$+\frac{2\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}}$$
$$+\frac{2(-1)^{2/3}\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}}$$
$$+\frac{2\sqrt[3]{-1}\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{5/3}\sqrt[3]{b}}$$
$$+\frac{2\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}}$$
$$+\frac{2(-1)^{2/3}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}}$$
$$+\frac{\sin(c+dx)}{3abx^2}-\frac{\sin(c+dx)}{3bx^2(a+bx^3)}$$

```
[In] Int[Sin[c + d*x]/(a + b*x^3)^2,x]
```

```
[Out] ((-1)^(2/3)*d*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(1/
3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(4/3)*b^(2/3)) + (d*Cos[c - (a^(1/3)*d)/
b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(4/3)*b^(2/3)) - ((-1
)^(1/3)*d*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*a
^(1/3)*d)/b^(1/3) + d*x])/(9*a^(4/3)*b^(2/3)) + (2*CosIntegral[(a^(1/3)*d)/
b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(9*a^(5/3)*b^(1/3)) - (2*(-1)^
(1/3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)
*a^(1/3)*d)/b^(1/3)])/(9*a^(5/3)*b^(1/3)) + (2*(-1)^(2/3)*CosIntegral[((-1)
^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(
9*a^(5/3)*b^(1/3)) + Sin[c + d*x]/(3*a*b*x^2) - Sin[c + d*x]/(3*b*x^2*(a +
b*x^3)) + (2*(-1)^(1/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral
[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(5/3)*b^(1/3)) + ((-1)^(2/3)*d
*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)
/b^(1/3) - d*x])/(9*a^(4/3)*b^(2/3)) + (2*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinI
ntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3)) - (d*Sin[c - (a^(1/
3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(4/3)*b^(2/3))
+ (2*(-1)^(2/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(
2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3)) + ((-1)^(1/3)*d*Sin[c -
 ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3)
 + d*x])/(9*a^(4/3)*b^(2/3))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Sim
p[x^(-n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))), x] + (-Dist[
(-n + 1)/(b*n*(p + 1)), Int[((a + b*x^n)^(p + 1)*Sin[c + d*x])/x^n, x], x]
- Dist[d/(b*n*(p + 1)), Int[x^(-n + 1)*(a + b*x^n)^(p + 1)*Cos[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1] && IGtQ[n, 2]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3427

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sin(c+dx)}{3bx^2\left(a+bx^3\right)}-\frac{2\int\frac{\sin(c+dx)}{x^3(a+bx^3)}\,dx}{3b}+\frac{d\int\frac{\cos(c+dx)}{x^2(a+bx^3)}\,dx}{3b}\\
&= -\frac{\sin(c+dx)}{3bx^2\left(a+bx^3\right)}-\frac{2\int\left(\frac{\sin(c+dx)}{ax^3}-\frac{b\sin(c+dx)}{a(a+bx^3)}\right)dx}{3b}+\frac{d\int\left(\frac{\cos(c+dx)}{ax^2}-\frac{bx\cos(c+dx)}{a(a+bx^3)}\right)dx}{3b}\\
&= -\frac{\sin(c+dx)}{3bx^2\left(a+bx^3\right)}+\frac{2\int\frac{\sin(c+dx)}{a+bx^3}\,dx}{3a}-\frac{2\int\frac{\sin(c+dx)}{x^3}\,dx}{3ab}-\frac{d\int\frac{x\cos(c+dx)}{a+bx^3}\,dx}{3a}+\frac{d\int\frac{\cos(c+dx)}{x^2}\,dx}{3ab}
\end{aligned}$$

$$= -\frac{d\cos(c+dx)}{3abx} + \frac{\sin(c+dx)}{3abx^2} - \frac{\sin(c+dx)}{3bx^2(a+bx^3)} + \frac{2\int \left(-\frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{3a} - \frac{d\int \left(-\frac{\cos(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}\cos(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}\cos(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{3a} - \frac{d\int \frac{\cos(c+dx)}{x^2}dx}{3ab} - \frac{d^2\int \frac{\sin(c+dx)}{x}dx}{3ab}$$

$$= \frac{\sin(c+dx)}{3abx^2} - \frac{\sin(c+dx)}{3bx^2(a+bx^3)} - \frac{2\int \frac{\sin(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}dx}{9a^{5/3}} - \frac{2\int \frac{\sin(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}dx}{9a^{5/3}} - \frac{2\int \frac{\sin(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}dx}{9a^{5/3}} + \frac{d\int \frac{\cos(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{9a^{4/3}\sqrt[3]{b}} - \frac{\left(\sqrt[3]{-1}d\right)\int \frac{\cos(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}dx}{9a^{4/3}\sqrt[3]{b}} + \frac{\left((-1)^{2/3}d\right)\int \frac{\cos(c+dx)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}dx}{9a^{4/3}\sqrt[3]{b}} + \frac{d^2\int \frac{\sin(c+dx)}{x}dx}{3ab} - \frac{\left(d^2\cos(c)\right)\int \frac{\sin(dx)}{x}dx}{3ab} - \frac{\left(d^2\sin(c)\right)\int \frac{\cos(dx)}{x}dx}{3ab}$$

$$= -\frac{d^2\,\text{CosIntegral}(dx)\sin(c)}{3ab} + \frac{\sin(c+dx)}{3abx^2} - \frac{\sin(c+dx)}{3bx^2\,(a+bx^3)} - \frac{d^2\cos(c)\text{Si}(dx)}{3ab}$$

$$+ \frac{(d^2\cos(c))\int \frac{\sin(dx)}{x}\,dx}{3ab} - \frac{\left(2\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{9a^{5/3}}$$

$$+ \frac{\left(d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{4/3}\sqrt[3]{b}}$$

$$+ \frac{\left(2\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{9a^{5/3}}$$

$$- \frac{\left(\sqrt[3]{-1}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{9a^{4/3}\sqrt[3]{b}}$$

$$- \frac{\left(2\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{9a^{5/3}}$$

$$+ \frac{\left((-1)^{2/3}d\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{9a^{4/3}\sqrt[3]{b}}$$

$$+ \frac{(d^2\sin(c))\int \frac{\cos(dx)}{x}\,dx}{3ab} - \frac{\left(2\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{9a^{5/3}}$$

$$- \frac{\left(d\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{4/3}\sqrt[3]{b}}$$

$$- \frac{\left(2\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{9a^{5/3}}$$

$$- \frac{\left(\sqrt[3]{-1}d\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{9a^{4/3}\sqrt[3]{b}}$$

$$- \frac{\left(2\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{9a^{5/3}}$$

$$\left((-1)^{2/3}d\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\cdots}\,dx$$

$$= \frac{(-1)^{2/3} d \cos\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}} - dx\right)}{9a^{4/3}b^{2/3}}$$

$$+ \frac{d \cos\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}} + dx\right)}{9a^{4/3}b^{2/3}}$$

$$- \frac{\sqrt[3]{-1} d \cos\left(c - \frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}} + dx\right)}{9a^{4/3}b^{2/3}}$$

$$+ \frac{2\,\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}} + dx\right) \sin\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}}$$

$$- \frac{2\sqrt[3]{-1}\,\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}} - dx\right) \sin\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}}$$

$$+ \frac{2(-1)^{2/3}\,\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}} + dx\right) \sin\left(c - \frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}} + \frac{\sin(c + dx)}{3abx^2}$$

$$- \frac{\sin(c + dx)}{3bx^2(a + bx^3)} + \frac{2\sqrt[3]{-1}\cos\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}} - dx\right)}{9a^{5/3}\sqrt[3]{b}}$$

$$+ \frac{(-1)^{2/3} d \sin\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}} - dx\right)}{9a^{4/3}b^{2/3}}$$

$$+ \frac{2\cos\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}} + dx\right)}{9a^{5/3}\sqrt[3]{b}} - \frac{d \sin\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}} + dx\right)}{9a^{4/3}b^{2/3}}$$

$$+ \frac{2(-1)^{2/3}\cos\left(c - \frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}} + dx\right)}{9a^{5/3}\sqrt[3]{b}}$$

$$+ \frac{\sqrt[3]{-1} d \sin\left(c - \frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}} + dx\right)}{9a^{4/3}b^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 406, normalized size of antiderivative = 0.55

$$\int \frac{\sin(c+dx)}{(a+bx^3)^2}\,dx =$$

$$-\frac{(a+bx^3)\,\text{RootSum}\left[a+b\#1^3\&, \frac{-2i\cos(c+d\#1)\,\text{CosIntegral}(d(x-\#1))-2\,\text{CosIntegral}(d(x-\#1))\sin(c+d\#1)-2\cos(c+d\#}{}\right.}{}$$

```
[In] Integrate[Sin[c + d*x]/(a + b*x^3)^2,x]
```

```
[Out] -1/18*((a + b*x^3)*RootSum[a + b*#1^3 & , ((-2*I)*Cos[c + d*#1]*CosIntegral
[d*(x - #1)] - 2*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - 2*Cos[c + d*#1]*Si
nIntegral[d*(x - #1)] + (2*I)*Sin[c + d*#1]*SinIntegral[d*(x - #1)] + d*Cos
[c + d*#1]*CosIntegral[d*(x - #1)]*#1 - I*d*CosIntegral[d*(x - #1)]*Sin[c +
 d*#1]*#1 - I*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c + d*#1]*
SinIntegral[d*(x - #1)]*#1)/#1^2 & ] + (a + b*x^3)*RootSum[a + b*#1^3 & , (
(2*I)*Cos[c + d*#1]*CosIntegral[d*(x - #1)] - 2*CosIntegral[d*(x - #1)]*Sin
[c + d*#1] - 2*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - (2*I)*Sin[c + d*#1]*
SinIntegral[d*(x - #1)] + d*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1 + I*d*
CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1 + I*d*Cos[c + d*#1]*SinIntegral[d*
(x - #1)]*#1 - d*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1)/#1^2 & ] - 6*b*x
*Sin[c + d*x])/(a*b*(a + b*x^3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.53 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.34

method	result
derivativedivides	$d^5 \left(\frac{\sin(dx+c)\left(\frac{dx+c}{3a\,d^3}-\frac{c}{3a\,d^3}\right)}{a\,d^3-c^3b+3b\,c^2(dx+c)-3bc(dx+c)^2+b(dx+c)^3} + \frac{2\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3\_Z^2bc+3c^2b\_Z+a\,d^3-c^3b\right)} \frac{-\mathrm{Si}\left(-dx+\ldots\right.}{\ldots}\right.}{9a\,d^3b\ldots}\right.$
default	$d^5 \left(\frac{\sin(dx+c)\left(\frac{dx+c}{3a\,d^3}-\frac{c}{3a\,d^3}\right)}{a\,d^3-c^3b+3b\,c^2(dx+c)-3bc(dx+c)^2+b(dx+c)^3} + \frac{2\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3\_Z^2bc+3c^2b\_Z+a\,d^3-c^3b\right)} \frac{-\mathrm{Si}\left(-dx+\ldots\right.}{\ldots}\right.}{9a\,d^3b\ldots}\right.$
risch	$-\frac{d^2\left(\sum_{\_R1=\mathrm{RootOf}\left(-3i\_Z^2bc-id^3a+ib\,c^3+b\_Z^3-3c^2b\_Z\right)} \frac{\left(i\_R1+c-2i\right)e^{-\_R1}\mathrm{Ei}_1\left(-idx-ic+\_R1\right)}{2ic\_R1-\_R1^2+c^2}\right)}{18ab} - \frac{d^2\left(\ldots\right.}{\ldots}$

```
[In] int(sin(d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^5*(sin(d*x+c)*(1/3/a/d^3*(d*x+c)-1/3*c/a/d^3)/(a*d^3-c^3*b+3*b*c^2*(d*x+c
)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)+2/9/a/d^3/b*sum(1/(_R1^2-2*_R1*c+c^2)*(-Si(-
d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*
_Z*b*c^2+a*d^3-b*c^3))+1/9/a/d^3/b*sum(1/(-_RR1+c)*(Si(-d*x+_RR1-c)*sin(_RR
1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3
-b*c^3)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 669, normalized size of antiderivative = 0.91

$$\int \frac{\sin(c+dx)}{(a+bx^3)^2}\,dx$$

$$= \frac{12\,adx\sin\left(dx+c\right)+\left(\left(bx^3+\sqrt{3}(-i\,bx^3-i\,a)+a\right)\left(\frac{i\,ad^3}{b}\right)^{\frac{2}{3}}+2\left(bx^3-\sqrt{3}(-i\,bx^3-i\,a)+a\right)\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\right)\mathrm{E}\ldots}{\ldots}$$

```
[In] integrate(sin(d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(12*a*d*x*sin(d*x + c) + ((b*x^3 + sqrt(3)*(-I*b*x^3 - I*a) + a)*(I*a*
d^3/b)^(2/3) + 2*(b*x^3 - sqrt(3)*(-I*b*x^3 - I*a) + a)*(I*a*d^3/b)^(1/3))*
Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3
)*(I*sqrt(3) + 1) - I*c) + ((b*x^3 + sqrt(3)*(-I*b*x^3 - I*a) + a)*(-I*a*d^
3/b)^(2/3) + 2*(b*x^3 - sqrt(3)*(-I*b*x^3 - I*a) + a)*(-I*a*d^3/b)^(1/3))*E
i(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3
```

```
)*(I*sqrt(3) + 1) + I*c) + ((b*x^3 + sqrt(3)*(I*b*x^3 + I*a) + a)*(I*a*d^3/
b)^(2/3) + 2*(b*x^3 - sqrt(3)*(I*b*x^3 + I*a) + a)*(I*a*d^3/b)^(1/3))*Ei(-I
*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(-I*
sqrt(3) + 1) - I*c) + ((b*x^3 + sqrt(3)*(I*b*x^3 + I*a) + a)*(-I*a*d^3/b)^(
2/3) + 2*(b*x^3 - sqrt(3)*(I*b*x^3 + I*a) + a)*(-I*a*d^3/b)^(1/3))*Ei(I*d*x
 + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*sq
rt(3) + 1) + I*c) - 2*((b*x^3 + a)*(-I*a*d^3/b)^(2/3) + 2*(b*x^3 + a)*(-I*a
*d^3/b)^(1/3))*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3))
- 2*((b*x^3 + a)*(I*a*d^3/b)^(2/3) + 2*(b*x^3 + a)*(I*a*d^3/b)^(1/3))*Ei(-I
*d*x + (I*a*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)))/(a^2*b*d*x^3 + a^3*
d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{(a+bx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)/(b*x**3+a)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{(a+bx^3)^2}\,dx = \int \frac{\sin(dx+c)}{(bx^3+a)^2}\,dx$$

```
[In] integrate(sin(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate(sin(d*x + c)/(b*x^3 + a)^2, x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{(a+bx^3)^2}\,dx = \int \frac{\sin(dx+c)}{(bx^3+a)^2}\,dx$$

```
[In] integrate(sin(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
[Out] integrate(sin(d*x + c)/(b*x^3 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{(a+bx^3)^2}\,dx = \int \frac{\sin(c+d\,x)}{(b\,x^3+a)^2}\,dx$$

```
[In] int(sin(c + d*x)/(a + b*x^3)^2,x)
[Out] int(sin(c + d*x)/(a + b*x^3)^2, x)
```

3.106 $\int \frac{\sin(c+dx)}{x(a+bx^3)^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 693

$$\int \frac{\sin(c+dx)}{x\left(a+bx^3\right)^2}\,dx = \frac{\sqrt[3]{-1}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{5/3}\sqrt[3]{b}} - \frac{d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{5/3}\sqrt[3]{b}} - \frac{(-1)^{2/3}d\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{5/3}\sqrt[3]{b}} + \frac{\operatorname{CosIntegral}(dx)\sin(c)}{a^2} - \frac{\operatorname{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^2} - \frac{\operatorname{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^2} - \frac{\operatorname{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^2} + \frac{\sin(c+dx)}{3abx^3} - \frac{\sin(c+dx)}{3bx^3\left(a+bx^3\right)} + \frac{\cos(c)\operatorname{Si}(dx)}{a^2} + \frac{\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^2} + \frac{\sqrt[3]{-1}d\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{5/3}\sqrt[3]{b}} - \frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^2} + \frac{d\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{5/3}\sqrt[3]{b}} - \frac{\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^2} + \frac{(-1)^{2/3}d\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{5/3}\sqrt[3]{b}}$$

```
[Out] -1/9*d*Ci(a^(1/3)*d/b^(1/3)+d*x)*cos(c-a^(1/3)*d/b^(1/3))/a^(5/3)/b^(1/3)+1
/9*(-1)^(1/3)*d*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cos(c+(-1)^(1/3)*a^(1/
3)*d/b^(1/3))/a^(5/3)/b^(1/3)-1/9*(-1)^(2/3)*d*Ci((-1)^(2/3)*a^(1/3)*d/b^(1
```

```
/3)+d*x)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(1/3)+cos(c)*Si(d*x)
/a^2-1/3*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/
3)+d*x)/a^2-1/3*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^2-1/3*
cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^
2+Ci(d*x)*sin(c)/a^2-1/3*Ci(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))
/a^2+1/9*d*Si(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(5/3)/b^(1/
3)-1/3*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1
/3))/a^2-1/9*(-1)^(1/3)*d*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c+(-1)^
(1/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(1/3)-1/3*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3
)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^2+1/9*(-1)^(2/3)*d*Si((-1)^(2/
3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(1/
3)+1/3*sin(d*x+c)/a/b/x^3-1/3*sin(d*x+c)/b/x^3/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.00, number of steps used = 41, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used

$= \{3424, 3426, 3378, 3384, 3380, 3383, 3427, 3415\}$

$$\int \frac{\sin(c+dx)}{x\left(a+bx^3\right)^2}\,dx = \frac{\sqrt[3]{-1}d\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{5/3}\sqrt[3]{b}}$$

$$-\frac{d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}}$$

$$-\frac{(-1)^{2/3}d\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}}$$

$$+\frac{\sqrt[3]{-1}d\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{5/3}\sqrt[3]{b}}$$

$$+\frac{d\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}}$$

$$+\frac{(-1)^{2/3}d\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}}$$

$$-\frac{\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^2}$$

$$-\frac{\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^2}$$

$$-\frac{\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^2}$$

$$+\frac{\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^2}$$

$$-\frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^2}$$

$$-\frac{\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^2}$$

$$+\frac{\sin(c)\,\text{CosIntegral}(dx)}{a^2}+\frac{\cos(c)\text{Si}(dx)}{a^2}-\frac{\sin(c+dx)}{3bx^3\left(a+bx^3\right)}+\frac{\sin(c+dx)}{3abx^3}$$

```
[In] Int[Sin[c + d*x]/(x*(a + b*x^3)^2),x]
```

```
[Out] ((-1)^(1/3)*d*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(1/
3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(5/3)*b^(1/3)) - (d*Cos[c - (a^(1/3)*d)/
b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3)) - ((-1
)^(2/3)*d*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*a
^(1/3)*d)/b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3)) + (CosIntegral[d*x]*Sin[c])/a
^2 - (CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/
(3*a^2) - (CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^
(1/3)*a^(1/3)*d)/b^(1/3)])/(3*a^2) - (CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^
(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*a^2) + Sin[c + d*x
]/(3*a*b*x^3) - Sin[c + d*x]/(3*b*x^3*(a + b*x^3)) + (Cos[c]*SinIntegral[d*
x])/a^2 + (Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*
a^(1/3)*d)/b^(1/3) - d*x])/(3*a^2) + ((-1)^(1/3)*d*Sin[c + ((-1)^(1/3)*a^(1
/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(5/
3)*b^(1/3)) - (Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3)
 + d*x])/(3*a^2) + (d*Sin[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/
b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3)) - (Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/
3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^2) + ((-1)^(2/3
)*d*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)
*d)/b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3415

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3424

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3427

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sin(c+dx)}{3bx^3(a+bx^3)} - \frac{\int \frac{\sin(c+dx)}{x^4(a+bx^3)}\,dx}{b} + \frac{d\int \frac{\cos(c+dx)}{x^3(a+bx^3)}\,dx}{3b} \\
&= -\frac{\sin(c+dx)}{3bx^3(a+bx^3)} - \frac{\int \left(\frac{\sin(c+dx)}{ax^4} - \frac{b\sin(c+dx)}{a^2x} + \frac{b^2x^2\sin(c+dx)}{a^2(a+bx^3)}\right)dx}{b} \\
&\quad + \frac{d\int \left(\frac{\cos(c+dx)}{ax^3} - \frac{b\cos(c+dx)}{a(a+bx^3)}\right)dx}{3b} \\
&= -\frac{\sin(c+dx)}{3bx^3(a+bx^3)} + \frac{\int \frac{\sin(c+dx)}{x}\,dx}{a^2} - \frac{\int \frac{\sin(c+dx)}{x^4}\,dx}{ab} \\
&\quad - \frac{b\int \frac{x^2\sin(c+dx)}{a+bx^3}\,dx}{a^2} - \frac{d\int \frac{\cos(c+dx)}{a+bx^3}\,dx}{3a} + \frac{d\int \frac{\cos(c+dx)}{x^3}\,dx}{3ab}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{d\cos(c+dx)}{6abx^2} + \frac{\sin(c+dx)}{3abx^3} - \frac{\sin(c+dx)}{3bx^3(a+bx^3)} \\
&- \frac{b\int \left(\frac{\sin(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sin(c+dx)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sin(c+dx)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right)dx}{a^2} \\
&- \frac{d\int \left(-\frac{\cos(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\cos(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{\cos(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{3a} \\
&- \frac{d\int \frac{\cos(c+dx)}{x^3}dx}{3ab} - \frac{d^2\int \frac{\sin(c+dx)}{x^2}dx}{6ab} + \frac{\cos(c)\int \frac{\sin(dx)}{x}dx}{a^2} + \frac{\sin(c)\int \frac{\cos(dx)}{x}dx}{a^2} \\
&= \frac{\text{CosIntegral}(dx)\sin(c)}{a^2} + \frac{\sin(c+dx)}{3abx^3} + \frac{d^2\sin(c+dx)}{6abx} - \frac{\sin(c+dx)}{3bx^3(a+bx^3)} \\
&+ \frac{\cos(c)\text{Si}(dx)}{a^2} - \frac{\sqrt[3]{b}\int \frac{\sin(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{3a^2} - \frac{\sqrt[3]{b}\int \frac{\sin(c+dx)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}dx}{3a^2} \\
&- \frac{\sqrt[3]{b}\int \frac{\sin(c+dx)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}dx}{3a^2} + \frac{d\int \frac{\cos(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}dx}{9a^{5/3}} + \frac{d\int \frac{\cos(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}dx}{9a^{5/3}} \\
&+ \frac{d\int \frac{\cos(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}dx}{9a^{5/3}} + \frac{d^2\int \frac{\sin(c+dx)}{x^2}dx}{6ab} - \frac{d^3\int \frac{\cos(c+dx)}{x}dx}{6ab}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\mathrm{CosIntegral}(dx)\sin(c)}{a^2} + \frac{\sin(c+dx)}{3abx^3} - \frac{\sin(c+dx)}{3bx^3(a+bx^3)} \\
&+ \frac{\cos(c)\mathrm{Si}(dx)}{a^2} + \frac{d^3\int \frac{\cos(c+dx)}{x}\,dx}{6ab} - \frac{(d^3\cos(c))\int \frac{\cos(dx)}{x}\,dx}{6ab} \\
&- \frac{\left(\sqrt[3]{b}\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^2} \\
&+ \frac{\left(d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{9a^{5/3}} \\
&+ \frac{\left(\sqrt[3]{b}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^2} \\
&+ \frac{\left(d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{9a^{5/3}} \\
&- \frac{\left(\sqrt[3]{b}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^2} \\
&+ \frac{\left(d\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{9a^{5/3}} \\
&+ \frac{(d^3\sin(c))\int \frac{\sin(dx)}{x}\,dx}{6ab} - \frac{\left(\sqrt[3]{b}\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^2} \\
&- \frac{\left(d\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{9a^{5/3}} \\
&- \frac{\left(\sqrt[3]{b}\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^2} \\
&+ \frac{\left(d\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{9a^{5/3}} \\
&- \frac{\left(\sqrt[3]{b}\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{d^3\cos(c)\,\text{CosIntegral}(dx)}{6ab} \\
&+ \frac{\sqrt[3]{-1}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{5/3}\sqrt[3]{b}} \\
&- \frac{d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{5/3}\sqrt[3]{b}} \\
&- \frac{(-1)^{2/3}d\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{5/3}\sqrt[3]{b}} \\
&+ \frac{\text{CosIntegral}(dx)\sin(c)}{a^2} - \frac{\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^2} \\
&- \frac{\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^2} \\
&- \frac{\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^2} \\
&+ \frac{\sin(c+dx)}{3abx^3} - \frac{\sin(c+dx)}{3bx^3(a+bx^3)} + \frac{\cos(c)\text{Si}(dx)}{a^2} \\
&+ \frac{d^3\sin(c)\text{Si}(dx)}{6ab} + \frac{\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^2} \\
&+ \frac{\sqrt[3]{-1}d\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{5/3}\sqrt[3]{b}} \\
&- \frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^2} + \frac{d\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{5/3}\sqrt[3]{b}} \\
&- \frac{\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^2} \\
&+ \frac{(-1)^{2/3}d\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{5/3}\sqrt[3]{b}} \\
&+ \frac{(d^3\cos(c))\int\frac{\cos(dx)}{x}\,dx}{6ab} - \frac{(d^3\sin(c))\int\frac{\sin(dx)}{x}\,dx}{6ab}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\sqrt[3]{-1}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{5/3}\sqrt[3]{b}} \\
&- \frac{d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{5/3}\sqrt[3]{b}} \\
&- \frac{(-1)^{2/3}d\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{5/3}\sqrt[3]{b}} \\
&+ \frac{\text{CosIntegral}(dx)\sin(c)}{a^2} - \frac{\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^2} \\
&- \frac{\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^2} \\
&- \frac{\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^2} + \frac{\sin(c+dx)}{3abx^3} \\
&- \frac{\sin(c+dx)}{3bx^3(a+bx^3)} + \frac{\cos(c)\text{Si}(dx)}{a^2} + \frac{\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^2} \\
&+ \frac{\sqrt[3]{-1}d\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{5/3}\sqrt[3]{b}} \\
&- \frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^2} + \frac{d\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{5/3}\sqrt[3]{b}} \\
&- \frac{\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^2} \\
&+ \frac{(-1)^{2/3}d\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{5/3}\sqrt[3]{b}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.40 (sec) , antiderivative size = 446, normalized size of antiderivative = 0.64

$$\int \frac{\sin(c+dx)}{x\left(a+bx^3\right)^2}\,dx$$

$$= \frac{-\frac{1}{2}i\text{RootSum}\left[a+b\#1^3\&,\cos(c+d\#1)\,\text{CosIntegral}(d(x-\#1))-i\,\text{CosIntegral}(d(x-\#1))\sin(c+d\#1\right.}{}$$

```
[In] Integrate[Sin[c + d*x]/(x*(a + b*x^3)^2),x]
```

```
[Out] ((-1/2*I)*RootSum[a + b*#1^3 & , Cos[c + d*#1]*CosIntegral[d*(x - #1)] - I*
CosIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*SinIntegral[d*(x -
#1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)] & ] + (I/2)*RootSum[a + b*#1^3
 & , Cos[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*(x - #1)]*Sin[
c + d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinInte
gral[d*(x - #1)] & ] - (a*d*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegr
al[d*(x - #1)] - I*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*
SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1^2 & ])/
(6*b) - (a*d*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)]
+ I*CosIntegral[d*(x - #1)]*Sin[c + d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(
x - #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1^2 & ])/(6*b) + (a*Cos[
d*x]*Sin[c])/(a + b*x^3) + 3*CosIntegral[d*x]*Sin[c] + (a*Cos[c]*Sin[d*x])/
(a + b*x^3) + 3*Cos[c]*SinIntegral[d*x])/(3*a^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.51 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.34

method	result
derivativedivides	$\frac{\sin(dx+c)d^3}{3a\left(a\,d^3-c^3b+3b\,c^2(dx+c)-3bc(dx+c)^2+b(dx+c)^3\right)} - \frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3\_Z^2bc+3c^2b\_Z+a\,d^3-c^3b\right)}\left(-\mathrm{Si}\left(-dx+\_\right.\right.}{3a^2}$
default	$\frac{\sin(dx+c)d^3}{3a\left(a\,d^3-c^3b+3b\,c^2(dx+c)-3bc(dx+c)^2+b(dx+c)^3\right)} - \frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3\_Z^2bc+3c^2b\_Z+a\,d^3-c^3b\right)}\left(-\mathrm{Si}\left(-dx+\_\right.\right.}{3a^2}$
risch	$-\frac{i\left(\sum_{\_R1=\mathrm{RootOf}\left(-3i\_Z^2bc-id^3a+ibc^3+b\_Z^3-3c^2b\_Z\right)}\frac{\left(-id^3a-6i\_R1bc+3b\_R1^2-3c^2b\right)e^{\_R1}\mathrm{Ei}_1\left(-idx-ic+\_R\right.}{-2ic\_R1+\_R1^2-c^2}\right.}{18a^2b}$

[In] int(sin(d*x+c)/x/(b*x^3+a)^2,x,method=_RETURNVERBOSE)

[Out] 1/3*sin(d*x+c)*d^3/a/(a*d^3-c^3*b+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)-1/3/a^2*sum(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/a^2*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))-1/9*d^3/a/b*sum(1/(_RR1^2-2*_RR1*c+c^2)*(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 576, normalized size of antiderivative = 0.83

$$\int \frac{\sin(c+dx)}{x\left(a+bx^3\right)^2}\,dx$$

$$= \frac{\left(-6i\,bx^3+\left(i\,bx^3-\sqrt{3}(bx^3+a)+i\,a\right)\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}-6i\,a\right)\mathrm{Ei}\left(-i\,dx+\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}(i\right.}}{}$$

[In] integrate(sin(d*x+c)/x/(b*x^3+a)^2,x, algorithm="fricas")

[Out] 1/36*((-6*I*b*x^3 + (I*b*x^3 - sqrt(3)*(b*x^3 + a) + I*a)*(I*a*d^3/b)^(1/3) - 6*I*a)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) + (6*I*b*x^3 + (-I*b*x^3 + sqrt(3)*(b*x^3 + a) - I*a)*(-I*a*d^3/b)^(1/3) + 6*I*a)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) + (-6*I*b*x^3 + (I*b*x^3 + sqrt(3)*(b*x^3 + a) + I*a)*(I*a*d^3/b)^(1/3) - 6*I*a)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) - I*c) + (6*I*b*x^3 + (-I*b*x^3 - sqrt(3)*(b*x^3 + a) - I*

```
a)*(-I*a*d^3/b)^(1/3) + 6*I*a)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3)
 - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) - 2*(-3*I*b*x^3 +
(-I*b*x^3 - I*a)*(-I*a*d^3/b)^(1/3) - 3*I*a)*Ei(I*d*x + (-I*a*d^3/b)^(1/3))
*e^(I*c - (-I*a*d^3/b)^(1/3)) - 2*(3*I*b*x^3 + (I*b*x^3 + I*a)*(I*a*d^3/b)^
(1/3) + 3*I*a)*Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3))
+ 36*(b*x^3 + a)*cos_integral(d*x)*sin(c) + 36*(b*x^3 + a)*cos(c)*sin_integ
ral(d*x) + 12*a*sin(d*x + c))/(a^2*b*x^3 + a^3)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{x(a+bx^3)^2}\,dx = \int \frac{\sin(c+dx)}{x(a+bx^3)^2}\,dx$$

```
[In] integrate(sin(d*x+c)/x/(b*x**3+a)**2,x)
[Out] Integral(sin(c + d*x)/(x*(a + b*x**3)**2), x)
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x(a+bx^3)^2}\,dx = \int \frac{\sin(dx+c)}{(bx^3+a)^2x}\,dx$$

```
[In] integrate(sin(d*x+c)/x/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate(sin(d*x + c)/((b*x^3 + a)^2*x), x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{x(a+bx^3)^2}\,dx = \int \frac{\sin(dx+c)}{(bx^3+a)^2x}\,dx$$

```
[In] integrate(sin(d*x+c)/x/(b*x^3+a)^2,x, algorithm="giac")
[Out] integrate(sin(d*x + c)/((b*x^3 + a)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x\,(a+bx^3)^2}\,dx = \int \frac{\sin\left(c+d\,x\right)}{x\,{\left(b\,x^3+a\right)}^2}\,dx$$

```
[In] int(sin(c + d*x)/(x*(a + b*x^3)^2),x)
[Out] int(sin(c + d*x)/(x*(a + b*x^3)^2), x)
```

3.107 $\int \frac{\sin(c+dx)}{x^2(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 712

$$\int \frac{\sin(c+dx)}{x^2(a+bx^3)^2}\,dx = \frac{d\cos(c)\,\text{CosIntegral}(dx)}{a^2} + \frac{d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^2} + \frac{d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^2} + \frac{d\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^2} + \frac{4\sqrt[3]{b}\,\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{7/3}} + \frac{4(-1)^{2/3}\sqrt[3]{b}\,\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{7/3}} - \frac{4\sqrt[3]{-1}\sqrt[3]{b}\,\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{7/3}} + \frac{\sin(c+dx)}{3abx^4} - \frac{4\sin(c+dx)}{3a^2x} - \frac{\sin(c+dx)}{3bx^4(a+bx^3)} - \frac{d\sin(c)\text{Si}(dx)}{a^2} - \frac{4(-1)^{2/3}\sqrt[3]{b}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{7/3}} + \frac{d\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^2} + \frac{4\sqrt[3]{b}\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{7/3}} - \frac{d\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^2} - \frac{4\sqrt[3]{-1}\sqrt[3]{b}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{7/3}} - \frac{d\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^2}$$

```
[Out] d*Ci(d*x)*cos(c)/a^2+1/9*d*Ci(a^(1/3)*d/b^(1/3)+d*x)*cos(c-a^(1/3)*d/b^(1/3
))/a^2+1/9*d*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cos(c+(-1)^(1/3)*a^(1/3)*
d/b^(1/3))/a^2+1/9*d*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*cos(c-(-1)^(2/3)*
a^(1/3)*d/b^(1/3))/a^2+4/9*(-1)^(2/3)*b^(1/3)*cos(c+(-1)^(1/3)*a^(1/3)*d/b^
(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(7/3)+4/9*b^(1/3)*cos(c-a^(1
/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^(7/3)-4/9*(-1)^(1/3)*b^(1/3)*cos
(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(7/
3)-d*Si(d*x)*sin(c)/a^2+4/9*b^(1/3)*Ci(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)
*d/b^(1/3))/a^(7/3)-1/9*d*Si(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3)
)/a^2+4/9*(-1)^(2/3)*b^(1/3)*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1
)^(1/3)*a^(1/3)*d/b^(1/3))/a^(7/3)-1/9*d*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d
*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^2-4/9*(-1)^(1/3)*b^(1/3)*Ci((-1)^
(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(7/3)-1/
9*d*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3)
)/a^2+1/3*sin(d*x+c)/a/b/x^4-4/3*sin(d*x+c)/a^2/x-1/3*sin(d*x+c)/b/x^4/(b*x
^3+a)
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 712, normalized size of antiderivative = 1.00, number of steps used = 47, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used

$= \{3424, 3426, 3378, 3384, 3380, 3383, 3427\}$

$$\int \frac{\sin(c+dx)}{x^2\left(a+bx^3\right)^2}\,dx = \frac{4\sqrt[3]{b}\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{7/3}}$$
$$+\frac{4(-1)^{2/3}\sqrt[3]{b}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{7/3}}$$
$$-\frac{4\sqrt[3]{-1}\sqrt[3]{b}\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{7/3}}$$
$$-\frac{4(-1)^{2/3}\sqrt[3]{b}\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{7/3}}$$
$$+\frac{4\sqrt[3]{b}\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{7/3}}$$
$$-\frac{4\sqrt[3]{-1}\sqrt[3]{b}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{7/3}}$$
$$+\frac{d\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^2}$$
$$+\frac{d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^2}$$
$$+\frac{d\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^2}$$
$$+\frac{d\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^2}$$
$$-\frac{d\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^2}$$
$$-\frac{d\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^2}$$
$$+\frac{d\cos(c)\,\text{CosIntegral}(dx)}{a^2}-\frac{d\sin(c)\text{Si}(dx)}{a^2}$$
$$-\frac{4\sin(c+dx)}{3a^2x}+\frac{\sin(c+dx)}{3abx^4}-\frac{\sin(c+dx)}{3bx^4\left(a+bx^3\right)}$$

```
[In] Int[Sin[c + d*x]/(x^2*(a + b*x^3)^2),x]
```

```
[Out] (d*Cos[c]*CosIntegral[d*x])/a^2 + (d*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)
]*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^2) + (d*Cos[c - (
a^(1/3)*d)/b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^2) + (d*Co
s[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^
(1/3) + d*x])/(9*a^2) + (4*b^(1/3)*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*S
in[c - (a^(1/3)*d)/b^(1/3)])/(9*a^(7/3)) + (4*(-1)^(2/3)*b^(1/3)*CosIntegra
l[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1
/3)])/(9*a^(7/3)) - (4*(-1)^(1/3)*b^(1/3)*CosIntegral[((-1)^(2/3)*a^(1/3)*d
)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(7/3)) + Sin
[c + d*x]/(3*a*b*x^4) - (4*Sin[c + d*x])/(3*a^2*x) - Sin[c + d*x]/(3*b*x^4*
(a + b*x^3)) - (d*Sin[c]*SinIntegral[d*x])/a^2 - (4*(-1)^(2/3)*b^(1/3)*Cos[
c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1
/3) - d*x])/(9*a^(7/3)) + (d*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinInt
egral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^2) + (4*b^(1/3)*Cos[c - (
a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(7/3)) - (
d*Sin[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a
^2) - (4*(-1)^(1/3)*b^(1/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinInte
gral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(7/3)) - (d*Sin[c - ((-1)^
(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]
)/(9*a^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3424

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3427

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sin(c+dx)}{3bx^4\left(a+bx^3\right)}-\frac{4\int\frac{\sin(c+dx)}{x^5(a+bx^3)}\,dx}{3b}+\frac{d\int\frac{\cos(c+dx)}{x^4(a+bx^3)}\,dx}{3b}\\
&= -\frac{\sin(c+dx)}{3bx^4\left(a+bx^3\right)}-\frac{4\int\left(\frac{\sin(c+dx)}{ax^5}-\frac{b\sin(c+dx)}{a^2x^2}+\frac{b^2x\sin(c+dx)}{a^2(a+bx^3)}\right)dx}{3b}\\
&\quad+\frac{d\int\left(\frac{\cos(c+dx)}{ax^4}-\frac{b\cos(c+dx)}{a^2x}+\frac{b^2x^2\cos(c+dx)}{a^2(a+bx^3)}\right)dx}{3b}\\
&= -\frac{\sin(c+dx)}{3bx^4\left(a+bx^3\right)}+\frac{4\int\frac{\sin(c+dx)}{x^2}\,dx}{3a^2}-\frac{4\int\frac{\sin(c+dx)}{x^5}\,dx}{3ab}-\frac{(4b)\int\frac{x\sin(c+dx)}{a+bx^3}\,dx}{3a^2}\\
&\quad-\frac{d\int\frac{\cos(c+dx)}{x}\,dx}{3a^2}+\frac{d\int\frac{\cos(c+dx)}{x^4}\,dx}{3ab}+\frac{(bd)\int\frac{x^2\cos(c+dx)}{a+bx^3}\,dx}{3a^2}
\end{aligned}$$

$$= -\frac{d\cos(c+dx)}{9abx^3} + \frac{\sin(c+dx)}{3abx^4} - \frac{4\sin(c+dx)}{3a^2x} - \frac{\sin(c+dx)}{3bx^4(a+bx^3)}$$

$$-\frac{(4b)\int\left(-\frac{\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{3a^2}$$

$$+\frac{(4d)\int\frac{\cos(c+dx)}{x}dx}{3a^2} - \frac{d\int\frac{\cos(c+dx)}{x^4}dx}{3ab}$$

$$+\frac{(bd)\int\left(\frac{\cos(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\cos(c+dx)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\cos(c+dx)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right)dx}{3a^2}$$

$$-\frac{d^2\int\frac{\sin(c+dx)}{x^3}dx}{9ab} - \frac{(d\cos(c))\int\frac{\cos(dx)}{x}dx}{3a^2} + \frac{(d\sin(c))\int\frac{\sin(dx)}{x}dx}{3a^2}$$

$$= -\frac{d\cos(c)\operatorname{CosIntegral}(dx)}{3a^2} + \frac{\sin(c+dx)}{3abx^4} + \frac{d^2\sin(c+dx)}{18abx^2}$$

$$-\frac{4\sin(c+dx)}{3a^2x} - \frac{\sin(c+dx)}{3bx^4(a+bx^3)} + \frac{d\sin(c)\operatorname{Si}(dx)}{3a^2}$$

$$+\frac{\left(4b^{2/3}\right)\int\frac{\sin(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{9a^{7/3}} - \frac{\left(4\sqrt[3]{-1}b^{2/3}\right)\int\frac{\sin(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}dx}{9a^{7/3}}$$

$$+\frac{\left(4(-1)^{2/3}b^{2/3}\right)\int\frac{\sin(c+dx)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}dx}{9a^{7/3}} + \frac{\left(\sqrt[3]{b}d\right)\int\frac{\cos(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{9a^2}$$

$$+\frac{\left(\sqrt[3]{b}d\right)\int\frac{\cos(c+dx)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}dx}{9a^2} + \frac{\left(\sqrt[3]{b}d\right)\int\frac{\cos(c+dx)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}dx}{9a^2} + \frac{d^2\int\frac{\sin(c+dx)}{x^3}dx}{9ab}$$

$$-\frac{d^3\int\frac{\cos(c+dx)}{x^2}dx}{18ab} + \frac{(4d\cos(c))\int\frac{\cos(dx)}{x}dx}{3a^2} - \frac{(4d\sin(c))\int\frac{\sin(dx)}{x}dx}{3a^2}$$

$= \text{Too large to display}$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.54 (sec) , antiderivative size = 445, normalized size of antiderivative = 0.62

$$\int\frac{\sin(c+dx)}{x^2(a+bx^3)^2}dx =$$

$$-\frac{(3a+4bx^3)\cos(dx)\sin(c) + (3a+4bx^3)\cos(c)\sin(dx) - \frac{1}{6}x(a+bx^3)\left(18d\cos(c)\operatorname{CosIntegral}(dx) + \mathrm{R}\right.}{}$$

```
[In] Integrate[Sin[c + d*x]/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -1/3*((3*a + 4*b*x^3)*Cos[d*x]*Sin[c] + (3*a + 4*b*x^3)*Cos[c]*Sin[d*x] - (
x*(a + b*x^3)*(18*d*Cos[c]*CosIntegral[d*x] + RootSum[a + b*#1^3 & , ((-4*I
)*Cos[c + d*#1]*CosIntegral[d*(x - #1)] - 4*CosIntegral[d*(x - #1)]*Sin[c +
 d*#1] - 4*Cos[c + d*#1]*SinIntegral[d*(x - #1)] + (4*I)*Sin[c + d*#1]*SinI
ntegral[d*(x - #1)] + d*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1 - I*d*CosI
ntegral[d*(x - #1)]*Sin[c + d*#1]*#1 - I*d*Cos[c + d*#1]*SinIntegral[d*(x -
 #1)]*#1 - d*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1)/#1 & ] + RootSum[a +
 b*#1^3 & , ((4*I)*Cos[c + d*#1]*CosIntegral[d*(x - #1)] - 4*CosIntegral[d*
(x - #1)]*Sin[c + d*#1] - 4*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - (4*I)*S
in[c + d*#1]*SinIntegral[d*(x - #1)] + d*Cos[c + d*#1]*CosIntegral[d*(x - #
1)]*#1 + I*d*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1 + I*d*Cos[c + d*#1]*S
inIntegral[d*(x - #1)]*#1 - d*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1)/#1
& ] - 18*d*Sin[c]*SinIntegral[d*x]))/6)/(a^2*x*(a + b*x^3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.78 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.40

method	result
derivativedivides	$d\left(-\frac{\sin(dx+c)\left(\frac{4b(dx+c)^3}{3a^2}-\frac{4cb(dx+c)^2}{a^2}+\frac{4c^2b(dx+c)}{a^2}+\frac{3a\,d^3-4c^3b}{3a^2}\right)}{dx\left(a\,d^3-c^3b+3b\,c^2(dx+c)-3bc(dx+c)^2+b(dx+c)^3\right)}+\frac{4\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3\_Z^2bc+3c^2b\_Z+a\,d^3-\right.}\right.}{}\right.$
default	$d\left(-\frac{\sin(dx+c)\left(\frac{4b(dx+c)^3}{3a^2}-\frac{4cb(dx+c)^2}{a^2}+\frac{4c^2b(dx+c)}{a^2}+\frac{3a\,d^3-4c^3b}{3a^2}\right)}{dx\left(a\,d^3-c^3b+3b\,c^2(dx+c)-3bc(dx+c)^2+b(dx+c)^3\right)}+\frac{4\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3\_Z^2bc+3c^2b\_Z+a\,d^3-\right.}\right.}{}\right.$
risch	$-\frac{d\,\mathrm{Ei}_1(-idx)\mathrm{e}^{ic}}{2a^2}-\frac{d\left(\sum_{\_R1=\mathrm{RootOf}\left(-3i\_Z^2bc-id^3a+ibc^3+b\_Z^3-3c^2b\_Z\right)}\frac{\left(-ic+\_R1-4\right)\mathrm{e}^{\_R1}\,\mathrm{Ei}_1\left(-idx-ic+\_R\right.}{-ic+\_R1}\right.}{18a^2}$

```
[In] int(sin(d*x+c)/x^2/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d*(-sin(d*x+c)*(4/3*b/a^2*(d*x+c)^3-4*c*b/a^2*(d*x+c)^2+4*c^2*b/a^2*(d*x+c)
+1/3*(3*a*d^3-4*b*c^3)/a^2)/d/x/(a*d^3-c^3*b+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^
2+b*(d*x+c)^3)+4/9/a^2*sum(1/(-_R1+c)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+
c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/9/a^2*
sum(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1),_RR1=RootOf(_Z^3*b-3
*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/a^2*(-Si(d*x)*sin(c)+Ci(d*x)*cos(c)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 720, normalized size of antiderivative = 1.01

$$\int \frac{\sin(c+dx)}{x^2\left(a+bx^3\right)^2}\,dx$$

$$= \frac{18\left(abd^3x^4+a^2d^3x\right)\cos\left(c\right)\mathrm{Ci}\left(dx\right)+\left(abd^3x^4+a^2d^3x-2\left(-i\,b^2x^4-i\,abx-\sqrt{3}(b^2x^4+abx)\right)\left(\frac{i\,ad^3}{b}\right)^{\frac{2}{3}}\right)}{}$$

```
[In] integrate(sin(d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(18*(a*b*d^3*x^4 + a^2*d^3*x)*cos(c)*cos_integral(d*x) + (a*b*d^3*x^4
+ a^2*d^3*x - 2*(-I*b^2*x^4 - I*a*b*x - sqrt(3)*(b^2*x^4 + a*b*x))*(I*a*d^3
/b)^(2/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*
d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) + (a*b*d^3*x^4 + a^2*d^3*x - 2*(I*b^2*x
^4 + I*a*b*x + sqrt(3)*(b^2*x^4 + a*b*x))*(-I*a*d^3/b)^(2/3))*Ei(I*d*x + 1/
2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3)
 + 1) + I*c) + (a*b*d^3*x^4 + a^2*d^3*x - 2*(-I*b^2*x^4 - I*a*b*x + sqrt(3)
*(b^2*x^4 + a*b*x))*(I*a*d^3/b)^(2/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I
*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) - I*c) + (a*b*d^3*
x^4 + a^2*d^3*x - 2*(I*b^2*x^4 + I*a*b*x - sqrt(3)*(b^2*x^4 + a*b*x))*(-I*a
*d^3/b)^(2/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-
I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) + (a*b*d^3*x^4 + a^2*d^3*x - 4*(-I
*b^2*x^4 - I*a*b*x)*(-I*a*d^3/b)^(2/3))*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I
*c - (-I*a*d^3/b)^(1/3)) + (a*b*d^3*x^4 + a^2*d^3*x - 4*(I*b^2*x^4 + I*a*b*
x)*(I*a*d^3/b)^(2/3))*Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^
(1/3)) - 18*(a*b*d^3*x^4 + a^2*d^3*x)*sin(c)*sin_integral(d*x) - 6*(4*a*b*d
^2*x^3 + 3*a^2*d^2)*sin(d*x + c))/(a^3*b*d^2*x^4 + a^4*d^2*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x^2\left(a+bx^3\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)/x**2/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x^2(a+bx^3)^2}\,dx = \int \frac{\sin(dx+c)}{(bx^3+a)^2x^2}\,dx$$

```
[In] integrate(sin(d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate(sin(d*x + c)/((b*x^3 + a)^2*x^2), x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{x^2(a+bx^3)^2}\,dx = \int \frac{\sin(dx+c)}{(bx^3+a)^2x^2}\,dx$$

```
[In] integrate(sin(d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="giac")
[Out] integrate(sin(d*x + c)/((b*x^3 + a)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x^2(a+bx^3)^2}\,dx = \int \frac{\sin(c+d\,x)}{x^2\,(b\,x^3+a)^2}\,dx$$

```
[In] int(sin(c + d*x)/(x^2*(a + b*x^3)^2),x)
[Out] int(sin(c + d*x)/(x^2*(a + b*x^3)^2), x)
```

3.108 $\int \frac{\sin(c+dx)}{x^3(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 800

$$
\begin{aligned}
\int \frac{\sin(c+dx)}{x^3\left(a+bx^3\right)^2}\,dx = &-\frac{d\cos(c+dx)}{2a^2x} \\
&-\frac{(-1)^{2/3}\sqrt[3]{b}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{7/3}} \\
&-\frac{\sqrt[3]{b}d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{7/3}} \\
&+\frac{\sqrt[3]{-1}\sqrt[3]{b}d\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{7/3}} \\
&-\frac{d^2\,\text{CosIntegral}(dx)\sin(c)}{2a^2} \\
&-\frac{5b^{2/3}\,\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{8/3}} \\
&+\frac{5\sqrt[3]{-1}b^{2/3}\,\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{8/3}} \\
&-\frac{5(-1)^{2/3}b^{2/3}\,\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{8/3}} \\
&+\frac{\sin(c+dx)}{3abx^5}-\frac{5\sin(c+dx)}{6a^2x^2}-\frac{\sin(c+dx)}{3bx^5\left(a+bx^3\right)}-\frac{d^2\cos(c)\text{Si}(dx)}{2a^2} \\
&-\frac{5\sqrt[3]{-1}b^{2/3}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{8/3}} \\
&-\frac{(-1)^{2/3}\sqrt[3]{b}d\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{7/3}} \\
&-\frac{5b^{2/3}\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{8/3}} \\
&+\frac{\sqrt[3]{b}d\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{7/3}} \\
&-\frac{5(-1)^{2/3}b^{2/3}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{8/3}} \\
&-\frac{\sqrt[3]{-1}\sqrt[3]{b}d\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{7/3}}
\end{aligned}
$$

```
[Out] -1/9*b^(1/3)*d*Ci(a^(1/3)*d/b^(1/3)+d*x)*cos(c-a^(1/3)*d/b^(1/3))/a^(7/3)-1
/9*(-1)^(2/3)*b^(1/3)*d*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cos(c+(-1)^(1/
3)*a^(1/3)*d/b^(1/3))/a^(7/3)+1/9*(-1)^(1/3)*b^(1/3)*d*Ci((-1)^(2/3)*a^(1/3
)*d/b^(1/3)+d*x)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(7/3)-1/2*d*cos(d*x+
c)/a^2/x-1/2*d^2*cos(c)*Si(d*x)/a^2+5/9*(-1)^(1/3)*b^(2/3)*cos(c+(-1)^(1/3)
*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(8/3)-5/9*b^(2/
3)*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^(8/3)-5/9*(-1)^(2/3
)*b^(2/3)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/
3)+d*x)/a^(8/3)-1/2*d^2*Ci(d*x)*sin(c)/a^2-5/9*b^(2/3)*Ci(a^(1/3)*d/b^(1/3)
+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(8/3)+1/9*b^(1/3)*d*Si(a^(1/3)*d/b^(1/3)+d
*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(7/3)+5/9*(-1)^(1/3)*b^(2/3)*Ci((-1)^(1/3)*a
^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(8/3)+1/9*(-1)^
(2/3)*b^(1/3)*d*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c+(-1)^(1/3)*a^(1
/3)*d/b^(1/3))/a^(7/3)-5/9*(-1)^(2/3)*b^(2/3)*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/
3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(8/3)-1/9*(-1)^(1/3)*b^(1/3)*
d*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/
a^(7/3)+1/3*sin(d*x+c)/a/b/x^5-5/6*sin(d*x+c)/a^2/x^2-1/3*sin(d*x+c)/b/x^5/
(b*x^3+a)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 800, normalized size of antiderivative = 1.00, number of steps used = 51, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used

$= \{3424, 3426, 3378, 3384, 3380, 3383, 3414, 3427\}$

$$\int \frac{\sin(c+dx)}{x^3\left(a+bx^3\right)^2}\,dx = -\frac{\text{CosIntegral}(dx)\sin(c)d^2}{2a^2} - \frac{\cos(c)\text{Si}(dx)d^2}{2a^2} - \frac{\cos(c+dx)d}{2a^2x}$$
$$- \frac{(-1)^{2/3}\sqrt[3]{b}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)d}{9a^{7/3}}$$
$$- \frac{\sqrt[3]{b}\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d}{9a^{7/3}}$$
$$+ \frac{\sqrt[3]{-1}\sqrt[3]{b}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d}{9a^{7/3}}$$
$$- \frac{(-1)^{2/3}\sqrt[3]{b}\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)d}{9a^{7/3}}$$
$$+ \frac{\sqrt[3]{b}\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d}{9a^{7/3}}$$
$$- \frac{\sqrt[3]{-1}\sqrt[3]{b}\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d}{9a^{7/3}}$$
$$- \frac{5b^{2/3}\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{8/3}}$$
$$+ \frac{5\sqrt[3]{-1}b^{2/3}\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{8/3}}$$
$$- \frac{5(-1)^{2/3}b^{2/3}\text{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{8/3}}$$
$$- \frac{\sin(c+dx)}{3bx^5\left(bx^3+a\right)} - \frac{5\sin(c+dx)}{6a^2x^2} + \frac{\sin(c+dx)}{3abx^5}$$
$$- \frac{5\sqrt[3]{-1}b^{2/3}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{8/3}}$$
$$- \frac{5b^{2/3}\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{8/3}}$$
$$- \frac{5(-1)^{2/3}b^{2/3}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{8/3}}$$

```
[In] Int[Sin[c + d*x]/(x^3*(a + b*x^3)^2),x]
```

```
[Out] -1/2*(d*Cos[c + d*x])/(a^2*x) - ((-1)^(2/3)*b^(1/3)*d*Cos[c + ((-1)^(1/3)*a
^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^
(7/3)) - (b^(1/3)*d*Cos[c - (a^(1/3)*d)/b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^
(1/3) + d*x])/(9*a^(7/3)) + ((-1)^(1/3)*b^(1/3)*d*Cos[c - ((-1)^(2/3)*a^(1/
3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(7/3
)) - (d^2*CosIntegral[d*x]*Sin[c])/(2*a^2) - (5*b^(2/3)*CosIntegral[(a^(1/3
)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(9*a^(8/3)) + (5*(-1)^(1/
3)*b^(2/3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^
(1/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(8/3)) - (5*(-1)^(2/3)*b^(2/3)*CosIntegral[
((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3
)])/(9*a^(8/3)) + Sin[c + d*x]/(3*a*b*x^5) - (5*Sin[c + d*x])/(6*a^2*x^2) -
 Sin[c + d*x]/(3*b*x^5*(a + b*x^3)) - (d^2*Cos[c]*SinIntegral[d*x])/(2*a^2)
 - (5*(-1)^(1/3)*b^(2/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegra
l[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(8/3)) - ((-1)^(2/3)*b^(1/3)*
d*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d
)/b^(1/3) - d*x])/(9*a^(7/3)) - (5*b^(2/3)*Cos[c - (a^(1/3)*d)/b^(1/3)]*Sin
Integral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(8/3)) + (b^(1/3)*d*Sin[c - (a^(1
/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(7/3)) - (5*(-
1)^(2/3)*b^(2/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^
(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(8/3)) - ((-1)^(1/3)*b^(1/3)*d*Sin[c
- ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3
) + d*x])/(9*a^(7/3))
```

<u>Rule 3378</u>

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

<u>Rule 3380</u>

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

<u>Rule 3383</u>

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

<u>Rule 3384</u>

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3424

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3427

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sin(c+dx)}{3bx^5\left(a+bx^3\right)}-\frac{5\int\frac{\sin(c+dx)}{x^6(a+bx^3)}\,dx}{3b}+\frac{d\int\frac{\cos(c+dx)}{x^5(a+bx^3)}\,dx}{3b}\\
&= -\frac{\sin(c+dx)}{3bx^5\left(a+bx^3\right)}-\frac{5\int\left(\frac{\sin(c+dx)}{ax^6}-\frac{b\sin(c+dx)}{a^2x^3}+\frac{b^2\sin(c+dx)}{a^2(a+bx^3)}\right)dx}{3b}\\
&\quad+\frac{d\int\left(\frac{\cos(c+dx)}{ax^5}-\frac{b\cos(c+dx)}{a^2x^2}+\frac{b^2x\cos(c+dx)}{a^2(a+bx^3)}\right)dx}{3b}\\
&= -\frac{\sin(c+dx)}{3bx^5\left(a+bx^3\right)}+\frac{5\int\frac{\sin(c+dx)}{x^3}\,dx}{3a^2}-\frac{5\int\frac{\sin(c+dx)}{x^6}\,dx}{3ab}-\frac{(5b)\int\frac{\sin(c+dx)}{a+bx^3}\,dx}{3a^2}\\
&\quad-\frac{d\int\frac{\cos(c+dx)}{x^2}\,dx}{3a^2}+\frac{d\int\frac{\cos(c+dx)}{x^5}\,dx}{3ab}+\frac{(bd)\int\frac{x\cos(c+dx)}{a+bx^3}\,dx}{3a^2}
\end{aligned}$$

$$= -\frac{d\cos(c+dx)}{12abx^4} + \frac{d\cos(c+dx)}{3a^2x} + \frac{\sin(c+dx)}{3abx^5} - \frac{5\sin(c+dx)}{6a^2x^2} - \frac{\sin(c+dx)}{3bx^5\,(a+bx^3)} - \frac{(5b)\int\left(-\frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\,\sqrt[3]{b}x\right)} - \frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\,\sqrt[3]{b}x\right)}\right)dx}{3a^2} + \frac{(5d)\int\frac{\cos(c+dx)}{x^2}dx}{6a^2} - \frac{d\int\frac{\cos(c+dx)}{x^5}dx}{3ab} + \frac{(bd)\int\left(-\frac{\cos(c+dx)}{3\sqrt[3]{a}\,\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}\cos(c+dx)}{3\sqrt[3]{a}\,\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\,\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}\cos(c+dx)}{3\sqrt[3]{a}\,\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\,\sqrt[3]{b}x\right)}\right)dx}{3a^2} + \frac{d^2\int\frac{\sin(c+dx)}{x}dx}{3a^2} - \frac{d^2\int\frac{\sin(c+dx)}{x^4}dx}{12ab}$$

$$= -\frac{d\cos(c+dx)}{2a^2x} + \frac{\sin(c+dx)}{3abx^5} + \frac{d^2\sin(c+dx)}{36abx^3} - \frac{5\sin(c+dx)}{6a^2x^2} - \frac{\sin(c+dx)}{3bx^5\,(a+bx^3)} + \frac{(5b)\int\frac{\sin(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}dx}{9a^{8/3}} + \frac{(5b)\int\frac{\sin(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\,\sqrt[3]{b}x}dx}{9a^{8/3}} + \frac{(5b)\int\frac{\sin(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\,\sqrt[3]{b}x}dx}{9a^{8/3}} - \frac{\left(b^{2/3}d\right)\int\frac{\cos(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{9a^{7/3}} + \frac{\left(\sqrt[3]{-1}b^{2/3}d\right)\int\frac{\cos(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\,\sqrt[3]{b}x}dx}{9a^{7/3}} - \frac{\left((-1)^{2/3}b^{2/3}d\right)\int\frac{\cos(c+dx)}{\sqrt[3]{a}-\sqrt[3]{-1}\,\sqrt[3]{b}x}dx}{9a^{7/3}} - \frac{\left(5d^2\right)\int\frac{\sin(c+dx)}{x}dx}{6a^2} + \frac{d^2\int\frac{\sin(c+dx)}{x^4}dx}{12ab} - \frac{d^3\int\frac{\cos(c+dx)}{x^3}dx}{36ab} + \frac{\left(d^2\cos(c)\right)\int\frac{\sin(dx)}{x}dx}{3a^2} + \frac{\left(d^2\sin(c)\right)\int\frac{\cos(dx)}{x}dx}{3a^2}$$

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Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.62 (sec) , antiderivative size = 470, normalized size of antiderivative = 0.59

$$\int \frac{\sin(c+dx)}{x^3\left(a+bx^3\right)^2}\,dx$$

$$= \frac{\text{RootSum}\left[a+b\#1^3\&, \frac{-5i\cos(c+d\#1)\,\text{CosIntegral}(d(x-\#1))-5\,\text{CosIntegral}(d(x-\#1))\sin(c+d\#1)-5\cos(c+d\#1)\text{Si}(d(x-\#1}{}\right.}{}$$

```
[In] Integrate[Sin[c + d*x]/(x^3*(a + b*x^3)^2),x]
```

```
[Out] (RootSum[a + b*#1^3 & , ((-5*I)*Cos[c + d*#1]*CosIntegral[d*(x - #1)] - 5*C
osIntegral[d*(x - #1)]*Sin[c + d*#1] - 5*Cos[c + d*#1]*SinIntegral[d*(x - #
```

```
1)] + (5*I)*Sin[c + d*#1]*SinIntegral[d*(x - #1)] + d*Cos[c + d*#1]*CosInte
gral[d*(x - #1)]*#1 - I*d*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1 - I*d*Co
s[c + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c + d*#1]*SinIntegral[d*(x -
 #1)]*#1)/#1^2 & ] + RootSum[a + b*#1^3 & , ((5*I)*Cos[c + d*#1]*CosIntegra
l[d*(x - #1)] - 5*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - 5*Cos[c + d*#1]*S
inIntegral[d*(x - #1)] - (5*I)*Sin[c + d*#1]*SinIntegral[d*(x - #1)] + d*Co
s[c + d*#1]*CosIntegral[d*(x - #1)]*#1 + I*d*CosIntegral[d*(x - #1)]*Sin[c
+ d*#1]*#1 + I*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c + d*#1]
*SinIntegral[d*(x - #1)]*#1)/#1^2 & ] - (3*(3*a*d*x*Cos[c + d*x] + 3*b*d*x^
4*Cos[c + d*x] + 3*d^2*x^2*(a + b*x^3)*CosIntegral[d*x]*Sin[c] + 3*a*Sin[c
+ d*x] + 5*b*x^3*Sin[c + d*x] + 3*d^2*x^2*(a + b*x^3)*Cos[c]*SinIntegral[d*
x]))/(x^2*(a + b*x^3)))/(18*a^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.93 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.39

method	result
risch	$-\frac{i d^2 \mathrm{Ei}_1(-idx)\mathrm{e}^{ic}}{4a^2} - \frac{i d^2 \left(\sum_{\_R1=\mathrm{RootOf}\left(-3i\_Z^2 bc - id^3 a + ibc^3 + b\_Z^3 - 3c^2 b\_Z\right)} \frac{\left(-ic+\_R1-5\right)\mathrm{e}^{\_R1}\mathrm{Ei}_1\left(-idx-ic+\_\right.}{-2ic\_R1+\_R1^2-c^2}\right.}{18a^2}$
derivativedivides	$d^2 \left(\frac{-\frac{\sin(dx+c)}{2d^2x^2} - \frac{\cos(dx+c)}{2dx} - \frac{\mathrm{Si}(dx)\cos(c)}{2} - \frac{\mathrm{Ci}(dx)\sin(c)}{2}}{a^2} - \frac{b d^3 \left(\frac{\sin(dx+c)\left(\frac{dx+c}{3a d^3} - \frac{c}{3a d^3}\right)}{a d^3 - c^3 b + 3bc^2(dx+c) - 3bc(dx+c)^2 + b(dx+c)^3} + \frac{2\left(\_R1=\right.}{} \right.}{} \right.$
default	$d^2 \left(\frac{-\frac{\sin(dx+c)}{2d^2x^2} - \frac{\cos(dx+c)}{2dx} - \frac{\mathrm{Si}(dx)\cos(c)}{2} - \frac{\mathrm{Ci}(dx)\sin(c)}{2}}{a^2} - \frac{b d^3 \left(\frac{\sin(dx+c)\left(\frac{dx+c}{3a d^3} - \frac{c}{3a d^3}\right)}{a d^3 - c^3 b + 3bc^2(dx+c) - 3bc(dx+c)^2 + b(dx+c)^3} + \frac{2\left(\_R1=\right.}{} \right.}{} \right.$

```
[In] int(sin(d*x+c)/x^3/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I*d^2/a^2*Ei(1,-I*d*x)*exp(I*c)-1/18*I*d^2/a^2*sum((-I*c+_R1-5)/(-2*I*
c*_R1+_R1^2-c^2)*exp(_R1)*Ei(1,_R1-I*d*x-I*c),_R1=RootOf(-3*I*_Z^2*b*c-I*d^
```

```
3*a+I*b*c^3+b*_Z^3-3*c^2*b*_Z))+1/4*I*d^2/a^2*Ei(1,I*d*x)*exp(-I*c)-1/18*I*
d^2/a^2*sum((-I*c+_R1+5)/(-2*I*c*_R1+_R1^2-c^2)*exp(-_R1)*Ei(1,I*d*x+I*c-_R
1),_R1=RootOf(-3*I*_Z^2*b*c-I*d^3*a+I*b*c^3+b*_Z^3-3*c^2*b*_Z))+1/2*(-b*d^4
*x^4-a*d^4*x)/a^2/x^2/(b*d^3*x^3+a*d^3)*cos(d*x+c)-1/6*(5*b*d^3*x^3+3*a*d^3
)/a^2/x^2/(b*d^3*x^3+a*d^3)*sin(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 898, normalized size of antiderivative = 1.12

$$\int \frac{\sin(c+dx)}{x^3\left(a+bx^3\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/36*(((b^2*x^5 + a*b*x^2 - sqrt(3)*(I*b^2*x^5 + I*a*b*x^2))*(I*a*d^3/b)^(
2/3) + 5*(b^2*x^5 + a*b*x^2 + sqrt(3)*(I*b^2*x^5 + I*a*b*x^2))*(I*a*d^3/b)^
(1/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/
b)^(1/3)*(I*sqrt(3) + 1) - I*c) + ((b^2*x^5 + a*b*x^2 - sqrt(3)*(I*b^2*x^5
+ I*a*b*x^2))*(-I*a*d^3/b)^(2/3) + 5*(b^2*x^5 + a*b*x^2 + sqrt(3)*(I*b^2*x^
5 + I*a*b*x^2))*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*s
qrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) + ((b^2*x^5 +
 a*b*x^2 - sqrt(3)*(-I*b^2*x^5 - I*a*b*x^2))*(I*a*d^3/b)^(2/3) + 5*(b^2*x^5
 + a*b*x^2 + sqrt(3)*(-I*b^2*x^5 - I*a*b*x^2))*(I*a*d^3/b)^(1/3))*Ei(-I*d*x
 + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt
(3) + 1) - I*c) + ((b^2*x^5 + a*b*x^2 - sqrt(3)*(-I*b^2*x^5 - I*a*b*x^2))*(
-I*a*d^3/b)^(2/3) + 5*(b^2*x^5 + a*b*x^2 + sqrt(3)*(-I*b^2*x^5 - I*a*b*x^2)
)*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^
(1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) - 2*((b^2*x^5 + a*b*x^2)*(-
I*a*d^3/b)^(2/3) + 5*(b^2*x^5 + a*b*x^2)*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + (-I
*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) - 2*((b^2*x^5 + a*b*x^2)*(I*a
*d^3/b)^(2/3) + 5*(b^2*x^5 + a*b*x^2)*(I*a*d^3/b)^(1/3))*Ei(-I*d*x + (I*a*d
^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)) + 18*(a*b*d^3*x^5 + a^2*d^3*x^2)*
cos_integral(d*x)*sin(c) + 18*(a*b*d^3*x^5 + a^2*d^3*x^2)*cos(c)*sin_integr
al(d*x) + 18*(a*b*d^2*x^4 + a^2*d^2*x)*cos(d*x + c) + 6*(5*a*b*d*x^3 + 3*a^
2*d)*sin(d*x + c))/(a^3*b*d*x^5 + a^4*d*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x^3\left(a+bx^3\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate(sin(d*x+c)/x**3/(b*x**3+a)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x^3\left(a+bx^3\right)^2}\,dx=\int \frac{\sin\left(dx+c\right)}{\left(bx^3+a\right)^2x^3}\,dx$$

```
[In] integrate(sin(d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate(sin(d*x + c)/((b*x^3 + a)^2*x^3), x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{x^3\left(a+bx^3\right)^2}\,dx=\int \frac{\sin\left(dx+c\right)}{\left(bx^3+a\right)^2x^3}\,dx$$

```
[In] integrate(sin(d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="giac")
[Out] integrate(sin(d*x + c)/((b*x^3 + a)^2*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x^3\left(a+bx^3\right)^2}\,dx=\int \frac{\sin\left(c+d\,x\right)}{x^3\,\left(b\,x^3+a\right)^2}\,dx$$

```
[In] int(sin(c + d*x)/(x^3*(a + b*x^3)^2),x)
[Out] int(sin(c + d*x)/(x^3*(a + b*x^3)^2), x)
```

3.109 $\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 772

$$\int \frac{x^3\sin(c+dx)}{(a+bx^3)^3}\,dx = \frac{d\cos(c+dx)}{18ab^2x} - \frac{d\cos(c+dx)}{18b^2x\,(a+bx^3)}$$
$$+ \frac{\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{5/3}b^{4/3}}$$
$$+ \frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54ab^2}$$
$$- \frac{\sqrt[3]{-1}\,\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{5/3}b^{4/3}}$$
$$+ \frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54ab^2}$$
$$+ \frac{(-1)^{2/3}\,\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{5/3}b^{4/3}}$$
$$+ \frac{d^2\,\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54ab^2}$$
$$+ \frac{\sin(c+dx)}{18ab^2x^2} - \frac{x\sin(c+dx)}{6b\,(a+bx^3)^2} - \frac{\sin(c+dx)}{18b^2x^2\,(a+bx^3)}$$
$$+ \frac{\sqrt[3]{-1}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^{5/3}b^{4/3}}$$
$$- \frac{d^2\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54ab^2}$$
$$+ \frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{27a^{5/3}b^{4/3}}$$
$$+ \frac{d^2\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{54ab^2}$$
$$+ \frac{(-1)^{2/3}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{27a^{5/3}b^{4/3}}$$
$$+ \frac{d^2\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{54ab^2}$$

```
[Out] 1/18*d*cos(d*x+c)/a/b^2/x-1/18*d*cos(d*x+c)/b^2/x/(b*x^3+a)-1/27*(-1)^(1/3)
*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/
a^(5/3)/b^(4/3)+1/54*d^2*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)
*a^(1/3)*d/b^(1/3)+d*x)/a/b^2+1/27*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^
(1/3)+d*x)/a^(5/3)/b^(4/3)+1/54*d^2*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b
^(1/3)+d*x)/a/b^2+1/27*(-1)^(2/3)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-
1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(4/3)+1/54*d^2*cos(c-(-1)^(2/3)*a
^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a/b^2+1/27*Ci(a^(1/3
)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)+1/54*d^2*Ci(a^(1/
3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a/b^2-1/27*(-1)^(1/3)*Ci((-1)^(1
/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4
/3)+1/54*d^2*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*
d/b^(1/3))/a/b^2+1/27*(-1)^(2/3)*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c
-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)+1/54*d^2*Ci((-1)^(2/3)*a^(1/
3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a/b^2+1/18*sin(d*x+c)
/a/b^2/x^2-1/6*x*sin(d*x+c)/b/(b*x^3+a)^2-1/18*sin(d*x+c)/b^2/x^2/(b*x^3+a)
```

Rubi [A] (verified)

Time = 1.77 (sec) , antiderivative size = 772, normalized size of antiderivative = 1.00, number of steps used = 71, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules

used = {3424, 3412, 3426, 3378, 3384, 3380, 3383, 3414, 3427, 3425}

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^3}\,dx = \frac{\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{5/3}b^{4/3}}$$
$$-\frac{\sqrt[3]{-1}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^{5/3}b^{4/3}}$$
$$+\frac{(-1)^{2/3}\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{5/3}b^{4/3}}$$
$$+\frac{\sqrt[3]{-1}\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^{5/3}b^{4/3}}$$
$$+\frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{5/3}b^{4/3}}$$
$$+\frac{(-1)^{2/3}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{5/3}b^{4/3}}$$
$$+\frac{d^2\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54ab^2}$$
$$+\frac{d^2\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54ab^2}$$
$$+\frac{d^2\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54ab^2}$$
$$-\frac{d^2\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54ab^2}$$
$$+\frac{d^2\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54ab^2}$$
$$+\frac{d^2\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54ab^2}-\frac{d\cos(c+dx)}{18b^2x\,(a+bx^3)}$$
$$+\frac{\sin(c+dx)}{18ab^2x^2}-\frac{\sin(c+dx)}{18b^2x^2\,(a+bx^3)}+\frac{d\cos(c+dx)}{18ab^2x}-\frac{x\sin(c+dx)}{6b\,(a+bx^3)^2}$$

```
[In] Int[(x^3*Sin[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] (d*Cos[c + d*x])/(18*a*b^2*x) - (d*Cos[c + d*x])/(18*b^2*x*(a + b*x^3)) + (
CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(27*a^
(5/3)*b^(4/3)) + (d^2*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/
3)*d)/b^(1/3)])/(54*a*b^2) - ((-1)^(1/3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)
/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(27*a^(5/3)*b^(4/3
)) + (d^2*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(
1/3)*a^(1/3)*d)/b^(1/3)])/(54*a*b^2) + ((-1)^(2/3)*CosIntegral[((-1)^(2/3)*
a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(27*a^(5
/3)*b^(4/3)) + (d^2*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c
 - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(54*a*b^2) + Sin[c + d*x]/(18*a*b^2*x^2
) - (x*Sin[c + d*x])/(6*b*(a + b*x^3)^2) - Sin[c + d*x]/(18*b^2*x^2*(a + b*
x^3)) + ((-1)^(1/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-
1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(5/3)*b^(4/3)) - (d^2*Cos[c + ((-
1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d
*x])/(54*a*b^2) + (Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(
1/3) + d*x])/(27*a^(5/3)*b^(4/3)) + (d^2*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIn
tegral[(a^(1/3)*d)/b^(1/3) + d*x])/(54*a*b^2) + ((-1)^(2/3)*Cos[c - ((-1)^(
2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])
/(27*a^(5/3)*b^(4/3)) + (d^2*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinInt
egral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(54*a*b^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Sim
p[x^(-n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))), x] + (-Dist[
(-n + 1)/(b*n*(p + 1)), Int[((a + b*x^n)^(p + 1)*Sin[c + d*x])/x^n, x], x]
- Dist[d/(b*n*(p + 1)), Int[x^(-n + 1)*(a + b*x^n)^(p + 1)*Cos[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1] && IGtQ[n, 2]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3424

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3425

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cos[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Cos[c + d*x], x], x] + Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Sin[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3427

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\frac{x\sin(c+dx)}{6b\left(a+bx^3\right)^2} + \frac{\int \frac{\sin(c+dx)}{\left(a+bx^3\right)^2}\,dx}{6b} + \frac{d\int \frac{x\cos(c+dx)}{\left(a+bx^3\right)^2}\,dx}{6b}$$

$$= -\frac{d\cos(c+dx)}{18b^2x\left(a+bx^3\right)} - \frac{x\sin(c+dx)}{6b\left(a+bx^3\right)^2} - \frac{\sin(c+dx)}{18b^2x^2\left(a+bx^3\right)} - \frac{\int \frac{\sin(c+dx)}{x^3\left(a+bx^3\right)}\,dx}{9b^2} - \frac{d^2\int \frac{\sin(c+dx)}{x\left(a+bx^3\right)}\,dx}{18b^2}$$

$$= -\frac{d\cos(c+dx)}{18b^2x\left(a+bx^3\right)} - \frac{x\sin(c+dx)}{6b\left(a+bx^3\right)^2} - \frac{\sin(c+dx)}{18b^2x^2\left(a+bx^3\right)} - \frac{\int \left(\frac{\sin(c+dx)}{ax^3} - \frac{b\sin(c+dx)}{a\left(a+bx^3\right)}\right)dx}{9b^2} - \frac{d^2\int \left(\frac{\sin(c+dx)}{ax} - \frac{bx^2\sin(c+dx)}{a\left(a+bx^3\right)}\right)dx}{18b^2}$$

$$= -\frac{d\cos(c+dx)}{18b^2x\left(a+bx^3\right)} - \frac{x\sin(c+dx)}{6b\left(a+bx^3\right)^2} - \frac{\sin(c+dx)}{18b^2x^2\left(a+bx^3\right)} - \frac{\int \frac{\sin(c+dx)}{x^3}\,dx}{9ab^2} + \frac{\int \frac{\sin(c+dx)}{a+bx^3}\,dx}{9ab} - \frac{d^2\int \frac{\sin(c+dx)}{x}\,dx}{18ab^2} + \frac{d^2\int \frac{x^2\sin(c+dx)}{a+bx^3}\,dx}{18ab}$$

$$= -\frac{d\cos(c+dx)}{18b^2x\left(a+bx^3\right)} + \frac{\sin(c+dx)}{18ab^2x^2} - \frac{x\sin(c+dx)}{6b\left(a+bx^3\right)^2} - \frac{\sin(c+dx)}{18b^2x^2\left(a+bx^3\right)} + \frac{\int \left(-\frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{9ab} - \frac{d\int \frac{\cos(c+dx)}{x^2}\,dx}{18ab^2} + \frac{d^2\int \left(\frac{\sin(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sin(c+dx)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sin(c+dx)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right)dx}{18ab} - \frac{\left(d^2\cos(c)\right)\int \frac{\sin(dx)}{x}\,dx}{18ab^2} - \frac{\left(d^2\sin(c)\right)\int \frac{\cos(dx)}{x}\,dx}{18ab^2}$$

$$= \frac{d\cos(c+dx)}{18ab^2x} - \frac{d\cos(c+dx)}{18b^2x\left(a+bx^3\right)} - \frac{d^2\,\text{CosIntegral}(dx)\sin(c)}{18ab^2} + \frac{\sin(c+dx)}{18ab^2x^2} - \frac{x\sin(c+dx)}{6b\left(a+bx^3\right)^2} - \frac{\sin(c+dx)}{18b^2x^2\left(a+bx^3\right)} - \frac{d^2\cos(c)\text{Si}(dx)}{18ab^2} - \frac{\int \frac{\sin(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{27a^{5/3}b} - \frac{\int \frac{\sin(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{27a^{5/3}b} - \frac{\int \frac{\sin(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{27a^{5/3}b} + \frac{d^2\int \frac{\sin(c+dx)}{x}\,dx}{18ab^2} + \frac{d^2\int \frac{\sin(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{54ab^{5/3}} + \frac{d^2\int \frac{\sin(c+dx)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{54ab^{5/3}} + \frac{d^2\int \frac{\sin(c+dx)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{54ab^{5/3}}$$

$$= \frac{d\cos(c+dx)}{18ab^2x} - \frac{d\cos(c+dx)}{18b^2x\,(a+bx^3)} - \frac{d^2\,\mathrm{CosIntegral}(dx)\sin(c)}{18ab^2} + \frac{\sin(c+dx)}{18ab^2x^2} - \frac{x\sin(c+dx)}{6b\,(a+bx^3)^2} - \frac{\sin(c+dx)}{18b^2x^2\,(a+bx^3)} - \frac{d^2\cos(c)\mathrm{Si}(dx)}{18ab^2} + \frac{(d^2\cos(c))\int\frac{\sin(dx)}{x}\,dx}{18ab^2}$$

$$- \frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\sin\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{27a^{5/3}b} + \frac{\left(d^2\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{54ab^{5/3}}$$

$$+ \frac{\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{27a^{5/3}b}$$

$$- \frac{\left(d^2\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{54ab^{5/3}}$$

$$- \frac{\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\sin\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{27a^{5/3}b}$$

$$+ \frac{\left(d^2\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\sin\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{54ab^{5/3}} + \frac{(d^2\sin(c))\int\frac{\cos(dx)}{x}\,dx}{18ab^2}$$

$$- \frac{\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\cos\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{27a^{5/3}b} + \frac{\left(d^2\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{54ab^{5/3}}$$

$$- \frac{\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{27a^{5/3}b}$$

$$+ \frac{\left(d^2\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{54ab^{5/3}}$$

$$- \frac{\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\cos\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{27a^{5/3}b}$$

$$+ \frac{\left(d^2\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\cos\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{54ab^{5/3}}$$

$$
\begin{aligned}
&= \frac{d\cos(c+dx)}{18ab^2x} - \frac{d\cos(c+dx)}{18b^2x\,(a+bx^3)} + \frac{\operatorname{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{5/3}b^{4/3}} \\
&+ \frac{d^2\operatorname{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54ab^2} \\
&- \frac{\sqrt[3]{-1}\operatorname{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{5/3}b^{4/3}} \\
&+ \frac{d^2\operatorname{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54ab^2} \\
&+ \frac{(-1)^{2/3}\operatorname{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{5/3}b^{4/3}} \\
&+ \frac{d^2\operatorname{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54ab^2} \\
&+ \frac{\sin(c+dx)}{18ab^2x^2} - \frac{x\sin(c+dx)}{6b\,(a+bx^3)^2} - \frac{\sin(c+dx)}{18b^2x^2\,(a+bx^3)} \\
&+ \frac{\sqrt[3]{-1}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^{5/3}b^{4/3}} \\
&- \frac{d^2\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54ab^2} \\
&+ \frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{27a^{5/3}b^{4/3}} + \frac{d^2\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{54ab^2} \\
&+ \frac{(-1)^{2/3}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{27a^{5/3}b^{4/3}} \\
&+ \frac{d^2\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{54ab^2}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.46 (sec) , antiderivative size = 457, normalized size of antiderivative = 0.59

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^3} dx = \frac{i\text{RootSum}\left[a+b\#1^3\&, \frac{2\cos(c+d\#1)\text{CosIntegral}(d(x-\#1))-2i\,\text{CosIntegral}(d(x-\#1))\sin(c+d\#1)-2i\cos(c+d\#1)\text{Si}(d(x-\#1))}{}\right.}{}$$

```
[In] Integrate[(x^3*Sin[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] (I*RootSum[a + b*#1^3 & , (2*Cos[c + d*#1]*CosIntegral[d*(x - #1)] - (2*I)*
CosIntegral[d*(x - #1)]*Sin[c + d*#1] - (2*I)*Cos[c + d*#1]*SinIntegral[d*(
x - #1)] - 2*Sin[c + d*#1]*SinIntegral[d*(x - #1)] + d^2*Cos[c + d*#1]*CosI
ntegral[d*(x - #1)]*#1^2 - I*d^2*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1^2
 - I*d^2*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1^2 - d^2*Sin[c + d*#1]*Sin
Integral[d*(x - #1)]*#1^2)/#1^2 & ] - I*RootSum[a + b*#1^3 & , (2*Cos[c + d
*#1]*CosIntegral[d*(x - #1)] + (2*I)*CosIntegral[d*(x - #1)]*Sin[c + d*#1]
+ (2*I)*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - 2*Sin[c + d*#1]*SinIntegral
[d*(x - #1)] + d^2*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1^2 + I*d^2*CosIn
tegral[d*(x - #1)]*Sin[c + d*#1]*#1^2 + I*d^2*Cos[c + d*#1]*SinIntegral[d*(
x - #1)]*#1^2 - d^2*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1^2)/#1^2 & ] +
(6*b*x*(d*x*(a + b*x^3)*Cos[c + d*x] + (-2*a + b*x^3)*Sin[c + d*x]))/(a + b
*x^3)^2)/(108*a*b^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.25 (sec) , antiderivative size = 1337, normalized size of antiderivative = 1.73

method	result	size
risch	Expression too large to display	1337
derivativedivides	Expression too large to display	2035
default	Expression too large to display	2035

```
[In] int(x^3*sin(d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/108*I/d/a^2/b*c^3*sum((-2*I*c*_R1+6*I*c+_R1^2-c^2-6*_R1+10)/(-2*I*c*_R1+_
R1^2-c^2)*exp(_R1)*Ei(1,_R1-I*d*x-I*c),_R1=RootOf(-3*I*_Z^2*b*c-I*d^3*a+I*b
*c^3+b*_Z^3-3*c^2*b*_Z))+1/36*I/d/a^2/b^2*c^2*sum((-2*I*b*_R1*c^2+4*I*b*_R1
^2+2*I*b*c^2+_R1^2*b*c+a*d^3-c^3*b-4*I*_R1*b+2*b*c*_R1+6*c*b)/(2*I*c*_R1-_R
1^2+c^2)*exp(_R1)*Ei(1,_R1-I*d*x-I*c),_R1=RootOf(-3*I*_Z^2*b*c-I*d^3*a+I*b*
```

```
c^3+b*_Z^3-3*c^2*b*_Z))+1/36*I/d/a^2/b^2*c*sum((I*_R1*a*d^3+2*I*_R1*b*c^3-8
*I*_R1^2*b*c-2*I*a*d^3+2*I*b*c^3-_R1^2*b*c^2-a*c*d^3+b*c^4+8*I*_R1*b*c-10*_
R1*b*c^2-2*c^2*b)/(2*I*c*_R1-_R1^2+c^2)*exp(_R1)*Ei(1,_R1-I*d*x-I*c),_R1=Ro
otOf(-3*I*_Z^2*b*c-I*d^3*a+I*b*c^3+b*_Z^3-3*c^2*b*_Z))-1/108*I/d/a^2/b^2*su
m((I*_R1*a*c*d^3+2*I*_R1*b*c^4-12*I*_R1^2*b*c^2-6*I*a*c*d^3+6*I*b*c^4+_R1^2
*a*d^3-_R1^2*b*c^3-a*c^2*d^3+c^5*b+12*I*b*_R1*c^2-18*_R1*b*c^3-2*a*d^3+2*c^
3*b)/(2*I*c*_R1-_R1^2+c^2)*exp(_R1)*Ei(1,_R1-I*d*x-I*c),_R1=RootOf(-3*I*_Z^
2*b*c-I*d^3*a+I*b*c^3+b*_Z^3-3*c^2*b*_Z))-1/108*I/d/a^2/b*c^3*sum((-2*I*c*_
R1-6*I*c+_R1^2-c^2+6*_R1+10)/(-2*I*c*_R1+_R1^2-c^2)*exp(-_R1)*Ei(1,I*d*x+I*
c-_R1),_R1=RootOf(-3*I*_Z^2*b*c-I*d^3*a+I*b*c^3+b*_Z^3-3*c^2*b*_Z))-1/36*I/
d/a^2/b^2*c^2*sum((-2*I*b*_R1*c^2-4*I*b*_R1^2-2*I*b*c^2+_R1^2*b*c+a*d^3-c^3
*b-4*I*_R1*b-2*b*c*_R1+6*c*b)/(2*I*c*_R1-_R1^2+c^2)*exp(-_R1)*Ei(1,I*d*x+I*
c-_R1),_R1=RootOf(-3*I*_Z^2*b*c-I*d^3*a+I*b*c^3+b*_Z^3-3*c^2*b*_Z))-1/36*I/
d/a^2/b^2*c*sum((I*_R1*a*d^3+2*I*_R1*b*c^3+8*I*_R1^2*b*c+2*I*a*d^3-2*I*b*c^
3-_R1^2*b*c^2-a*c*d^3+b*c^4+8*I*_R1*b*c+10*_R1*b*c^2-2*c^2*b)/(2*I*c*_R1-_R
1^2+c^2)*exp(-_R1)*Ei(1,I*d*x+I*c-_R1),_R1=RootOf(-3*I*_Z^2*b*c-I*d^3*a+I*b
*c^3+b*_Z^3-3*c^2*b*_Z))+1/108*I/d/a^2/b^2*sum((I*_R1*a*c*d^3+2*I*_R1*b*c^4
+12*I*_R1^2*b*c^2+6*I*a*c*d^3-6*I*b*c^4+_R1^2*a*d^3-_R1^2*b*c^3-a*c^2*d^3+c
^5*b+12*I*b*_R1*c^2+18*_R1*b*c^3-2*a*d^3+2*c^3*b)/(2*I*c*_R1-_R1^2+c^2)*exp
(-_R1)*Ei(1,I*d*x+I*c-_R1),_R1=RootOf(-3*I*_Z^2*b*c-I*d^3*a+I*b*c^3+b*_Z^3-
3*c^2*b*_Z))-1/18/a*d^2*(-b*d^5*x^5-a*d^5*x^2)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3
+a^2*d^6)*cos(d*x+c)+1/18/a*d^2*(b*d^4*x^4-2*a*d^4*x)/b/(b^2*d^6*x^6+2*a*b*
d^6*x^3+a^2*d^6)*sin(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 890, normalized size of antiderivative = 1.15

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^3} \, dx = \text{Too large to display}$$

```
[In] integrate(x^3*sin(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/108*((I*a*b^2*d^3*x^6 + 2*I*a^2*b*d^3*x^3 + I*a^3*d^3 + (b^3*x^6 + 2*a*b^
2*x^3 + a^2*b + sqrt(3)*(I*b^3*x^6 + 2*I*a*b^2*x^3 + I*a^2*b))*(I*a*d^3/b)^
(1/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/
b)^(1/3)*(I*sqrt(3) + 1) - I*c) + (-I*a*b^2*d^3*x^6 - 2*I*a^2*b*d^3*x^3 - I
*a^3*d^3 + (b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(3)*(I*b^3*x^6 + 2*I*a*b^2*
x^3 + I*a^2*b))*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*s
qrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) + (I*a*b^2*d^
3*x^6 + 2*I*a^2*b*d^3*x^3 + I*a^3*d^3 + (b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sq
rt(3)*(-I*b^3*x^6 - 2*I*a*b^2*x^3 - I*a^2*b))*(I*a*d^3/b)^(1/3))*Ei(-I*d*x
+ 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(
```

```
3) + 1) - I*c) + (-I*a*b^2*d^3*x^6 - 2*I*a^2*b*d^3*x^3 - I*a^3*d^3 + (b^3*x
^6 + 2*a*b^2*x^3 + a^2*b + sqrt(3)*(-I*b^3*x^6 - 2*I*a*b^2*x^3 - I*a^2*b))*
(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1
/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) + (-I*a*b^2*d^3*x^6 - 2*I*a^2
*b*d^3*x^3 - I*a^3*d^3 - 2*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*(-I*a*d^3/b)^(1/
3))*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) + (I*a*b^2*
d^3*x^6 + 2*I*a^2*b*d^3*x^3 + I*a^3*d^3 - 2*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b)
*(I*a*d^3/b)^(1/3))*Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1
/3)) + 6*(a*b^2*d^2*x^5 + a^2*b*d^2*x^2)*cos(d*x + c) + 6*(a*b^2*d*x^4 - 2*
a^2*b*d*x)*sin(d*x + c))/(a^2*b^4*d*x^6 + 2*a^3*b^3*d*x^3 + a^4*b^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*sin(d*x+c)/(b*x**3+a)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{x^3 \sin(dx+c)}{(bx^3+a)^3}\,dx$$

```
[In] integrate(x^3*sin(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
[Out] -1/2*(6*(cos(c)^2 + sin(c)^2)*d*x^2*sin(d*x + c) + ((d^2*x^3*cos(c) - 6*d*x
^2*sin(c) - 42*x*cos(c))*cos(d*x + c)^2 + (d^2*x^3*cos(c) - 6*d*x^2*sin(c)
- 42*x*cos(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + ((cos(c)^2 + sin(c)^2)*d^2*
x^3 - 42*(cos(c)^2 + sin(c)^2)*x)*cos(d*x + c) - 2*(((b^3*cos(c)^2 + b^3*si
n(c)^2)*d^3*x^9 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^3*x^6 + 3*(a^2*b*co
s(c)^2 + a^2*b*sin(c)^2)*d^3*x^3 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^3)*cos(d
*x + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d^3*x^9 + 3*(a*b^2*cos(c)^2 + a*
b^2*sin(c)^2)*d^3*x^6 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^3*x^3 + (a^3*
cos(c)^2 + a^3*sin(c)^2)*d^3)*sin(d*x + c)^2)*integrate(3/2*(18*a*d*x*sin(d
*x + c) + (3*a*d^2*x^2 + 112*b*x^3 - 14*a)*cos(d*x + c))/(b^4*d^3*x^12 + 4*
a*b^3*d^3*x^9 + 6*a^2*b^2*d^3*x^6 + 4*a^3*b*d^3*x^3 + a^4*d^3), x) - 2*(((b
^3*cos(c)^2 + b^3*sin(c)^2)*d^3*x^9 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d
^3*x^6 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^3*x^3 + (a^3*cos(c)^2 + a^3*
sin(c)^2)*d^3)*cos(d*x + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d^3*x^9 + 3*
(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^3*x^6 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c
```

```
)^2)*d^3*x^3 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^3)*sin(d*x + c)^2)*integrate
(3/2*(18*a*d*x*sin(d*x + c) + (3*a*d^2*x^2 + 112*b*x^3 - 14*a)*cos(d*x + c)
)/((b^4*d^3*x^12 + 4*a*b^3*d^3*x^9 + 6*a^2*b^2*d^3*x^6 + 4*a^3*b*d^3*x^3 +
a^4*d^3)*cos(d*x + c)^2 + (b^4*d^3*x^12 + 4*a*b^3*d^3*x^9 + 6*a^2*b^2*d^3*x
^6 + 4*a^3*b*d^3*x^3 + a^4*d^3)*sin(d*x + c)^2), x) + ((d^2*x^3*sin(c) + 6*
d*x^2*cos(c) - 42*x*sin(c))*cos(d*x + c)^2 + (d^2*x^3*sin(c) + 6*d*x^2*cos(
c) - 42*x*sin(c))*sin(d*x + c)^2)*sin(d*x + 2*c))/(((b^3*cos(c)^2 + b^3*sin
(c)^2)*d^3*x^9 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^3*x^6 + 3*(a^2*b*cos
(c)^2 + a^2*b*sin(c)^2)*d^3*x^3 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^3)*cos(d*
x + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d^3*x^9 + 3*(a*b^2*cos(c)^2 + a*b
^2*sin(c)^2)*d^3*x^6 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^3*x^3 + (a^3*c
os(c)^2 + a^3*sin(c)^2)*d^3)*sin(d*x + c)^2)
```

Giac [F]

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{x^3 \sin(dx+c)}{(bx^3+a)^3}\,dx$$

```
[In] integrate(x^3*sin(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
[Out] integrate(x^3*sin(d*x + c)/(b*x^3 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{x^3\,\sin(c+d\,x)}{(b\,x^3+a)^3}\,dx$$

```
[In] int((x^3*sin(c + d*x))/(a + b*x^3)^3,x)
[Out] int((x^3*sin(c + d*x))/(a + b*x^3)^3, x)
```

3.110 $\int \frac{x^2 \sin(c+dx)}{(a+bx^3)^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 777

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^3)^3}\,dx = \frac{d\cos(c+dx)}{18ab^2x^2} - \frac{d\cos(c+dx)}{18b^2x^2(a+bx^3)} - \frac{\sqrt[3]{-1}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^{5/3}b^{4/3}} + \frac{d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{27a^{5/3}b^{4/3}} + \frac{(-1)^{2/3}d\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{27a^{5/3}b^{4/3}} - \frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54a^{4/3}b^{5/3}} - \frac{(-1)^{2/3}d^2\,\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54a^{4/3}b^{5/3}} + \frac{\sqrt[3]{-1}d^2\,\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54a^{4/3}b^{5/3}} - \frac{\sin(c+dx)}{6b(a+bx^3)^2} + \frac{(-1)^{2/3}d^2\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54a^{4/3}b^{5/3}} - \frac{\sqrt[3]{-1}d\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^{5/3}b^{4/3}} - \frac{d^2\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{54a^{4/3}b^{5/3}} - \frac{d\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{27a^{5/3}b^{4/3}} + \frac{\sqrt[3]{-1}d^2\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{54a^{4/3}b^{5/3}} - \frac{(-1)^{2/3}d\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{27a^{5/3}b^{4/3}}$$

```
[Out] 1/27*d*Ci(a^(1/3)*d/b^(1/3)+d*x)*cos(c-a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)-1
/27*(-1)^(1/3)*d*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cos(c+(-1)^(1/3)*a^(1
/3)*d/b^(1/3))/a^(5/3)/b^(4/3)+1/27*(-1)^(2/3)*d*Ci((-1)^(2/3)*a^(1/3)*d/b^
(1/3)+d*x)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)+1/18*d*cos(d
*x+c)/a/b^2/x^2-1/18*d*cos(d*x+c)/b^2/x^2/(b*x^3+a)-1/54*(-1)^(2/3)*d^2*cos
(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(4
/3)/b^(5/3)-1/54*d^2*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^(
4/3)/b^(5/3)+1/54*(-1)^(1/3)*d^2*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1
)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(4/3)/b^(5/3)-1/54*d^2*Ci(a^(1/3)*d/b^(1/3
)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(4/3)/b^(5/3)-1/27*d*Si(a^(1/3)*d/b^(1/3)
+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)-1/54*(-1)^(2/3)*d^2*Ci((-1)^
(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(4/3)/b^
(5/3)+1/27*(-1)^(1/3)*d*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c+(-1)^(1
/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)+1/54*(-1)^(1/3)*d^2*Ci((-1)^(2/3)*a^
(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(4/3)/b^(5/3)-1/
27*(-1)^(2/3)*d*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/
3)*d/b^(1/3))/a^(5/3)/b^(4/3)-1/6*sin(d*x+c)/b/(b*x^3+a)^2
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 777, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used

$= \{3422, 3413, 3427, 3378, 3384, 3380, 3383, 3415, 3426\}$

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^3)^3}\,dx = -\frac{d^2 \sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54a^{4/3}b^{5/3}}$$
$$-\frac{(-1)^{2/3}d^2 \sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54a^{4/3}b^{5/3}}$$
$$+\frac{\sqrt[3]{-1}d^2 \sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54a^{4/3}b^{5/3}}$$
$$-\frac{\sqrt[3]{-1}d \cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^{5/3}b^{4/3}}$$
$$+\frac{d \cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{5/3}b^{4/3}}$$
$$+\frac{(-1)^{2/3}d \cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{5/3}b^{4/3}}$$
$$+\frac{(-1)^{2/3}d^2 \cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54a^{4/3}b^{5/3}}$$
$$-\frac{d^2 \cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54a^{4/3}b^{5/3}}$$
$$+\frac{\sqrt[3]{-1}d^2 \cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54a^{4/3}b^{5/3}}$$
$$-\frac{\sqrt[3]{-1}d \sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^{5/3}b^{4/3}}$$
$$-\frac{d \sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{5/3}b^{4/3}}$$
$$-\frac{(-1)^{2/3}d \sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{5/3}b^{4/3}}$$
$$+\frac{d\cos(c+dx)}{18ab^2x^2}-\frac{d\cos(c+dx)}{18b^2x^2(a+bx^3)}-\frac{\sin(c+dx)}{6b(a+bx^3)^2}$$

```
[In] Int[(x^2*Sin[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] (d*Cos[c + d*x])/(18*a*b^2*x^2) - (d*Cos[c + d*x])/(18*b^2*x^2*(a + b*x^3))
 - ((-1)^(1/3)*d*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^
(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(5/3)*b^(4/3)) + (d*Cos[c - (a^(1/3)
*d)/b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(5/3)*b^(4/3)) +
 ((-1)^(2/3)*d*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2
/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(5/3)*b^(4/3)) - (d^2*CosIntegral[(a^(
1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(54*a^(4/3)*b^(5/3)) -
 ((-1)^(2/3)*d^2*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c +
((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(54*a^(4/3)*b^(5/3)) + ((-1)^(1/3)*d^2*Cos
Integral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*
d)/b^(1/3)])/(54*a^(4/3)*b^(5/3)) - Sin[c + d*x]/(6*b*(a + b*x^3)^2) + ((-1
)^(2/3)*d^2*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)
*a^(1/3)*d)/b^(1/3) - d*x])/(54*a^(4/3)*b^(5/3)) - ((-1)^(1/3)*d*Sin[c + ((
-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) -
d*x])/(27*a^(5/3)*b^(4/3)) - (d^2*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[
(a^(1/3)*d)/b^(1/3) + d*x])/(54*a^(4/3)*b^(5/3)) - (d*Sin[c - (a^(1/3)*d)/b
^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(5/3)*b^(4/3)) + ((-1
)^(1/3)*d^2*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)
*a^(1/3)*d)/b^(1/3) + d*x])/(54*a^(4/3)*b^(5/3)) - ((-1)^(2/3)*d*Sin[c - ((
-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) +
d*x])/(27*a^(5/3)*b^(4/3))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3413

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Sim
p[x^(-n + 1)*(a + b*x^n)^(p + 1)*(Cos[c + d*x]/(b*n*(p + 1))), x] + (-Dist[
(-n + 1)/(b*n*(p + 1)), Int[((a + b*x^n)^(p + 1)*Cos[c + d*x])/x^n, x], x]
+ Dist[d/(b*n*(p + 1)), Int[x^(-n + 1)*(a + b*x^n)^(p + 1)*Sin[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1] && IGtQ[n, 2]
```

Rule 3415

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3422

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))),
x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3427

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\frac{\sin(c+dx)}{6b\,(a+bx^3)^2} + \frac{d\int \frac{\cos(c+dx)}{(a+bx^3)^2}\,dx}{6b}$$

$$= -\frac{d\cos(c+dx)}{18b^2x^2\,(a+bx^3)} - \frac{\sin(c+dx)}{6b\,(a+bx^3)^2} - \frac{d\int \frac{\cos(c+dx)}{x^3(a+bx^3)}\,dx}{9b^2} - \frac{d^2\int \frac{\sin(c+dx)}{x^2(a+bx^3)}\,dx}{18b^2}$$

$$= -\frac{d\cos(c+dx)}{18b^2x^2(a+bx^3)} - \frac{\sin(c+dx)}{6b(a+bx^3)^2} - \frac{d\int\left(\frac{\cos(c+dx)}{ax^3} - \frac{b\cos(c+dx)}{a(a+bx^3)}\right)dx}{9b^2} - \frac{d^2\int\left(\frac{\sin(c+dx)}{ax^2} - \frac{bx\sin(c+dx)}{a(a+bx^3)}\right)dx}{18b^2}$$

$$= -\frac{d\cos(c+dx)}{18b^2x^2(a+bx^3)} - \frac{\sin(c+dx)}{6b(a+bx^3)^2} - \frac{d\int\frac{\cos(c+dx)}{x^3}dx}{9ab^2} + \frac{d\int\frac{\cos(c+dx)}{a+bx^3}dx}{9ab} - \frac{d^2\int\frac{\sin(c+dx)}{x^2}dx}{18ab^2} + \frac{d^2\int\frac{x\sin(c+dx)}{a+bx^3}dx}{18ab}$$

$$= \frac{d\cos(c+dx)}{18ab^2x^2} - \frac{d\cos(c+dx)}{18b^2x^2(a+bx^3)} + \frac{d^2\sin(c+dx)}{18ab^2x} - \frac{\sin(c+dx)}{6b(a+bx^3)^2} + \frac{d\int\left(-\frac{\cos(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\cos(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{\cos(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{9ab} + \frac{d^2\int\frac{\sin(c+dx)}{x^2}dx}{18ab^2} + \frac{d^2\int\left(-\frac{\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{18ab} - \frac{d^3\int\frac{\cos(c+dx)}{x}dx}{18ab^2}$$

$$= \frac{d\cos(c+dx)}{18ab^2x^2} - \frac{d\cos(c+dx)}{18b^2x^2(a+bx^3)} - \frac{\sin(c+dx)}{6b(a+bx^3)^2} - \frac{d\int\frac{\cos(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}dx}{27a^{5/3}b} - \frac{d\int\frac{\cos(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}dx}{27a^{5/3}b} - \frac{d\int\frac{\cos(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}dx}{27a^{5/3}b} - \frac{d^2\int\frac{\sin(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{54a^{4/3}b^{4/3}} + \frac{\left(\sqrt[3]{-1}d^2\right)\int\frac{\sin(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}dx}{54a^{4/3}b^{4/3}} - \frac{\left((-1)^{2/3}d^2\right)\int\frac{\sin(c+dx)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}dx}{54a^{4/3}b^{4/3}} + \frac{d^3\int\frac{\cos(c+dx)}{x}dx}{18ab^2} - \frac{(d^3\cos(c))\int\frac{\cos(dx)}{x}dx}{18ab^2} + \frac{(d^3\sin(c))\int\frac{\sin(dx)}{x}dx}{18ab^2}$$

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Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.31 (sec) , antiderivative size = 449, normalized size of antiderivative = 0.58

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^3)^3}\,dx = \frac{id\text{RootSum}\left[a+b\#1^3\&, \frac{-2i\cos(c+d\#1)\,\text{CosIntegral}(d(x-\#1))-2\,\text{CosIntegral}(d(x-\#1))\sin(c+d\#1)-2\cos(c+d\#1)\text{Si}(d(x-\cdots}{}\right.}{}$$

```
[In] Integrate[(x^2*Sin[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] (I*d*RootSum[a + b*#1^3 & , ((-2*I)*Cos[c + d*#1]*CosIntegral[d*(x - #1)] -
 2*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - 2*Cos[c + d*#1]*SinIntegral[d*(x
 - #1)] + (2*I)*Sin[c + d*#1]*SinIntegral[d*(x - #1)] + d*Cos[c + d*#1]*Cos
Integral[d*(x - #1)]*#1 - I*d*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1 - I*
d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c + d*#1]*SinIntegral[d*
(x - #1)]*#1)/#1^2 & ] - I*d*RootSum[a + b*#1^3 & , ((2*I)*Cos[c + d*#1]*Co
sIntegral[d*(x - #1)] - 2*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - 2*Cos[c +
 d*#1]*SinIntegral[d*(x - #1)] - (2*I)*Sin[c + d*#1]*SinIntegral[d*(x - #1)
] + d*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1 + I*d*CosIntegral[d*(x - #1)
]*Sin[c + d*#1]*#1 + I*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c
 + d*#1]*SinIntegral[d*(x - #1)]*#1)/#1^2 & ] + (6*b*Cos[d*x]*(d*x*(a + b*x
^3)*Cos[c] - 3*a*Sin[c]))/(a + b*x^3)^2 - (6*b*(3*a*Cos[c] + d*x*(a + b*x^3
)*Sin[c])*Sin[d*x])/(a + b*x^3)^2)/(108*a*b^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.74 (sec) , antiderivative size = 918, normalized size of antiderivative = 1.18

method	result	size
risch	Expression too large to display	918
derivativedivides	Expression too large to display	1396
default	Expression too large to display	1396

```
[In] int(x^2*sin(d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/108*I/a^2/b*c^2*sum((-2*I*c*_R1+6*I*c+_R1^2-c^2-6*_R1+10)/(-2*I*c*_R1+_R
1^2-c^2)*exp(_R1)*Ei(1,_R1-I*d*x-I*c),_R1=RootOf(-3*I*_Z^2*b*c-I*d^3*a+I*b*
c^3+b*_Z^3-3*c^2*b*_Z))-1/54*I/a^2/b^2*c*sum((-2*I*b*_R1*c^2+4*I*b*_R1^2+2*
I*b*c^2+_R1^2*b*c+a*d^3-c^3*b-4*I*_R1*b+2*b*c*_R1+6*c*b)/(2*I*c*_R1-_R1^2+c
^2)*exp(_R1)*Ei(1,_R1-I*d*x-I*c),_R1=RootOf(-3*I*_Z^2*b*c-I*d^3*a+I*b*c^3+b
```

```
*_Z^3-3*c^2*b*_Z))-1/108*I/a^2/b^2*sum((I*_R1*a*d^3+2*I*_R1*b*c^3-8*I*_R1^2
*b*c-2*I*a*d^3+2*I*b*c^3-_R1^2*b*c^2-a*c*d^3+b*c^4+8*I*_R1*b*c-10*_R1*b*c^2
-2*c^2*b)/(2*I*c*_R1-_R1^2+c^2)*exp(_R1)*Ei(1,_R1-I*d*x-I*c),_R1=RootOf(-3*
I*_Z^2*b*c-I*d^3*a+I*b*c^3+b*_Z^3-3*c^2*b*_Z))+1/108*I/a^2/b*c^2*sum((-2*I*
c*_R1-6*I*c+_R1^2-c^2+6*_R1+10)/(-2*I*c*_R1+_R1^2-c^2)*exp(-_R1)*Ei(1,I*d*x
+I*c-_R1),_R1=RootOf(-3*I*_Z^2*b*c-I*d^3*a+I*b*c^3+b*_Z^3-3*c^2*b*_Z))+1/54
*I/a^2/b^2*c*sum((-2*I*b*_R1*c^2-4*I*b*_R1^2-2*I*b*c^2+_R1^2*b*c+a*d^3-c^3*
b-4*I*_R1*b-2*b*c*_R1+6*c*b)/(2*I*c*_R1-_R1^2+c^2)*exp(-_R1)*Ei(1,I*d*x+I*c
-_R1),_R1=RootOf(-3*I*_Z^2*b*c-I*d^3*a+I*b*c^3+b*_Z^3-3*c^2*b*_Z))+1/108*I/
a^2/b^2*sum((I*_R1*a*d^3+2*I*_R1*b*c^3+8*I*_R1^2*b*c+2*I*a*d^3-2*I*b*c^3-_R
1^2*b*c^2-a*c*d^3+b*c^4+8*I*_R1*b*c+10*_R1*b*c^2-2*c^2*b)/(2*I*c*_R1-_R1^2+
c^2)*exp(-_R1)*Ei(1,I*d*x+I*c-_R1),_R1=RootOf(-3*I*_Z^2*b*c-I*d^3*a+I*b*c^3
+b*_Z^3-3*c^2*b*_Z))+1/18*(a*b*d^7*x^4+a^2*d^7*x)/a^2/b/(b^2*d^6*x^6+2*a*b*
d^6*x^3+a^2*d^6)*cos(d*x+c)-1/6*d^6/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*s
in(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 935, normalized size of antiderivative = 1.20

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*sin(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/216*(((-I*b^2*x^6 - 2*I*a*b*x^3 - I*a^2 - sqrt(3)*(b^2*x^6 + 2*a*b*x^3 +
a^2))*(I*a*d^3/b)^(2/3) - 2*(I*b^2*x^6 + 2*I*a*b*x^3 + I*a^2 - sqrt(3)*(b^2
*x^6 + 2*a*b*x^3 + a^2))*(I*a*d^3/b)^(1/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/
3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) + ((I*
b^2*x^6 + 2*I*a*b*x^3 + I*a^2 + sqrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*(-I*a*
d^3/b)^(2/3) - 2*(-I*b^2*x^6 - 2*I*a*b*x^3 - I*a^2 + sqrt(3)*(b^2*x^6 + 2*a
*b*x^3 + a^2))*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sq
rt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) + ((-I*b^2*x^6
 - 2*I*a*b*x^3 - I*a^2 + sqrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*(I*a*d^3/b)^(
2/3) - 2*(I*b^2*x^6 + 2*I*a*b*x^3 + I*a^2 + sqrt(3)*(b^2*x^6 + 2*a*b*x^3 +
a^2))*(I*a*d^3/b)^(1/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))
*e^(1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) - I*c) + ((I*b^2*x^6 + 2*I*a*b*x
^3 + I*a^2 - sqrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*(-I*a*d^3/b)^(2/3) - 2*(-
I*b^2*x^6 - 2*I*a*b*x^3 - I*a^2 - sqrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*(-I*
a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(
-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) - 2*((I*b^2*x^6 + 2*I*a*b*x^3 + I
*a^2)*(-I*a*d^3/b)^(2/3) + 2*(I*b^2*x^6 + 2*I*a*b*x^3 + I*a^2)*(-I*a*d^3/b)
^(1/3))*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) - 2*((-
```

```
I*b^2*x^6 - 2*I*a*b*x^3 - I*a^2)*(I*a*d^3/b)^(2/3) + 2*(-I*b^2*x^6 - 2*I*a*
b*x^3 - I*a^2)*(I*a*d^3/b)^(1/3))*Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I*c -
(I*a*d^3/b)^(1/3)) - 36*a^2*sin(d*x + c) + 12*(a*b*d*x^4 + a^2*d*x)*cos(d*x
 + c))/(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*sin(d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{x^2 \sin(dx+c)}{(bx^3+a)^3}\,dx$$

```
[In] integrate(x^2*sin(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*((cos(c)^2 + sin(c)^2)*d*x^2*cos(d*x + c) + 7*(cos(c)^2 + sin(c)^2)*x*
sin(d*x + c) + ((d*x^2*cos(c) - 7*x*sin(c))*cos(d*x + c)^2 + (d*x^2*cos(c)
- 7*x*sin(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b^3*cos(c)^2 + b^3*sin(
c)^2)*d^2*x^9 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^2*x^6 + 3*(a^2*b*cos(
c)^2 + a^2*b*sin(c)^2)*d^2*x^3 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^2)*cos(d*x
 + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d^2*x^9 + 3*(a*b^2*cos(c)^2 + a*b^
2*sin(c)^2)*d^2*x^6 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^2*x^3 + (a^3*co
s(c)^2 + a^3*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(-1/2*(9*a*d*x*cos(d*x
 + c) - 7*(8*b*x^3 - a)*sin(d*x + c))/(b^4*d^2*x^12 + 4*a*b^3*d^2*x^9 + 6*a
^2*b^2*d^2*x^6 + 4*a^3*b*d^2*x^3 + a^4*d^2), x) + 2*(((b^3*cos(c)^2 + b^3*s
in(c)^2)*d^2*x^9 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^2*x^6 + 3*(a^2*b*c
os(c)^2 + a^2*b*sin(c)^2)*d^2*x^3 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^2)*cos(
d*x + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d^2*x^9 + 3*(a*b^2*cos(c)^2 + a
*b^2*sin(c)^2)*d^2*x^6 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^2*x^3 + (a^3
*cos(c)^2 + a^3*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(-1/2*(9*a*d*x*cos(
d*x + c) - 7*(8*b*x^3 - a)*sin(d*x + c))/((b^4*d^2*x^12 + 4*a*b^3*d^2*x^9 +
 6*a^2*b^2*d^2*x^6 + 4*a^3*b*d^2*x^3 + a^4*d^2)*cos(d*x + c)^2 + (b^4*d^2*x
^12 + 4*a*b^3*d^2*x^9 + 6*a^2*b^2*d^2*x^6 + 4*a^3*b*d^2*x^3 + a^4*d^2)*sin(
d*x + c)^2), x) + ((d*x^2*sin(c) + 7*x*cos(c))*cos(d*x + c)^2 + (d*x^2*sin(
c) + 7*x*cos(c))*sin(d*x + c)^2)*sin(d*x + 2*c))/(((b^3*cos(c)^2 + b^3*sin(
c)^2)*d^2*x^9 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^2*x^6 + 3*(a^2*b*cos(
```

```
c)^2 + a^2*b*sin(c)^2)*d^2*x^3 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^2)*cos(d*x
 + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d^2*x^9 + 3*(a*b^2*cos(c)^2 + a*b^
2*sin(c)^2)*d^2*x^6 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^2*x^3 + (a^3*co
s(c)^2 + a^3*sin(c)^2)*d^2)*sin(d*x + c)^2)
```

Giac [F]

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{x^2 \sin(dx+c)}{(bx^3+a)^3}\,dx$$

```
[In] integrate(x^2*sin(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*sin(d*x + c)/(b*x^3 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{x^2 \sin(c+d\,x)}{(b\,x^3+a)^3}\,dx$$

```
[In] int((x^2*sin(c + d*x))/(a + b*x^3)^3,x)
```

```
[Out] int((x^2*sin(c + d*x))/(a + b*x^3)^3, x)
```

3.111 $\int \frac{x \sin(c+dx)}{(a+bx^3)^3} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 1141

$$\int \frac{x\sin(c+dx)}{(a+bx^3)^3}\,dx = \frac{d\cos(c+dx)}{18ab^2x^3} - \frac{d\cos(c+dx)}{18b^2x^3(a+bx^3)} - \frac{2d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^2b} - \frac{2d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{27a^2b} - \frac{2d\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{27a^2b} - \frac{2\operatorname{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{7/3}b^{2/3}} + \frac{d^2\operatorname{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54a^{5/3}b^{4/3}} - \frac{2(-1)^{2/3}\operatorname{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{7/3}b^{2/3}} - \frac{\sqrt[3]{-1}d^2\operatorname{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54a^{5/3}b^{4/3}} + \frac{2\sqrt[3]{-1}\operatorname{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{7/3}b^{2/3}} + \frac{(-1)^{2/3}d^2\operatorname{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54a^{5/3}b^{4/3}} - \frac{\sin(c+dx)}{18ab^2x^4} + \frac{2\sin(c+dx)}{9a^2bx} - \frac{\sin(c+dx)}{6bx(a+bx^3)^2} + \frac{\sin(c+dx)}{18b^2x^4(a+bx^3)} + \frac{2(-1)^{2/3}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^{7/3}b^{2/3}} + \frac{\sqrt[3]{-1}d^2\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54a^{5/3}b^{4/3}} - \frac{2d\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^2b} - \frac{2\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{27a^{7/3}b^{2/3}} + \frac{d^2\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\cdots}$$

```
[Out] -2/27*d*Ci(a^(1/3)*d/b^(1/3)+d*x)*cos(c-a^(1/3)*d/b^(1/3))/a^2/b-2/27*d*Ci(
(-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^2/b
-2/27*d*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(
1/3))/a^2/b+1/18*d*cos(d*x+c)/a/b^2/x^3-1/18*d*cos(d*x+c)/b^2/x^3/(b*x^3+a)
-2/27*(-1)^(2/3)*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)
*d/b^(1/3)+d*x)/a^(7/3)/b^(2/3)-1/54*(-1)^(1/3)*d^2*cos(c+(-1)^(1/3)*a^(1/3
)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(4/3)-2/27*cos
(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^(7/3)/b^(2/3)+1/54*d^2*co
s(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(4/3)+2/27*(-1)^
(1/3)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d
*x)/a^(7/3)/b^(2/3)+1/54*(-1)^(2/3)*d^2*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))
*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(4/3)-2/27*Ci(a^(1/3)*d/b^(
1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(7/3)/b^(2/3)+1/54*d^2*Ci(a^(1/3)*d/b^
(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)+2/27*d*Si(a^(1/3)*d/b^(
1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^2/b-2/27*(-1)^(2/3)*Ci((-1)^(1/3)*a^(1
/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(7/3)/b^(2/3)-1/54
*(-1)^(1/3)*d^2*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/
3)*d/b^(1/3))/a^(5/3)/b^(4/3)+2/27*d*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*
sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^2/b+2/27*(-1)^(1/3)*Ci((-1)^(2/3)*a^(
1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(7/3)/b^(2/3)+1/5
4*(-1)^(2/3)*d^2*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1
/3)*d/b^(1/3))/a^(5/3)/b^(4/3)+2/27*d*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*
sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^2/b-1/18*sin(d*x+c)/a/b^2/x^4+2/9*sin
(d*x+c)/a^2/b/x-1/6*sin(d*x+c)/b/x/(b*x^3+a)^2+1/18*sin(d*x+c)/b^2/x^4/(b*x
^3+a)
```

Rubi [A] (verified)

Time = 2.00 (sec) , antiderivative size = 1141, normalized size of antiderivative = 1.00, number of steps used = 89, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used

$= \{3424, 3426, 3378, 3384, 3380, 3383, 3427, 3425, 3414\}$

$$
\begin{aligned}
\int \frac{x\sin(c+dx)}{(a+bx^3)^3}\,dx = {} & \frac{\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d^2}{54a^{5/3}b^{4/3}} \\
& - \frac{\sqrt[3]{-1}\,\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d^2}{54a^{5/3}b^{4/3}} \\
& + \frac{(-1)^{2/3}\,\text{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d^2}{54a^{5/3}b^{4/3}} \\
& + \frac{\sqrt[3]{-1}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)d^2}{54a^{5/3}b^{4/3}} \\
& + \frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d^2}{54a^{5/3}b^{4/3}} \\
& + \frac{(-1)^{2/3}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d^2}{54a^{5/3}b^{4/3}} \\
& - \frac{\cos(c+dx)d}{18b^2x^3(bx^3+a)} + \frac{\cos(c+dx)d}{18ab^2x^3} \\
& - \frac{2\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)d}{27a^2b} \\
& - \frac{2\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d}{27a^2b} \\
& - \frac{2\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d}{27a^2b} \\
& - \frac{2\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)d}{27a^2b} \\
& + \frac{2\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d}{27a^2b} \\
& + \frac{2\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d}{27a^2b} \\
& - \frac{2\,\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{7/3}b^{2/3}} \\
& - \frac{2(-1)^{2/3}\,\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{7/3}b^{2/3}} \\
& + \frac{2\sqrt[3]{-1}\,\text{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{7/3}b^{2/3}}
\end{aligned}
$$

```
[In] Int[(x*Sin[c + d*x])/(a + b*x^3)^3,x]

[Out] (d*Cos[c + d*x])/(18*a*b^2*x^3) - (d*Cos[c + d*x])/(18*b^2*x^3*(a + b*x^3))
 - (2*d*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(1/3)*a^(
1/3)*d)/b^(1/3) - d*x])/(27*a^2*b) - (2*d*Cos[c - (a^(1/3)*d)/b^(1/3)]*CosI
ntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^2*b) - (2*d*Cos[c - ((-1)^(2/3)*a
^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a
^2*b) - (2*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/
3)])/(27*a^(7/3)*b^(2/3)) + (d^2*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin
[c - (a^(1/3)*d)/b^(1/3)])/(54*a^(5/3)*b^(4/3)) - (2*(-1)^(2/3)*CosIntegral
[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/
3)])/(27*a^(7/3)*b^(2/3)) - ((-1)^(1/3)*d^2*CosIntegral[((-1)^(1/3)*a^(1/3)
*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(54*a^(5/3)*b^(
4/3)) + (2*(-1)^(1/3)*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin
[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(27*a^(7/3)*b^(2/3)) + ((-1)^(2/3)*d^
2*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(
1/3)*d)/b^(1/3)])/(54*a^(5/3)*b^(4/3)) - Sin[c + d*x]/(18*a*b^2*x^4) + (2*S
in[c + d*x])/(9*a^2*b*x) - Sin[c + d*x]/(6*b*x*(a + b*x^3)^2) + Sin[c + d*x
]/(18*b^2*x^4*(a + b*x^3)) + (2*(-1)^(2/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b
^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(7/3)*b^(2
/3)) + ((-1)^(1/3)*d^2*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[
((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a^(5/3)*b^(4/3)) - (2*d*Sin[c +
((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3)
- d*x])/(27*a^2*b) - (2*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d
)/b^(1/3) + d*x])/(27*a^(7/3)*b^(2/3)) + (d^2*Cos[c - (a^(1/3)*d)/b^(1/3)]*
SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(54*a^(5/3)*b^(4/3)) + (2*d*Sin[c -
 (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^2*b) +
(2*(-1)^(1/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/
3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(7/3)*b^(2/3)) + ((-1)^(2/3)*d^2*Cos[c
- ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3
) + d*x])/(54*a^(5/3)*b^(4/3)) + (2*d*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3
)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a^2*b)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3424

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3425

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cos[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Cos[c + d*x], x], x] + Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Sin[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3427

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
```

```
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\frac{\sin(c+dx)}{6bx\left(a+bx^3\right)^2} - \frac{\int \frac{\sin(c+dx)}{x^2(a+bx^3)^2}\,dx}{6b} + \frac{d\int \frac{\cos(c+dx)}{x(a+bx^3)^2}\,dx}{6b}$$

$$= -\frac{d\cos(c+dx)}{18b^2x^3\left(a+bx^3\right)} - \frac{\sin(c+dx)}{6bx\left(a+bx^3\right)^2} + \frac{\sin(c+dx)}{18b^2x^4\left(a+bx^3\right)} + \frac{2\int \frac{\sin(c+dx)}{x^5(a+bx^3)}\,dx}{9b^2} - \frac{d\int \frac{\cos(c+dx)}{x^4(a+bx^3)}\,dx}{18b^2} - \frac{d\int \frac{\cos(c+dx)}{x^4(a+bx^3)}\,dx}{6b^2} - \frac{d^2\int \frac{\sin(c+dx)}{x^3(a+bx^3)}\,dx}{18b^2}$$

$$= -\frac{d\cos(c+dx)}{18b^2x^3\left(a+bx^3\right)} - \frac{\sin(c+dx)}{6bx\left(a+bx^3\right)^2} + \frac{\sin(c+dx)}{18b^2x^4\left(a+bx^3\right)} + \frac{2\int \left(\frac{\sin(c+dx)}{ax^5} - \frac{b\sin(c+dx)}{a^2x^2} + \frac{b^2x\sin(c+dx)}{a^2(a+bx^3)}\right)dx}{9b^2} - \frac{d\int \left(\frac{\cos(c+dx)}{ax^4} - \frac{b\cos(c+dx)}{a^2x} + \frac{b^2x^2\cos(c+dx)}{a^2(a+bx^3)}\right)dx}{18b^2} - \frac{d\int \left(\frac{\cos(c+dx)}{ax^4} - \frac{b\cos(c+dx)}{a^2x} + \frac{b^2x^2\cos(c+dx)}{a^2(a+bx^3)}\right)dx}{6b^2} - \frac{d^2\int \left(\frac{\sin(c+dx)}{ax^3} - \frac{b\sin(c+dx)}{a(a+bx^3)}\right)dx}{18b^2}$$

$$= -\frac{d\cos(c+dx)}{18b^2x^3\left(a+bx^3\right)} - \frac{\sin(c+dx)}{6bx\left(a+bx^3\right)^2} + \frac{\sin(c+dx)}{18b^2x^4\left(a+bx^3\right)} + \frac{2\int \frac{x\sin(c+dx)}{a+bx^3}\,dx}{9a^2} + \frac{2\int \frac{\sin(c+dx)}{x^5}\,dx}{9ab^2} - \frac{2\int \frac{\sin(c+dx)}{x^2}\,dx}{9a^2b} - \frac{d\int \frac{x^2\cos(c+dx)}{a+bx^3}\,dx}{18a^2} - \frac{d\int \frac{x^2\cos(c+dx)}{a+bx^3}\,dx}{6a^2} - \frac{d\int \frac{\cos(c+dx)}{x^4}\,dx}{18ab^2} - \frac{d\int \frac{\cos(c+dx)}{x^4}\,dx}{6ab^2} + \frac{d\int \frac{\cos(c+dx)}{x}\,dx}{18a^2b} + \frac{d\int \frac{\cos(c+dx)}{x}\,dx}{6a^2b} - \frac{d^2\int \frac{\sin(c+dx)}{x^3}\,dx}{18ab^2} + \frac{d^2\int \frac{\sin(c+dx)}{a+bx^3}\,dx}{18ab}$$

$$
\begin{aligned}
&= \frac{2d\cos(c+dx)}{27ab^2x^3} - \frac{d\cos(c+dx)}{18b^2x^3\,(a+bx^3)} - \frac{\sin(c+dx)}{18ab^2x^4} + \frac{d^2\sin(c+dx)}{36ab^2x^2} \\
&+ \frac{2\sin(c+dx)}{9a^2bx} - \frac{\sin(c+dx)}{6bx\,(a+bx^3)^2} + \frac{\sin(c+dx)}{18b^2x^4\,(a+bx^3)} \\
&+ \frac{2\int \left(-\frac{\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right) dx}{9a^2} \\
&- \frac{d\int \left(\frac{\cos(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\cos(c+dx)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\cos(c+dx)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right) dx}{18a^2} \\
&- \frac{d\int \left(\frac{\cos(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\cos(c+dx)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\cos(c+dx)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right) dx}{6a^2} \\
&+ \frac{d\int \frac{\cos(c+dx)}{x^4}\,dx}{18ab^2} - \frac{(2d)\int \frac{\cos(c+dx)}{x}\,dx}{9a^2b} + \frac{d^2\int \frac{\sin(c+dx)}{x^3}\,dx}{54ab^2} + \frac{d^2\int \frac{\sin(c+dx)}{x^3}\,dx}{18ab^2} \\
&+ \frac{d^2\int \left(-\frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right) dx}{18ab} \\
&- \frac{d^3\int \frac{\cos(c+dx)}{x^2}\,dx}{36ab^2} + \frac{(d\cos(c))\int \frac{\cos(dx)}{x}\,dx}{18a^2b} + \frac{(d\cos(c))\int \frac{\cos(dx)}{x}\,dx}{6a^2b} \\
&- \frac{(d\sin(c))\int \frac{\sin(dx)}{x}\,dx}{18a^2b} - \frac{(d\sin(c))\int \frac{\sin(dx)}{x}\,dx}{6a^2b}
\end{aligned}
$$

$$= \frac{d\cos(c+dx)}{18ab^2x^3} + \frac{d^3\cos(c+dx)}{36ab^2x} - \frac{d\cos(c+dx)}{18b^2x^3(a+bx^3)} + \frac{2d\cos(c)\,\text{CosIntegral}(dx)}{9a^2b} - \frac{\sin(c+dx)}{18ab^2x^4} - \frac{d^2\sin(c+dx)}{108ab^2x^2} + \frac{2\sin(c+dx)}{9a^2bx} - \frac{\sin(c+dx)}{6bx(a+bx^3)^2} + \frac{\sin(c+dx)}{18b^2x^4(a+bx^3)} - \frac{2d\sin(c)\text{Si}(dx)}{9a^2b} - \frac{2\int\frac{\sin(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{27a^{7/3}\sqrt[3]{b}} + \frac{\left(2\sqrt[3]{-1}\right)\int\frac{\sin(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}dx}{27a^{7/3}\sqrt[3]{b}} - \frac{\left(2(-1)^{2/3}\right)\int\frac{\sin(c+dx)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}dx}{27a^{7/3}\sqrt[3]{b}} - \frac{d\int\frac{\cos(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{54a^2b^{2/3}} - \frac{d\int\frac{\cos(c+dx)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}dx}{54a^2b^{2/3}} - \frac{d\int\frac{\cos(c+dx)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}dx}{54a^2b^{2/3}} - \frac{d\int\frac{\cos(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{18a^2b^{2/3}} - \frac{d\int\frac{\cos(c+dx)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}dx}{18a^2b^{2/3}} - \frac{d\int\frac{\cos(c+dx)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}dx}{18a^2b^{2/3}} - \frac{d^2\int\frac{\sin(c+dx)}{x^3}dx}{54ab^2} - \frac{d^2\int\frac{\sin(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}dx}{54a^{5/3}b} - \frac{d^2\int\frac{\sin(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}dx}{54a^{5/3}b} - \frac{d^2\int\frac{\sin(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}dx}{54a^{5/3}b} + \frac{d^3\int\frac{\cos(c+dx)}{x^2}dx}{108ab^2} + \frac{d^3\int\frac{\cos(c+dx)}{x^2}dx}{36ab^2} + \frac{d^4\int\frac{\sin(c+dx)}{x}dx}{36ab^2} - \frac{(2d\cos(c))\int\frac{\cos(dx)}{x}dx}{9a^2b} + \frac{(2d\sin(c))\int\frac{\sin(dx)}{x}dx}{9a^2b}$$

$$= \text{Too large to display}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.41 (sec) , antiderivative size = 698, normalized size of antiderivative = 0.61

$$\int \frac{x\sin(c+dx)}{(a+bx^3)^3}\,dx = -\frac{\text{RootSum}\left[a+b\#1^3\&, \frac{-iad^2\cos(c+d\#1)\,\text{CosIntegral}(d(x-\#1))-ad^2\,\text{CosIntegral}(d(x-\#1))\sin(c+d\#1)-ad^2\cos(c+d\#1)\text{S}}{}\right.}{}$$

```
[In] Integrate[(x*Sin[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] -1/108*(RootSum[a + b*#1^3 & , ((-I)*a*d^2*Cos[c + d*#1]*CosIntegral[d*(x -
 #1)] - a*d^2*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - a*d^2*Cos[c + d*#1]*S
inIntegral[d*(x - #1)] + I*a*d^2*Sin[c + d*#1]*SinIntegral[d*(x - #1)] - (4
*I)*b*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1 - 4*b*CosIntegral[d*(x - #1)
]*Sin[c + d*#1]*#1 - 4*b*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 + (4*I)*b
*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1 + 4*b*d*Cos[c + d*#1]*CosIntegral
```

```
[d*(x - #1)]*#1^2 - (4*I)*b*d*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1^2 -
(4*I)*b*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1^2 - 4*b*d*Sin[c + d*#1]*
SinIntegral[d*(x - #1)]*#1^2)/#1^2 & ] + RootSum[a + b*#1^3 & , (I*a*d^2*Co
s[c + d*#1]*CosIntegral[d*(x - #1)] - a*d^2*CosIntegral[d*(x - #1)]*Sin[c +
 d*#1] - a*d^2*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - I*a*d^2*Sin[c + d*#1
]*SinIntegral[d*(x - #1)] + (4*I)*b*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#
1 - 4*b*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1 - 4*b*Cos[c + d*#1]*SinInt
egral[d*(x - #1)]*#1 - (4*I)*b*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1 + 4
*b*d*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1^2 + (4*I)*b*d*CosIntegral[d*(
x - #1)]*Sin[c + d*#1]*#1^2 + (4*I)*b*d*Cos[c + d*#1]*SinIntegral[d*(x - #1
)]*#1^2 - 4*b*d*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1^2)/#1^2 & ] - (6*b
*Cos[d*x]*(a*d*(a + b*x^3)*Cos[c] + b*x^2*(7*a + 4*b*x^3)*Sin[c]))/(a + b*x
^3)^2 - (6*b*(b*x^2*(7*a + 4*b*x^3)*Cos[c] - a*d*(a + b*x^3)*Sin[c])*Sin[d*
x])/(a + b*x^3)^2)/(a^2*b^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.38 (sec) , antiderivative size = 610, normalized size of antiderivative = 0.53

method	result
risch	$\frac{idc\left(\sum_{\_R1=\mathrm{RootOf}\left(-3i\_Z^2bc-id^3a+ibc^3+b\_Z^3-3c^2b\_Z\right)}\frac{\left(-2ic\_R1+\_R1^2-c^2+6ic-6\_R1+10\right)e^{-\_R1}\mathrm{Ei}_1\left(-idx-ic+\right.}{-2ic\_R1+\_R1^2-c^2}\right.}{108a^2b}$
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int(x*sin(d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/108*I*d/a^2/b*c*sum((-2*I*c*_R1+6*I*c+_R1^2-c^2-6*_R1+10)/(-2*I*c*_R1+_R1
^2-c^2)*exp(_R1)*Ei(1,_R1-I*d*x-I*c),_R1=RootOf(-3*I*_Z^2*b*c-I*d^3*a+I*b*c
^3+b*_Z^3-3*c^2*b*_Z))+1/108*I*d/a^2/b^2*sum((-2*I*b*_R1*c^2+4*I*b*_R1^2+2*
I*b*c^2+_R1^2*b*c+a*d^3-c^3*b-4*I*_R1*b+2*b*c*_R1+6*c*b)/(2*I*c*_R1-_R1^2+c
^2)*exp(_R1)*Ei(1,_R1-I*d*x-I*c),_R1=RootOf(-3*I*_Z^2*b*c-I*d^3*a+I*b*c^3+b
*_Z^3-3*c^2*b*_Z))-1/108*I*d/a^2/b*c*sum((-2*I*c*_R1-6*I*c+_R1^2-c^2+6*_R1+
10)/(-2*I*c*_R1+_R1^2-c^2)*exp(-_R1)*Ei(1,I*d*x+I*c-_R1),_R1=RootOf(-3*I*_Z
^2*b*c-I*d^3*a+I*b*c^3+b*_Z^3-3*c^2*b*_Z))-1/108*I*d/a^2/b^2*sum((-2*I*b*_R
1*c^2-4*I*b*_R1^2-2*I*b*c^2+_R1^2*b*c+a*d^3-c^3*b-4*I*_R1*b-2*b*c*_R1+6*c*b
)/(2*I*c*_R1-_R1^2+c^2)*exp(-_R1)*Ei(1,I*d*x+I*c-_R1),_R1=RootOf(-3*I*_Z^2*
b*c-I*d^3*a+I*b*c^3+b*_Z^3-3*c^2*b*_Z))+1/18*d^4/a*(b*d^3*x^3+a*d^3)/b/(b^2
*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*cos(d*x+c)-1/18*d*(-4*b*d^5*x^5-7*a*d^5*x^2
)/a^2/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*sin(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 1319, normalized size of antiderivative = 1.16

$$\int \frac{x\sin(c+dx)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x*sin(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/216*((8*a*b^2*d^3*x^6 + 16*a^2*b*d^3*x^3 + 8*a^3*d^3 + 4*(I*b^3*x^6 + 2*
I*a*b^2*x^3 + I*a^2*b + sqrt(3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*(I*a*d^3/b
)^(2/3) - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(3)*(I*a*b^2*d^3
*x^6 + 2*I*a^2*b*d^3*x^3 + I*a^3*d^3))*(I*a*d^3/b)^(1/3))*Ei(-I*d*x + 1/2*(
I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1)
 - I*c) + (8*a*b^2*d^3*x^6 + 16*a^2*b*d^3*x^3 + 8*a^3*d^3 + 4*(-I*b^3*x^6 -
 2*I*a*b^2*x^3 - I*a^2*b - sqrt(3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*(-I*a*d
^3/b)^(2/3) - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(3)*(I*a*b^2
*d^3*x^6 + 2*I*a^2*b*d^3*x^3 + I*a^3*d^3))*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1
/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3
) + 1) + I*c) + (8*a*b^2*d^3*x^6 + 16*a^2*b*d^3*x^3 + 8*a^3*d^3 + 4*(I*b^3*
x^6 + 2*I*a*b^2*x^3 + I*a^2*b - sqrt(3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*(I
*a*d^3/b)^(2/3) - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(3)*(-I*
a*b^2*d^3*x^6 - 2*I*a^2*b*d^3*x^3 - I*a^3*d^3))*(I*a*d^3/b)^(1/3))*Ei(-I*d*
x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(-I*sqr
t(3) + 1) - I*c) + (8*a*b^2*d^3*x^6 + 16*a^2*b*d^3*x^3 + 8*a^3*d^3 + 4*(-I*
b^3*x^6 - 2*I*a*b^2*x^3 - I*a^2*b + sqrt(3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b)
)*(-I*a*d^3/b)^(2/3) - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(3)
*(-I*a*b^2*d^3*x^6 - 2*I*a^2*b*d^3*x^3 - I*a^3*d^3))*(-I*a*d^3/b)^(1/3))*Ei
(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*
(-I*sqrt(3) + 1) + I*c) + 2*(4*a*b^2*d^3*x^6 + 8*a^2*b*d^3*x^3 + 4*a^3*d^3
+ 4*(I*b^3*x^6 + 2*I*a*b^2*x^3 + I*a^2*b)*(-I*a*d^3/b)^(2/3) + (a*b^2*d^3*x
^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + (-I*a*d^3/b)
^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) + 2*(4*a*b^2*d^3*x^6 + 8*a^2*b*d^3*x^3
 + 4*a^3*d^3 + 4*(-I*b^3*x^6 - 2*I*a*b^2*x^3 - I*a^2*b)*(I*a*d^3/b)^(2/3) +
 (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*(I*a*d^3/b)^(1/3))*Ei(-I*d*x +
 (I*a*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)) - 12*(a^2*b*d^3*x^3 + a^3*
d^3)*cos(d*x + c) - 12*(4*a*b^2*d^2*x^5 + 7*a^2*b*d^2*x^2)*sin(d*x + c))/(a
^3*b^3*d^2*x^6 + 2*a^4*b^2*d^2*x^3 + a^5*b*d^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x\sin(c+dx)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x*sin(d*x+c)/(b*x**3+a)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x\sin(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{x\sin(dx+c)}{(bx^3+a)^3}\,dx$$

```
[In] integrate(x*sin(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
[Out] -1/2*((cos(c)^2 + sin(c)^2)*x*cos(d*x + c) + (x*cos(d*x + c)^2*cos(c) + x*c
os(c)*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b^3*cos(c)^2 + b^3*sin(c)^2)*d*
x^9 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x^6 + 3*(a^2*b*cos(c)^2 + a^2*b
*sin(c)^2)*d*x^3 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d)*cos(d*x + c)^2 + ((b^3*
cos(c)^2 + b^3*sin(c)^2)*d*x^9 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x^6
+ 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d*x^3 + (a^3*cos(c)^2 + a^3*sin(c)^2)
*d)*sin(d*x + c)^2)*integrate(1/2*(8*b*x^3 - a)*cos(d*x + c)/(b^4*d*x^12 +
4*a*b^3*d*x^9 + 6*a^2*b^2*d*x^6 + 4*a^3*b*d*x^3 + a^4*d), x) + 2*(((b^3*cos
(c)^2 + b^3*sin(c)^2)*d*x^9 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x^6 + 3
*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d*x^3 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d)
*cos(d*x + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d*x^9 + 3*(a*b^2*cos(c)^2
+ a*b^2*sin(c)^2)*d*x^6 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d*x^3 + (a^3*
cos(c)^2 + a^3*sin(c)^2)*d)*sin(d*x + c)^2)*integrate(1/2*(8*b*x^3 - a)*cos
(d*x + c)/((b^4*d*x^12 + 4*a*b^3*d*x^9 + 6*a^2*b^2*d*x^6 + 4*a^3*b*d*x^3 +
a^4*d)*cos(d*x + c)^2 + (b^4*d*x^12 + 4*a*b^3*d*x^9 + 6*a^2*b^2*d*x^6 + 4*a
^3*b*d*x^3 + a^4*d)*sin(d*x + c)^2), x) + (x*cos(d*x + c)^2*sin(c) + x*sin(
d*x + c)^2*sin(c))*sin(d*x + 2*c))/(((b^3*cos(c)^2 + b^3*sin(c)^2)*d*x^9 +
3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x^6 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c
)^2)*d*x^3 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d)*cos(d*x + c)^2 + ((b^3*cos(c)
^2 + b^3*sin(c)^2)*d*x^9 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x^6 + 3*(a
^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d*x^3 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d)*si
n(d*x + c)^2)
```

Giac [F]

$$\int \frac{x\sin(c+dx)}{(a+bx^3)^3}\,dx=\int \frac{x\sin(dx+c)}{(bx^3+a)^3}\,dx$$

```
[In] integrate(x*sin(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
[Out] integrate(x*sin(d*x + c)/(b*x^3 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sin(c+dx)}{(a+bx^3)^3}\,dx=\int \frac{x\,\sin(c+d\,x)}{(b\,x^3+a)^3}\,dx$$

```
[In] int((x*sin(c + d*x))/(a + b*x^3)^3,x)
[Out] int((x*sin(c + d*x))/(a + b*x^3)^3, x)
```

3.112 $\int \frac{\sin(c+dx)}{(a+bx^3)^3} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 1161

$$
\begin{aligned}
\int \frac{\sin(c+dx)}{(a+bx^3)^3}\,dx &= \frac{d\cos(c+dx)}{18ab^2x^4}-\frac{d\cos(c+dx)}{18a^2bx}-\frac{d\cos(c+dx)}{18b^2x^4(a+bx^3)} \\
&+\frac{(-1)^{2/3}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{7/3}b^{2/3}} \\
&+\frac{d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{7/3}b^{2/3}} \\
&-\frac{\sqrt[3]{-1}d\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{7/3}b^{2/3}} \\
&+\frac{5\,\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{8/3}\sqrt[3]{b}} \\
&-\frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54a^2b} \\
&-\frac{5\sqrt[3]{-1}\,\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{8/3}\sqrt[3]{b}} \\
&-\frac{d^2\,\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54a^2b} \\
&+\frac{5(-1)^{2/3}\,\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{8/3}\sqrt[3]{b}} \\
&-\frac{d^2\,\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54a^2b} \\
&-\frac{\sin(c+dx)}{9ab^2x^5}+\frac{5\sin(c+dx)}{18a^2bx^2}-\frac{\sin(c+dx)}{6bx^2(a+bx^3)^2}+\frac{\sin(c+dx)}{9b^2x^5(a+bx^3)} \\
&+\frac{5\sqrt[3]{-1}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^{8/3}\sqrt[3]{b}} \\
&+\frac{d^2\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54a^2b} \\
&+\frac{(-1)^{2/3}d\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{7/3}b^{2/3}} \\
&+\frac{5\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{27a^{8/3}\sqrt[3]{b}} \\
&\quad d^2\cos\left(c-\frac{\sqrt[3]{a}d}{}\right)\text{Si}\left(\frac{\sqrt[3]{a}d}{}+dx\right)
\end{aligned}
$$

```
[Out] -5/27*(-1)^(1/3)*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)
*d/b^(1/3)+d*x)/a^(8/3)/b^(1/3)-1/54*d^2*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3)
)*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^2/b+1/9*d*Ci(a^(1/3)*d/b^(1/3)+d*
x)*cos(c-a^(1/3)*d/b^(1/3))/a^(7/3)/b^(2/3)+1/18*d*cos(d*x+c)/a/b^2/x^4-1/1
8*d*cos(d*x+c)/a^2/b/x-1/18*d*cos(d*x+c)/b^2/x^4/(b*x^3+a)-1/54*d^2*cos(c-a
^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^2/b+5/27*(-1)^(2/3)*cos(c-(-1
)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(8/3)/b^(
1/3)-1/54*d^2*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b
^(1/3)+d*x)/a^2/b-1/54*d^2*Ci(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3
))/a^2/b-1/9*d*Si(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(7/3)/b
^(2/3)-5/27*(-1)^(1/3)*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3
)*a^(1/3)*d/b^(1/3))/a^(8/3)/b^(1/3)-1/54*d^2*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/
3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^2/b+5/27*(-1)^(2/3)*Ci((-1)^(
2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(8/3)/b^(
1/3)-1/54*d^2*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)
*d/b^(1/3))/a^2/b-1/9*(-1)^(2/3)*d*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*si
n(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(7/3)/b^(2/3)+1/9*(-1)^(2/3)*d*Ci((-1)^
(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(7/3)/b^
(2/3)-1/9*(-1)^(1/3)*d*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*cos(c-(-1)^(2/3
)*a^(1/3)*d/b^(1/3))/a^(7/3)/b^(2/3)+1/9*(-1)^(1/3)*d*Si((-1)^(2/3)*a^(1/3)
*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(7/3)/b^(2/3)+5/27*co
s(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^(8/3)/b^(1/3)+5/27*Ci(a^
(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(8/3)/b^(1/3)-1/9*sin(d*x+c
)/a/b^2/x^5+5/18*sin(d*x+c)/a^2/b/x^2-1/6*sin(d*x+c)/b/x^2/(b*x^3+a)^2+1/9*
sin(d*x+c)/b^2/x^5/(b*x^3+a)
```

Rubi [A] (verified)

Time = 2.29 (sec) , antiderivative size = 1161, normalized size of antiderivative = 1.00, number of steps used = 99, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules

used = {3412, 3424, 3426, 3378, 3384, 3380, 3383, 3414, 3427, 3425}

$$
\begin{aligned}
\int \frac{\sin(c+dx)}{(a+bx^3)^3}\,dx = &-\frac{\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d^2}{54a^2b} \\
&-\frac{\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d^2}{54a^2b} \\
&-\frac{\text{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d^2}{54a^2b} \\
&+\frac{\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)d^2}{54a^2b} \\
&-\frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d^2}{54a^2b} \\
&-\frac{\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d^2}{54a^2b} \\
&-\frac{\cos(c+dx)d}{18a^2bx}-\frac{\cos(c+dx)d}{18b^2x^4\left(bx^3+a\right)}+\frac{\cos(c+dx)d}{18ab^2x^4} \\
&+\frac{(-1)^{2/3}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)d}{9a^{7/3}b^{2/3}} \\
&+\frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d}{9a^{7/3}b^{2/3}} \\
&-\frac{\sqrt[3]{-1}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d}{9a^{7/3}b^{2/3}} \\
&+\frac{(-1)^{2/3}\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)d}{9a^{7/3}b^{2/3}} \\
&-\frac{\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d}{9a^{7/3}b^{2/3}} \\
&+\frac{\sqrt[3]{-1}\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d}{9a^{7/3}b^{2/3}} \\
&+\frac{5\,\text{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{8/3}\sqrt[3]{b}} \\
&-\frac{5\sqrt[3]{-1}\,\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{8/3}\sqrt[3]{b}} \\
&+ 5(-1)^{2/3}\,\text{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)
\end{aligned}
$$

```
[In] Int[Sin[c + d*x]/(a + b*x^3)^3,x]
```

```
[Out] (d*Cos[c + d*x])/(18*a*b^2*x^4) - (d*Cos[c + d*x])/(18*a^2*b*x) - (d*Cos[c
+ d*x])/(18*b^2*x^4*(a + b*x^3)) + ((-1)^(2/3)*d*Cos[c + ((-1)^(1/3)*a^(1/3
)*d)/b^(1/3)]*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(7/3)
*b^(2/3)) + (d*Cos[c - (a^(1/3)*d)/b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1/3)
 + d*x])/(9*a^(7/3)*b^(2/3)) - ((-1)^(1/3)*d*Cos[c - ((-1)^(2/3)*a^(1/3)*d)
/b^(1/3)]*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(7/3)*b^(
2/3)) + (5*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/
3)])/(27*a^(8/3)*b^(1/3)) - (d^2*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin
[c - (a^(1/3)*d)/b^(1/3)])/(54*a^2*b) - (5*(-1)^(1/3)*CosIntegral[((-1)^(1/
3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(27*a
^(8/3)*b^(1/3)) - (d^2*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Si
n[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(54*a^2*b) + (5*(-1)^(2/3)*CosIntegr
al[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(
1/3)])/(27*a^(8/3)*b^(1/3)) - (d^2*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/
3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(54*a^2*b) - Sin[c + d*x
]/(9*a*b^2*x^5) + (5*Sin[c + d*x])/(18*a^2*b*x^2) - Sin[c + d*x]/(6*b*x^2*(
a + b*x^3)^2) + Sin[c + d*x]/(9*b^2*x^5*(a + b*x^3)) + (5*(-1)^(1/3)*Cos[c
+ ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3
) - d*x])/(27*a^(8/3)*b^(1/3)) + (d^2*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3
)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a^2*b) + ((-1)^(2
/3)*d*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/
3)*d)/b^(1/3) - d*x])/(9*a^(7/3)*b^(2/3)) + (5*Cos[c - (a^(1/3)*d)/b^(1/3)]
*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(8/3)*b^(1/3)) - (d^2*Cos[c
- (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(54*a^2*b) -
 (d*Sin[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9
*a^(7/3)*b^(2/3)) + (5*(-1)^(2/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*S
inIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(8/3)*b^(1/3)) - (d
^2*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*
d)/b^(1/3) + d*x])/(54*a^2*b) + ((-1)^(1/3)*d*Sin[c - ((-1)^(2/3)*a^(1/3)*d
)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(7/3)*b^
(2/3))
```

<u>Rule 3378</u>

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

<u>Rule 3380</u>

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

<u>Rule 3383</u>

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Sim
p[x^(-n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))), x] + (-Dist[
(-n + 1)/(b*n*(p + 1)), Int[((a + b*x^n)^(p + 1)*Sin[c + d*x])/x^n, x], x]
- Dist[d/(b*n*(p + 1)), Int[x^(-n + 1)*(a + b*x^n)^(p + 1)*Cos[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1] && IGtQ[n, 2]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3424

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3425

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cos[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Cos[c + d*x], x], x] + Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Sin[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
```

```
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3427

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sin(c+dx)}{6bx^2\left(a+bx^3\right)^2}-\frac{\int\frac{\sin(c+dx)}{x^3(a+bx^3)^2}\,dx}{3b}+\frac{d\int\frac{\cos(c+dx)}{x^2(a+bx^3)^2}\,dx}{6b}\\
&= -\frac{d\cos(c+dx)}{18b^2x^4\left(a+bx^3\right)}-\frac{\sin(c+dx)}{6bx^2\left(a+bx^3\right)^2}+\frac{\sin(c+dx)}{9b^2x^5\left(a+bx^3\right)}+\frac{5\int\frac{\sin(c+dx)}{x^6(a+bx^3)}\,dx}{9b^2}\\
&\quad-\frac{d\int\frac{\cos(c+dx)}{x^5(a+bx^3)}\,dx}{9b^2}-\frac{(2d)\int\frac{\cos(c+dx)}{x^5(a+bx^3)}\,dx}{9b^2}-\frac{d^2\int\frac{\sin(c+dx)}{x^4(a+bx^3)}\,dx}{18b^2}\\
&= -\frac{d\cos(c+dx)}{18b^2x^4\left(a+bx^3\right)}-\frac{\sin(c+dx)}{6bx^2\left(a+bx^3\right)^2}+\frac{\sin(c+dx)}{9b^2x^5\left(a+bx^3\right)}\\
&\quad+\frac{5\int\left(\frac{\sin(c+dx)}{ax^6}-\frac{b\sin(c+dx)}{a^2x^3}+\frac{b^2\sin(c+dx)}{a^2(a+bx^3)}\right)dx}{9b^2}\\
&\quad-\frac{d\int\left(\frac{\cos(c+dx)}{ax^5}-\frac{b\cos(c+dx)}{a^2x^2}+\frac{b^2x\cos(c+dx)}{a^2(a+bx^3)}\right)dx}{9b^2}\\
&\quad-\frac{(2d)\int\left(\frac{\cos(c+dx)}{ax^5}-\frac{b\cos(c+dx)}{a^2x^2}+\frac{b^2x\cos(c+dx)}{a^2(a+bx^3)}\right)dx}{9b^2}\\
&\quad-\frac{d^2\int\left(\frac{\sin(c+dx)}{ax^4}-\frac{b\sin(c+dx)}{a^2x}+\frac{b^2x^2\sin(c+dx)}{a^2(a+bx^3)}\right)dx}{18b^2}\\
&= -\frac{d\cos(c+dx)}{18b^2x^4\left(a+bx^3\right)}-\frac{\sin(c+dx)}{6bx^2\left(a+bx^3\right)^2}+\frac{\sin(c+dx)}{9b^2x^5\left(a+bx^3\right)}+\frac{5\int\frac{\sin(c+dx)}{a+bx^3}\,dx}{9a^2}\\
&\quad+\frac{5\int\frac{\sin(c+dx)}{x^6}\,dx}{9ab^2}-\frac{5\int\frac{\sin(c+dx)}{x^3}\,dx}{9a^2b}-\frac{d\int\frac{x\cos(c+dx)}{a+bx^3}\,dx}{9a^2}-\frac{(2d)\int\frac{x\cos(c+dx)}{a+bx^3}\,dx}{9a^2}\\
&\quad-\frac{d\int\frac{\cos(c+dx)}{x^5}\,dx}{9ab^2}-\frac{(2d)\int\frac{\cos(c+dx)}{x^5}\,dx}{9ab^2}+\frac{d\int\frac{\cos(c+dx)}{x^2}\,dx}{9a^2b}+\frac{(2d)\int\frac{\cos(c+dx)}{x^2}\,dx}{9a^2b}\\
&\quad-\frac{d^2\int\frac{x^2\sin(c+dx)}{a+bx^3}\,dx}{18a^2}-\frac{d^2\int\frac{\sin(c+dx)}{x^4}\,dx}{18ab^2}+\frac{d^2\int\frac{\sin(c+dx)}{x}\,dx}{18a^2b}
\end{aligned}$$

$$= \frac{d\cos(c+dx)}{12ab^2x^4} - \frac{d\cos(c+dx)}{3a^2bx} - \frac{d\cos(c+dx)}{18b^2x^4(a+bx^3)} - \frac{\sin(c+dx)}{9ab^2x^5}$$
$$+ \frac{d^2\sin(c+dx)}{54ab^2x^3} + \frac{5\sin(c+dx)}{18a^2bx^2} - \frac{\sin(c+dx)}{6bx^2(a+bx^3)^2} + \frac{\sin(c+dx)}{9b^2x^5(a+bx^3)}$$
$$+ \frac{5\int \left(-\frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{9a^2}$$
$$- \frac{d\int \left(-\frac{\cos(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}\cos(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}\cos(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{9a^2}$$
$$- \frac{(2d)\int \left(-\frac{\cos(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}\cos(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}\cos(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{9a^2}$$
$$+ \frac{d\int \frac{\cos(c+dx)}{x^5}dx}{9ab^2} - \frac{(5d)\int \frac{\cos(c+dx)}{x^2}dx}{18a^2b}$$
$$- \frac{d^2\int \left(\frac{\sin(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sin(c+dx)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sin(c+dx)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right)dx}{18a^2}$$
$$+ \frac{d^2\int \frac{\sin(c+dx)}{x^4}dx}{36ab^2} + \frac{d^2\int \frac{\sin(c+dx)}{x^4}dx}{18ab^2} - \frac{d^2\int \frac{\sin(c+dx)}{x}dx}{9a^2b} - \frac{(2d^2)\int \frac{\sin(c+dx)}{x}dx}{9a^2b}$$
$$- \frac{d^3\int \frac{\cos(c+dx)}{x^3}dx}{54ab^2} + \frac{(d^2\cos(c))\int \frac{\sin(dx)}{x}dx}{18a^2b} + \frac{(d^2\sin(c))\int \frac{\cos(dx)}{x}dx}{18a^2b}$$

$$
\begin{aligned}
&= \frac{d\cos(c+dx)}{18ab^2x^4} + \frac{d^3\cos(c+dx)}{108ab^2x^2} - \frac{d\cos(c+dx)}{18a^2bx} - \frac{d\cos(c+dx)}{18b^2x^4\left(a+bx^3\right)} \\
&\quad + \frac{d^2\,\text{CosIntegral}(dx)\sin(c)}{18a^2b} - \frac{\sin(c+dx)}{9ab^2x^5} - \frac{d^2\sin(c+dx)}{108ab^2x^3} \\
&\quad + \frac{5\sin(c+dx)}{18a^2bx^2} - \frac{\sin(c+dx)}{6bx^2\left(a+bx^3\right)^2} + \frac{\sin(c+dx)}{9b^2x^5\left(a+bx^3\right)} + \frac{d^2\cos(c)\text{Si}(dx)}{18a^2b} \\
&\quad - \frac{5\int \frac{\sin(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{27a^{8/3}} - \frac{5\int \frac{\sin(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{27a^{8/3}} - \frac{5\int \frac{\sin(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{27a^{8/3}} \\
&\quad + \frac{d\int \frac{\cos(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{7/3}\sqrt[3]{b}} + \frac{(2d)\int \frac{\cos(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{7/3}\sqrt[3]{b}} - \frac{\left(\sqrt[3]{-1}d\right)\int \frac{\cos(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{27a^{7/3}\sqrt[3]{b}} \\
&\quad - \frac{\left(2\sqrt[3]{-1}d\right)\int \frac{\cos(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{27a^{7/3}\sqrt[3]{b}} + \frac{\left((-1)^{2/3}d\right)\int \frac{\cos(c+dx)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{27a^{7/3}\sqrt[3]{b}} \\
&\quad + \frac{\left(2(-1)^{2/3}d\right)\int \frac{\cos(c+dx)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{27a^{7/3}\sqrt[3]{b}} - \frac{d^2\int \frac{\sin(c+dx)}{x^4}\,dx}{36ab^2} + \frac{\left(5d^2\right)\int \frac{\sin(c+dx)}{x}\,dx}{18a^2b} \\
&\quad - \frac{d^2\int \frac{\sin(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{54a^2b^{2/3}} - \frac{d^2\int \frac{\sin(c+dx)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{54a^2b^{2/3}} - \frac{d^2\int \frac{\sin(c+dx)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{54a^2b^{2/3}} \\
&\quad + \frac{d^3\int \frac{\cos(c+dx)}{x^3}\,dx}{108ab^2} + \frac{d^3\int \frac{\cos(c+dx)}{x^3}\,dx}{54ab^2} + \frac{d^4\int \frac{\sin(c+dx)}{x^2}\,dx}{108ab^2} - \frac{\left(d^2\cos(c)\right)\int \frac{\sin(dx)}{x}\,dx}{9a^2b} \\
&\quad - \frac{\left(2d^2\cos(c)\right)\int \frac{\sin(dx)}{x}\,dx}{9a^2b} - \frac{\left(d^2\sin(c)\right)\int \frac{\cos(dx)}{x}\,dx}{9a^2b} - \frac{\left(2d^2\sin(c)\right)\int \frac{\cos(dx)}{x}\,dx}{9a^2b} \\
&= \text{Too large to display}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.32 (sec) , antiderivative size = 675, normalized size of antiderivative = 0.58

$$
\int \frac{\sin(c+dx)}{\left(a+bx^3\right)^3}\,dx
$$

$$
= \frac{-\frac{i\text{RootSum}\left[a+b\#1^3\&, \frac{-10\cos(c+d\#1)\,\text{CosIntegral}(d(x-\#1))+10i\,\text{CosIntegral}(d(x-\#1))\sin(c+d\#1)+10i\cos(c+d\#1)\text{Si}(d(x-\#1))+10\sin(c\cdots}{\cdots}\right.}{\cdots}}{\cdots}
$$

```
[In] Integrate[Sin[c + d*x]/(a + b*x^3)^3,x]
```

```
[Out] (((-I)*RootSum[a + b*#1^3 & , (-10*Cos[c + d*#1]*CosIntegral[d*(x - #1)] +
(10*I)*CosIntegral[d*(x - #1)]*Sin[c + d*#1] + (10*I)*Cos[c + d*#1]*SinInte
gral[d*(x - #1)] + 10*Sin[c + d*#1]*SinIntegral[d*(x - #1)] - (6*I)*d*Cos[c
 + d*#1]*CosIntegral[d*(x - #1)]*#1 - 6*d*CosIntegral[d*(x - #1)]*Sin[c + d
```

```
*#1]*#1 - 6*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 + (6*I)*d*Sin[c + d*
#1]*SinIntegral[d*(x - #1)]*#1 + d^2*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*
#1^2 - I*d^2*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1^2 - I*d^2*Cos[c + d*#
1]*SinIntegral[d*(x - #1)]*#1^2 - d^2*Sin[c + d*#1]*SinIntegral[d*(x - #1)]
*#1^2)/#1^2 & ])/b + (I*RootSum[a + b*#1^3 & , (-10*Cos[c + d*#1]*CosIntegr
al[d*(x - #1)] - (10*I)*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - (10*I)*Cos[
c + d*#1]*SinIntegral[d*(x - #1)] + 10*Sin[c + d*#1]*SinIntegral[d*(x - #1)
] + (6*I)*d*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1 - 6*d*CosIntegral[d*(x
 - #1)]*Sin[c + d*#1]*#1 - 6*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 - (
6*I)*d*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1 + d^2*Cos[c + d*#1]*CosInte
gral[d*(x - #1)]*#1^2 + I*d^2*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1^2 +
I*d^2*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1^2 - d^2*Sin[c + d*#1]*SinInt
egral[d*(x - #1)]*#1^2)/#1^2 & ])/b - (6*x*Cos[d*x]*(d*x*(a + b*x^3)*Cos[c]
 - (8*a + 5*b*x^3)*Sin[c]))/(a + b*x^3)^2 + (6*x*((8*a + 5*b*x^3)*Cos[c] +
d*x*(a + b*x^3)*Sin[c])*Sin[d*x])/(a + b*x^3)^2)/(108*a^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.05 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.29

method	result
risch	$-\frac{i d^2 \left(\sum_{\_R1=\text{RootOf}\left(-3i\_Z^2bc-id^3a+ibc^3+b\_Z^3-3c^2b\_Z\right)} \frac{\left(-2ic\_R1+\_R1^2-c^2+6ic-6\_R1+10\right) e^{\_R1} \text{Ei}_1\left(-ida\right.}{-2ic\_R1+\_R1^2-c^2}\right.}{108a^2b}$
derivativedivides	$d^8\left(-\frac{\sin(dx+c)\left(8acd^3-8ad^3(dx+c)-5bc^4+20bc^3(dx+c)-30bc^2(dx+c)^2+20bc(dx+c)^3-5b(dx+c)^4\right)}{18a^2d^6\left(ad^3-c^3b+3bc^2(dx+c)-3bc(dx+c)^2+b(dx+c)^3\right)^2}-\frac{}{18a^2d^6(}\right.$
default	$d^8\left(-\frac{\sin(dx+c)\left(8acd^3-8ad^3(dx+c)-5bc^4+20bc^3(dx+c)-30bc^2(dx+c)^2+20bc(dx+c)^3-5b(dx+c)^4\right)}{18a^2d^6\left(ad^3-c^3b+3bc^2(dx+c)-3bc(dx+c)^2+b(dx+c)^3\right)^2}-\frac{}{18a^2d^6(}\right.$

```
[In] int(sin(d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/108*I*d^2/a^2/b*sum((-2*I*c*_R1+6*I*c+_R1^2-c^2-6*_R1+10)/(-2*I*c*_R1+_R
1^2-c^2)*exp(_R1)*Ei(1,_R1-I*d*x-I*c),_R1=RootOf(-3*I*_Z^2*b*c-I*d^3*a+I*b*
c^3+b*_Z^3-3*c^2*b*_Z))+1/108*I*d^2/a^2/b*sum((-2*I*c*_R1-6*I*c+_R1^2-c^2+6
*_R1+10)/(-2*I*c*_R1+_R1^2-c^2)*exp(-_R1)*Ei(1,I*d*x+I*c-_R1),_R1=RootOf(-3
*I*_Z^2*b*c-I*d^3*a+I*b*c^3+b*_Z^3-3*c^2*b*_Z))+1/18*d^2*(-b*d^5*x^5-a*d^5*
```

```
x^2)/a^2/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*cos(d*x+c)+1/18*d^2*(5*b*d^4*x
^4+8*a*d^4*x)/a^2/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*sin(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 1223, normalized size of antiderivative = 1.05

$$\int \frac{\sin(c+dx)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/108*((-I*a*b^2*d^3*x^6 - 2*I*a^2*b*d^3*x^3 - I*a^3*d^3 + 3*(b^3*x^6 + 2*a
*b^2*x^3 + a^2*b - sqrt(3)*(I*b^3*x^6 + 2*I*a*b^2*x^3 + I*a^2*b))*(I*a*d^3/
b)^(2/3) + 5*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(3)*(-I*b^3*x^6 - 2*I*a*b
^2*x^3 - I*a^2*b))*(I*a*d^3/b)^(1/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I
*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) + (I*a*b^2*d
^3*x^6 + 2*I*a^2*b*d^3*x^3 + I*a^3*d^3 + 3*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b -
 sqrt(3)*(I*b^3*x^6 + 2*I*a*b^2*x^3 + I*a^2*b))*(-I*a*d^3/b)^(2/3) + 5*(b^3
*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(3)*(-I*b^3*x^6 - 2*I*a*b^2*x^3 - I*a^2*b)
)*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e
^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) + (-I*a*b^2*d^3*x^6 - 2*I*a
^2*b*d^3*x^3 - I*a^3*d^3 + 3*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(3)*(-I*b
^3*x^6 - 2*I*a*b^2*x^3 - I*a^2*b))*(I*a*d^3/b)^(2/3) + 5*(b^3*x^6 + 2*a*b^2
*x^3 + a^2*b - sqrt(3)*(I*b^3*x^6 + 2*I*a*b^2*x^3 + I*a^2*b))*(I*a*d^3/b)^(
1/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)
^(1/3)*(-I*sqrt(3) + 1) - I*c) + (I*a*b^2*d^3*x^6 + 2*I*a^2*b*d^3*x^3 + I*a
^3*d^3 + 3*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(3)*(-I*b^3*x^6 - 2*I*a*b^2
*x^3 - I*a^2*b))*(-I*a*d^3/b)^(2/3) + 5*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sq
rt(3)*(I*b^3*x^6 + 2*I*a*b^2*x^3 + I*a^2*b))*(-I*a*d^3/b)^(1/3))*Ei(I*d*x +
 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt
(3) + 1) + I*c) + (I*a*b^2*d^3*x^6 + 2*I*a^2*b*d^3*x^3 + I*a^3*d^3 - 6*(b^3
*x^6 + 2*a*b^2*x^3 + a^2*b)*(-I*a*d^3/b)^(2/3) - 10*(b^3*x^6 + 2*a*b^2*x^3
+ a^2*b)*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*
d^3/b)^(1/3)) + (-I*a*b^2*d^3*x^6 - 2*I*a^2*b*d^3*x^3 - I*a^3*d^3 - 6*(b^3*
x^6 + 2*a*b^2*x^3 + a^2*b)*(I*a*d^3/b)^(2/3) - 10*(b^3*x^6 + 2*a*b^2*x^3 +
a^2*b)*(I*a*d^3/b)^(1/3))*Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3
/b)^(1/3)) - 6*(a*b^2*d^2*x^5 + a^2*b*d^2*x^2)*cos(d*x + c) + 6*(5*a*b^2*d*
x^4 + 8*a^2*b*d*x)*sin(d*x + c))/(a^3*b^3*d*x^6 + 2*a^4*b^2*d*x^3 + a^5*b*d
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)/(b*x**3+a)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{\sin(dx+c)}{(bx^3+a)^3}\,dx$$

```
[In] integrate(sin(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
[Out] integrate(sin(d*x + c)/(b*x^3 + a)^3, x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{\sin(dx+c)}{(bx^3+a)^3}\,dx$$

```
[In] integrate(sin(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
[Out] integrate(sin(d*x + c)/(b*x^3 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{\sin(c+d\,x)}{(b\,x^3+a)^3}\,dx$$

```
[In] int(sin(c + d*x)/(a + b*x^3)^3,x)
[Out] int(sin(c + d*x)/(a + b*x^3)^3, x)
```

3.113 $\int \frac{\sin(c+dx)}{x\left(a+bx^3\right)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 1163

$$
\begin{aligned}
\int \frac{\sin(c+dx)}{x\left(a+bx^3\right)^3}\,dx &= \frac{d\cos(c+dx)}{18ab^2x^5} - \frac{d\cos(c+dx)}{18a^2bx^2} - \frac{d\cos(c+dx)}{18b^2x^5\left(a+bx^3\right)} \\
&+ \frac{4\sqrt[3]{-1}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^{8/3}\sqrt[3]{b}} \\
&- \frac{4d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{27a^{8/3}\sqrt[3]{b}} \\
&- \frac{4(-1)^{2/3}d\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{27a^{8/3}\sqrt[3]{b}} \\
&+ \frac{\operatorname{CosIntegral}(dx)\sin(c)}{a^3} - \frac{\operatorname{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^3} \\
&+ \frac{d^2\operatorname{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54a^{7/3}b^{2/3}} \\
&- \frac{\operatorname{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^3} \\
&+ \frac{(-1)^{2/3}d^2\operatorname{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54a^{7/3}b^{2/3}} \\
&- \frac{\operatorname{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^3} \\
&- \frac{\sqrt[3]{-1}d^2\operatorname{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54a^{7/3}b^{2/3}} \\
&- \frac{\sin(c+dx)}{6ab^2x^6} + \frac{\sin(c+dx)}{3a^2bx^3} - \frac{\sin(c+dx)}{6bx^3\left(a+bx^3\right)^2} + \frac{\sin(c+dx)}{6b^2x^6\left(a+bx^3\right)} \\
&+ \frac{\cos(c)\operatorname{Si}(dx)}{a^3} + \frac{\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^3} \\
&- \frac{(-1)^{2/3}d^2\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54a^{7/3}b^{2/3}} \\
&+ \frac{4\sqrt[3]{-1}d\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^{8/3}\sqrt[3]{b}} \\
&- \frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^3} \\
&\quad d^2\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)
\end{aligned}
$$

```
[Out] -4/27*d*Ci(a^(1/3)*d/b^(1/3)+d*x)*cos(c-a^(1/3)*d/b^(1/3))/a^(8/3)/b^(1/3)+
1/18*d*cos(d*x+c)/a/b^2/x^5-1/18*d*cos(d*x+c)/a^2/b/x^2-1/18*d*cos(d*x+c)/b
^2/x^5/(b*x^3+a)+1/54*d^2*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x
)/a^(7/3)/b^(2/3)+1/54*d^2*Ci(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3
))/a^(7/3)/b^(2/3)+4/27*d*Si(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3)
)/a^(8/3)/b^(1/3)-1/3*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^
(1/3)*d/b^(1/3)+d*x)/a^3-1/3*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+
d*x)/a^3-1/3*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^
(1/3)+d*x)/a^3-1/3*Ci(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^3-1
/3*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))
/a^3-1/3*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^
(1/3))/a^3+1/54*(-1)^(2/3)*d^2*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)
^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(7/3)/b^(2/3)-4/27*(-1)^(1/3)*d*Si(-(-1)^(1
/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(8/3)/b^(1
/3)+4/27*(-1)^(1/3)*d*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cos(c+(-1)^(1/3)
*a^(1/3)*d/b^(1/3))/a^(8/3)/b^(1/3)-4/27*(-1)^(2/3)*d*Ci((-1)^(2/3)*a^(1/3)
*d/b^(1/3)+d*x)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(8/3)/b^(1/3)-1/54*(-
1)^(1/3)*d^2*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^
(1/3)+d*x)/a^(7/3)/b^(2/3)+1/54*(-1)^(2/3)*d^2*Ci((-1)^(1/3)*a^(1/3)*d/b^(1
/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(7/3)/b^(2/3)-1/54*(-1)^(1/3
)*d^2*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/
3))/a^(7/3)/b^(2/3)+4/27*(-1)^(2/3)*d*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*
sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(8/3)/b^(1/3)+cos(c)*Si(d*x)/a^3+Ci(d
*x)*sin(c)/a^3-1/6*sin(d*x+c)/a/b^2/x^6+1/3*sin(d*x+c)/a^2/b/x^3-1/6*sin(d*
x+c)/b/x^3/(b*x^3+a)^2+1/6*sin(d*x+c)/b^2/x^6/(b*x^3+a)
```

Rubi [A] (verified)

Time = 2.41 (sec) , antiderivative size = 1163, normalized size of antiderivative = 1.00, number of steps used = 110, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules

used = {3424, 3426, 3378, 3384, 3380, 3383, 3427, 3415, 3425}

$$\int \frac{\sin(c+dx)}{x\left(a+bx^3\right)^3}\,dx = \frac{\operatorname{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d^2}{54a^{7/3}b^{2/3}}$$

$$+\frac{(-1)^{2/3}\operatorname{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d^2}{54a^{7/3}b^{2/3}}$$

$$-\frac{\sqrt[3]{-1}\operatorname{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d^2}{54a^{7/3}b^{2/3}}$$

$$-\frac{(-1)^{2/3}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)d^2}{54a^{7/3}b^{2/3}}$$

$$+\frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d^2}{54a^{7/3}b^{2/3}}$$

$$-\frac{\sqrt[3]{-1}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d^2}{54a^{7/3}b^{2/3}}$$

$$-\frac{\cos(c+dx)d}{18b^2x^5\left(bx^3+a\right)}-\frac{\cos(c+dx)d}{18a^2bx^2}+\frac{\cos(c+dx)d}{18ab^2x^5}$$

$$+\frac{4\sqrt[3]{-1}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)d}{27a^{8/3}\sqrt[3]{b}}$$

$$-\frac{4\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d}{27a^{8/3}\sqrt[3]{b}}$$

$$-\frac{4(-1)^{2/3}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d}{27a^{8/3}\sqrt[3]{b}}$$

$$+\frac{4\sqrt[3]{-1}\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)d}{27a^{8/3}\sqrt[3]{b}}$$

$$+\frac{4\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d}{27a^{8/3}\sqrt[3]{b}}$$

$$+\frac{4(-1)^{2/3}\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d}{27a^{8/3}\sqrt[3]{b}}$$

$$+\frac{\operatorname{CosIntegral}(dx)\sin(c)}{a^3}-\frac{\operatorname{CosIntegral}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^3}$$

$$-\frac{\operatorname{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^3}$$

$$\operatorname{CosIntegral}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\ldots}\right)\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\ldots}\right)$$

```
[In] Int[Sin[c + d*x]/(x*(a + b*x^3)^3),x]
```

```
[Out] (d*Cos[c + d*x])/(18*a*b^2*x^5) - (d*Cos[c + d*x])/(18*a^2*b*x^2) - (d*Cos[
c + d*x])/(18*b^2*x^5*(a + b*x^3)) + (4*(-1)^(1/3)*d*Cos[c + ((-1)^(1/3)*a^
(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^
(8/3)*b^(1/3)) - (4*d*Cos[c - (a^(1/3)*d)/b^(1/3)]*CosIntegral[(a^(1/3)*d)/
b^(1/3) + d*x])/(27*a^(8/3)*b^(1/3)) - (4*(-1)^(2/3)*d*Cos[c - ((-1)^(2/3)*
a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*
a^(8/3)*b^(1/3)) + (CosIntegral[d*x]*Sin[c])/a^3 - (CosIntegral[(a^(1/3)*d)
/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*a^3) + (d^2*CosIntegral[(a
^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(54*a^(7/3)*b^(2/3))
 - (CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a
^(1/3)*d)/b^(1/3)])/(3*a^3) + ((-1)^(2/3)*d^2*CosIntegral[((-1)^(1/3)*a^(1/
3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(54*a^(7/3)*b
^(2/3)) - (CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^
(2/3)*a^(1/3)*d)/b^(1/3)])/(3*a^3) - ((-1)^(1/3)*d^2*CosIntegral[((-1)^(2/3
)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(54*a^
(7/3)*b^(2/3)) - Sin[c + d*x]/(6*a*b^2*x^6) + Sin[c + d*x]/(3*a^2*b*x^3) -
Sin[c + d*x]/(6*b*x^3*(a + b*x^3)^2) + Sin[c + d*x]/(6*b^2*x^6*(a + b*x^3))
 + (Cos[c]*SinIntegral[d*x])/a^3 + (Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]
*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^3) - ((-1)^(2/3)*d
^2*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*
d)/b^(1/3) - d*x])/(54*a^(7/3)*b^(2/3)) + (4*(-1)^(1/3)*d*Sin[c + ((-1)^(1/
3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(
27*a^(8/3)*b^(1/3)) - (Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)
/b^(1/3) + d*x])/(3*a^3) + (d^2*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a
^(1/3)*d)/b^(1/3) + d*x])/(54*a^(7/3)*b^(2/3)) + (4*d*Sin[c - (a^(1/3)*d)/b
^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(8/3)*b^(1/3)) - (Cos
[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(
1/3) + d*x])/(3*a^3) - ((-1)^(1/3)*d^2*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/
3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(54*a^(7/3)*b^(2/3))
 + (4*(-1)^(2/3)*d*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1
)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(8/3)*b^(1/3))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3415

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3424

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3425

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cos[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Cos[c + d*x], x], x] + Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Sin[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3427

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
```

```
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sin(c+dx)}{6bx^3(a+bx^3)^2} - \frac{\int \frac{\sin(c+dx)}{x^4(a+bx^3)^2}\,dx}{2b} + \frac{d\int \frac{\cos(c+dx)}{x^3(a+bx^3)^2}\,dx}{6b} \\
&= -\frac{d\cos(c+dx)}{18b^2x^5(a+bx^3)} - \frac{\sin(c+dx)}{6bx^3(a+bx^3)^2} + \frac{\sin(c+dx)}{6b^2x^6(a+bx^3)} + \frac{\int \frac{\sin(c+dx)}{x^7(a+bx^3)}\,dx}{b^2} \\
&\quad - \frac{d\int \frac{\cos(c+dx)}{x^6(a+bx^3)}\,dx}{6b^2} - \frac{(5d)\int \frac{\cos(c+dx)}{x^6(a+bx^3)}\,dx}{18b^2} - \frac{d^2\int \frac{\sin(c+dx)}{x^5(a+bx^3)}\,dx}{18b^2} \\
&= -\frac{d\cos(c+dx)}{18b^2x^5(a+bx^3)} - \frac{\sin(c+dx)}{6bx^3(a+bx^3)^2} + \frac{\sin(c+dx)}{6b^2x^6(a+bx^3)} \\
&\quad + \frac{\int \left(\frac{\sin(c+dx)}{ax^7} - \frac{b\sin(c+dx)}{a^2x^4} + \frac{b^2\sin(c+dx)}{a^3x} - \frac{b^3x^2\sin(c+dx)}{a^3(a+bx^3)}\right)dx}{b^2} \\
&\quad - \frac{d\int \left(\frac{\cos(c+dx)}{ax^6} - \frac{b\cos(c+dx)}{a^2x^3} + \frac{b^2\cos(c+dx)}{a^2(a+bx^3)}\right)dx}{6b^2} \\
&\quad - \frac{(5d)\int \left(\frac{\cos(c+dx)}{ax^6} - \frac{b\cos(c+dx)}{a^2x^3} + \frac{b^2\cos(c+dx)}{a^2(a+bx^3)}\right)dx}{18b^2} \\
&\quad - \frac{d^2\int \left(\frac{\sin(c+dx)}{ax^5} - \frac{b\sin(c+dx)}{a^2x^2} + \frac{b^2x\sin(c+dx)}{a^2(a+bx^3)}\right)dx}{18b^2} \\
&= -\frac{d\cos(c+dx)}{18b^2x^5(a+bx^3)} - \frac{\sin(c+dx)}{6bx^3(a+bx^3)^2} + \frac{\sin(c+dx)}{6b^2x^6(a+bx^3)} + \frac{\int \frac{\sin(c+dx)}{x}\,dx}{a^3} \\
&\quad + \frac{\int \frac{\sin(c+dx)}{x^7}\,dx}{ab^2} - \frac{\int \frac{\sin(c+dx)}{x^4}\,dx}{a^2b} - \frac{b\int \frac{x^2\sin(c+dx)}{a+bx^3}\,dx}{a^3} - \frac{d\int \frac{\cos(c+dx)}{a+bx^3}\,dx}{6a^2} \\
&\quad - \frac{(5d)\int \frac{\cos(c+dx)}{a+bx^3}\,dx}{18a^2} - \frac{d\int \frac{\cos(c+dx)}{x^6}\,dx}{6ab^2} - \frac{(5d)\int \frac{\cos(c+dx)}{x^6}\,dx}{18ab^2} + \frac{d\int \frac{\cos(c+dx)}{x^3}\,dx}{6a^2b} \\
&\quad + \frac{(5d)\int \frac{\cos(c+dx)}{x^3}\,dx}{18a^2b} - \frac{d^2\int \frac{x\sin(c+dx)}{a+bx^3}\,dx}{18a^2} - \frac{d^2\int \frac{\sin(c+dx)}{x^5}\,dx}{18ab^2} + \frac{d^2\int \frac{\sin(c+dx)}{x^2}\,dx}{18a^2b}
\end{aligned}
$$

$$= \frac{4d\cos(c+dx)}{45ab^2x^5} - \frac{2d\cos(c+dx)}{9a^2bx^2} - \frac{d\cos(c+dx)}{18b^2x^5(a+bx^3)} - \frac{\sin(c+dx)}{6ab^2x^6} + \frac{d^2\sin(c+dx)}{72ab^2x^4} + \frac{\sin(c+dx)}{3a^2bx^3} - \frac{d^2\sin(c+dx)}{18a^2bx} - \frac{\sin(c+dx)}{6bx^3(a+bx^3)^2} + \frac{\sin(c+dx)}{6b^2x^6(a+bx^3)}$$

$$- \frac{b\int\left(\frac{\sin(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sin(c+dx)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sin(c+dx)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right)dx}{a^3}$$

$$- \frac{d\int\left(-\frac{\cos(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\cos(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{\cos(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{6a^2}$$

$$- \frac{(5d)\int\left(-\frac{\cos(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\cos(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{\cos(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{18a^2}$$

$$+ \frac{d\int\frac{\cos(c+dx)}{x^6}dx}{6ab^2} - \frac{d\int\frac{\cos(c+dx)}{x^3}dx}{3a^2b}$$

$$- \frac{d^2\int\left(-\frac{\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{18a^2}$$

$$+ \frac{d^2\int\frac{\sin(c+dx)}{x^5}dx}{30ab^2} + \frac{d^2\int\frac{\sin(c+dx)}{x^5}dx}{18ab^2} - \frac{d^2\int\frac{\sin(c+dx)}{x^2}dx}{12a^2b} - \frac{(5d^2)\int\frac{\sin(c+dx)}{x^2}dx}{36a^2b}$$

$$- \frac{d^3\int\frac{\cos(c+dx)}{x^4}dx}{72ab^2} + \frac{d^3\int\frac{\cos(c+dx)}{x}dx}{18a^2b} + \frac{\cos(c)\int\frac{\sin(dx)}{x}dx}{a^3} + \frac{\sin(c)\int\frac{\cos(dx)}{x}dx}{a^3}$$

$$= \frac{d\cos(c+dx)}{18ab^2x^5} + \frac{d^3\cos(c+dx)}{216ab^2x^3} - \frac{d\cos(c+dx)}{18a^2bx^2} - \frac{d\cos(c+dx)}{18b^2x^5(a+bx^3)} + \frac{\text{CosIntegral}(dx)\sin(c)}{a^3} - \frac{\sin(c+dx)}{6ab^2x^6} - \frac{d^2\sin(c+dx)}{120ab^2x^4} + \frac{\sin(c+dx)}{3a^2bx^3} + \frac{d^2\sin(c+dx)}{6a^2bx} - \frac{\sin(c+dx)}{6bx^3(a+bx^3)^2} + \frac{\sin(c+dx)}{6b^2x^6(a+bx^3)} + \frac{\cos(c)\text{Si}(dx)}{a^3} - \frac{\sqrt[3]{b}\int \frac{\sin(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{3a^3} - \frac{\sqrt[3]{b}\int \frac{\sin(c+dx)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}dx}{3a^3} - \frac{\sqrt[3]{b}\int \frac{\sin(c+dx)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}dx}{3a^3} + \frac{d\int \frac{\cos(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}dx}{18a^{8/3}} + \frac{d\int \frac{\cos(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}dx}{18a^{8/3}} + \frac{d\int \frac{\cos(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}dx}{18a^{8/3}} + \frac{(5d)\int \frac{\cos(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}dx}{54a^{8/3}} + \frac{(5d)\int \frac{\cos(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}dx}{54a^{8/3}} + \frac{(5d)\int \frac{\cos(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}dx}{54a^{8/3}} - \frac{d^2\int \frac{\sin(c+dx)}{x^5}dx}{30ab^2} + \frac{d^2\int \frac{\sin(c+dx)}{x^2}dx}{6a^2b} + \frac{d^2\int \frac{\sin(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{54a^{7/3}\sqrt[3]{b}} - \frac{\left(\sqrt[3]{-1}d^2\right)\int \frac{\sin(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}dx}{54a^{7/3}\sqrt[3]{b}} + \frac{\left((-1)^{2/3}d^2\right)\int \frac{\sin(c+dx)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}dx}{54a^{7/3}\sqrt[3]{b}} + \frac{d^3\int \frac{\cos(c+dx)}{x^4}dx}{120ab^2} + \frac{d^3\int \frac{\cos(c+dx)}{x^4}dx}{72ab^2} - \frac{d^3\int \frac{\cos(c+dx)}{x}dx}{12a^2b} - \frac{(5d^3)\int \frac{\cos(c+dx)}{x}dx}{36a^2b} + \frac{d^4\int \frac{\sin(c+dx)}{x^3}dx}{216ab^2} + \frac{(d^3\cos(c))\int \frac{\cos(dx)}{x}dx}{18a^2b} - \frac{(d^3\sin(c))\int \frac{\sin(dx)}{x}dx}{18a^2b}$$

= Too large to display

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.55 (sec) , antiderivative size = 2109, normalized size of antiderivative = 1.81

$$\int \frac{\sin(c+dx)}{x(a+bx^3)^3}\,dx = \text{Result too large to show}$$

```
[In] Integrate[Sin[c + d*x]/(x*(a + b*x^3)^3),x]
```

```
[Out] (-6*a^2*b*d*x*Cos[c + d*x] - 6*a*b^2*d*x^4*Cos[c + d*x] - (18*I)*b*(a + b*x
^3)^2*RootSum[a + b*#1^3 & , Cos[c + d*#1]*CosIntegral[d*(x - #1)] - I*CosI
ntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*SinIntegral[d*(x - #1)]
 - Sin[c + d*#1]*SinIntegral[d*(x - #1)] & ] + (18*I)*b*(a + b*x^3)^2*RootS
um[a + b*#1^3 & , Cos[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*(
x - #1)]*Sin[c + d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c +
d*#1]*SinIntegral[d*(x - #1)] & ] - 6*a^3*d*RootSum[a + b*#1^3 & , (Cos[c +
```

```
 d*#1]*CosIntegral[d*(x - #1)] - I*CosIntegral[d*(x - #1)]*Sin[c + d*#1] -
I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinIntegral[d*(x -
#1)])/#1^2 & ] - 12*a^2*b*d*x^3*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIn
tegral[d*(x - #1)] - I*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*
#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1^2 &
 ] - 6*a*b^2*d*x^6*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x -
 #1)] - I*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*SinIntegr
al[d*(x - #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1^2 & ] - 6*a^3*d*
RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegr
al[d*(x - #1)]*Sin[c + d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Si
n[c + d*#1]*SinIntegral[d*(x - #1)])/#1^2 & ] - 12*a^2*b*d*x^3*RootSum[a +
b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*(x - #1
)]*Sin[c + d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*
SinIntegral[d*(x - #1)])/#1^2 & ] - 6*a*b^2*d*x^6*RootSum[a + b*#1^3 & , (C
os[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*(x - #1)]*Sin[c + d*
#1] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinIntegral[d
*(x - #1)])/#1^2 & ] - I*a^3*d*RootSum[a + b*#1^3 & , ((-2*I)*Cos[c + d*#1]
*CosIntegral[d*(x - #1)] - 2*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - 2*Cos[
c + d*#1]*SinIntegral[d*(x - #1)] + (2*I)*Sin[c + d*#1]*SinIntegral[d*(x -
#1)] + d*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1 - I*d*CosIntegral[d*(x -
#1)]*Sin[c + d*#1]*#1 - I*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Si
n[c + d*#1]*SinIntegral[d*(x - #1)]*#1)/#1^2 & ] - (2*I)*a^2*b*d*x^3*RootSu
m[a + b*#1^3 & , ((-2*I)*Cos[c + d*#1]*CosIntegral[d*(x - #1)] - 2*CosInteg
ral[d*(x - #1)]*Sin[c + d*#1] - 2*Cos[c + d*#1]*SinIntegral[d*(x - #1)] + (
2*I)*Sin[c + d*#1]*SinIntegral[d*(x - #1)] + d*Cos[c + d*#1]*CosIntegral[d*
(x - #1)]*#1 - I*d*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1 - I*d*Cos[c + d
*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#
1)/#1^2 & ] - I*a*b^2*d*x^6*RootSum[a + b*#1^3 & , ((-2*I)*Cos[c + d*#1]*Co
sIntegral[d*(x - #1)] - 2*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - 2*Cos[c +
 d*#1]*SinIntegral[d*(x - #1)] + (2*I)*Sin[c + d*#1]*SinIntegral[d*(x - #1)
] + d*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1 - I*d*CosIntegral[d*(x - #1)
]*Sin[c + d*#1]*#1 - I*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c
 + d*#1]*SinIntegral[d*(x - #1)]*#1)/#1^2 & ] + I*a^3*d*RootSum[a + b*#1^3
& , ((2*I)*Cos[c + d*#1]*CosIntegral[d*(x - #1)] - 2*CosIntegral[d*(x - #1)
]*Sin[c + d*#1] - 2*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - (2*I)*Sin[c + d
*#1]*SinIntegral[d*(x - #1)] + d*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1 +
 I*d*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1 + I*d*Cos[c + d*#1]*SinIntegr
al[d*(x - #1)]*#1 - d*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1)/#1^2 & ] +
(2*I)*a^2*b*d*x^3*RootSum[a + b*#1^3 & , ((2*I)*Cos[c + d*#1]*CosIntegral[d
*(x - #1)] - 2*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - 2*Cos[c + d*#1]*SinI
ntegral[d*(x - #1)] - (2*I)*Sin[c + d*#1]*SinIntegral[d*(x - #1)] + d*Cos[c
 + d*#1]*CosIntegral[d*(x - #1)]*#1 + I*d*CosIntegral[d*(x - #1)]*Sin[c + d
*#1]*#1 + I*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c + d*#1]*Si
nIntegral[d*(x - #1)]*#1)/#1^2 & ] + I*a*b^2*d*x^6*RootSum[a + b*#1^3 & , (
(2*I)*Cos[c + d*#1]*CosIntegral[d*(x - #1)] - 2*CosIntegral[d*(x - #1)]*Sin
```

```
[c + d*#1] - 2*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - (2*I)*Sin[c + d*#1]*
SinIntegral[d*(x - #1)] + d*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1 + I*d*
CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1 + I*d*Cos[c + d*#1]*SinIntegral[d*
(x - #1)]*#1 - d*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1)/#1^2 & ] + 108*a
^2*b*CosIntegral[d*x]*Sin[c] + 216*a*b^2*x^3*CosIntegral[d*x]*Sin[c] + 108*
b^3*x^6*CosIntegral[d*x]*Sin[c] + 54*a^2*b*Sin[c + d*x] + 36*a*b^2*x^3*Sin[
c + d*x] + 108*a^2*b*Cos[c]*SinIntegral[d*x] + 216*a*b^2*x^3*Cos[c]*SinInte
gral[d*x] + 108*b^3*x^6*Cos[c]*SinIntegral[d*x])/(108*a^3*b*(a + b*x^3)^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.09 (sec) , antiderivative size = 363, normalized size of antiderivative = 0.31

method	result
derivativedivides	$\frac{\sin(dx+c)d^3\left(3a\,d^3-2c^3b+6b\,c^2(dx+c)-6bc(dx+c)^2+2b(dx+c)^3\right)}{6a^2\left(a\,d^3-c^3b+3b\,c^2(dx+c)-3bc(dx+c)^2+b(dx+c)^3\right)^2}-\frac{\cos(dx+c)d^4x}{18a^2\left(a\,d^3-c^3b+3b\,c^2(dx+c)-3bc(dx+c)^2+b(dx+\right.}$
default	$\frac{\sin(dx+c)d^3\left(3a\,d^3-2c^3b+6b\,c^2(dx+c)-6bc(dx+c)^2+2b(dx+c)^3\right)}{6a^2\left(a\,d^3-c^3b+3b\,c^2(dx+c)-3bc(dx+c)^2+b(dx+c)^3\right)^2}-\frac{\cos(dx+c)d^4x}{18a^2\left(a\,d^3-c^3b+3b\,c^2(dx+c)-3bc(dx+c)^2+b(dx+\right.}$
risch	$-\frac{i\left(\sum_{\_R1=\mathrm{RootOf}\left(-3i\_Z^2bc-id^3a+ibc^3+b\_Z^3-3c^2b\_Z\right)}\frac{\left(i\_R1a\,d^3+ac\,d^3-8id^3a-36i\_R1bc+18b\_R1^2-18c^2b\right)e^{-R}}{-2ic\_R1+\_R1^2-c^2}\right.}{108b\,a^3}$

```
[In] int(sin(d*x+c)/x/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*sin(d*x+c)*d^3*(3*a*d^3-2*c^3*b+6*b*c^2*(d*x+c)-6*b*c*(d*x+c)^2+2*b*(d*
x+c)^3)/a^2/(a*d^3-c^3*b+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)^2-1/1
8*cos(d*x+c)*d^4*x/a^2/(a*d^3-c^3*b+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+
c)^3)+1/a^3*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))+1/54/b/a^3*sum((a*d^3+18*_R1*b-
18*b*c)/(-_R1+c)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=Root
Of(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-4/27*d^3/a^2/b*sum(1/(_RR1^2-
2*_RR1*c+c^2)*(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=Roo
tOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 1113, normalized size of antiderivative = 0.96

$$\int \frac{\sin(c+dx)}{x\left(a+bx^3\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)/x/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/216*((-36*I*b^2*x^6 - 72*I*a*b*x^3 - 36*I*a^2 + (I*b^2*x^6 + 2*I*a*b*x^3
+ I*a^2 + sqrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*(I*a*d^3/b)^(2/3) - 8*(-I*b^
2*x^6 - 2*I*a*b*x^3 - I*a^2 + sqrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*(I*a*d^3
/b)^(1/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*
d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) + (36*I*b^2*x^6 + 72*I*a*b*x^3 + 36*I*a
^2 + (-I*b^2*x^6 - 2*I*a*b*x^3 - I*a^2 - sqrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2
))*(-I*a*d^3/b)^(2/3) - 8*(I*b^2*x^6 + 2*I*a*b*x^3 + I*a^2 - sqrt(3)*(b^2*x
^6 + 2*a*b*x^3 + a^2))*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3
)*(-I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) + (-36
*I*b^2*x^6 - 72*I*a*b*x^3 - 36*I*a^2 + (I*b^2*x^6 + 2*I*a*b*x^3 + I*a^2 - s
qrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*(I*a*d^3/b)^(2/3) - 8*(-I*b^2*x^6 - 2*I
*a*b*x^3 - I*a^2 - sqrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*(I*a*d^3/b)^(1/3))*
Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)
*(-I*sqrt(3) + 1) - I*c) + (36*I*b^2*x^6 + 72*I*a*b*x^3 + 36*I*a^2 + (-I*b^
2*x^6 - 2*I*a*b*x^3 - I*a^2 + sqrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*(-I*a*d^
3/b)^(2/3) - 8*(I*b^2*x^6 + 2*I*a*b*x^3 + I*a^2 + sqrt(3)*(b^2*x^6 + 2*a*b*
x^3 + a^2))*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3
) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) - 2*(-18*I*b^2*x^
6 - 36*I*a*b*x^3 - 18*I*a^2 + (-I*b^2*x^6 - 2*I*a*b*x^3 - I*a^2)*(-I*a*d^3/
b)^(2/3) + 8*(-I*b^2*x^6 - 2*I*a*b*x^3 - I*a^2)*(-I*a*d^3/b)^(1/3))*Ei(I*d*
x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) - 2*(18*I*b^2*x^6 + 36
*I*a*b*x^3 + 18*I*a^2 + (I*b^2*x^6 + 2*I*a*b*x^3 + I*a^2)*(I*a*d^3/b)^(2/3)
 + 8*(I*b^2*x^6 + 2*I*a*b*x^3 + I*a^2)*(I*a*d^3/b)^(1/3))*Ei(-I*d*x + (I*a*
d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)) + 216*(b^2*x^6 + 2*a*b*x^3 + a^2
)*cos_integral(d*x)*sin(c) + 216*(b^2*x^6 + 2*a*b*x^3 + a^2)*cos(c)*sin_int
egral(d*x) - 12*(a*b*d*x^4 + a^2*d*x)*cos(d*x + c) + 36*(2*a*b*x^3 + 3*a^2)
*sin(d*x + c))/(a^3*b^2*x^6 + 2*a^4*b*x^3 + a^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x(a+bx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)/x/(b*x**3+a)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sin(c+dx)}{x(a+bx^3)^3}\,dx = \int \frac{\sin(dx+c)}{(bx^3+a)^3x}\,dx$$

```
[In] integrate(sin(d*x+c)/x/(b*x^3+a)^3,x, algorithm="maxima")
[Out] integrate(sin(d*x + c)/((b*x^3 + a)^3*x), x)
```

Giac [F]

$$\int \frac{\sin(c+dx)}{x(a+bx^3)^3}\,dx = \int \frac{\sin(dx+c)}{(bx^3+a)^3x}\,dx$$

```
[In] integrate(sin(d*x+c)/x/(b*x^3+a)^3,x, algorithm="giac")
[Out] integrate(sin(d*x + c)/((b*x^3 + a)^3*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{x(a+bx^3)^3}\,dx = \int \frac{\sin(c+d\,x)}{x\,(b\,x^3+a)^3}\,dx$$

```
[In] int(sin(c + d*x)/(x*(a + b*x^3)^3),x)
[Out] int(sin(c + d*x)/(x*(a + b*x^3)^3), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                    end if
                else #result contains complex but optimal is not
                    if debug then
                            print("result contains complex but optimal is not");
                    fi;
                    return "C","Result contains complex when optimal does not.";
                fi;
            else # result do not contain complex
                 # this assumes optimal do not as well. No check is needed here.
                if debug then
                       print("result do not contain complex, this assumes optimal do not as well")
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    if debug then
                        print("leaf_count_result<=2*leaf_count_optimal");
                    fi;
                    return "A"," ";
                else
                    if debug then
                        print("leaf_count_result>2*leaf_count_optimal");
                    fi;
                    return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                    convert(leaf_count_result,string),"$ vs. $2(",
                                    convert(leaf_count_optimal,string),")=",convert(2*leaf_count
                fi;
            fi;
        else #ExpnType(result) > ExpnType(optimal)
            if debug then
                print("ExpnType(result) > ExpnType(optimal)");
            fi;
            return "C",cat("Result contains higher order function than in optimal. Order ",
                              convert(ExpnType_result,string)," vs. order ",
                              convert(ExpnType_optimal,string),".");
        fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):



    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
   if result.has(I):
      if optimal.has(I): #both result and optimal complex
         if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
         else:
            grade = "B"
            grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
      else: #result contains complex but optimal is not
         grade = "C"
         grade_annotation ="Result contains complex when optimal does not."
   else: # result do not contain complex, this assumes optimal do not as well
      if leaf_count_result <= 2*leaf_count_optimal:
         grade = "A"
         grade_annotation ="none"
      else:
         grade = "B"
         grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
   grade = "C"
   grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```